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Identity as a Mathematical Thinker

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I argue in this article that identity as a mathematical thinker develops through self-directed learning within a supportive community of practice. The dynamic nature of identity as a mathematical thinker is illustrated by considering the experiences of primary pre-service teachers who undertook a mathematics and technology subject in their undergraduate education degree. The pre-service teachers developed their identity as self-regulating learners through setting goals, planning, organising, recording, self-evaluating, and structuring their learning environment. Affective issues, such as resilience, confidence, and ownership, were also significant in identity formation. The mathematical social context provided by the tutorial group and use of technology impacted on self-regulation and social identity and hence on pre-service teachers' identity as mathematical thinkers.

This paper explores theoretically the ways in which pre-service teachers may develop their identity as mathematical thinkers. The basis of the argument is that teacher education students need to become self-regulated, confident learners with a sense of ownership of their mathematical problem solving in order to identify as mathematical thinkers. The social milieu and control of the social processes involved in learning plays a significant role in this development. Figure 1 provides a simplified view of this argument.

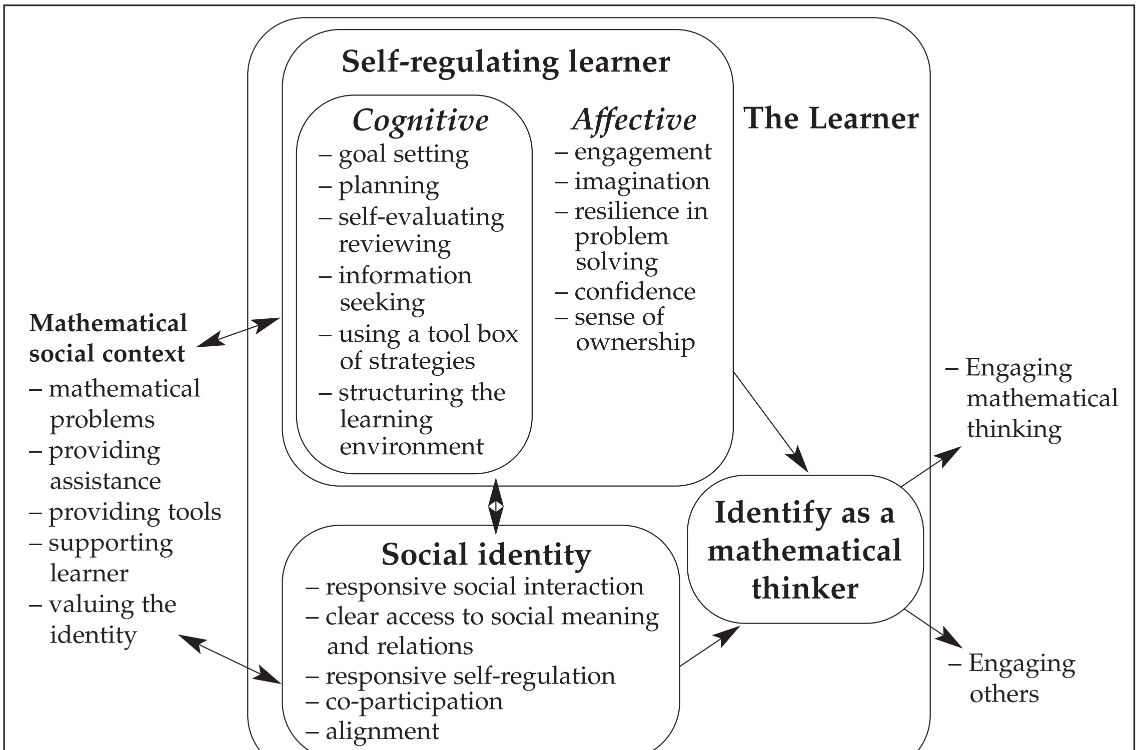

Figure 1. Developing identity as a mathematical thinker.

Theoretical Framework

Figure 1 represents the following theoretical discussion on identity as a mathematical thinker. Davis (1999) argues that identity is an enactive, dynamic, interactive ever-changing state of being. It is the doing of mathematics in a social context that is the identity. Social context surrounds and interacts with the learner whose affective learning controls the cognitive processing involved in becoming a self-regulating learner alongside and interacting with their social identity. The social context of learning influences the way a person thinks and feels about mathematical learning. For example, the form of questions (teachers and students), the expectations of the classroom, the available materials and others in the learning space impact on the individual's cognitive and affective processing (Owens & Clements, 1998).

Cognition involves conceptual and imagistic processing as well as heuristic processing during problem solving, while affect controls cognitive processing (Goldin, 1992, 2000). For example, acceptance of open-ended questions may impact on students' willingness to attempt this kind of question as well as their knowledge of how to attempt such questions. Positive beliefs about being a problem solver develop as problems are successfully undertaken with cognitive strategy assistance or 'aha' moments drawing the problem solver back to positive attitudes (Goldin, 2000). Imagination creates new images of the mathematics and the learner as a mathematical thinker in a social context (Owens, 1997; Wenger, 1998). This imagery is a dynamic tool for giving concepts meaning (Davis, 1999). For example, a person can imagine a triangle changing shape or turning around. Imagery also allows one to visualise oneself as an actor solving the problem (Owens & Clements, 1998; Presmeg, 1986). Imagination precedes action and provides the opportunity to perceive different possibilities. Imagination goes hand-in-hand with risk taking (Owens, 1998; Wenger, 1998) as learners attempt activities beyond their current repertoire of procedures and feelings of familiarity and comfort. Imagination also goes hand-in-hand with resilience (Goldin, 1992; Zimmerman, 1990) whereby a person will persist to solve a problem despite discomfort. The idea of seeing oneself as an actor in solving the problem is more transient than Sfard and Prusak's (2005) view of learning as moving a person from their actual identity to their designated identity. It is a part of each problem solving experience that builds up to a belief in oneself as a problem solver (Goldin, 1992, 2000). This learning is part of the dynamic formation of identity and is not solely directed by the cultural knowledge and expectation of being a mathematical thinker. The identity is a state of becoming; it is constantly evolving (in Davis', 1999, terms) in a less goal-directed manner than suggested by Sfard and Prusak (2005).

The individual psychological aspects of the self-regulating learner interact with the individual social identity (Macmillan, 1998), which is also evolving (Davis, 1999). Both the psychologically developing and the socio-culturally developing aspects contribute to formation of identity as a mathematical thinker. The social context that encourages mathematical problem solving will still impact and surround this identity. Hence, the identity is fluid depending on space (those people, power-relationships and environments around them) and time (dependent on experience). Identity may be relatively stable but it may develop and change at different velocities depending on the circumstances. An 'aha' experience may change the identity quickly, whereas thinking about the number and variety of problems that one has solved over a period of time may move a person more slowly towards an identity as a mathematical thinker. Problem solving within a community will also change one's identity when identity as a mathematical thinker is valued and tacit knowledge develops about how to think and communicate mathematically. Furthermore, in different contexts, such as a regimented drill-based classroom, identity as a mathematical thinker may not be as evident as in another type of classroom or social setting (Owens, 1997).

It is possible to link Figure 1 to the social theory of learning presented by Wenger (1998). In Wenger's terms, learning as doing (practice) results in the psychological aspects of the self-regulating learner but this is still influenced by the social context (Owens, 1999). The social identity encompasses learning as belonging (community) and involves continuous aspects of social interaction with the context of learning. This identity also incorporates learning as experience (meaning) by which language between members of the community provides the meaning for problem solving and learning. Learning as becoming is the resultant dynamic identity as a mathematical thinker.

Identity as a mathematical thinker influences the social context since the learner engages in mathematical activity and engages with others. Furthermore, pre-service teachers are expected to plan to engage students in their own classrooms in mathematical thinking and thus fulfill the position taken by Wenger regarding the importance of identity for creative teaching. "Being engaged to the fullest of one's identity is the source of creativity required for participation" (Wenger, cited in Kagan, 2004, p. 30) in both the community of practice and "outside that community" (p. 33). Knowledge presented in a lecture is valued less than an "experience … that … often involves feeling like an integral part of a community" of practice (Kahan, 2004, p. 31). Kahan (2004, p. 36) concluded his paper by reminding readers that Wenger said "a person's identity is engagement in the world". Identity is thus far more unique and complex than might be implied by the phrase "identity as a mathematical thinker": identity is dynamic, evolving and enacted (Davis, 1999; Wenger, 1998).

Method

Research Setting and Data Collection

This explanation of the development of a person's identity as a mathematical thinker is illustrated by presenting data collected from 49 students enrolled in a first year subject of a primary teacher education degree. Other papers (e.g., Walshaw, 2004) discuss the development of pre-service teacher identity as a mathematics teacher rather than as a mathematical thinker; the latter aspect of teacher preparation is addressed in this article.

The philosophy behind the subject was that primary school teachers of mathematics need to experience first hand how to solve mathematical problems and construct mathematical knowledge through problem solving if they are to be effective teachers. The challenge was to ensure pre-service teachers have these experiences and have positive responses to these experiences and hence develop an identity as a mathematical thinker to take into their own primary classrooms. Many of the pre-service teachers were mature-aged students, which impacted on the dynamics of the classroom as they brought different background experiences and degrees of confidence in doing mathematics.

The data included descriptions of class activities, records of assignment topics, observations, and anonymous self-reports provided mid-way through the semester. Observations of these pre-service teachers in a subject from the third year provided further insight for the developing theory on identity. They had been informed that the data were being collected to report on the subject to the mathematics education community, including an international conference and publication (Owens, 2005). They could choose to complete the self-report evaluation or not. Upon further reflection on this data, my continued participation in the subject, and reflection on my earlier research (Owens, 1996; Owens & Clements, 1998; Owens, Perry, Conroy, Geoghegan, & Howe, 1998) and on my continued reading in education, I developed the theory presented in this article. The data are used to authenticate this theoretical perspective. Identity became a focal issue as my earlier research had showed cultural identity was a significant influence on students' problem solving (Owens, 1999).

Design of the Mathematics and Technology Subject

In order to provide the background context for the discussion, the following information is provided on the subject undertaken by the pre-service teachers. The subject was based on problem solving and integrates mathematics with technology. It is the first mathematics subject in a primary teacher education degree and deemed equivalent to a basic mathematics subject for the final year of high school. Students are expected to:

- demonstrate the purposes and functions of mathematics in the community;
- use basic statistics in reporting societal phenomena;
- utilise a constructivist approach to mathematics learning;
- use calculators and computers for investigating mathematics;
- report their own discoveries and constructs within mathematical investigations;
- create spreadsheets to solve mathematical problems, and create charts and graphs to illustrate mathematical relationships;
- use the internet to explore and demonstrate mathematical concepts (Charles Sturt University Subject Outline, approved 2002).

There was an emphasis on real-life problem solving through a community project and investigating by looking for patterns and relationships in number, space, and measurement. The pre-service teachers were regularly expected to solve problems without worked examples although techniques were discussed and technology tools were provided and demonstrated. The subject began by highlighting mathematics used in everyday contexts through newspaper articles and by pre-service teachers relating their own mathematical experiences. They participated in enjoyable mathematical data gathering activities. Emphasis was placed on deciding the nature of analysis of the data and interpreting the results. Finding patterns was challenging, socially engaging and enhanced by computer usage. The problems were often open-ended and relationships between different problems were established as the class delved further into the patterns that they were noticing. Tables were recommended as a way of assisting the problem solver to notice patterns. In both data analysis and pattern recognition computer programs like Microsoft Excel were used.

Each of these pre-service teachers was able to pass both assessment tasks — the community project and an open investigation task — and subsequently to write an essay on learning through investigation and construction of knowledge, and then to prepare learning activities and a program that they could justify in terms of their essay. Despite this achievement, this paper is not about mathematical ability but rather the more encompassing view of learning that has not only cognitive perspectives but also affective and social features. The remainder of this paper looks at the various aspects of the theoretical perspective outlined above, integrating the data and further literature enabling an expansion of our understanding of identity.

Findings and Discussion

Developing the Self-Regulating Learner — Cognitive Aspects

Self-directed learners engage in higher order thinking to solve problems through goal setting and planning, self-evaluating and reviewing, organising and transforming, being resilient in problem solving, seeking information and keeping records, structuring the learning environment and seeking social assistance, and rehearsing and memorising (Zimmerman, 1990). Some of these characteristics involve affective aspects of learning. These learners are more aware of their tool box of strategies and when to utilise them (Lidner & Harris, 1993, cited in Wilson, 1997). Each of these aspects of the self-regulating learner is discussed below with illustrations from the implementation of the first year mathematics subject.

Goal setting and planning

The community project required pre-service teachers to decide on their own project, although they were encouraged to consider their own rural communities. They selected a wide range of topics. These topics are listed together if they relate to similar aspects of living.

- *Work and finance related topics* included the progress of shares from selected companies and a survey of some business people on-line about their share buying and selling; the on-line uses and needs of small businesses in a rural city; available loans for farmers; tourism trends in a rural city; perceptions of logos by country businesses.

- *Family expenses and activities* included water, telephone and electricity usage; holiday expenses; car purchasing; insurance rates for comprehensive car insurance; people's choices of public and private health care; effects of registration costs for football; musical instrument usage.
- *Rural comparisons* included the reasons for nurses leaving the profession in a rural city; spouses working off the farm; purchasing appliances or shopping in a small town versus a rural city; price differences of laptops between a capital city and a rural city; cost, variety and quality of baby products from chemist shops and supermarkets; petrol prices over a week at different service stations; attractions of local supermarkets; beer costs in the city and the country; facilities in clubs and pubs; the effect of shopper dockets on a rural town's petrol buying; availability of child care services in a country city versus a large coastal city; attitudes to private and public schooling of rural parents; parents' perception of country teaching; early intervention in a country city and a capital city; travel for medical facilities; crime rates in a rural city; income tax in a country business — gender differences; fast food outlet usage in rural city and rural town.
- *Animal and farm related industries* included the returns on following horses from specific thoroughbreds and ridden by specific jockeys; horse breeding costs; beef prices at different sale yards; sheep costs; people's knowledge and attitudes about genetically modified foods; knowledge and use of different wheat varieties; wheat sales, rainfalls and droughts over the last 10 years; 25 year rainfall trends.
- *Personal well-being* included the impact of smoking parents on their children's decisions about smoking; eating breakfast; coffee drinking by students; recycling in a rural town; eating out in restaurants in a rural city; health levels for different ages in a rural community; age for attitudes towards mathematics changing.

This first aspect of self-regulation is illustrated by the pre-service teachers setting goals and planning their own project through their choice of topic.

Self-evaluation and reviewing plans

Pre-service teachers were given minimal guidance on how to write an interim report on their progress. They were asked to provide their focus question and contributing questions, explain how they would collect data, submit information and consent letters that they would use if collecting data from human participants, give the column headings for a Microsoft Excel spreadsheet for entering data, and outline their plans for analysing the data. At this early stage, as their lecturer I was not an external evaluator but a peer along with the rest of the class, assisting the pre-service teachers to evaluate their own progress. Frequently this self-evaluation led them to revise their plans. They were given an outline of a report and they could access web pages on report writing provided by Student Services at the University.

Information seeking, structuring the learning environment, and social assistance

Pre-service teachers were shown how to handle specific technical difficulties when they arose and how to use the *Help* function for Microsoft Excel. They were encouraged to share ideas that the lecturer may not have considered. This provided a model for information seeking and gaining social assistance. On-line tutorials on Microsoft Excel were available from the library and the subject's on-line resources. Pre-service teachers were in control of whether they accessed this material or not. The message soon spread that the on-line help was useful but pre-service teachers sometimes needed to be shown how to access the materials.

Pre-service teachers regularly accessed the internet to find and solve problems. Often the lecturing staff provided appropriate links, making the purpose for using the internet materials more important than the procedure. For example, pre-service teachers searched for data and information about why calculators should be used in schools, and to find out about Pascal's triangle, tessellations and numerous other topics.

Using a tool box of strategies

Pre-service teachers were also being immersed in my view that mathematical thinking is about noticing patterns and relationships. They looked for patterns in numbers, used repeated addition on the calculator, tackled problems that were best started with drawing or acting out, working backward, simplifying the problem, or solving logically or by try-and-modify tactics. Some pre-service teachers were given a new vision of mathematics providing them with an opportunity to set their own goals about themselves as mathematical problem solvers. Without these strategies and social support, they would struggle to solve a problem when they were not provided with answers or a teacher-set procedure to follow, or when they saw no purpose for a problem. Pre-service teachers' evolving identities as problem solvers were challenged and they may have faltered without the tool box of strategies and social support.

Developing the Self-Regulating Learner — Affective Aspects

Lebow (1993, cited in Wilson, 1997) suggested that attending to the affective domain of learning with information and communication technologies (ICTs) as a tool would reduce the dominance of mastery learning. Affective strategies include personalising the learning, helping students to develop skills to become self-regulating, balancing control of the learning with personal goals of the learner, and providing for success. In the teacher education subject, real-life examples assisted pre-service teachers to personalise the learning and activities were designed so that they used information to assist with achieving their goals.

"Since the key to meaningful learning is ownership of a problem or learning goal, it is important to present learners with an interesting, relevant, and engaging problem to solve or project to complete" (Jonassen, Peck, & Wilson, 1999, p. 196). This strategy engaged pre-service teachers with solving mathematics problems. Nevertheless the problems were kept as indefinite and open as possible to encourage deep thought and imagination about the process.

Grabinger, Dunlap, and Heath (1993, cited in Skaalid, nd), in line with Jonassen et al. (1999), suggested that realistic environments for active learning (REAL) reflect the constructivist orientation by allowing students to determine what they need to learn, enabling them to manage their own learning activities and to contribute to each other's learning, helping them develop metacognitive awareness and creating a non-threatening setting for learning (Skaalid, nd). These authors also pointed out the importance for meaningful learning of making maximum use of students' existing knowledge and providing multiple ways to learn the content, using activities to promote higher level thinking, encouraging the review of multiple perspectives, encouraging creative and flexible problem solving and providing a mechanism for students to present their learning. When the pre-service teachers used Microsoft Excel for their self-selected community-based projects they learned efficiently about statistics while solving a problem with social consequences. The reality of the problems impacted favourably on the affective domain. The projects reflected the literature summarised above by allowing pre-service teachers to manage their own learning environment and use their existing knowledge. They shared their focus questions and problems that arose in collecting their data, entering the data into spreadsheets, and analysing their results. Each week's activities resulted in pre-service teachers sharing ideas for solving problems, mathematical strategies, newspaper reports or new tools found on the internet. They also shared their frustrations and elations as they tackled new problems. Having-a-go and sharing the problems they encountered in a safe environment was critical and often required careful reading of body language on the part of the lecturer to recognise when it was appropriate to encourage a pre-service teacher to share a difficulty with a peer, a small group, or the rest of the class. Although pre-service teachers worked on their own computers, they were often paired to work together or they chose the classmates with whom they communicated while problem solving. This made the classroom lively and interactive.

Resilience in problem solving

As an affective aspect of learning, resilience leads to positive beliefs about oneself as a mathematical problem solver using technology (Goldin, 1992). During the semester, pre-service teachers were becoming more familiar with Microsoft Excel, types of data, types of and purposes for graphing, and people and reports that used mathematics. They rehearsed a number of simple Excel functions before using the COUNTIF function for obtaining frequencies. This rehearsal was embedded in problem solving rather than drill exercises (Southwell, 2005). During class we used a few formulae and made a few graphs but it was only when pre-service teachers rehearsed through trials to produce meaningful and attractive graphs for their project that they really identified with using Microsoft Excel as a tool. Many realised the power of using Microsoft Excel and were then more willing to persist with it. They had to be resilient in order to cope with the awkward formatting required by the computer dialogue boxes.

Developing confidence

Confidence is also an aspect of the affective domain. Mature-aged students especially need to have some success, feel in control, value the mode of learning, and be realistic about the causes of difficulties in order to develop self-efficacy with ICTs and mathematics (building on Wilson, 1997). Thus, the teacher educator needs to be aware of the specific skills required for using the computer (Lidner & Harris, cited in Wilson, 1997). The pre-service teachers using databases and web searches for gathering information needed to be shown how to identify problems, how to use the systems, and how to evaluate and synthesise their information and data.

About ten percent of the pre-service teachers (mostly mature-aged) were quite anxious about the subject at the start, but by the end of the second day they were much relieved that they handled both the Microsoft Excel and the mathematics. Moreover, the tutorial group was soon seen as a community helping each other so that the pre-service teachers' confidence grew and some of them really flourished with their community projects. Interestingly, these pre-service teachers self-regulated their assistance by asking people they knew in the community not only to complete their surveys but to assist if they were struggling with some of the mathematics and Microsoft Excel. Nevertheless, they still remained in control of their learning and continued to develop confidence. These anxious pre-service teachers realised they could think mathematically, apply mathematics and technology, and succeed. Comments from these pre-service teachers included:

> I think I've learnt a lot in this subject. I was very apprehensive about this because of my experiences with maths in high school, but it wasn't so bad after all.

> I was very concerned that I would not be able to keep up with the class. But I find that (the lecturer) makes everything seem easier and I feel a lot more confident at attempting new things and I feel elated when I can understand what the lesson is about.

One of the activities used Microsoft Excel to aid building magic squares (square grids of numbers in which the rows, columns and diagonals all add to the same number). This lesson introduced pre-service teachers to using Microsoft Excel tools to calculate sums. After trial and error and sharing of initial strategies (such as placing only one large number in each row and column and putting 5 in the centre for the magic square with numbers 1 to 9) the pre-service teachers generated ways to make more magic squares. Once they had an idea, then they really pursued the task, trying a range of ideas. The Microsoft Excel program saved time and its use was appreciated. Although a few pre-service teachers felt uneasy at not finding solutions as quickly as the rest of the class, one commented:

> My husband and I had a real tussle with the mouse to see who could out-compete the other. Excitement was felt when number patterns in the magic squares were found, and making other number patterns easier to work out.

This quote also shows how the pre-service teacher's identity impacted on her social context. The role of technology in developing a sense of identity as a mathematical thinker cannot be underestimated.

Technology provided a semantic network of presentations of concepts (Jonassen, Peck, & Wilson, 1999, p. 166) that was partially controlled by the pre-service teachers but provided some of the thinking needed for solving the problem and gave a degree of feedback on their actions. In this study, the calculations and graphical displays converted numbers into alternate representations from which pre-service teachers noticed other mathematical constructs. If they used the technology well, then the results were evident in the computer display providing reinforcement for their thinking and actions.

The role of the technology in supporting mathematical learning at times involved only straightforward observation of the representation by the pre-service teachers, but at other times they needed to observe conceptual properties or act on the representation (Thomas & Hong, 2001). Technology became a partner supporting learning and providing a means of interaction within the pre-service teachers' problem-solving contexts (Goos, Galbraith, Renshaw, & Geiger, 2000). Constructed knowledge in turn was embedded in meaningful tasks that were presented in the technological environment. In this subject, constructed knowledge resulted from interactions with the technological environment that provided a visual representation of the knowledge. Technology as a partner meant learning with technology was of a higher quality than may have been expected without technology. In some cases, it was speed in doing calculations or making trials that assisted the learning and pattern recognition while the change of representation (tabular and graphical) meant other relationships were noticed and concepts were made meaningful. For example, pre-service teachers perceived averages and spread of data in different ways.

Developing Ownership of One's Identity

In this subject many types of problems were given to illustrate how mathematical thinkers use a variety of strategies to solve problems. These were problems that the subject lecturers (the author and Bob Dengate) had collected over the years. Pre-service teachers became happy to share their incomplete ideas and to share their diversity of diagrams and other representations as they were constructing their mathematics (Mergel, 1998). Many turned to the computer and started drawing and recording rather than working on paper.

The second assessment task was an investigation. An example of these investigations is "A rectangle of squares is cracked along a diagonal.

Investigate." (Bastow, Hughes, Kissane & Morlock, 1984). For this task, each investigation was given to a pair of pre-service teachers and the idea of working by themselves was soon gone. As one commented:

> I felt more at ease and started learning different strategies that I would never have thought before. It's a way of looking at things from a different perspective.

One question on the subject self-report survey asked the pre-service teachers what they thought problem solving was. A selection of responses indicates their brief but developing notion of problem solving as follows:

> Looking for a way to solve a problem and come up with an answer, taking into consideration many different ways a problem can be looked at/thought about.
>
> Looking at all the possibilities to a question and through trial and error coming up with a valid solution.
>
> Analysis, deeper thought, broad thinking.
>
> Working out how to approach a question in a fashion that sets you up to find an appropriate answer.
>
> Being able to be given a problem and research it to find answers to it. Investigation is looking at a question and analysing different aspects of it.
>
> Problem solving is to find out a strategy or strategies to solve a problem. It not only happens in mathematical problems but in real life. It is faced with us [sic] every day no matter how small or large. Investigation is a way of using thinking strategies to find a solution. The strategies must be noted how the answer was achieved.
>
> Getting information together sorting through it and investigating the information.

These responses indicate a reasonable view of problem solving and a degree of ownership.

Some pre-service teachers felt frustrated by the open-endedness of the investigation but each was able to present a Microsoft PowerPoint presentation of their results and processes for investigating. Some pre-service teachers placed their presentation on the web, indicating their pride in their investigative ability. In the third year of their course, the pre-service teachers wrote an essay on investigating in mathematics. Interestingly, some selected their own investigations as illustrations of the power of investigating for mathematics learning. They were able to articulate why investigating is a powerful way of learning mathematics as well as the key idea in mathematical thinking. Being able to articulate the nature of mathematical investigation indicated their identities as mathematical thinkers. This is in line with Sfard and Prusak's (2005) definition of identity as narrative but I am suggesting narrative is an indicator of identity rather than the identity itself.

Developing Social Identity

It is interesting to compare the response of one class in two different sessions. On one occasion, some pre-service teachers were absent from a tutorial class. These students often generated initial ideas for solving problems. When they were away, the class did not easily generate initial approaches to the problems and struggled to enjoy the lesson and succeed. It would seem that the development of identity as a mathematical thinker was embedded in a social identity (Lee & Robbins, 1998); that is, identity formation is a social process as well as a cognitive and affective process. Clearly, some pre-service teachers were already developing a mathematical problem solving identity, and interactions between this group and those who were less mathematically confident created participation structures that strengthened the community of practice (Goos & Bennison, 2002).

The pre-service teachers' role in the classroom, whether wittingly or unwittingly, was to provide responsive social interactions (Macmillan, 1998). Drawing on the theoretical perspectives of Lave and Wenger (1991), Bourdieu (1977) and Foucault (1980), Macmillan (1998) provided a model for socio-regulative meanings including self-regulation and identity development (Figure 2). The model suggests that knowledge is interdependent with power relations.

Responsiveness in problem solving was a major way forward in the development of problem solving skills. Cognitive and affective processing

Restrictive social interactions	**Responsive social interactions**		**Identity development**
Unclear or unreasonable statements or explanations	Clear or reasoned statements or explanations	→	Clear access to social meaning and relations
Discouraging responsible self-regulation (dependence, passive acceptance)	Encouraging responsible self-regulation (independence, assertiveness)	→	Responsible self-regulation
Non-negotiable regulation (exclusion, resistance, closure, rule-domination)	Negotiable regulation (modelling, assistance, observation, improvisation)	→	Co-participation

Figure 2. Social interaction and identity development (Macmillan, 1998, p. 115).

resulted in a cognitive and affective response (modifying equipment, expressing a feeling, sharing a recording) that impacted on the social context (the community and equipment), which in turn impacted on the individual's processing of the problem (Owens & Clements, 1998). The responsiveness was both a social interaction and a result of self-regulated thinking. The tutorial groups, formed by pre-service teachers of different ages and from different places, became more cohesive as they struggled with problem solving and responsive social interactions. Pre-service teachers enjoyed the camaraderie of the class and the challenges they were facing together.

Identity as a Mathematical Thinker

Pre-service teachers began from different positions in their development as mathematical thinkers. For some, past experiences like success in school mathematics meant that they began confidently to try problems, although some floundered as they could not solve the problems by relying on a formula that they knew they had learned at school but could not remember. For these pre-service teachers, the social identity of being a mathematical thinker had to be modified. In some ways, the novelty of the problems meant that pre-service teachers began on a level playing field in which all had a chance to succeed. Class cooperation and respect for each other ebbed and flowed at different times but there was generally a supportive environment that provided the social context for the development of both self-regulated learning and social identity within the mathematics classroom. The pre-service teachers' mathematical identities were expressed both in the classroom as they participated in weekly problem solving and later when they prepared lessons for students to learn through problem solving and investigation and they prepared their essays on investigating and problem solving as a way of learning to construct mathematical knowledge. Their own identities were developing each week and were then shared with others in the tutorial sessions and later with school children. Pre-service teachers' written comments, oral comments, and observed behaviour in the classroom were all manifestations of their identities as mathematical thinkers as defined in terms of the development of self-regulation and social identity.

Conclusion

Social theories of learning (e.g., Wenger, 1998) and constructivist theories of learning focussing on the self-regulating learner (Jonasson, Peck, & Wilson, 1999; Zimmerman, 1990) and on representation in problem solving (Goldin, 1992) are developed in this article into a coherent theory of identity as a mathematical thinker. Theories of responsiveness (Macmillan, 1998; Owens, 1993) predicate how social and psychological learning are drawn together when discussing the theory of identity as a mathematical thinker.

Pre-service teachers' identities as mathematical thinkers developed during a series of classroom experiences centred on mathematical problem solving and on solving their own community-based problems. The discussion shows that identity as a mathematical thinker arose from a combination of learning experiences, social interactions and technological supports. The real-life problems placed the pre-service teachers into an environment that was challenging but supported in various ways. The environment engaged them and assisted them to persist with the problem situations. Consequently the pre-service teachers established an identity as self-regulating learners, setting their own goals, organising and controlling their learning environment, using a toolbox of strategies for problem solving and self-evaluating. The mathematical class context encouraged a social identity through responsive mathematical interactions. These two identities formed an identity as a mathematical thinker that is dynamic and set within a social context.

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