Bob's Shoe Store: Modelling Task Implementation and Five Practices for Orchestrating Productive Mathematical Discussions

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Received May 2023 II Accepted July 2024

One of the constant challenges facing the teacher educator is connecting theory with practice in a meaningful way, while operating within the constraints of limited tutorial time and requirements to cover an ever-increasing range of course topics. While pre-service teachers (PSTs) are exposed to a variety of classroom experiences on placement, there is no guarantee that they will observe modelling of mathematical teaching approaches that are advocated in their university courses. This article reports on a teaching activity conducted with a class of second-year undergraduate Bachelor of Education PSTs, who were studying the first mathematics pedagogy unit in their course. During a lesson, the teacher educator modelled how to teach with challenging tasks, and how to incorporate Smith and Stein's *Five Practices* into their teaching approach. This teaching account would be of relevance and interest to other teacher educators who are looking to engage their PSTs in authentic classroom experiences to extend and complement practicum experiences.

Keywords · mathematics teacher education research · challenging tasks · 5 practices · pedagogy

Background and Context

Challenging tasks are designed to get students to think and engage in problem solving behaviours. According to Sullivan (2018), challenging tasks require students to plan their approach, record the steps in their solutions, explain their strategies, and justify their thinking to the teacher and other students. Smith and Stein (2018), however, warned against reducing the complexity of tasks through giving too much information, and Sullivan (2018) recommended introducing tasks with minimal instruction. When implementing challenging tasks with students, the approach is to initially provide the same task to all students, with minimal guidance, and encourage them to attempt the task individually and persist to solve it. Enabling prompts can be provided to students who are having difficulty progressing through the task, with extending prompts provided to students who finish the task quickly (Sullivan, 2018). This approach fosters productive struggle and helps to develop dispositions such as persistence and resilience (for more information see, e.g., Sullivan et al., 2023; Russo & Hopkins, 2019).

The teaching with challenging tasks approach can be used in conjunction with the "five practices for orchestrating productive mathematical discussions" (Smith & Stein, 2018). The five practices: *Anticipating, Monitoring, Selecting, Sequencing,* and *Connecting*, are designed to facilitate purposeful mathematical discussion. Before giving the students the task, the teacher is expected to *anticipate* likely student responses, and may even prepare enabling and extending prompts to cater for this. Once students are working on the task, the teacher *monitors* students' approaches and offers prompts if required. While monitoring, the teacher is also looking to *select* students to share their strategies and solutions, which are *sequenced* to maximise the learning for the whole class, and to allow *connections* to key mathematical ideas to be made (see Table 1 for an overview of the five practices and examples of use). It is the modelling of these two approaches that are described in this article.

Table 1.
Overview of the Five Practices and Examples of use in Lessons

Practice	Description	Example
Anticipating	Likely student responses to a challenging mathematical task.	Some PSTs will add the monetary amounts of $100 and $25 together without considering all the transactions.
Monitoring	Students' actual responses to the task while working on the task.	MTE observed PSTs who were not making progress with task and offered an enabling prompt; MTE asked clarifying questions of individuals and provided clarification when necessary; extending prompts were given to PSTs who correctly finished the task.
Selecting	Particular students selected to present their work during whole-class discussion.	Solution strategies identified that were representative of the whole class's responses and/or showed divergent thinking.
Sequencing	Sharing of student responses in a specific order.	Sequenced according to most common incorrect response, followed by another common response that could have been interpreted as correct, followed by a correct response with detailed working out
Connecting	Comparing different students' responses and connecting the responses to mathematical ideas	Liam and Beth's solutions both showed examples of using addition and subtraction, and making connections with profit and loss (see Figures 1 & 2)

Note: PSTs: pre-service teachers; MTE: mathematics teacher educator

The lesson outlined in this article was conducted with a class of 24 pre-service teachers (PSTs), who were enrolled in an undergraduate Bachelor of Education primary course. They were in their second year of the four-year long course, and this was their first mathematics pedagogy unit (Early Childhood and Primary Mathematics), which was undertaken in Semester 2. The PSTs had all completed four weeks of school placement in Semester 1 of their second year. The lesson was conducted by the author, an experienced mathematics teacher educator (MTE), who had taught the unit for eight years.

The Challenging Task

The problem selected for the activity was "Bob's Shoe Store" (source unknown). I had used the problem before in a variety of professional learning and teaching contexts and it always provoked a lot of discussion and disagreement.

> *A lady came into Bob's shoe store and bought a $75 pair of shoes. She gave Bob a $100 note. Having no change, Bob went to Jan next door, exchanged the $100 note for smaller notes and gave the lady $25 change and the shoes. Later that day, Jan came into the shoe store saying that the $100 note was fake. Bob apologised and, having more money in the till, gave Jan $100 for the fake note. How much money has Bob lost over this?*

Modelling the Five Practices

In keeping with Sullivan's (2018) recommendation, I gave each PST a copy of the problem with minimal guidance on how to complete the task. PSTs were asked to work individually on the problem and to record their working out. They could then talk about their answers with their table groups. *Anticipating* their likely responses, I had prepared an enabling prompt for those who may experience difficulty with attempting or solving the problem and an extending prompt for those PSTs who may complete the problem quickly. Note that both prompts contain a variation of the fundamental mathematics inherent in the problem, which is essentially profit and loss, addition and subtraction.

Enabling prompt: If the shoes cost $50 and the lady gave Bob $100, how much change would Bob give her? (Encourage acting out the transaction using "play money").

Extending prompt: A man bought a horse for $50 and sold it for $60. He then bought the horse back for $70 and sold it again for $80. How much money did the man make or lose? (adapted from Burns, 2015)

As expected, some PSTs finished the task quickly but did not necessarily achieve the correct answer. The problem was deliberately chosen as although there was only one correct answer, past experience with using this problem generated some common incorrect responses including $75, $100, $125 and $200. Having anticipated PSTs' likely responses, I used the second phase of the lesson where PSTs worked on and/or talked about their responses to the task, to model the *Monitoring* practice, which is the practice of paying attention to the thinking of the students as they worked on the task, either individually or collectively (Smith & Stein, 2018). During this time, I gave an enabling prompt and an extending prompt to two PSTs in order to provide a teaching point when it came time to reflect on the lesson at the end of the class.

The next step after providing guidance as needed and observing the PSTs' different solution strategies, was *Selecting* the solutions to focus on. This was followed by *Sequencing* the solutions in a coherent way that was likely to advance the mathematical understanding of the group (Smith & Stein, 2018). In this instance, I selected three PSTs' solution strategies (see Figures 1, 2, & 3) and confirmed with them that they were willing to share with the whole group. Alternatively, I could have prepared illustrative anonymous responses beforehand and asked the PSTs to sequence them and discuss the reasons for this. The former strategy provided a more authentic learning experience that approximated what would occur in the classroom.

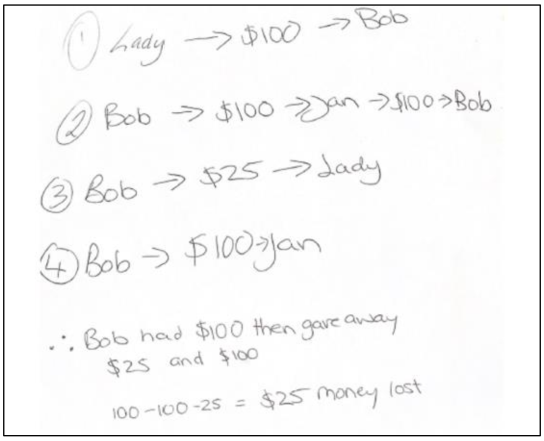

Figure 1. Beth's solution.

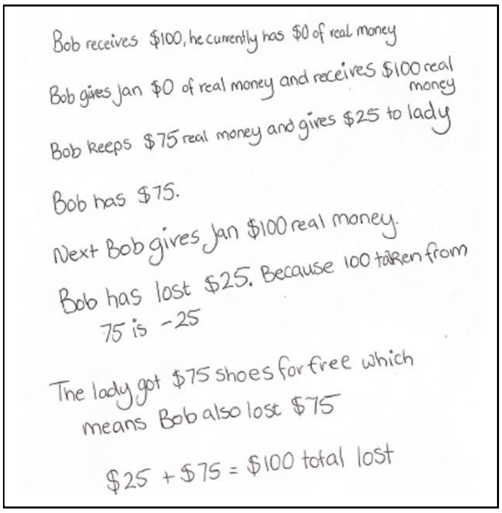
Bob receives $100, he currently has $0 of real money
Bob gives Jan $0 of real money and receives $100 real money
Bob keeps $75 real money and gives $25 to lady
Bob has $75.
Next Bob gives Jan $100 real money.
Bob has lost $25. Because 100 taken from 75 is -25
The lady got $75 shoes for free which means Bob also lost $75
$25 + $75 = $100 total lost

Figure 2. Liam's solution.

I think that he has lost $125- $100 to Jan and $25 to the lady.
He kept the $75- which was real money for the shoes.
He changed the fake $100 the lady gave him for real money with Jan.
Bob then gave the customer $25. Later Bob then gave Jan $100 back.
Therefore Bob loses $125.

Figure 3. Rosie's solution.

Selecting and sequencing are critical because they give the teacher the choice of what is to be shared. As a teacher educator, I wanted to make the PSTs aware of the distinction between calling for volunteers to share and the purposeful sharing as advocated by Smith and Stein (2018,) who stated, "By asking for volunteers to present, teachers relinquish control over the conversation and leave themselves and their students at the mercy of the student who they have placed on centre stage" (p. 44). Although the student's justification and reasoning may drive the conversation, the teacher maintains control over the mathematics and misconceptions to be highlighted.

Choosing the order for sequencing requires pedagogical content knowledge, in that the aim is to make the mathematics accessible to students and to build a mathematically coherent story line (Smith & Stein, 2018). For example, if Liam's solution was shared first, it might have been challenging for some students to understand as although it was a common incorrect answer, it was not a common approach and involved the use of negative numbers. Having said that, beginning with the most used strategy may not always be the best approach, particularly if misconceptions surface through incorrect solutions. In this case, I chose to sequence Rosie's solution, followed by Liam's and then Beth's. Because the problem generated various answers, I wanted to present some examples of incorrect responses before presenting a correct response.

Rosie's response was chosen to share first, as a number of other PSTs showed similar thought processes in that they did not take into account, that of the $100 returned to Jan, $75 of it was her original money. Liam's solution is interesting and represents another common incorrect response. Rather than focusing on identifying how much money Bob lost, Liam (and others) factored in the value of the shoes when determining loss. This distinction provided the opportunity for productive discussion around what the question was actually asking. While Beth's solution was correct, some PSTs were still not convinced, but modelling through acting out the situation assisted them to see the breakdown of the transactions. By discussing with the PSTs the rationale behind the sequencing of these particular solutions and inviting them to sequence a different set of work samples, I was able to explicitly model and teach this practice.

The fifth practice, *Connecting,* is arguably the most challenging of the five practices because teachers need to draw on their mathematical content knowledge and pedagogical content knowledge (Hill et al., 2008) to focus on mathematical meaning and relationships. Importantly teachers need to bridge the gap between what students know and what mathematics they need to learn (Smith & Stein, 2018). The goal was to promote what Ma (2010) termed, "profound understanding of fundamental mathematics" (PUFM) to describe an understanding of mathematics that is deep, broad, and thorough. In relation to Connecting, a teacher with PUFM is able to make connections among mathematical concepts and procedures. This is of benefit because connectionist teachers have been identified as being highly effective (Askew et al., 1997).

When modelling the practice of Connecting, I explicitly focused on the mathematics in the solutions, such as the operations of addition and subtraction, and the concept of profit and loss. Together with modelling how a classroom teacher may use students' solutions to make connections, I also made public the work of an MTE by explaining and justifying the learning approaches and decisions made. In this respect, I was demonstrating what Rowland et al. (2009) termed contingency knowledge but enacting it within the context of a mathematics teacher education classroom, rather than a primary classroom. This enactment also required a kind of meta-knowledge, which could be described as "knowledge for teaching knowledge for teaching mathematics" (Goos & Beswick, 2021, p. 3).

Discussion and Implications

Bob's Shoe Store problem was an appropriate challenging task to engage PSTs in problem solving and for the MTE to model the approach to teaching with challenging tasks and the five practices. The task constituted a problem in that there was no obvious path to the solution, it allowed for a range of strategies to be used and required an investment of time and energy to solve it. In essence, it was a low threshold high ceiling task (LTHC) (NRICH, 2013), as everyone could get started and everyone could get "stuck" (for more on LTHC tasks, see NRICH, 2013). Observations made during the monitoring phase indicated that many of the PSTs experienced confusion, some got stuck but most persevered and were able to generate at least one solution to the problem. Enabling prompts were used to support the PSTs who did become stuck, with the prompts serving a dual purpose: as a scaffold to help the PSTs achieve success with the task; and modelling of what, how, and when to incorporate enabling prompts.

The extensive body of research on teacher knowledge and PST knowledge has identified that teachers need knowledge of the content area and knowledge of how to make the content accessible to learners (Pedagogical Content Knowledge). The MTEs require this knowledge, but they are also required to hold knowledge different from that of the classroom teacher. Chapman (2021) terms this, knowledge of mathematics teacher education (KMTEd) where the content is teacher education, instead of mathematics. Enactment of this knowledge could include, for example, mathematics teaching practices and skills, teacher professional identity, professional noticing, and classroom preparation. Modelling teaching approaches, such as the one described in this article, require MTEs to simultaneously and flexibly focus on both school mathematics teachers' PCK and MTEs' PCK (Chick & Beswick, 2018). Furthermore, the modelling of the approach needs to be accompanied by explicit unpacking of why the MTE performed particular actions in order to make the links between theory and practice obvious for the PSTs.

This article has provided an account of a teaching episode that is illustrative of the practice that MTEs engage in when preparing PSTs to teach mathematics. It is relevant to other MTEs who have limited opportunity to access accounts of what MTEs' work looks like in practice. The implications are that PSTs benefit from seeing recommended teaching approaches enacted in their tutorials, with the benefits enhanced when the MTE explicitly provides a rationale for the approach. There is no guarantee that PSTs will see examples of good practice while on placement, and the MTE should possess the required expertise and knowledge to deepen PSTS' understanding of how theory impacts on practice.

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Acknowledgements

Thank you to Kim Beswick for introducing me to 'Bob's Shoe Store Problem' (original source unknown)

Ethical approval

Ethical approval for the research was granted by the author's institution, involving the collection of class data as part of scholarship of teaching and learning activities. Participants had the option of withdrawing data.

Competing interests

The authors declare there are no competing interests.

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