Facilities for Mathematics Teachers' Learning from Professional Development Programmes: A Qualitative Systematic Review

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It is indicated that research on professional development programmes (PDPs) for mathematics teachers mostly concerns whether a programme affects the teachers' practice or student learning, while the teachers' learning is treated as a *black box*. Calls have been made for a shared body of knowledge on teachers' professional learning and what supports such learning. In this article, 84 articles from mathematics education journals are examined with respect to facilities and catalysts for teacher learning from PDPs for mathematics teachers. Two main types of PDPs were identified—those that target *teacher knowledge growth* and those that target *changed instruction*—and it was determined that the types of facilities for catalysing learning linked to them differ. Typical of PDPs targeting *teacher knowledge growth* is that they emphasise *knowledge of content and teaching*, *knowledge of content and students* and *specialised content knowledge*; following *insight* and/or *body of knowledge* as facilities for catalysing the learning. Typical of PDPs targeting *changed instruction* is that they use more time and have a larger time span than PDPs targeting teacher knowledge growth, emphasise *knowledge of content and teaching*; and follow *strategies* by *doing/experimenting* when establishing them in the classroom as facilities for catalysing the learning.

Introduction

Participating in professional development programmes (PDPs) for teachers is assumed to increase teachers' knowledge and/or change their instruction sustainable over time, as a means to improve student learning (Desimone, 2009). A prerequisite for this assumption is that the teachers are affected by the activities facilitated by the PDP; that they learn from it. This obvious prerequisite—teachers learning from PDPs—may be the most difficult aspect of PDPs to measure (Desimone, 2009). Not surprising, research on PDPs for mathematics teachers mostly concerns whether a programme seemingly affects teachers' practice or student learning, while the teachers' learning is treated as a "black box" (Goldsmith et al., 2014). Thus, the view of how, and from what, teachers learn through PDPs, which reasonably constitutes an essential starting point for the PDP initiative, seems to have been only explored moderately. This article aims to contribute to this sparsely researched area by identifying and describing different facilities of PDPs that are assumed to function as features that catalyse mathematics teachers' learning from the PDPs. This is operationalised through a qualitative systematic review of 84 articles from journals of mathematics education, focusing on the following research questions:

Q1: *What assumed features and catalysts for teacher learning are detectable in professional development programmes for mathematics teachers?*

Q2: *How are different features and catalysts for learning connected to the target of the professional development programmes?*

Relevant Research

Historically, studies on PDPs mainly focused on teacher satisfaction and changed attitudes, rather than teacher learning or student achievement (Frechtling et al., 1995; Guskey, 2000) or changes in teachers' classroom behaviour (Kennedy, 2019; Kragler et al., 2014). About two decades ago, research on PDPs for mathematics teachers took more account of teachers' professionalism and opinions regarding professional development (Kragler et al., 2014), but mostly addressing whether a programme had seemingly affected teacher practice or student learning while still leaving out the teachers' learning (Goldsmith et al., 2014). Research on PDPs "… [takes] empirical relationships between forms of activity or task (e.g., being activity based), structures for learning (e.g., collaboration between teachers), location (e.g., situated in practice), and so on, and some measure of change to *be* teacher learning" (Opfer & Pedder, 2011, p. 377). However, temporary changes in behaviour or self-evaluated experiences of progress due to a PDP do not necessarily equal learning.

The concept of learning is complicated and multifaceted and can be described and understood in many ways and on many levels (Säljö, 2015). From the wide range of different theories on learning, this study adopted the idea that all learning theories are in some way underpinned by at least one of two metaphors for learning: the acquisition and participation metaphors (Sfard, 1998). An acquisition view of learning "imply gaining ownership over some kind of self-sustained entity" (Sfard, 1998, p. 5), often termed as knowledge, conception, notion, schema, constructions, meaning, or facts which are often connected to words denoting the individuals' actions of making these entities their own, including reception, internalisation, appropriation, accumulation, and transmission. The participation metaphor for learning can be "viewed as a process of becoming a part of a greater whole" (Sfard, 1998, p. 6), in which prominence is given to the part-whole mutual relation, in contrast to the acquisition metaphor in which the individual mind and what "goes into it" is emphasised. Learning in the participatory metaphor is pronounced by situatedness, contextuality and social mediation in which practice, discourse and communication are leading words, emphasising the learner as participating in activities rather than accumulating private knowledge. The participation and acquisition metaphors for learning, however, are not to be seen as contrasts, but rather two sides of one coin. The learner is participating in situations for knowledge intake, and "testing and verifying" the in-taken knowledge by participating in social situations.

Despite this complexity in the concept of learning and indicated lack of empirically based research focusing on teachers' learning from PDPs, some models do conceptualise what can function as catalysts to establish that learning (e.g., Clarke & Hollingsworth, 2002; Desimone, 2009; Gregoire, 2003; Guskey, 2000; Opfer & Pedder, 2011, Shulman & Shulman, 2004). Typically, such models centre around some activity that is intended to promote learning for the teacher (e.g., Clarke & Hollingsworth, 2002; Kazemi & Franke, 2004; Vermunt & Endedijk, 2011), and in most cases, the "doing" of the activity is followed by a reflection on the doing part, in which the doing is assumed to function as the "eye-opening event" for the teacher, while the reflection is assumed to function as the catalyst for learning the content (e.g., Castellan, 2012; Korthagen, 2017). In this article, the goal is to identify signs of such theoretical conceptualisations of facilities for and catalysts of teacher learning from PDPs, and the aim of the study is to identify and describe different facilities of PDPs that are assumed to function as features that catalyse mathematics teachers' learning from the PDPs. This is achieved by describing different aspects of PDPs that are assumed to function as facilities or catalysts for mathematics teachers' learning from the PDPs. As a starting point, it is assumed that an activity, which may be reading texts to gain knowledge, teaching a lesson by following a specific lesson plan, cooperating with colleagues in assessing students' work or reflecting upon an illustration of practice, for example a videotaped lesson, is commonly described as having a target (what/who is expected to change from the suggested activity), a content (what to learn from the activity), a form (what to do and how to do it, according to the description of the activity) and is implemented within sufficient time frames according to the activity requirements (Brehmer & Ryve, 2018).

In describing activities that facilitate catalysts for teacher learning in PDPs, the *target* of the PDP can be understood as *what/who is expected to change/improve* due to the PDP. Even if the final goal of

PDPs for teachers in some sense is to improve student learning, the PDP itself might target some change in teaching instruction and/or growth in teacher knowledge as a means to achieve student learning (Desimone, 2009). For categorising the PDP's target, Desimone's (2009) model—including the three different targets—teacher knowledge growth, changed instruction, and improved student learning—is useful as a well-established conceptual framework for studying effects of PDPs. The *content* of a PDP for mathematics teachers can be described in terms of *mathematical knowledge for teaching* (Ball et al., 2008), and categorised as promoting learning concerning *subject matter knowledge* or *pedagogical content knowledge* and subcategories of these. The *form* of the activities suggested by the PDP can be categorised by *what to do* and *how to do it*. Kwakman (2003) uses a simple model for categorising *what to do* by separating reading, doing/experimenting, reflecting and/or cooperating, while Kennedy (2016) offers a frame for categorising *how to do it* (e.g., by following prescriptions for how to act and/or build one's knowledge base without any prescribed actions to take). The *duration* of the PDP can be measured in number of hours is used for the PDP (time spent) and over what timespan these hours are spread (time spread) (Desimone, 2009). Thus, these frameworks that describe the target, content, form, and duration of activities in PDPs together constitute a collection of features that are assumed to facilitate and affect teacher learning from PDPs.

Despite such measurable features that may facilitate teacher learning from PDPs, Goldsmith et al. (2014) noted in their review of mathematics teacher professional learning that the same professional learning opportunities have different impact on different individuals. They mention examples such as teachers' stance towards curriculum use, mathematical knowledge, and teachers' beliefs about mathematics teaching and learning, but do not explicitly mention that different targets of the PDPs may constitute different learning opportunities and may appeal differently to different individuals. It is, however, reasonable to believe that the learning process in a PDP targeting teachers "knowledge growth" (acquisition learning metaphor) differ from a PDP targeting "change in teacher instruction" (acquisition learning metaphor), which in turn may appeal to different teachers differently, and as an affect constitute different learning opportunities for different individuals. Further, they noted that research on professional learning for teachers tends to focus on PDPs' effectiveness (without revealing what "effective" means or how it is measured) and few studies focus on processes and mechanisms of teacher learning. From this, they concluded that the field of mathematics education needs to "build a shared body of knowledge about the nature of teachers' learning and the catalysts that support it" (Goldsmith et al., p. 25).

Catalysts that support learning are often described in PDPs as activities that are meant to function as means to support the teachers' learning from the PDP content (e.g., Clarke & Hollingsworth, 2002; Kazemi & Franke, 2004; Vermunt & Endedijk, 2011). The term *catalyst* is not specifically defined, however. It has been described as "the thing" that establishes connections that result in new knowledge (Castellan, 2012), and has been used in terms of how some activity can catalyse learning (e.g., Bolander et al., 2020; Castellan, 2012; Goldsmith et al., 2014; Korthagen, 2017), but it has not been explicitly described what catalysing *is* or signifies; only that it is something that *implies* learning. This "thing" can thus be understood as some eye-opening event that originates from some activity, or any feature that facilitates learning opportunities, from a participation metaphor for learning perspective. From this perspective, it seems impossible to point out the actual learning catalyst; the very "thing" that establishes the cognitive connection in the learner as described in the acquisition metaphor for learning. Instead, one can point out the activities/features that are interpreted as facilitating plausible catalysts for learning, the activity that provides the eye-opening moment. Thus, catalysts for teacher learning are typically not explicitly mentioned in reports on teacher PDPs. Rather, suppositions of how, or from what, teachers learn are revealed based on learning facilities of the programme, the activities from which the learning is assumed to originate. How such activities, assumed for teacher learning, can be detected in articles is further elaborated on next.

Methodology

For this study, a qualitative systematic review methodology (Grant & Booth, 2009) is used for selecting, analysing, and mapping articles describing PDPs for mathematics teachers. It is a method for investigating the findings from qualitative studies, which "looks for 'themes' or 'constructs' that lie in or across individual qualitative studies" (p. 99) and is "interpretative in broadening understanding of a particular phenomenon" (Booth, 2006, p. 422). In this study, the goal is to investigate reports from qualitative studies on PDPs for mathematics teachers to focus on themes and constructs connected to teacher learning from these PDPs, to interpret and better understand how and from what such learning is facilitated by the PDP. The methodology is elaborated on below, but before describing the different methods, it is relevant to establish how in this article one should understand and relate to signs and descriptions of teacher learning and gain ideas of how to identify and describe activities in PDPs that are assumed to function as catalysts for learning.

As described, this article adopts the participation and acquisition metaphors for learning (Sfard, 1998) as for how to view the complicated and multifaceted concept of learning. From this view on learning, and as the aim of this review is to identify and describe different facilities of PDPs that are assumed to function as features that catalyses mathematics teachers' learning from the PDPs, it is reasonable to include activities that fit both the idea of learning as acquisition and as participation in the analysis of the articles. Thus, it is rational to focus the analysis on activities in the PDPs, equivalent to for example practicing, teaching, discussions, and collaboration (participation metaphor), as well as reading, assessing, reflecting, and structuring (acquisition metaphor) and combinations of these. To recognise different facilities assumed to catalyse learning in PDPs for mathematics teachers, the analytical approach starts with the idea that what one teaches reflects an idea of how the intended learner learns.

As an example, the notion that students are expected to learn new mathematical content through problem-solving rather than being served facts from teachers, books, or other sources (e.g., Simon, 1995; Smith & Stein, 2011) is based on a *teaching idea*. This idea is based on research indicating that students learn through struggling, communicating, and creating their own solutions rather than imitating some existing solution schema, which is manifested in the teaching approach *teaching through problem-solving*. The rationale for adopting this teaching approach is a research-based assumption that it is more beneficial for students to learn from this type of teaching than through other approaches (Simon, 1995). As for this teaching idea for students, the analysis approach in this study is to assume that the teaching idea in a PDP for teachers reflects ideas of how teachers are assumed to learn. Such teaching ideas are manifested in some activity. For example, the idea *teaching through problem-solving* could be manifested in an activity in which the teacher teaches a lesson that is designed as a problem-solving lesson. In this case the teaching idea, which reflects an educated assumption of how to learn something, is detectable through which activity is described in the PDP. Thus, to detect facilities for catalysing teacher learning from PDPs, this study looks for activities that are assumed to manifest teaching ideas in articles reporting on PDPs for mathematics teachers. Further, to be able to analyse the detected activities, it is noted: what/who is expected to change from the activity (i.e., the target); what to learn from the activity (i.e. the content); what to do and how to do it according to the activity's description (i.e. the form) and timeframes for the PDP (duration).

The methodological approach to doing this is elaborated on in the following three sections: 1) *selecting and reading articles*, in which the workflow is presented and positions in the selection of articles is argued for; 2) *the mapping*, in which it is described how the articles were analysed to extract and categorise plausible facilities for teacher learning; and 3) *analysing the mapping*, in which it is described how the mapping is analysed to identify and describe facilities and catalysts for teacher learning.

Selecting and Reading Articles

In this study, articles were selected from the following journals: *Journal of Mathematics Teacher Education, International Journal of Science and Mathematics Education, Mathematics Teacher Education and Development, Journal for Research in Mathematics Education, Educational Studies in Mathematics, Mathematics Education Research Journal, Journal of Mathematical Behavior, Mathematical Thinking and Learning,* and *Zentralblatt für Didaktik der Mathematik* (*ZDM-Mathematics Education* [ZDM]).

The selection of journals was based on three reasons: 1) a search through the databases *Web of Science, ERIC, Google Scholar, Scopus, Math Educ* and *Wilson Web* based on the phrases/words mathematics teacher knowledge, learning, change, and professional development in different combinations revealed that relevant hits for the purpose of this study stemmed predominantly from the above-mentioned journals; 2) these journals represent a variety of contexts; and 3) they are established peer-reviewed journals in mathematics education. Every paper in all volumes of the selected journals published between 2009 and 2022 (January) were inspected. This time span was chosen because Goldsmith et al. (2014) performed their review up to the end of 2008, and this study is focused on investigating what has happened to the "black box of teacher learning" since then. However, as for qualitative systematic reviews in general, the goal was not to accomplish a complete summary of all articles reporting on a certain PDP (Grant & Booth, 2009). Rather, with the purpose of answering the research questions, articles were collected as described above without the intention of covering the whole field. Upon finding an article the title and abstract were read first, searching for the words professional development, learning, teachers, change, and knowledge, as well as synonyms for them such as PD, in-service teacher, mathematics teacher, teaching mathematics, teacher change and knowledge growth. The criteria for including an article were that it should include an accomplished PDP (i.e., time-framed organised education programme for in-service teachers conducted within their working hours); comprise in-service teachers teaching mathematics (not pre-service teachers or teachers not teaching mathematics); and describe something about the participant teachers' activities for learning from the PDP (e.g., practicing, teaching, discussing, collaborating, reading, assessing, reflecting and structuring) and not, as in some cases, what the PDP's facilitator had learned from launching a programme. Theoretically based articles were not excluded. Only empirically based articles that fulfilled these criteria were found. The reading of these papers' titles and abstracts resulted in the selection of 129 papers for further analysis.

The Mapping

The first step in categorising the articles in the mapping process was to read them in their entirety. During this reading, 45 articles were excluded due to reasons such as: concerning pre-service teachers; not describing fully the PDP; only describing preparations for a PDP; only describing teachers' perceptions of PDP; or focusing on the teacher educators rather than the mathematics teachers. The 84 resulting articles are distributed across the journals as presented in Table 1.

Table 1.
Distribution of articles analysed by journal

Journal	Number of articles
Journal of Mathematics Teacher Education	23
International Journal of Science and Mathematics Education	8
Mathematics Teacher Education and Development	12
Journal for Research in Mathematics Education	2
Educational Studies in Mathematics	2
Mathematics Education Research Journal	8
Journal of Mathematical Behavior	3
Mathematical Thinking and Learning	1
Zentralblatt für Didaktik der Mathematik (ZDM)	25

In mapping the PDPs' indicated facilities and catalysts assumed for teacher learning in the articles, the *target*, the *content*, the *form* and the *duration* of the PDP activities were noted, which are elaborated on separately below.

Target

The PDP *target,* is here understood as answering the question, "What is expected to change due to the programme?" A PDP may focus on one or several of three categories of change: a) teacher knowledge growth, which can be thought of in terms of the acquisition metaphor for learning; b) change in teacher instruction, which can be associated with the participation metaphor for learning (Sfard, 1998); and c) improved student learning. It was fruitful to map the PDP target, as Research Question 2 of the study was to examine whether different catalysts for teacher learning were facilitated depending on the PDP target.

As an example of the mapping process, the PDP described in Geiger et al. (2016) was interpreted as targeting *teacher knowledge growth* as they aimed to "examine the potential of video-catalysed reflective practice for supporting ongoing teacher professional learning in numeracy" (p. 457). Thus, the target was to increase the participating teachers' knowledge in numeracy, which fits the acquisition metaphor (Sfard, 1998). Another example is, the PDP described in Lotter et al. (2014), which was interpreted as targeting *changed instruction* as that programme was constructed "with (1) content instruction through inquiry lessons, (2) practice teaching to middle school students, and (3) coach-led reflection improved their understanding of inquiry-based practices and the quality of their classroom inquiry implementation" (p. 1). The aim of the PDP was thus to introduce the participants to a specific way of teaching, which was interpreted as targeting a change in the participants' instruction. This can be interpreted as following the participation metaphor for learning as the activity in the PDP was focusing on teachers' participating in structured teaching sequences. There were also some PDPs that were interpreted as having more than one target. For example, Orrill and Kittleson (2015) was interpreted as targeting both *teacher knowledge growth* and *changed instruction* as they "examine one teacher's opportunities to develop a coherent understanding of proportional situations through connection making in professional development (PD) and the ways in which those experiences were evidenced in her own classroom practice teaching the same task from PD" (p. 273). The interpretation was based on the way that the targets examine the teacher's understanding of proportional situations (teacher knowledge growth, acquisition metaphor) and show simultaneously how this was evidenced in the teaching of the subject (changed instruction, participation metaphor).

To sum up, the target of the PDP is here understood as answering what or who is the primary object of change; the teachers' knowledge growth (fitting the acquisition metaphor), the teachers' instruction (fitting the participation metaphor) or the students' learning. Next, the content to acquire or participate or engage with was mapped.

Content

The content of the PDPs, thus what type of knowledge the teachers were expected to acquire or engage with during the PDP, was mapped by the framework of Ball et al. (2008)—*mathematical knowledge for teaching*. It was noted whether the content was connected to one or more of the categories *common content knowledge* (CCK), *specialised content knowledge* (SCK), *horizon content knowledge* (HCK), *knowledge of content and students* (KCS), *knowledge of content and teaching* (KCT), and *knowledge of content and curriculum* (KCC).

As an example, Geiger et al. (2016) were investigating, "What changes to teachers' state of awareness of their own teaching practice are possible in short-term professional learning programmes?" (p. 458), which focused on developing the teachers' awareness in their teaching practice of a specific subject. Thus, this PDP was interpreted as focusing on developing the participants' KCT. As an example in which two types of content were interpreted, Orrill and Kittleson (2015) focused on both how teachers "develop a coherent understanding of proportional understanding" (p. 273), which was interpreted as SCK, and "the ways in which those experiences were evidenced in her own classroom practice teaching the same task from PD" (p. 273), which was interpreted as focusing on KCT.

Form

Two types of form were categorised: what to do and how to do it.

What to do: Here *what to do* is understood as *activities for learning*. For this matter, *learning* is regarded as an always active process, whether the "action" is visible and/or cognitive. *What to do* was mapped by following Kwakman (2003), in which a PDP activity can be divided into: *reading* in order to collect new personal input such as data, knowledge or information; *doing/experimenting* to gain new experiences and apply new ideas; *reflection* in order to recognise, think through and change routine behaviour; and *cooperating*, which through collaboration provides teachers with new ideas and feedback.

As an example, An and Wu (2012) focused on teacher learning "through grading homework, assessing and analyzing misconceptions" (p. 717), which was interpreted as *reflecting* according to Kwakman (2003). The participants' learning process stemmed from reflecting on the misconceptions detected rather than the grading itself, which would have been interpreted as doing/experimenting. Even if there were elements of the other categories, especially doing/experimenting, the learning factor of the PDP was on reflecting on students' misconceptions. An example in which the PDP was interpreted as focusing on all four forms of what to do is Orrill and Kittleson (2015). In this study, the participants first explored proportional reasoning (reading), then they were teaching this subject (doing/experimenting) and followed this up by reflecting on the teaching (reflection) collaboratively (cooperating). In this example, all four forms of what to do were interpreted to be equally focused in the PDP and crucial for the teacher learning aspect of it.

How to do it: To map *how* the teachers are expected to be active, Kennedy's (2016) four categories of how to act upon the content as presented in a PDP was used. A PDP can be *prescriptive* with explicitly described actions to take, like following a recipe. If there is a described rationale for the recipe, it is called a *strategy*. Here the teacher is expected not only to act, but also to understand the rationale for the action. If no particular action is suggested for the teacher, who is merely provided with knowledge aiming to offer support through "aha moments", this is called *insight*; or, if only knowledge is provided, this is called *body of knowledge*.

In Koichu et al. (2016) the focus on *how to do it* was interpreted as *body of knowledge* as the study was examining "how variations in task design may affect mathematics teachers' learning experiences" (p. 349). Thus, the PDP focused on increasing the teacher's body of knowledge rather than providing aha-moments (insight), describing actions to take (prescriptive), or providing rationale for some action to take (strategy). An example of a PDP that was interpreted as more than one *form* of *how to do it*, is Bruns et al. (2017). In that study, the participants were guided to aha-moments (insight) in how important teachers' own mathematical knowledge is for providing opportunities for students to learn mathematics. Further, the PDP aimed to "expand and deepen their basic mathematical knowledge" (p. 82, body of knowledge).

Duration

Duration was noted in terms of both amount of time (the number of hours the teachers spend on the PDP, according to recommendations of Desimone (2009), more or less than 20 hours), and span of time (how long the PDP lasts, more or less than one semester or 5 months).

An example from the mapping is Orrill and Kittleson (2015) in which 42 hours (more than 20 hours) was spent on the PDP, spread over 14 weeks (less than 5 months). This was explicitly and very precisely described, but in other articles, such as Lotter et al. (2012), it was described more implicitly. Here one had to summarise the time spent and spread for different parts of the PDP to conclude that it in total used 86 hours (more than 20 hours) and that time was spread over approximately one year (more than 5 months).

Summary of the mapping

The mapping process is summarised by the following example. In the Orrill and Kittleson (2015) study, the participants first explored proportional reasoning, then taught the subject, which was followed up by reflecting on the teaching collaboratively. In this example, the *target* was mapped as both teacher knowledge growth (acquisition metaphor) and changed instruction (participation metaphor) as the

participants were both exploring proportional reasoning for themselves (growth in knowledge) followed by teaching the subject (changed instruction). The *content* to learn was mapped as SCK because the teachers were expected to understand the concept of proportionality, and as KCT because they were supposed to learn teaching strategies for proportional reasoning. According to *form*, the participants first explored proportional reasoning (reading), then they were teaching this subject (doing/experimenting) and followed this up by reflecting on the teaching (reflection) collaboratively (cooperating). Thus, all four forms of *what to do were* interpreted to be equally focused in the PDP and crucial for the teachers' learning of it. In terms of *how to do it*, aha-moments were created when the participants explored proportional reasoning, which was mapped as *insight*, and then the participants followed described and rationalised actions to take when teaching the content, mapped as *strategy*. Finally, *duration* was mapped as 42 hours *spent* on the PDP, *spread* over 14 weeks. Thus, Orrill and Kittleson (2015) was mapped as described in Table 2. The shaded areas are the features of the PDP identified.

Table 2.
Analysis of Orrill and Kittleson (2015)

Target	Teacher knowledge growth		Changed instruction		Improved student learning	
Content	CCK	SCK	HCK	KCS	KCT	KCC
Form (what)	Reading	Doing/exp	Reflection	Cooperating		
Form (how)	Prescriptive	Strategy	Insight	Body of knowledge		
Duration (spent)	< 20 hours	≥ 20 hours				
Duration (spread)	< 1 semester	≥ 1 semester				

Altogether, 84 articles focusing on PDPs for mathematics teachers were collected and analysed as the example in Table 2. Different frameworks to map the PDPs' target, content, form, and duration were used and the mapping should be seen as a collection of assumed features facilitating catalysts for teacher learning through PDPs. Table 4 in the Results section summarises the mapping. Next, the method for analysing the mapping is presented.

Analysing the Mapping

The third phase of the methodological process was an analysis of the mapping together with the analysis of each article. In this process, the parameter *target* of the PDP was *isolated* and connected to the mapped *content, form,* and *duration* for one such target at a time. As an overview example of the procedural rationale, 54 of the 84 articles described PDPs with the *target* changed instruction. In this process, only these 54 articles were handled and for these the *content, form* and *duration* of the PDPs were noted from the analysis. More precisely, for these 54 articles targeting changed instruction, the *content* was mapped in terms of what to know*, form* in terms of what to do and how to do it, and *duration* in terms of time span and spent. In this analysis, tables as illustrated in Table 3 were constructed for all articles analysed.

Table 3.
Example of Analysis of Content, Form and Duration Connected to the Target Changed Instruction

Target	Changed instruction					
Content	CCK	SCK	HCK	KCS	KCT	KCC
Form (what)	Reading	Doing/exp.	Reflection	Cooperating		
Form (how)	Prescriptive	Strategy	Insight	Body of knowledge		
Duration (spent)	< 20 hours	≥ 20 hours				
Duration (spread)	< 1 semester	≥ 1 semester				

Fifty-four tables similar to Table 3 were created and filled with information for each analysed PDP targeting changed instruction. In this process, the content KCT was represented in 47 of these 54 tables, the form doing/experimenting was represented in 51 of the 54 tables and strategy was represented in

35 of the 54 tables. Regarding duration, not all articles provided such information. Only 22 mentioned time spent, of which 18 used more than 20 hours and 37 mentioned times spread of which 26 included more than one semester. This approach, which made it possible to compare facilities for catalysing teacher learning with departure in PDPs with different targets, was then repeated for the targets, *teacher knowledge growth* and *improved student learning*. Thus, the rationale for and aim of this procedure was to detect contingent differences in types of facilities and catalysts used in PDPs, for different types of PDPs. More precisely, the aim was to investigate whether there were different learning catalysts facilitated by PDPs with the target *teacher knowledge growth* compared to PDPs with the target *changed instruction* or *improved student learning*. Further details of the analysis are provided in the *Results* section.

Results

The results are presented in two main sections: *Distribution of programme target, content, forms, and duration*; and *Characterising two types of PDPs based on the target*. The distribution of the categories is presented in Table 4. This is followed by a characterisation of two types of PDPs based on which target they aim to develop, identified through the mapping and examination of the articles.

Distribution of Programme Target, Content, Forms, and Duration

The analyses of the 84 articles revealed that 54 of them (64%) targeted teachers' changed instruction, 38 teacher knowledge growth (45%), and only five targeted improved student learning (6%). The fact that the sum of these categories exceeds 100% is due to some articles having been categorised into more than one target, which is the case for several categories in the mapping. Further, it was noted that the content of KCT (see Table 4 for explanation of abbreviations) was the most frequently occurring content in the articles analysed. SCK and KCS were also recurrent in many PDPs, while content focusing on CCK, HCK, and KCC was recurrent to a much lesser extent. In terms of *form: which actions to take*, reflection and doing/experimenting were the most commonly recurrent and reading was used to a slightly lesser extent, while cooperating was the least recurring form. Further, for *how to do it*, *strategy* and *prescriptive* were more frequently used than was *insight* and *body of knowledge*. As regards duration, more PDPs spent more than 20 hours than less than 20 hours; and regarding time span, more PDPs used more than one semester than less than one semester.

Table 4.

Mapping of Target, Content, Form and Duration of the Articles Analysed

	Category	*n*	%
Target	Teacher knowledge growth	38	45
	Changed instruction	54	64
	Improved student learning	5	6
Content	Common content knowledge (CCK)	10	12
	Specialised content knowledge (SCK)	25	30
	Horizon content knowledge (HCK)	2	2
	Knowledge of content and students (KCS)	29	35
	Knowledge of content and teaching (KCT)	62	74
	Knowledge of content and curriculum (KCC)	7	8
Form	<u>Which action to take</u>		
	Reading	43	51
	Doing/experimenting	65	77
	Reflection	66	79
	Cooperating	32	38
	<u>How to do it</u>		
	Prescriptive	21	25
	Strategy	34	40
	Insight	31	37
	Body of knowledge	19	23
Duration	Time spent * < 20 hours	7	23
	≥ 20 hours	23	77
	Time span** < 1 semester (5 months)	19	31
	≥ 1 semester (5 months)	42	69

Note: Regarding duration, *specified in 30 articles and **specified in 61 articles.

Characterising Two Types of PDPs Based on the Target

Two types of PDPs were characterised based on their target as identified from the mapping and examination of the 84 articles: one type with the target *teacher knowledge growth*, and one with the target *changed instruction*. As there were only five articles describing a PDP with the target *improved student learning*, there was not enough data to adequately characterise this type of PDP.

Characterising PDPs targeting teacher knowledge growth

Characterising a PDP targeting teacher knowledge growth (*n* = 38) was done by looking at all combinations of content, form and duration of such PDPs, by using Table 3 for each article analysed as targeting teacher knowledge growth. This shows (in numbers) that PDPs targeting teacher knowledge growth consist of:

a) *Content*—teacher knowledge growth was comprised mostly of *KCT* (*n* = 28, 74%) and *KCS* (*n* = 14, 37%). Also, of the 25 PDPs containing focus on SCK (see Table 4), 17 of them targeted teacher knowledge growth.

b) *Form*—several actions were often taken within the same PDP, but reflection (*n* = 31, 82%) was the most characteristic. How to be active concentrated on *insight/knowledge* (*n* = 29, 76%).

c) *Duration*—of the 38 articles, 28 commented on time span and 17 on time spent. It was a slight predominance of more than one semester (16 of the 28 = 57%) and more than 20 hours spent (12 of the 17 = 71%).

Based on the analysis of these 38 articles, a characterisation of a PDP targeting *teacher knowledge growth* was built. Such PDPs typically emphasised *KCT* and *KCS,* the majority of which also included a focus on *SCK.* KCT and KCS are included in what Ball et al. (2008) described as *pedagogical content*

knowledge, in contrast to *subject matter knowledge*, in which SCK is included. This content was typically managed through *reflection* (Kwakman, 2003) and typically promotes *insight* and/or *body of knowledge* (Kennedy, 2016), rather than prescriptions or strategies. As for duration, more than 20 hours of time spent was typical, but the time span could vary.

Characterising PDPs targeting changed instruction

Characterising a PDP targeting changed instruction (*n* = 54) was done by looking at all combinations of the content, form, and duration of such PDPs, by using Table 3 for each article analysed as targeting changed instruction. This shows (in numbers) that PDPs targeting changed instruction consists of:

a) *Content*—in PDPs that focused on changed instruction, the most represented content was *KCT* (*n* = 47, 87%).
b) *Form* was mostly represented through *prescriptive* and/or *strategy* (*n* = 35, 65%) regarding how to be active. Further, all types of actions to take (reading, cooperating, reflecting, or doing/experimenting) were represented, but the most conspicuous facilitator for learning was doing/experimenting (*n* = 51, 94%).
c) *Duration*—26 of the 37 (70%) articles mentioned "time span" of more than one semester, and 18 of the 22 (82%) articles mentioned "time spent" was more than 20 hours.

Based on the analysis of these 54 articles, a characterisation of a PDP targeting *changed instruction* was built. Such a PDP typically emphasises *KCT* (Ball et al., 2008). This content is typically expressed in the PDP as *strategies* (Kennedy, 2016) to follow when *doing/experimenting* (Kwakman, 2003); that is, establishing it in the classroom. Thus, doing/experimenting by following strategies from the PDP was commonly used facilitators for learning the content in PDPs targeting changed instruction. The classroom practice was then typically followed by reflection. Further, PDPs targeting changed instruction commonly used more than 20 hours spread over at least one semester. Compared to PDPs targeting teacher knowledge growth, time span seemed to be considered more important in PDPs targeting changed instruction.

Next, the resulting mapping and facilities for catalysing teacher learning from these two types of PDPs, based on their targets, are discussed.

Discussion

The methodology applied for analysing facilities on teacher learning from PDPs, which can be seen as an applicable model for future similar analyses, is based on mapping and analysing different features in PDPs, assumed to affect teacher learning from it; target, content, form, and duration (Brehmer & Ryve, 2018; Desimone, 2009). The target of a PDP entails what it is aimed at: teacher knowledge growth, changing teacher instruction or improving student learning. The target, however, may not connect directly to facilities for teacher learning from the PDP. The content concerns *what* to learn, but not *how* to learn it. It is reasonable to assume that the content of a PDP is based on the PDP's target. That is, a PDP aiming to increase teacher knowledge can be assumed to have a different content than one aiming to change a teacher's classroom instruction. Also, it is reasonable to assume that what activity is suggested to learn the content, is different in a PDP aiming to increase teacher knowledge compared to one aiming to change teacher classroom instruction. But even if a PDP's content can be connected to its target, and in some sense affect which activities are suggested to learn the content, it cannot itself constitute the learning. One learns the content, but the content itself does not facilitate or catalyse the acquisition of the intent of learning the content. It is within the activities in which the content is acted upon, that the learning effect lies. Thus, in this study, activities were analysed in terms of *what* to do and *how* to do it, that is, the *form* of the PDP. This methodological approach leads to a result that can be summarised as: a PDP's content is chosen based on who its target is; its activity is based on the content to be learnt; and it is within the PDP's activities that the learning of the content occurs, facilitated by what to do and how to do it. The time spent on a PDP, and the duration this time is spread, might affect the learning of the content. Time, however, is not regarded as a facilitator or catalyst for learning as it is not in itself establishing the acquisition of knowledge nor facilitating participating in activities.

Nevertheless, time can provide opportunities for knowledge acquisition—that is, the more time and span of time one spends thinking about something, the more opportunities there are for an eye-opening moment; but the time itself is not regarded as the eye-opener and maker of knowledge acquisition.

In the analytical frame, assumed features for facilitating catalysts for teacher learning from PDPs (Desimone, 2009), based on 84 articles describing PDPs for mathematics teachers were mapped. The resulting mapping (Table 4) displays the frequency of critical aspects of PDPs for mathematics teachers (Desimone, 2009), the target, content, form, and duration of the analysed PDPs. Most of the PDPs analysed targeted *changed instruction* and some targeted *teacher knowledge growth*, while only a few directly targeted *improved student learning*. KCS as well as KCT (Ball et al., 2008) were dominant in terms of what content to learn, and the content was often facilitated by *strategies* to follow and understand and/or *insights* to receive (Kennedy, 2016) and by *doing/experimenting* and/or *reflecting* (Kwakman, 2004). Also, most PDPs lasted more than one semester and used more than 20 hours.

What do these results indicate in terms of what learning opportunities PDPs provide and what facilitate this learning? Firstly, according to the *target* of a PDP, the results resonate with Desimone (2009) in that teachers are assumed to increase their knowledge and/or change their instruction, as a means to improve student learning. Even if the final goal of PDPs for teachers is to improve student learning, virtually all the analysed PDPs focused on increasing teacher knowledge and changed instruction. Secondly, more PDPs orient the *content* to learn toward pedagogical content knowledge (KCS and KCT) than toward subject matter knowledge (CCK and SCK). Even if subject matter knowledge is considered highly important for mathematics teachers (Hill et al., 2005), the results indicate that, in PDPs for mathematics teachers, learning to teach the subject is emphasised more than learning the subject itself. Thirdly, the results shows that a predominantly part of the analysed PDPs spend more than 20 hours spread over at least one semester for a PDP, which is in line with recommendations in Desimone's (2009) framework. Fourthly, regarding *form*, the results show that the content is mostly facilitated by following and understanding strategies as well as having insights (Kennedy, 2016), and the predominant activities for learning the content are doing/experimenting and reflecting (Kwakman, 2004).

Goldsmith et al. (2014) view teacher learning from PDPs as changes in knowledge, beliefs and/or practice, and different activities as catalysts for this learning. In this view, they indicated that reflecting is *the* activity for catalysing learning, which resonates with others' suppositions as to what catalyses learning from a PDP (e.g., Castellan, 2012; Korthagen, 2010, 2017). Goldsmith et al. highlighted the activity itself, for example, teaching a lesson according to prescribed instructions or reading some instructional text, as "opening the eyes" of the teacher and the following *reflection* on the completed lesson or the reading of the text is described as shaping the actual learning of the content. Thus, it seems to be within the act of reflection that the *catalysing* of the learning lies.

In most of the PDPs analysed in this study, teachers reflected upon different activities. For instance, they reflected individually on a videotaped lesson, together with others on experiences from reading some text, or after following some instruction in teaching a lesson. Although different eye-opening actions were facilitated, most PDPs included some following reflection as a concluding action to take. Thus, a conclusion based on the analysis of the 84 articles, is that the *reflection* on the activity is considered crucial for the learning of the activity. In other words, whatever features for facilitating teacher learning from PDPs, a following reflection seems to be considered conclusive for the learning of the content and might thus be considered what is assumed to catalyse the learning from an activity in a PDP for mathematics teachers. Based on this concluding assumption, two main issues are considered relevant to investigate further.

Firstly, in line with the ideas of Korthagen (2017), more research seems to be needed on the connection between what to learn, what activities best suit acquisition of the intended content, *how* to reflect on these activities, and what guidelines for reflection are the most convenient and effective for supporting teachers to best catalyse their learning of the content. Secondly, because of the vague definitions available of what "*catalysing learning*" means, there is a need for a more rigorous inquiry into, and ultimately a definition of, the concept of learning catalysts. From this study, it seems relevant

to include the idea of viewing learning as acquisition or participation in how to better define the concept of learning catalysts. Facilities for learning considered as eye-openers for the teachers, may be seen as aligning with the participation metaphor for learning, and the desired learning from the facilitated activity is then acquired from reflection. Thus, in trying to define catalysts for learning, one might consider the view of learning as acquisition, rather than as participation, as a guiding metaphor for learning as crucial. Regardless of what "catalysing learning" means and how different catalysts are connected to different PDP content, the notion of how different programme targets seem to use different facilities for catalysing learning can be interesting for PDP design and/or research.

In comparing PDPs with the target *teacher knowledge growth* and those with the target *changed instruction*, a connection between what content to learn, and what is assumed to facilitate the learning was noted. This result raises questions concerning core critical features emphasised in various models for teachers' PDPs (e.g., Clarke & Hollingsworth, 2002; Desimone, 2009; Gregoire, 2003), especially in relation to what content to learn and what facilities are preferred for catalysing the learning of the content. The results imply that PDPs with the target *changed instruction*, emphasising KCT (Ball et al., 2008), were assumed to be facilitated by doing/experimenting followed by reflection (Kwakman, 2004) for what to do, and following strategies (Kennedy, 2016) for how to do it. PDPs with the target *teacher knowledge growth*, emphasising both KCT and KCS as well as included the majority of the PDPs that focused on SCK (Ball et al., 2008), were facilitated through reflection (Kwakman 2004) and by insight/knowledge (Kennedy, 2016). The PDP's target and content are thus indicated to be connected to what is assumed to facilitate the learning of the content. Learning the content SCK and KCS, which is towards the acquisition metaphor for learning, is indicated to be facilitated by insight/knowledge and reflection, while learning the content KCT, which is towards the participation metaphor for learning, is facilitated mostly by strategies and doing/experimenting followed by reflection. Thus, the core critical feature "content focus", emphasised in various models for teachers' PDPs (e.g., Clarke & Hollingsworth, 2002; Desimone, 2009; Gregoire, 2003), is indicated to be considered to affect the core critical feature "activities for learning." This seems reasonable, as the learning process in, for example, a PDP that promotes orchestrating whole-class discussions, likely differs from that in a PDP that facilitates knowledge about students' cognitive development at a certain age. Such *content effects* on core critical features are not extensively elaborated on in various change models of teacher PDPs (e.g., Clarke & Hollingsworth, 2002; Desimone, 2009; Gregoire, 2003), therefore, may be incorporated into the development and future use of such models.

According to the result of the content's impact on facilities for learning, it is further reasonable to add to the idea of Goldsmith et al. (2014) that the same PDP have different impact on different individuals. They mention examples of explanations for this finding, such as teachers' stance towards curriculum use, and their mathematical knowledge and beliefs about mathematics teaching and learning, but do not mention that different targets and content of the PDPs may constitute different learning opportunities and may appeal differently to different individuals. The indicated connection between the target of the PDP, the content to learn and what is assumed to facilitate this learning is concluded to possibly affect teacher learning opportunities from PDPs in terms of affecting different individuals differently. It thus seems reasonable to claim that further studies are required to examine how the content affects what facilities are preferable to learn the content, and how this may be a part of explaining why different individuals are affected differently from the same PDP.

To sum up, based on the analysis of 84 journal articles and their descriptions of PDPs, different features considered as facilities for teacher learning from PDPs have been identified and quantified. These facilities may be viewed as representing core critical features emphasised in various models for teachers' PDPs (e.g., Clarke & Hollingsworth, 2002; Desimone, 2009; Gregoire, 2003), and thus the results of this study may contribute to their potential development, especially in terms of how the target and content of a PDP may affect which activities and catalysts for learning are promoted and how reflection seems to be considered *the* catalysing activity. Foremost, the results indicate that differences in what different individuals gain from a PDP may be affected by not only teachers' stance towards curriculum use, but also their mathematical knowledge and beliefs about mathematics teaching and learning (Goldsmith et al., 2014), as well as the target and content of the PDP. Further, the results indicate that

for PDPs targeting knowledge growth, as well as PDPs targeting instructional change, and no matter what content to learn from the PDP, reflection is viewed as being used as catalysing the teacher learning. This indicates that teacher learning from PDPs, whether "learning" is viewed from an acquisition or participation metaphor, is believed to be catalysed from the learner's *reflection* on some activity, which is somehow not new but is in line with and adds strength to others' assumptions and findings (e.g., Castellan, 2012; Goldsmith et al. 2014; Korthagen, 2010, 2017) and may be used in future clarification of the concept "learning catalysts." In total, based on the results and conclusions in this discussion, this article can be seen as a glimpse into the "black box" of teacher learning from PDPs (Goldsmith et al., 2014) and as a contribution to the building and strengthening of "a shared body of knowledge about the nature of teachers' learning and the catalysts that support it" (p. 25).

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Competing interests

The authors declare there are no competing interests

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Appendix

The 84 articles presented here constitute the empirical material for this study.

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