

How to Feel About and Learn Mathematics: Therapeutic Intervention and Attentiveness

Immaculate Namukasa George Gadanidis Michelle Cordy
The University of Western Ontario

In mathematics teacher education, tasks that centre on doing mathematics are used for a variety of purposes, including learning new mathematics. In our research, we focus on doing mathematics as a therapeutic intervention. Many pre-service teachers in our program narrate impoverished mathematics experiences. We engage pre-service elementary school teachers in non-routine problem solving and examine how this affects their experiences with mathematics. Specifically, we focus on change in affective responses as a precursor to development in mathematical thinking and as an indicator of potential changes in practice. Our study shows that doing mathematics evokes changes in how teachers think and feel about doing, learning, and teaching mathematics.

Developing teachers' mathematical thinking and pedagogical understanding is a collective, complex project that spans many contexts. A prospective teacher's pre-conceived notions about the teaching and learning of mathematics (Ball, 1990; Cooney, 1999; Pajares, 1992), along with their mathematical experiences, might not be conducive to productive mathematics teaching. In this paper we report on a one-year mathematics methods course in which pre-service teachers *do* school mathematics tasks, specifically non-routine problem solving. We view pre-service teachers' participation in doing school mathematics as *experiential therapeutic intervention*. The research question was: In what ways does doing mathematics with pre-service teachers affect their relationship with mathematics?

This article has four parts. First, we relate our work on the use of problem solving for therapeutic intervention to the existing body of research on teacher characteristics, particularly affective responses. Second, we describe some non-routine mathematics problems and the mathematical activity they can evoke. Third, we describe our research and practice program along with our procedures for data collection and analysis. Fourth, we examine how mathematical exploration affects pre-service teachers' mathematical experiences. We conclude the paper by looking at further directions for doing mathematics in ways we have dubbed *warm mathematics*.

Framework

Background

There is a large body of research on how teacher characteristics (e.g., Koehler & Grouws, 1992), knowledge (e.g., Ball 1990; Fennema & Franke, 1992), conceptions (e.g., Thompson, 1992), and affect, especially belief systems (e.g., Cooney 1994; McLeod, 1992), influence teaching practices. Impoverished school mathematics experiences have left many pre-service teachers with strong *negative* affective

responses about mathematics (Cooney, Shealy, & Arvold, 1998; McLeod, 1992; Wolodko, Willson, & Johnson, 2003). Even the few pre-service teachers who narrate enjoyable experiences appear to hold less-than-sound views about mathematics knowing, teaching, and learning, views that are counter to current developments in mathematics teaching (Brown & Borko, 1992; Calderhead & Robson, 1991; Cooney, 1999; Fennema & Franke, 1992; Pajares, 1992; Wilson, Cooney, & Stinson, 2005). Pre-service teachers may well have adequate mathematics knowledge (to varying degrees) and be capable of mathematical insight. The problem is that their mathematical experiences are not, to use Fennema and Franke's (1992) perspective, well organised to inform or be informed by teacher education and by classroom practice. Negative relationships with mathematics such as mathematics anxiety and avoidance behaviours also arise from out-of-class experiences such as home and business mathematics (Evans, 2000, 2002; Herbert & Furner, 1997; Wedege, 2002). Mathematics education researchers conceive that triggering re-organisation or interruption of the flow of experience is one of the major roles of teacher education courses (Ball, 1990; Cooney, 1994; Fennema & Franke, 1992; Pajares, 1992; Thompson, 1992).

Many researchers understand affect as an internal representational system that interacts with cognitive systems (DeBellis & Goldin, 2006), as a charge attached to cognitive aspects (Evans, 2000), and as states, predispositions, preferences, or personal commitments (Beswick, 2005; DeBellis & Goldin, 2006; Leder & Grootenboer, 2005). We find that affective responses to mathematics are not only individual and internal. They also vary with social, institutional, and cultural factors.

The interaction of *negative* affect with learning, let alone with teaching, is complex. Affect interacts with factors such as school level, gender, race, culture, and modes of teaching. It correlates with achievement in the following layered ways:

Negative affective responses such as avoidance and fear are difficult to separate (DeBellis & Goldin, 2006; Evans, 2000). Positive attitudes to mathematics relate to lower mathematics anxiety (Hembree, 1990). There are significant correlations between students' beliefs—such as the belief that mathematics requires natural ability—and their performance (Elliot, 1990).

- Negative affective responses do not always have negative interactions with learning. Intense affect like emotion, whether negative (e.g., I am stuck) or positive (Aha!), are major motivating factors during problem solving (Hannula, 2006; McLeod, 1994). Positive short-term affect might not guarantee a long-term positive relationship with mathematics (DeBellis & Goldin, 2006; Hannula, 2006).
- Mathematical anxiety and performance have an inverted-U relationship; beyond a certain level, anxiety is debilitating (Evans, 2000) even for gifted students (Herbert & Furner, 1997).
- Reform practices do not always lower students' anxiety. Many anxious students do better in familiar expository lessons than in discovery lessons (Herbert & Furner, 1997; McLeod, 1994).

- Mathematical anxiety can be derived from social interactions. It is related to test and to general anxiety (Hembree, 1990). Mathematical anxiety levels even in adults are related to the anxiety levels of their parents (Evans, 2000; Herbert & Furner, 1997; Scott, 2005).
- At the university level, elementary school teachers show the most mathematical anxiety (Hembree, 1990). Attitude and achievement correlation are more significant at the secondary and post-secondary levels (Ma & Kishor, 1997).
- Females report higher mathematics anxiety than males (Hembree, 1990; Herbert & Furner, 1997). Inhibited anxiety is common among male students (Evans, 2000). Some students, especially girls, exhibit mock-anxiety (Evans, 2000). Negative affective responses are more negatively correlated to achievement for girls than for boys (Herbert & Furner, 1997; Ma & Kishor, 1997). Hembree (1990) maintains that pre-college level girls cope better with anxiety.
- Japanese students show a greater dislike for mathematics, even when their achievement is higher than that of American students on international tests (McLeod, 1992). In the United States the correlation between attitudes and achievement is stronger for Asian students than for African-American and White students (Ma & Kishor, 1997).

Relationships between students' affective and cognitive factors cannot be depicted in a simple linear causal manner (McLeod, 1992; Thompson, 1992). Studies reveal inconsistencies among beliefs themselves (Cooney 1994; Pajares, 1992), and between negative affective responses and mathematical performance (Ma & Kishor, 1997).

Fennema and Franke (1992) observe that teachers are reflective and thoughtful individuals. To Evans (2002), they are also social and affective individuals. We add that they are *experiential* individuals. "Many of a teacher's beliefs and views seem to originate in and be shaped by experiences" (Thompson, 1992, p. 139). Pajares asserts that whereas beliefs that are formed from experiences appear to be more resistant, "learning and inquiry are dependent on prior beliefs" (Pajares, 1992, p. 320). Beliefs affect task behaviour (Scott, 2005). They "appear to act as filters through which teachers interpret and ascribe meanings" (Thompson, 1992, pp. 138-139). Thus, teachers' school experiences—through their affective responses toward mathematics—strongly shape future teacher education experiences (Calderhead & Robson, 1991).

To many researchers, teachers' beliefs about mathematics are a very strong influence on mathematics teaching practices (Barkatsas & Malone, 2005; Beswick, 2005; Pajares, 1992; Raymond, 1997; Thompson, 1992). A few studies reveal inconsistencies between teachers' beliefs and their classroom practices (see Beswick (2005) for a review). Beliefs and teaching practices, like many human systems, have a complex, two-way causal relationship that is contextually determined (Thompson, 1992). We agree with Wilson and Thornton (2005) that "anxiety among pre-service teachers must be overcome as they become effective teachers of mathematics" (p. 792).

We hypothesise that changing experiences might ultimately lead to changed practices as affective factors get changed. Focusing on developing teachers' beliefs or affective responses in general is a central goal of teacher education (Barkatsas & Malone, 2005). A re-organisation in experiences of learning school mathematics might be what is needed to start the process of changing classroom practices. Despite their professed beliefs, teachers' classroom practices after they graduate are greatly affected by teaching cultures (Brown & Borko, 1992; Buzeika, 1999; Ensor, 1998; Raymond, 1997), conflicting priorities (Skott, 1999), social context (Thompson, 1992), and teacher socialisation (Brown & Borko, 1992). For this reason, a deeper re-organisation akin to accommodation (as opposed to assimilation) is needed. Many elementary pre-service teachers need therapeutic intervention.

Warm Mathematics as Therapeutic Intervention

Few studies in mathematics teacher education have looked at teacher change in therapeutic terms (e.g., Hannula, Liljedahl, Kaasila, & Rosken, 2007; Wilson & Thornton, 2005). Researchers say that central to changing teachers are tasks that cause self-doubt, or tasks that deliberately challenge so as to elicit some form of disequilibrium or cognitive conflict (Barkatsas & Malone, 2005; Thompson, 1992). Wilson and Thornton (2005) used bibliotherapy—reading literature on mathematical emotional turmoil—to trigger improvement in pre-service primary teachers' self-image. Herbert and Furner (1997) used bibliotherapy with secondary gifted students.

We use mathematics tasks, or problem solving to offer pre-service teachers mathematics experiences that help them experience mathematics in warm ways. Pre-service teachers begin to see mathematics as different from the stereotypical cold, rigid, individual endeavour that requires special natural abilities.ⁱ Non-routine problems can be used as a source of mathematical experiences that might change how pre-service teachers feel and think about mathematics. Put differently, experiential therapeuticⁱⁱ intervention in mathematics is the use of non-routine school mathematics tasks to prompt affective and cognitive mathematical change. It is experiential rather than clinical therapy (Hembree, 1990). It is a form of classroom and curricular intervention that is targeted to a whole group rather than to individual pre-service teachers (Hembree, 1990).

DeBellis and Goldin (2006) say, "intimate mathematical experiences include emotional feelings of warmth, excitement ... affection They build a bond and can be harnessed for successful mathematics learning" (p. 137). Intimate

i Our reference to warm experiences is not related to the use of cold-cool-hot to differentiate affective responses according to intensity, stability, time-scale and the role cognition plays in the affective response (DeBellis and Goldin, 2006; Evans, 2002; McLeod, 1992). According to this model, attitudes, values, and beliefs (Pajares, 1992), are cold, stable, and global. Frustration, confidence, moods, feelings, and emotional states, at the other extreme, are hot and fluid (McLeod, 1992). Morals and motivations lie in between.

ii In psychotherapy, experiential therapy is used to refer to therapy in which activities such as dance and sport are used.

mathematical experiences are similar to what we describe as *warm mathematics*. Imagine mathematics activity as the personified behaviour of “cold” or “warm” molecules. Warm molecules, unlike cold molecules, are not set in their ways. They are fluid and move about into new environments. They are quick to interact, to react to change. They exchange energy.

In the sections that follow, we detail our use of warm mathematics in a pre-service mathematics education course. We have found that warm mathematics benefits teachers in three significant ways. It is a source of experiential therapeutic intervention, it promotes mathematical attentiveness, and it has the potential to nurture pedagogical responsiveness. In this article, we have chosen to focus on the first two benefits, experiential therapeutic intervention and mathematical attentiveness.

Non-Routine Mathematical Tasks

Tasks that center on the content of mathematics are used for a variety of purposes. Doing school mathematics tasks with pre-service teachers may be seen as a way of exploring the mathematics teachers need to know and encouraging them to be curious about mathematics (Ball, 2002; Simmt, Davis, Gordon, & Towers, 2003), or as a way of modelling how to teach problem solving (Anderson, 2005). Doing mathematics that gradually exposes pre-service teachers to concepts that caused them distress in the past, so as to teach them to cope meta-cognitively with the distress, is called psychological desensitisation (Hembree, 1990; Herbert & Furner, 1997). Although psychological desensitisation is an experientially based intervention (Thompson, 1992), it is an out-of-class intervention. As such, it might not affect mathematical thinking and classroom practices. It focuses on therapy without focusing on modelling teaching practices and motivating mathematical thinking. We emphasise that a different mathematics has to be modelled. For us, the purpose of doing warm mathematics with pre-service teachers, in addition to intervening therapeutically, is to help them (re)-experience school mathematics. In particular, (re)-experiencing school mathematics in a way that makes pre-service teachers aware of the potential impact of warm mathematical experiences on mathematics learning (Ingleton & O’Regan, 2002) and in a way that intrinsically motivates them to consider alternatives to current practices (Barkatsas & Malone, 2005; Scott, 2005; Thompson, 1992) is important.

In our research, we offer pre-service teachers problem-based mathematical tasks for exploration. Since learning is influenced by the tasks given (Watson & Mason, 2007), we specify that the tasks should be variable-entry (Lampert, 1991; Simmt, 2000), non-routine (Polya, 1945/1973), non-standard (Schoenfeld, 1985), or rich learning tasks (Flewelling & Higginson, 2001) so as to go beyond mere practice of procedures. The tasks need to invoke mathematical insight and focus on mathematical structure (Lampert, 1991). We offer worthwhile tasks (Jones, 2004) that are all-at-once mathematically attracting and structuring (Namukasa, 2005). To the extent that we design these problems to facilitate change, they might be seen as critical events (Confrey, 1998). Experiences with such tasks go

beyond procedural ways of doing mathematics; they involve moments of insight and engagement with big and connected mathematical ideas. Here is a sampling of the warm mathematics tasks:

- **Making Ten:** If the answer is ten, what was the question? What (data or graph) patterns emerge when we solve questions like $[\] + [\] = 10$ and consider similar equations like $[\] + [\] = 6$, $[\] - [\] = 10$, or $[\] \times [\] = 12$? The Making Ten question has many answers; it vividly highlights that one may begin with the answer and investigate possible questions. It also insightfully connects number to data, graphing, patterning, and algebra (Gadanidis, 2004).
- **Consecutive Terms:** We adopted this task from Mason, Burton, and Stacey (1982/1985). Some numbers can be expressed as the sum of a string of consecutive positive integers, for example, $6 = 1 + 2 + 3$, $18 = 3 + 4 + 5 + 6$, or $15 = 7 + 8$. What numbers have this property? We include the Consecutive Terms task mainly because pre-service teachers who engage in it experience many patterns and see the need to organise their work or to begin with a simpler case.
- **Dog Pen:** This problem involves using a constant perimeter, 24 metres, and finding the largest possible rectangular pen. It involves systematic exploration of area optimisation for given dimensions. As pre-service teachers explore the Dog Pen problem they notice relationships between area and perimeter, between areas of varied rectangular dimensions and between area of regular shapes, and they engage in graphing these relationships.
- **Handshakes:** This well-known problem of determining the number of handshakes when a group of people meets involves a multitude of solution forms and strategies. Some of the solutions are geometrical and others algebraic. It also connects different strands of mathematics and has the potential to focus attention on relations among pictorial, algebraic, and numerical representation.
- **Rolling Dice:** Which dice sum will “win” the race when two dice are rolled 20 times? We organise this exploration of probability around the predict-observe-explain format. The exploration involves both a game and a technological simulation.

Subsequent tasks may repeat some of the attributes of previous tasks, but it is crucial that each new task introduces some new aspects of doing, teaching, and learning mathematics. The five tasks described here highlight central mathematical structures and processes including patterning, varied strategies, multitude of solution forms, and connecting different strands of mathematics. In preparing the tasks, we tried to find problems that were critical, non-routine, and generally *helpful* for evoking change with respect to particular learners. These tasks are also appropriate for elementary school students (Gadanidis, Namukasa & Moghaddam, in press; Namukasa, 2005).

Mathematical Activity

Consider the classical task: How many handshakes happen at a party of, say, 8 guests, if all guests shake hands with each other? This task is variable-entry, non-routine, and helpful for evoking change. Pre-service teachers in this study usually responded to the Handshake question right away by saying, "There are $8 + 8$, 16 handshakes." Some added, "Easy problem." But even before a neighbouring pre-service teacher interrupted, many pre-service teachers were quick to correct themselves. "Wait, those are too few handshakes."

One pre-service teacher wrote this about his engagement:

My first reaction to this task epitomises my errant and rigid approach to math problems. "Oh," I said, "since everybody shakes everybody else's hand, there's going to be 64 handshakes." Then I realise there is one less handshake (me-me) so I arrive at the final solution, $8 \times 7 = 56$ but I'm wrong. Someone then tells me a different sum. They explain, "Some are repeated, so we have to subtract those." (Emphasis in original)

Such responses marked the beginning of motivated engagement. The task was engaging and the majority did not find it alienating. They found the small numerical values involved inviting. Many could draw from their experiences to engage in the activity. Some related the task to experiences with league tables or genetics problems. Some pre-service teachers used counting strategies. Many could visualise, act, or narrate the solution. A few saw it as a problem of permutations and combinations, and series. By the end of the activity, a majority of pre-service teachers could find the solution for larger numerical values. Many found a generalisation and some responded to the prompt about similar tasks with a list of tasks.

The use of non-routine problems is central in order not to constrain mathematical activity to the procedural mathematics (Brousseau & Gibel, 2005) that a majority of pre-service teachers find boring. Teachers with different levels of comfort with mathematics have a variety of ways of engaging in a non-routine problem. It is critical that a problem be variable-entry for a group of pre-service teachers whose initial emotional response to mathematics is fear. Variable-entry also models diverse engagement among learners. As they successfully engage in mathematics, elementary pre-service teachers' self-concept with respect to mathematics improves.

In terms of a mathematical analysis, the Handshake task has the potential of extending towards algebra that is more sophisticated in the following ways:

- finding a formula for n guests ($n(n-1)/2$)
- extending to geometrical thinking, networks, and graph theory (relating it to number of diagonals and vertices of a polygon)
- using a spreadsheet (applying a function formula), recursive functions ($H_n = H_{n-1} + n - 1$), T tables and a table of values, or a table of leagues
- using coordinate geometry to graph the function (a graphical relationships using the ordered pair (guests, number of handshakes))
- exploring the number additions or series ($1 + 2 + 3 + \dots + (n - 2) + (n - 1)$), using summation notation (Σ), and relating the sums to triangular numbers

- using permutation and combinatorial thinking (nC_2) and, for some, recalling the factorial notation.

In other writing, we have focused on three of the other tasks (Gadanidis, Namukasa & Moghaddam, in press). These tasks, taken together as a mini-course in conjunction with discussion and reflection activities we describe below, appear to offer pre-service teachers a space to re-experience mathematical thinking as learners (Thompson, 1992) and to re-examine their attitudes and beliefs about mathematics. Brown and Borko (1992) observe, “unless novice teachers experience good mathematics as students ... it will be difficult for them to implement and maintain good teaching in their own classrooms” (p. 227).

Method

Research Setting

Our elementary teacher education program is a one-year after-degree Bachelor of Education with an enrolment of over 400 pre-service teachers. A majority of the candidates are women. Very few have majored in mathematics for their first degree. Many have struggled with mathematics. A few have university education and job experiences in mathematics-intensive fields. Many narrate experiencing what we call *cold* mathematics. Consequently, they have varied affective responses to mathematics.

The mathematics methods course we offered the 2004-2005 pre-service teachers had three components: 1) eight bi-weekly in-class workshops of two hours each, 2) eight bi-weekly online modules, and 3) eight bi-weekly mathematics sessions of one hour each. The in-class workshops focused on developing appropriate instructional and assessment strategies for the provincial school curriculum. The online modules focused on selected practice-based issues addressed in the mathematics research community. The mathematics sessions focused on doing mathematics.

The mathematics sessions had three dimensions: a) solving a mathematical task, b) reflecting on the task to consider classroom implications, and c) articulating personal affective responses. In a large lecture room, 410 to 440 pre-service teachers worked in groups of two to four. We gave each group materials such as mathematics manipulatives, calculators, and activity sheets. In many sessions, solution strategies were shared in the whole group. In the last five minutes of each session, pre-service teachers completed and handed in the *learned and felt form*, which has three entries: a) one thing I learned, b) one thing I felt, and c) one question that I have.ⁱⁱⁱ At the beginning of each session, we provided and briefly shared summary sheets of their reflections from the previous session.^{iv} We created a website to provide extensions, interactive explorations, and discussions. There was an assessment component, where in the

iii This also served as a way of tracking attendance.

iv The learned and felt forms, along with online discussions and essay considerations, were critical triggers for considering and sharing pedagogical issues, especially the advantages for learners of using good teaching methods.

last workshop of the course pre-service teachers had thirty minutes to write an essay on one of the mathematical problems they had explored in the sessions.^v

Data Collection

To answer the research question we conducted an interpretive analysis of data from 309 pre-service teachers who consented to participate in our study. The data set included end-of-task reflections (the learned and felt forms), online collaborative discussions, end-of-year culminating mathematics writing, and a transcript from a follow-up focus group interview. Most research on the role of tasks in changing teachers' affective and cognitive structures has involved surveys, interviews, journals of pre-service teachers' reflections, memory work, self-reporting techniques, or personal narratives. Thompson (1992) cautions about the use of verbal data to study teacher views, suggesting that "data about their mathematical behaviour as they encounter tasks ... would be useful" (p. 135). Pajares (1992) contends that any inquiry into teachers' affective responses should infer these responses from what teachers say and intend (profess) as well as from what they do (practice). Mathematical affect can be expressed using sounds (umm, euwww), words (confused), revealing behaviours (a smile or a frown), or defensive behaviours (denial of feelings about mathematics) (DeBellis & Goldin, 2005; Evans, 2000).

We chose as our main source of data the pre-service teachers' written responses, taken from immediate experience with the tasks rather than about past mathematics (Mason, 1994). We collected what Mason (1994) calls brief-but-vivid quotes from the learned and felt forms of data. Using these quotes, we used *interpretive inquiry* to understand pre-service teachers' reflections about changes in their relationships with mathematics or about some form of disequilibrium and newness in their experiences (van Manen, 1998). We focused on changes in affective responses and on views about learning and teaching mathematics.

Findings and Discussion

We identified nine significant themes in the experiences of the participants, as shown in Table 1. Three themes show how doing mathematics offers pre-service teachers an opportunity to experience the pleasure and warmth of doing mathematics—experiential therapeutic intervention. Four themes centre on ways in which rich mathematical activities encourage pre-service teachers to think and attend mathematically—mathematical attentiveness. Two themes illuminate how teachers consider ways of enhancing students' learning experiences—pedagogical responsiveness. In Table 1 we show the correlation between quotes that strongly supported a theme and the essay task the pre-service teachers wrote about. This numerical detail may be helpful in designing a collection of tasks that supports growth in one or more areas.

^v The use of an essay form assessment was one way for us to disrupt and potentially restructure students' perceptions of mathematics.

Table 1
Themes from the pre-service teachers' mathematics essays

	Task/Theme	Making Ten	Consecutive Terms	Hand-shake	Rolling Dice	Dog Pen	TOTAL Quotes
Therapeutic Intervention	Different Ways	11	–	3	–	2	16
	Re-learning	3	–	–	–	–	3
	Reconciliation	7	1	1	2	2	13
Mathematical Attentiveness	Many Solutions	4	–	6	–	1	11
	Patterns	4	9	2	1	2	18
	Mathematics, in general	1	1	1	1	–	4
	Tools	3	5	6	1	3	18
Pedagogical Responsiveness	Teaching	9	1	15	4	2	31
	Learning	13	2	6	2	3	26
	TOTAL	55	19	40	11	15	140

For this paper we use illustrative quotes only from pre-service teachers being prepared to teach grades 4 to 10—ages 9 to 15 (rather than K to 8—ages 5 to 13) because two of the authors have taught at that level and can draw on their own experiences to validate data and analyses. This reduced the data set from 309 to 160 participants. We have only used accounts from the final mathematics essay. The ongoing learned and felt accounts are similar to the essay accounts except that on the learned and felt forms the pre-service teachers express their affective responses with more intensity than in the essays. For instance, in the learned and felt reflections pre-service teachers gave comments like, “Mesmerised! The patterns ... really exciting.” About the same activity (Consecutive Terms) and the same theme (patterning) in the culminating essay pre-service teachers said, “Patterns started to evolve for me” and “[Discovering patterns] is a ... fulfilling sense of understanding.” The essay comments gravitate towards more global and stable affective responses—expressions of developing values and attitudes about mathematics (Evans, 2002; Leder & Grootenboer, 2005; McLeod, 1992).^{vi} As well, the essay comments offer reflections on one task

vi One might argue that it is problematic to use graded assignments — the culminating essays — as a source of data, because the learned and felt forms would offer a more unbiased source of data. It is also possible that the students who did not consent to participate in the study were mainly those who did not have great things to say about the sessions. Both these situations do not appear to apply to our research. Comments on the learnt/felt forms were similar to those on the essay. Analysed data from 309 students is representative of whole group's—440 students—responses.

in relation to the rest of the tasks. In the next three sections, we further explore the role of warm mathematics explorations in shaping pre-service teachers' attitudes toward mathematics.

Experiential Therapeutic Intervention

In terms of therapeutic intervention, warm mathematics appears to work through eliciting memories, motivating engagement, evoking insight, allowing pre-service teachers to reflect on their feelings, as well as restoring their broken relationship with mathematics. Our approach to mathematics therapy is designed to normalise pre-service teachers' negative mathematical responses. We say that it is their unique school and out-of-school experiences that contribute to their negative mathematical responses. Thus, offering pre-service teachers warm mathematics (involving critical or radical events) is likely to evoke positive affect or an understanding of a need for change in teaching and learning practices. Three themes related to therapeutic intervention were evident in pre-service teachers' essays: different ways of doing mathematics, re-learning elementary school mathematics, and reconciliation.

Different ways of doing mathematics. Many comments revealed that the majority of pre-service teachers had not experienced alternatives to traditional mathematics. They often expressed positive feelings about warm mathematics. Here is an example of what they said: "Oh I never thought about doing it that way ... I don't remember in elementary school that there was a lot of different ways of doing things." Pre-service teachers enacted new metaphors for mathematics. "It is like being a detective trying to 'crack' the case." "[It] defied traditional ways of thinking." "I no longer think of math as simply being a 'cut and dried' right or wrong subject." New metaphors for mathematics might be taken as evidence of changes in values and conceptions about mathematics. These comments suggest that pre-service teachers were (re-)motivated to do mathematics.

A few participants expressed negative emotions. Having been successful with an approach based on worksheets, rules, and textbooks, they felt no need to break away from comfortable old experiences. A different way of doing mathematics comes with challenges. "Quite frustrating ... we were not given actual formulas." Assisting these teachers to see a need for change might require different experiences. Cooney (1994) and Cooney, Shealy, and Arvold (1998) would say that such students are less adoptive; their beliefs about mathematics as a "cut and dried" subject are impermeable. Raymond (1997) and Pajares (1992) would explain that their beliefs in traditional mathematics are evidential and core (not peripheral or surface) and are therefore resistant to change. Taylor (2003) refers to this resistance to contemporary teaching as a legitimate conflict for meaning that needs time and critical events to be resolved. One pre-service teacher noted, "At first, I was extremely apprehensive, I did not want to break away from the traditional approach However, as I began to work with the problem and with my peers, I quickly saw the benefits of reversing the system—I was actually enjoying myself" (emphasis in original).

Re-learning elementary school mathematics. Some pre-service teachers at first felt nervous about participating. It had been a while since they had done mathematics. Engaging in mathematical activity brought back negative memories and called forth intense negative emotions for some. Having large group sessions added to the initial anxiety. But doing mathematical activities appeared to offer these pre-service teachers an opportunity to re-learn school mathematics in ways that are connected and warm. One commented, "... so many mathematical concepts came back to me, such as slopes, ordered pairs, integers, and algebraic equations." Reviewing mathematics in a non-routine context appeared to boost the confidence level of many pre-service teachers.

Reconciliation. The initial intention of the mathematics sessions was to offer warm and alternative experiences of mathematics to our pre-service teachers with the purpose of triggering re-organisation of experience, especially for those who were fearful of mathematics. After a couple of sessions pre-service teachers' affective responses to mathematics, evidenced by feelings of liking and increased confidence, became positive. "When I entered ... in September it hadn't occurred to me that I would be teaching math. I was an Art specialist, why would I teach math? ... So I was surprised and amazed when, here at a University level math course we were playing with dice ... and I got it!" (emphasis in original). "I got my non-math mind working".

Pre-service teachers' fear of learning mathematics had transferred into a fear of teaching mathematics. The mathematics sessions were helpful in dealing with their self-concept with respect to mathematics. "It made me feel that this type of 'liberating' teaching can also be used cross-curricular ... I feel that the subject is no longer a mythological beast, but an area of interest." Whereas "mythological beast" may indicate an intense fear, "liberating" suggests a feeling of relief and empowerment. "Instead of being taught math, I was taught how to feel about it as a student and how to teach it as a teacher. This is exactly what I needed—an attitude adjustment". This pre-service teacher's comment shows that the sessions resulted in attitude adjustment. A reduction in negative mathematical affect influences learning mathematics in positive ways (McLeod, 1992).

There was evidence that through writing, summarising, and sharing of learned and felt forms, pre-service teachers learned to value, express, and respond to their own and each other's affective responses. Ingleton and O'Regan (2002) observe that it is important that teachers are offered the opportunity to redress their negative affective responses to mathematics so that they do not cycle them. Scott (2005) concludes that experiences that are more recent influence pre-service teachers' affect.

Mathematical Attentiveness

Four of the themes we identified in pre-service teachers' essays about their experience with mathematics tasks centre on ways in which rich mathematical activities encourage pre-service teachers to think and attend mathematically. The four themes are: many problems have many possible solutions; mathematics

involves noticing patterns; mathematics can be seen differently; and tools and organisers are helpful when doing mathematics.

Many possible solutions. There was evidence that in the mathematical exploration pre-service teachers were experiencing a mathematics that is not procedure-dominated. Many pre-service teachers reported positive emotions about experiencing a task with many solution strategies. They commented, “[O]ne thing I liked [about the sessions was] the many possible outcomes. ... To understand that there’s so many pathways” and, “As an elementary math student, I struggled ... during the Handshake problem, I realised why! I was always attempting to do my math exactly in the same method my teacher taught, whether or not it made sense.” This last pre-service teacher’s way of doing mathematics had been limited to mathematics as a mechanistic subject (Barkatsas & Malone, 2005). She attributes her struggle with mathematics to the desire to reiterate the teacher’s method. Another pre-service teacher observed after the Handshake session that it was confusing and frustrating to have more than one solution method, which is evidence of initial negative affect or of resistance.

Patterns in mathematics. Pre-service teachers got a glimpse of mathematics as a science of patterning (NRC, 1989). As with multiple solution strategies, many pre-service teachers’ affective responses about patterning were positive: “One of the biggest things that has helped me with my math ... is finding the Eureka connection that mathematics is really all about patterns.” “I was seeing ... patterns on my own.” “Algebra, which is based on patterning, has not been introduced to me that way in the past.” This is evidence that warm mathematics offers to students an opportunity to experience mathematics in dynamic ways (Nisbet & Warren, 2000). Perhaps this has the potential to challenge their traditional views (Barkatsas & Malone, 2005), in this case about algebra.

About mathematics. Many pre-service teachers commented on their images of mathematics in general. “This course has taught me that math can be very engaging and teachable ... students need to learn that thinking and experimenting is more advantageous than just practising formulas.” “This ... exemplifies how creative math can be.” In these comments, we see a change in attitude towards mathematics that involves thinking, experimenting, and creativity.

Tools and organisers. Pre-service teachers commented about the use of tools and organisers during the sessions. For the Consecutive Terms task, for instance, we provided linking cubes and graphical organisers such as worksheets with blank T-tables or table of values to be completed and with guiding and observation questions. A couple of pre-service teachers who believed that post-primary (ages 9 and above) teaching and learning of mathematics does not involve physical tools voiced their resistance to the use of “props.” Many of these reconsidered their belief, some gradually: “I did not feel stranded or in the dark when the chart and manipulatives were introduced—from this personal experience I discovered their effectiveness.” “As teachers, we ... need to encourage students to not be afraid to take risks, as I was. I did not want to

commit to sketching on the graph paper because of the fear of being wrong. However, by jotting and sketching, patterns may be noticed and aid the students in solving a problem." That some pre-service teachers do not commit to writing or sketching for fear of being wrong is a debilitating fear. It limits opportunities for mathematical thinking. "[Using manipulatives] is not something I did in school but something that I now recognise is central." This comment reflects a change in this person's views about learning and teaching mathematics.

Pedagogical Responsiveness

In terms of consequential gains, doing mathematics offered pre-service teachers opportunities to consider and articulate useful teaching and learning experiences. These experiences were from a learner's point of view. We explicitly modelled teaching strategies and occasionally commented about grade-level appropriateness and issues of modification and assessment. Two themes in the pre-service teachers' essays illuminate how they considered ways of enhancing students' learning experiences during classroom practice: appropriate teaching strategies and reflection about learning. Due to limitations of space, we do not discuss these two themes in detail. Through doing mathematics themselves, pre-service teachers appeared to become more aware of the potential impact of better mathematical experiences on mathematics learning (Ingleton & O'Regan, 2002). They gained direct models for helpful ways of teaching as well as some feedback on how such learning experiences could be helpful to learners. Also, this appeared to demonstrate to them the need for and potential benefits of change in mathematics teaching (Barkatsas & Malone, 2005; Scott, 2005). Further study on this theme is needed in a project that looks at pre-service teachers' practices during their practicum or that studies the role of warm mathematics with in-service teachers.

Conclusions and Implications

After the first activity, Making Ten, some pre-service teachers began to wish that their school mathematics program had involved the insightful and warm experiences they had in this course. Many even felt angry that they had not been "given a foundation of understanding earlier." One task per one-hour session is not a lot of time when the focus is on problem solving, and some pre-service teachers felt rushed by the finality of the sessions—this is another common source of mathematical anxiety (Herbert & Furner, 1997). There were moments of frustration when the pre-service teachers found a concept or way of thinking particularly difficult, for example, finding an algebraic expression. They sometimes experienced mental blocks. Specific anxieties, such as tools and organiser anxiety, had to be dealt with in the earlier sessions. For a few teachers who had deeper fears of mathematics, we wished we had had more contact time, smaller groups sessions, or interventions outside of the classroom. Negative affect in the form of resistance to alternative ways of doing mathematics was another issue. A few pre-service teachers still felt they needed to be explicitly

taught the solutions and strategies, perhaps rightly so. Novel modes of working such as “explain your thinking” may provoke anxiety in some students, including fast learners (Herbert & Furner, 1997). We continue to ponder ways of helping pre-service teachers to handle the tension and pain they experience when their core beliefs about mathematics are challenged. Gomez-Chacon (2000) observed that a *negative* affective emotion coupled with a productive meta-affect may motivate further engagement.

Our study might be seen as an attempt to implement and research a teacher education component that resolves inconsistencies between various beliefs. Barkatsas and Malone (2005), Beswick (2005), and Thompson (1992) noted such inconsistencies between (a) espoused beliefs and practiced beliefs; and (b) contemporary beliefs about teaching and learning that pre-service teachers gain during teacher education and traditional beliefs that pre-service teachers might have formed through their school experiences. Pajares (1992) noted that these inconsistencies might arise from teacher education methodology and from varying belief structures, whereas Barkatsas and Malone (2005), Beswick (2005), and Raymond (1997) maintain that inconsistencies arise from school contexts and constraints. Our work suggests that including warm mathematics in a teacher education program may facilitate consistency between what pre-service teachers profess to be their teaching beliefs and what they practise in the classroom.

We have inferred pre-service teachers’ affective responses mainly from what pre-service teachers wrote (i.e., by looking for affect-expressive words). Our research did not focus on evidence of use during practicum teaching (Pajares, 1992). There is a need for longer-term and cross-sectional studies to examine the practice implications of warm mathematics.

Our work is about reshaping the experiences of pre-service teachers of mathematics who may never have experienced rich and warm mathematics. We maintain that preparing elementary teachers for teaching involves addressing both cognitive and affective concerns in ways that have a potential to influence practice. Doing warm mathematics offers pre-service teachers the opportunity to (re)-experience and to feel positively about learning mathematics. For us, it is central that pre-service teachers be offered starting points for positive, warm affective responses and intimate relations with mathematics.

References

- Anderson, J. (2005). Implementing problem solving in mathematics classrooms: What support do teachers want? In P. Clarkson, A. Downton, D. Gronn, M. Horne, A. McDonough, R. Pierce, & A. Roche (Eds.), *Building connections: Research, theory and practice* (Proceedings of the 28th annual conference of the Mathematics Education Research Group of Australasia, pp. 89-96). Sydney: MERGA. Retrieved October 16, 2007, from <http://www.merga.net.au/publications/>
- Ball, D. L. (1990). Breaking with experience in learning to teach mathematics: The role of a pre-service methods course. *For the Learning of Mathematics*, 10(2), 10-16.
- Ball, D. L. (2002). Knowing mathematics for teaching: Relations between research and practice. *Mathematics and Education Reform Newsletter*, 14(3), 1-5.

- Barkatsas, A. T., & Malone, J. (2005). A typology of mathematics teachers' beliefs about teaching and learning mathematics and instructional practices. *Mathematics Education Research Journal*, 17(2), 69-90.
- Beswick, K. (2005). The beliefs/practice connection in broadly defined contexts. *Mathematics Education Research Journal*, 17(2), 39-68.
- Brousseau, G., & Gibel, P. (2005). Didactical handling of students' reasoning processes in problem solving situations. *Educational Studies in Mathematics*, 59, 13-58.
- Brown, A. A., & Borko, H. (1992). Becoming a mathematics teacher. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 209-239). New York: Macmillan.
- Buzeika, A. (1999). Teachers' doubts about invented algorithms. In O. Zaslavsky (Ed.), *Proceedings of the 23rd conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 161-168). Haifa, Israel: PME.
- Calderhead, J., & Robson, M. (1991). Images of teaching: Student teachers' early conceptions of classroom practice. *Teaching and Teacher Education*, 7(1), 1-8.
- Confrey, J. (1998). Building mathematical structure within: A conjecture driven teaching. In S. B. Porenson, K. R. Dawkins, M. Blanton, W. N. Coulombe, J. Kolb, K. Norwood, & L. Stiff (Eds.), *Proceedings of the 20th annual general meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 39-50). Columbus, OH: ERIC Clearinghouse.
- Cooney, T. J. (1994). Research and teacher education: In search of common ground. *Journal for Research in Mathematics Education*, 25, 608-636.
- Cooney, T. J. (1999). Conceptualizing teachers' ways of knowing. *Educational Studies in Mathematics*, 38(1-3), 163-87.
- Cooney, T. J., Shealy, B. E., & Arvold, B. (1998). Conceptualizing belief structures of pre-service secondary mathematics teachers. *Journal for Research in Mathematics Education*, 29(3), 306-333.
- DeBellis, V., & Goldin, G. (2006). Affect and meta-affect in mathematical problem solving: A representational perspective. *Educational Studies in Mathematics*, 63(2), 131-147.
- Elliot, J. C. (1990). Affect and mathematics achievement of nontraditional college students. *Journal for Research in Mathematics Education*, 21(2), 160-165.
- Ensor, P. (1998). Teachers' beliefs and the problem of the social. In A. Olivier & K. Newstead (Eds.), *Proceedings of the 22nd conference of the International Group for the Psychology of Mathematics Education* (pp. 280-287). Stellenbosch, South Africa: PME.
- Evans, J. (2000). *Adults' mathematical thinking: A study of numerate practices*. London: Routledge.
- Evans, J. (2002). Developing research conceptions of emotion among adult learners of mathematics. *Literacy and Numeracy Studies*, 11(2), 79-94.
- Fennema, E., & Franke, M. L. (1992). Teachers' knowledge and its impact. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 147-164). New York: Macmillan.
- Flewelling, G., & Higginson, H. (2001). *Realizing of vision of tomorrow's mathematics classroom: A handbook on rich learning tasks*. Kingston: MSTE.
- Gadanidis, G. (2004). The pleasure of attention and insight. *Mathematics Teaching*, 186(1), 10-13.
- Gadanidis, G., Namukasa, I., & Moghaddam, A. (in press). *Mathematics-for-teachers online: Facilitating conceptual shifts in elementary teachers' views of mathematics*. Bolema (Brazil).
- Gomez-Chacon, I. M. (2000). Affective influences in the knowledge of mathematics. *Educational Studies in Mathematics*, 43, 149-168.

- Hannula, M. (2006). Motivation in mathematics: Goals reflected in emotions. *Educational Studies in Mathematics*, 63(2), 165-178
- Hannula, S. M., Liljedahl, P., Kaasila, R., & Rosken, B. (2007). Researching relief of mathematics anxiety among pre-service elementary school teachers. In J. Woo, H. Lew, K. Park, & D. Seo (Eds.), *Proceedings of the 31st conference of the International Group for the Psychology of Mathematics Education* (pp. 153-162). Seoul, Korea: PME.
- Hembree, R. (1990). The nature, effects and relief of mathematics anxiety. *Journal for Research in Mathematics Education*, 21(1), 33-46.
- Herbert, T. P., & Furner, J. M. (1997). Helping high ability students overcome mathematics anxiety through bibliotherapy. *Journal of Secondary Gifted Education*, 8(4), 164-179.
- Ingleton, C., & O'Regan, K. (2002). Recounting mathematical experiences: Emotions in mathematical learning. *Literacy and Numeracy Journal*, 11(2), 95-107.
- Jones, G. (2004). The impact of 20 years of research. In B. Perry, G. Anthony, & C. Diezmann (Eds.), *Research in mathematics education in Australasia 2000-2003* (pp. 339-388). Flaxton, QLD: Post Pressed.
- Koehler, M. S., & Grouws, D. A. (1992). Mathematics teaching practices and their effects. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 115-126). New York: Macmillan.
- Lampert, M. (1991). Connecting mathematical teaching and learning. In E. Fennema, T. P. Carpenter, & A. J. Lamon (Eds.), *Integrating research on teaching and learning mathematics* (pp. 121-152). Albany: State University of New York.
- Leder, G., & Grootenboer, P. (2005). Affect and mathematics education. *Mathematics Education Research Journal*, 17(2), 1-8.
- Ma, X., & Kishor, N. (1997). Assessing the relationship between attitude toward mathematics and achievement in mathematics: A meta-analysis. *Journal for Research in Mathematics Education*, 28(1), 26-47.
- Mason, J. (1994). Researching from the inside in mathematics education: Locating an I-you relationship. In J. da Ponte, & J. Matos (Eds.), *Proceedings of the 18th conference of the International Group for the Psychology of Mathematics Education* (pp. 176-193). Lisbon, Portugal: PME.
- Mason, J., Burton, L., & Stacey, K. (1982/1985). *Thinking mathematically (revised edition)*. Wokingham: Addison-Wesley.
- McLeod, D. B. (1992). Research on affect in mathematics education: A reconceptualization. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 575-596). New York: Macmillan.
- McLeod, D. B. (1994). Research on affect and mathematics learning in the JRME: 1970 to the present. *Journal for Research in Mathematics Education*, 25(6), 637-647.
- Namukasa, K. I. (2005). *Attending in mathematics: A dynamic view about students' thinking*. Unpublished doctoral dissertation, University of Alberta, Alberta.
- NRC (1989). *Everybody counts: A report to the nation on the future of mathematics education*. Washington, DC: National Academy.
- Nisbet, S., & Warren, E. (2000). Primary teachers' beliefs and practices with respect to mathematics, teaching and assessing mathematics and factors that influence these beliefs. *Mathematics Teacher Education and Development*, 2, 34-47.
- Pajares, M. F. (1992). Teachers' beliefs and educational research: Cleaning up a messy construct. *Review of Educational Research*, 62(2), 307-332.
- Polya, G. (1945/1973). *How to solve it: A new aspect of mathematics (2nd edition)*. Princeton, New Jersey: University Press.
- Raymond, A. M. (1997). Inconsistency between a beginning elementary school teacher's mathematics beliefs and teaching practice. *Journal for Research in Mathematics Education*, 28(5), 550-576.

- Schoenfeld, A. H. (1985). *Mathematical problem solving*. New York: Academic Press.
- Scott, A. L. (2005). Pre-service teachers' experiences and the influences on their intentions for teaching primary school mathematics. *Mathematics Education Research Journal*, 17(3), 62-90.
- Simmt, E. (2000). *Mathematical knowing in action*. Unpublished doctoral dissertation, University of Alberta.
- Simmt, E., Davis, B., Gordon, L., & Towers, J. (2003). Teachers' mathematics: Cautious obligations. In N. A. Pateman, B. J. Dougherty, & J. T. Zilliox (Eds.), *Proceedings of the 27th conference of the International Group for the Psychology of Mathematics Education held jointly with the 25th conference of PME-NA* (Vol. 1, pp. 175-182). Honolulu, Hawaii: PME.
- Skott, J. (1999). The multiple motives of teaching activity and the role of the teacher's school mathematical images. In O. Zaslavsky (Ed.), *Proceedings of the 23rd conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 209-216). Haifa, Israel: PME.
- Taylor, A. R. (2003). Transforming pre-service teachers' understanding of mathematics: Dialogue, Bakhtin and open-mindedness. *Teaching in Higher Education*, 8(3), 333-344.
- Thompson, A. G. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 127-146). New York: Macmillan.
- van Manen, M. (1998). *Researching lived experience: Human science for an action sensitive pedagogy* (2nd edition). London, Ontario: Althouse.
- Watson, A., & Mason, J. (2007). Taken-as-shared: A review of common assumptions about mathematical tasks in teacher education. *Journal of Mathematics Teacher Education*, 10(4-6), 205-215.
- Wedge, T. (2002). "Mathematics – That's what I can't do": People's affective and social relationship with mathematics. *Literacy and Numeracy Journal*, 11(2), 63-78.
- Wilson, P. S., Cooney, T. J., & Stinson, D. W. (2005). What constitutes good mathematics teaching and how it develops: Nine high school teachers' perspectives. *Journal of Mathematics Teacher Education*, 8(2), 83-111.
- Wilson, S., & Thornton, S. (2005). 'I am really not alone in this anxiety': Bibliotherapy and pre-service primary teachers' self-image as mathematicians. In P. Clarkson, A. Downton, D. Gronn, M. Horne, A. McDonough, R. Pierce, & A. Roche (Eds.), *Building connections: Research, theory and practice* (Proceedings of the 28th annual conference of the Mathematics Education Research Group of Australasia, pp. 791-798). Retrieved October 16, 2007, from <http://www.merga.net.au/publications/conf.php>.
- Wolodko, B., Willson, K., & Johnson, R. (2003). Preservice teachers' perceptions of mathematics: Metaphors as a vehicle for exploring. *Teaching Children Mathematics*, 10(4), 224-230.

Authors

Immaculate Namukasa, Faculty of Education, University of Western Ontario, 1137 Western Road, London, Ontario, Canada, N6G 1G7. Email: <inamukas@uwo.ca>

George Gadanidis, Faculty of Education, University of Western Ontario, 1137 Western Road, London, Ontario, Canada, N6G 1G7. Email: <ggadanid@uwo.ca>

Michelle Cordy, 31-1294 Byron Baseline Road, London, Ontario, Canada, N6K 4G8. Email: <m.cordy@tvdsb.on.ca>