Using the Co-creation of Practice-based Lessons to Build Teachers' Capacity to Facilitate Mathematical Discussions

Barbara King
Florida International University

Carmen Petrick Smith
University of Vermont

Jeehyun Park
Florida International University

Shemail Fatima
Florida International University

Elizabeth Forde
State University of New York at New Paltz

Received: 11 May 2021 Accepted: 21 October 2022

Mathematics teacher educators frequently provide in-service teachers (ISTs) opportunities to practice teaching within the university classroom. While these opportunities are valuable, some question whether they focus too narrowly on learning a particular procedure for teaching instead of learning why and when to use that procedure. To address this concern, we engaged ISTs enrolled in a graduate degree program in the co-creation of practice-based lessons. This study examines what and how ISTs learned during the co-creation process. The results indicate that ISTs improved their understanding of facilitating classroom discussions in three crucial areas: asking conceptual questions, building connections, and using talk moves. Qualitative interview data revealed that ISTs learned through IST collaborations, instructor-IST interactions, and creation versus participation. Lastly, we discuss the teacher's intricate role in the co-creation process.

Keywords ·mathematics teacher education research · practice-based teacher education · approximations of practice · co-creation of learning and teaching · mathematical discussions

Introduction

Practice-based teacher education (PBTE) has gained momentum, leading teacher educators to spend significantly more class time on the enactment of teaching (Ball & Forzani, 2009; Grossman & McDonald, 2008). A core element of PBTE is using approximations of practice, which are opportunities for teachers (both pre-service and in-service) to try out specific teaching activities in a supportive, controlled environment while receiving real-time feedback (Grossman et al., 2009a). The need for practice-based learning opportunities is due to the complex nature of a school classroom. Imagine a teacher's challenge in trying to facilitate a productive discussion. Every student brings a distinct identity and combination of knowledge, understanding, and experience to the classroom. Each time a student participates in class, the teacher must customise a response based on that unique contribution (Lampert, 2005). Because of the complexity involved in facilitating discourse, building pre-service and in-service teachers' theoretical understanding is not enough; they need opportunities to practice leading discussions in safe, controlled environments (Grossman et al., 2009b; Janssen et al., 2015).

Grossman et al. (2009a) introduced a Framework for Teaching Practice that involves three phases: representing the teaching practice, decomposing the practice into smaller parts, and approximating the practice through supervised and controlled teaching opportunities (e.g., teaching to peers). Grossman et al.'s Framework, and in particular, the use of approximations of teaching, have shown significant promise in helping teachers improve their ability to organise and lead classroom discussions, as well as other teaching practices (Kavanagh et al., 2020; Schutz et al., 2019; Tyminski et al., 2014). Some question, however, whether this approach helps them replicate a procedure without understanding when and why the skill should be used and how to apply it in various situations (Janssen et al., 2015; Kennedy, 2016). While we certainly see Grossman et al.'s Framework for Teaching Practice as a valuable tool for building teachers' ability to lead classroom discussions, we take seriously the critics who call for building more understanding around core teaching practices. We wondered whether an effective way to improve ISTs' classroom discussion skills is to engage ISTs in the co-creation of what, in this paper, we will refer to as practice-based lessons that follow Grossman et al.'s framework and include opportunities for representation, decomposition, and approximation of practice. By co-creation, we mean the opportunity for ISTs and the university instructor to work side-by-side to develop practice-based lessons. Therefore, we set out to examine what and how ISTs who are enrolled in a graduate-level degree program learned during the co-creation of practice-based lessons. The following research questions guided our work,

(1) What do ISTs learn about facilitating effective mathematical discussions during the co-creation of a practice-based lesson? and
(2) How do ISTs learn during the co-creation of a practice-based lesson?

Literature Review

This study draws on two research frameworks. The first is Grossman et al.'s (2009a) Framework for Teaching Practice, and the second is Cook-Sather et al.'s (2014) work on the co-creation of learning and teaching. These frameworks guided our decision to engage ISTs in the co-creation of practice-based lessons.

Framework for Teaching Practice

Historically, teacher preparation programs used experiences in real K–12 classrooms as exclusive opportunities for practicing teaching while limiting course instruction in college classrooms to the examination of educational theories (Grossman et al., 2009b). Past research has documented the difficulty of applying the theories taught during coursework to an actual classroom setting (Janssen et al., 2015; Korthagen & Wubbels, 2001). As a result, there is a strong movement within mathematics teacher education to include opportunities for practice within education coursework to bridge the gap between theory and teaching. Grossman et al. (2009a) provided a framework for thinking about teaching practice involving three pedagogies of practice: representations, decompositions, and approximations.

Representations of practice allow students to see teaching represented, for example participating in a lesson that highlights a specific practice or watching a video of the practice being represented. Decomposition of practice involves breaking down practices into fundamental parts. Finally, approximations of practice provide opportunities for novices to engage in practices similar to the work of teaching but in supportive, controlled environments. There is evidence in mathematics teacher education research that using all three pedagogies of practice enhances in-service and pre-service teachers learning. In a recent study, Ghousseini and Herbst (2016) found that using all three was necessary to build a full range of understandings about managing classroom discussions. While Janssen et al. (2015) agreed all three practices are essential, they argued for flexibility in the way they are implemented and cautioned against exclusively following the set order of representation, followed by decomposition, followed by approximation.

One approach that supports the use of representations, decompositions, and approximations is to redesign methods courses around a set of high-leverage practices (Ball & Forzani, 2009; Kazemi et al., 2007; McDonald et al., 2013). High-leverage practices are practices that are central to the work of teaching, such as managing group work or posing questions about content (Ball & Forzani, 2009). Centring the content around high-leverage practices provides a structure for education courses that enable mathematics education instructors to decompose complex teaching tasks into various instructional routines and provide students with opportunities to practice these routines.

When developing approximations of practice, there are several factors to consider. First, Grossman et al. (2009a) highlight the idea of authenticity as a critical component to consider. They conceptualise a continuum from less to more authentic, with more authentic approximations involving settings, artifacts, and activities that are more like actual teaching practices. Working in classrooms with real students offers a more authentic setting, while less authentic settings might involve letter writing or simulations of teaching. In some approximations, the work is highly structured, such as in rehearsals, allowing the instructor to provide support and feedback more easily. During rehearsals, the instructor can pause the approximation of practice, provide time for reflection, address possible confusion, or extend students' thinking about the situation before restarting the practice (Schutz et al., 2018). The real-time, interactive process is not only beneficial to the student enacting the teaching but to all the students in the class. In more authentic settings, such as elementary classrooms, it is more challenging to administer in-the-moment feedback. In these situations, instructors can debrief after the approximation making visible vital aspects of teaching.

Facilitating Mathematical Discussions

One high-leverage practice that has been the focus of a great deal of research on approximations is facilitating an effective mathematical discussion (Boerst et al., 2011; Franke et al., 2009; Sleep, 2012). This practice is challenging as it requires "the integration and coordination of several knowledge bases, practices, and skills" (Tyminski et al., 2014, p. 464), such as eliciting student thinking, orienting students to each other's ideas, filtering important information, and drawing students' attention to important mathematical connections. Approximations can range from practicing a smaller piece of a practice, such as practicing how to elicit students' thinking (Franke et al., 2009; Shaughnessy & Boerst, 2017), to enacting the entire practice, for instance, facilitating a discussion. In education courses, an entire practice is often decomposed into smaller parts; these chunks are examined and enacted and then recomposed by putting the parts back together into the full practice (Tyminski et al., 2014).

Facilitating classroom discussions has garnered considerable attention in mathematics teacher education research and PBTE (Ghousseini, 2009; Kazemi et al., 2009; Tyminski et al., 2014). For example, Ghousseini (2009) found prospective elementary teachers benefited from engaging in discourse routines that centre around particular aspects of mathematical discussions, such as revoicing, pressing for explanations, and connecting students' ideas. Kazemi et al. (2009) found that their approximations helped prospective elementary teachers reconsider the role of errors and move from thinking of errors as negatives to thinking of them as opportunities for further learning. Tyminski et al. (2014) found that approximations helped prospective elementary teachers align the pedagogical decisions made while planning a discussion with their goals for the discussion. The research on approximations demonstrates the pivotal role they can play in preparing education students for facilitating effective mathematical discussions.

Despite the vast amount of research advocating for PBTE, some question this approach (Philip et al., 2019; Zeichner, 2012). For example, Kennedy (2016) argues that we need to "pay less attention to the visible behaviours of teaching and more attention to the purposes that are served by those behaviours" (p. 6). In her argument, she suggests we approach teacher education by starting with five core challenges faced in teaching, for example eliciting student participation and containing student behaviour. Then, the job of instructors is to help their students think about how teachers' actions are related to these challenges. This approach would put more emphasis on why a teacher might take a specific action instead of how to take that action. In line with this thinking, Janssen et al. (2015) critique the assumption that developing a teaching skill leads to developing the will to use that skill in teaching. Furthermore, they argue that PBTE's focus on following instructional routines limits teachers' ability to adapt and extend these routines, i.e., to help teachers be more responsive to students.

To address these concerns, Kavanagh et al. (2020) designed a study focused on using approximations to support ISTs' responsiveness to students' ideas. The authors explored instructors from four subject areas as they engaged ISTs in the implementation of approximations. Two key findings from this work impact the design of the current study. First, they found that it is possible to use approximations and keep the purpose central to the work instead of focusing on the technique. One way instructors did this was to "hold up students' ideas as decision-points for teachers instead of holding up teachers' moves as techniques to be practiced until they become automatic" (Kavanagh et al., 2020, p. 104). Second, they found that instructors who provided little structure or guidance about how the ISTs should enact the approximation (e.g., the choice of topic, the learning target, and the discussion structure) were less likely to successfully practice the agreed-upon goal of responding to students' ideas during the approximation. This finding suggests the importance of collaboration between instructors and ISTs when designing approximations.

In the current study, we build on past research in this area by providing ISTs with the opportunity to create and implement practice-based lessons. Instead of giving ISTs free rein to design these lessons, we drew from the literature on the co-creation of learning and teaching, suggesting that the instructor and ISTs engage in the lesson development together. We hypothesised that placing ISTs in the role of co-creator would help them understand the purpose surrounding their approximations and build their skills to enact the core strategies from the lessons they developed in their teaching. Next, we discuss the literature around the co-creation of learning and teaching.

Co-creation of Learning and Teaching

Rarely do students attending colleges and universities take an active role in creating the activities that guide the learning in a class; most learning opportunities involve passive engagement in activities designed by others (Mann, 2008). As Bovill et al. (2016) explain, "co-creation of learning and teaching occurs when staff and students work collaboratively with one another to create components of curricula and/or pedagogical approaches" (p. 196). There are many ways to engage college students in co-creation, to name a few: a student could be selected as a co-researcher, act as a consultant to provide a student's perspective, or work as a representative who contributes to university decision-making. In this study, though, we focus on a whole-class approach involving ISTs as pedagogical co-creators (or co-designers) where "the purpose, approaches, and outcomes of learning and teaching are jointly negotiated and there is a shared responsibility for learning" (Bovill, 2020, p. 1026). In classrooms where students are pedagogical co-creators, students contribute to the purpose of the work and the design of instructional strategies and course outcomes, thus promoting student motivation, agency, and empowerment (Bovill, 2020).

Past research, detailed in Cook-Sather et al.'s (2014) landmark book on student-faculty partnerships, documented three beneficial outcomes that consistently arise during the co-creation of learning and teaching: engagement, awareness, and enhancement. Individuals in college classes who are involved in co-creation tend to be more engaged in the process of learning compared to those in traditional settings who are more likely to focus on the outcomes of learning. Secondly, college students engaged in co-creation develop greater metacognitive awareness, most notably seeing their role as students in a new light. Lastly, co-creation leads college students to take a more active role as learners and take on more responsibility for their own learning. As a result, co-creation transforms their role. This role is no longer passive or merely doing; it is now to develop a metacognitive awareness about what is being done (Cook-Sather et al., 2014). A heightened focus and awareness help individuals look beyond carrying out actions, and to think deeply about why such actions are needed (Martinez, 2006).

Another positive outcome, and a critical element of the co-creation of learning and teaching, is positive student-teacher relationships (Bovill, 2020). As Breen and Littlejohn (2000) explain, "teachers may need to come to see their own plans for classroom work as simply proposals ... which learners have the right to reformulate, elaborate upon, or even reject" (p. 277). In co-creation, the responsibility for learning is shared; therefore, building trusting and respectful student-teacher relationships is necessary (Bovill, 2020; Cook-Sather et al., 2014). This requires a shift on the teacher's part, from viewing students as consumers of knowledge to viewing them as members of a learning community (Bergmark & Westman, 2016).

Such a shift can be challenging for college students and their teachers. Cook-Sather (2014) argues that grasping the nature of student-faculty partnerships of the kind investigated in this study can be difficult. In fact, both college students and their teachers may experience intellectual and emotional discomfort as this new way of learning and teaching affects their understanding of their past learning experiences. It is necessary to recognise that such a shift to see learning and teaching as a partnership may take time, and the involved parties may regress to their "earlier status," i.e., their role in a traditional classroom (Meyer & Land, 2006).

Guided by this research, we sought to engage a group of ISTs enrolled in a graduate-level course in the co-creation of practice-based lessons. During the co-creation process, the ISTs played a central role in co-creation by selecting the topics for the lessons, designing the representations, decompositions, and approximations that made up the practice-based lessons, and offering feedback to their peers. The co-created practice-based lessons were ultimately presented to other teachers working in nearby schools. We hypothesised ISTs would benefit from co-creation and sought to explore what and how they learned during the process. Additional details about the co-creation process are provided in the section below.

Research Methods

Participants and Context

We designed a qualitative research study to investigate what and how ISTs learned during the co-creation of a practice-based lesson with seven ISTs enrolled in a graduate-level mathematics education course at a large, public university in the Southeastern United States. The class followed a hybrid format where in-person meetings occurred every two weeks for two hours and forty minutes. Small group meetings or extended online assignments were used during the weeks without a face-to-face meeting. The first author was the lead instructor for the course. Seven ISTs (see Table 1 for participant information) in the class, six female and one male, were given the opportunity to participate in the study, and all seven agreed. Three of the ISTs who participated were elementary school teachers, and four were secondary mathematics teachers.

Table 1
Pseudonyms for Each Participant and Other Information

Participant Name (Pseudonym)	Gender	Age of Students Taught	Years of Experience
Evelyn	Female	Elementary (Grades K-5)	More than 5 years
Yolanda	Female	Elementary (Grades K-5)	More than 5 years
Amanda	Female	Elementary (Grades K-5)	More than 5 years
Marvin	Male	Middle School (Grades 6-8)	5 years or less
Kelsey	Female	Middle School (Grades 6-8)	5 years or less
Gretchen	Female	High School (Grades 9-12)	5 years or less
Roberta	Female	High School (Grades 9-12)	More than 5 years

The course where the study took place was designed to be the final course in a series of five graduate-level mathematics education courses offered to graduate students in mathematics education, most of whom are ISTs. The three over-arching goals of the mathematics education graduate program are (1) preparing teachers to use a problem-based approach to instruction, (2) building teachers' mathematics pedagogical content knowledge, and (3) developing teachers for leadership. Throughout the early courses in the program, ISTs participated in mathematics education lessons based on Grossman et al.'s (2009a) Framework for Teaching Practice which involves using representations, decompositions, and approximations. In the course where this study took place, ISTs were asked to move from being participants in these types of lessons to co-creating this type of lesson.

The Use of Co-creation in the Course

This course used co-creation to develop the content focus and curricular materials. These materials were developed through a partnership between the ISTs and the instructor. The instructor thought deeply about her role in the co-creation process and sought to act as a facilitator throughout the course, working to support but not determine the focus and design of the lessons. Heron (1999) discussed three ways to enact facilitation: the hierarchical mode, the co-operative mode, and the autonomous mode. The instructor in this study set out to use the co-operative mode. As Heron (1999) explained,

> Here you share your power over the learning process and manage the different dimensions with the group. You enable and guide the group to become more self-directing in the various forms of learning by conferring with them and prompting them. You work with group members to decide on the programme, to give meaning to experiences, to confront resistances, and so on. In this process, you share your own view which, though influential, is not final but one among many. (p. 8)

As such, the instructor saw her role as one in partnership with the ISTs, attempting to share power without giving complete autonomy or taking complete control. Despite her efforts, the instructor acknowledges that her voice may not be viewed as equal. One potential reason for this is ISTs may view the instructor's words as more valuable due to her experience and expertise or because of the long-standing tradition that the instructor is the authority in the classroom (Cook-Sather et al., 2014). Despite this challenge, the instructor was committed to using the co-operative mode throughout the course.

In practice, using the co-operative mode meant that the instructor worked with ISTs in all stages of the co-creation. For example, as the class worked to identify topics for the lesson plans the instructor and ISTs discussed potential topics openly. This included discussing mathematics education literature and analysing data from interviews conducted by ISTs with colleagues. During these interviews, ISTs gathered information on areas where their colleagues wanted to improve their teaching and on the types of instruction they preferred when learning about teaching. After completing this work, the class ultimately voted on which topics should guide the lesson plans. The instructor was also active during the two cycles of the lesson plan development. During this time, the instructor helped support ISTs by posing questions, directing ISTs to mathematics education literature, and asking them to consider the relationship between the literature and their lesson.

Content of the Course

The first part of the course focused on examining the literature on PBTE to prepare ISTs for the type of lesson they would ultimately develop. Next, the class investigated mathematics education research and collaborated to select lesson topics. The class decided that their lessons would centre around developing skills needed for facilitating an effective mathematics discussion. The class divided this broad topic into three sub-topics: using Talk Moves to create a culture where students regularly speak, listen, and respond to one another (Chapin et al., 2013; Michaels & O'Connor, 2015); presenting a variety of solution strategies (including ones that are not mathematically sound); and focusing on making connections within the structure of the Five Practices for Orchestrating Productive Mathematical Discussions (Smith & Stein, 2011). The eventual goal was for ISTs to present their lessons to peers during a professional development session. The co-creation process involved two cycles to prepare for this, each including developing a lesson, presenting it to the class, receiving feedback, and revising the lesson. Once the two cycles were completed, ISTs submitted a final lesson plan before presenting it to peers.

In addition to the seven ISTs who participated in the study, three additional students were taking the class who were each seeking a doctoral degree in mathematics education. The data from these three students were not used in the study for two main reasons. First, the study aimed to examine ISTs learning about facilitating effective discussions and how this learning took place. Therefore, we wanted to limit our sample to ISTs who currently work as K-12 teachers. Second, the doctoral students were part of the research team. They participated in the study design, were part of meetings in which we discussed the literature supporting the study and conducted interviews with the ISTs after the course was completed.

Therefore, we decided including their data was a conflict of interest. While taking the class, the three doctoral students participated the same way as all the ISTs, were assigned grades from the instructor, and were mindful of their positions as participant researchers.

The doctoral students worked to maintain an equal role during all small group and whole class collaborations. This included listening first before sharing their thinking and posing questions when needed. The three doctoral students had taken multiple classes with the ISTs who participated in the study. In these classes, a culture was created where everyone's voice was heard and valued, so providing space for all members of the class to speak was not new. Furthermore, none of the doctoral students had K-12 mathematics teaching experience in the United States. So, while they inarguably brought a unique set of expertise and their status as doctoral students was known by everyone in the class, they were in many ways learning from the ISTs about teaching in the United States and how to design professional development to reach the ISTs' peers. The questions asked by the doctoral students were often questions to seek understanding as opposed to questions intended to elicit a desired response.

Data Collection

To understand what and how ISTs learned while developing the lessons, we collected data from two sources: assessments and interviews. The first data source included a pre-assessment administered before the co-creation of the lesson and a post-assessment administered after the submission of the final lesson plan. The assessments were designed to examine ISTs' development of an effective mathematics discussion, and the data were used to help us understand what ISTs learned about facilitating effective discussions during the co-creation process. On each assessment, ISTs were provided with detailed information about a mathematics lesson. This information provided a context for the lesson, including the grade level, the learning focused on in previous lessons, and the Common Core mathematics standard (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) addressed during the lesson. Next, a description of the lesson in action was provided. This included: (1) the problem used to guide instruction, (2) the learning outcomes that the teacher was trying to develop in this lesson (see Figure 1), (3) three student solution methods, and (4) a sequence for presenting the three solutions. Using the information provided, ISTs were asked to construct a written dialogue of a whole-class discussion with the IST acting as the teacher and hypothetical students participating in the discussion. The ISTs wrote the dialogues in the format of a script specifying what the teacher would say or do and how the imaginary students would respond. We intentionally provided detailed information about the context so that ISTs could focus on the discussion only and not on the decisions leading up to the discussion, such as selecting and sequencing the possible solution strategies. By constraining the situation, it allowed us to concentrate on how the discussion was facilitated and analyse whether ISTs' use of specific mathematics discussion moves improved from the pre-assessment to the post-assessment. By discussion moves, we mean deliberate actions taken by the ISTs to “mediate, participate in, or influence” the mathematical discussion in the classroom (Krussel et al., 2004, p. 307).

Pre-Assessment	Post-Assessment
Problem that Guided the Lesson: *Julie gave Mandy* $2\frac{1}{2}$ *candy bars. Mandy gave* $\frac{1}{4}$ *of what she got to Carolina. How much candy bar did Carolina get?* **Target Learning Outcomes:** $2\frac{1}{2}$ *is the same as* $2+\frac{1}{2}$. $\frac{1}{4}$ x $2\frac{1}{2}$ *can be solved using the distributive property, such that* $\frac{1}{4}$ x $(2 + \frac{1}{2}) = \frac{1}{4}$ x $2 + \frac{1}{4}$ x $\frac{1}{2}$	**Problem that Guided the Lesson:** *At a restaurant, the waiter brings six sub sandwiches for nine children to share. If the subs are shared equally (with none left over), how much sub will each child get?* **Target Learning Outcomes:** *A fraction can be decomposed in more than one way. For example,* $\frac{6}{9}=\frac{1}{9}+\frac{1}{9}+\frac{1}{9}+\frac{1}{9}+\frac{1}{9}+\frac{1}{9}$ *, and* $\frac{6}{9}=\frac{1}{9}+\frac{1}{9}+\frac{2}{9}+\frac{2}{9}$. *To build understanding that* $\frac{6}{9}$ and $\frac{2}{3}$ *are equivalent fractions, such that* $\frac{2}{3}$ x $\frac{3}{3}$ = $\frac{6}{9}$

Figure 1. Problem and learning outcomes for each assessment.

To help us understand how ISTs learned, the researchers conducted semi-structured interviews with each of the seven ISTs at the end of the course. Using semi-structured interviews allowed us to prepare the interview questions beforehand, which enabled us to gather more focused answers to the research questions while also providing the opportunity to ask follow-up questions based on ISTs responses (DiCicco-Bloom & Crabtree, 2006; Savin-Baden & Howell-Major, 2013). An interview protocol also allowed us to examine the assessments for each IST and ask targeted questions about their answers. The interviews lasted from 40 to 90 minutes and took place in various locations chosen by the participants; some were conducted in participants' classrooms, while others took place on the phone. All interviews were audio-recorded and transcribed verbatim using Temi software, then checked and edited by the researchers for accuracy.

Data Analysis

Research Question 1

To explore research question one, examining what ISTs learned about facilitating mathematical discussions during the co-design process, we began by analysing the pre-and post-assessments using conceptual analysis (Carley, 1990). To begin, the first and second authors examined the written dialogues of hypothetical whole-class discussions created by ISTs on the pre-and post-assessments to identify the aspects of mathematical discussions that were central to their written dialogues. Then, we connected these to the research on effective mathematics discussions and used this to help create a set of codes. Next, we independently coded a subset of the assessment data using the *Discussion Move* codes. After coding, the researchers discussed similarities and differences in their coding, defined each code, and selected examples for each code (see Table 2). In all, there were nine codes in three different categories. Using the final codes, the two researchers independently coded the assessment data and compared the results. All differences in coding were discussed until a consensus was reached.

Table 2
Discussion Move Codes with their Descriptions and an Example

Code	Description	Example
Questioning		
Conceptual Questions	Questions that focus on conceptual, rather than, procedural thinking. Inquiring about the nature of things.	Why did you choose thirds instead of ninths?
Making Connections[1]		
Describing the big idea	Students' understanding of general rules, facts, and definitions and using these appropriately to describe the big idea.	Another strategy that I would like to highlight is that Hannah represented two and one-half as (2 + 1/2).
Knowing why a procedure works	Students' understanding of why a specific procedure works and the ability to apply it to solve math problems.	How did you know to shade in a quarter of each candy bar?
Multiple representations	Pointing out how the same idea is represented in two different representations.	Who can explain where in Zari's drawing we can see the one-eighth that Hannah got in her multiplication?
Using Talk Moves		
Orienting students to each other's work	Asking students to add on to another student's thinking or to evaluate someone else's work.	How can we use Joey's idea to connect Alfred's and D'Andre's ideas?
Pressing students for explanations	Asking students to explain their thinking (either about a specific aspect of what was said or in general) or asking for evidence.	Can you explain why you broke apart two and one-half?
Rephrasing	Asking a student to explain another student's thinking.	Karina, can you explain in your own words what Carlos just said?
Revoicing	The teacher revoices a student's thinking and determines whether the revoice captured their thinking accurately.	So, what you're saying is that you saw a common factor and divided the numerator and denominator by the common factor?
Waiting	The teacher provides time for students to think. This could involve wait time or providing time to write or talk with a partner.	Giving time for Carlos to think.

Note: [1] The definitions for the three conceptual connection categories came from Gil et al. (2019).

Within our conception of Discussion Moves, we identified three sub-categories representing ways ISTs used discussion moves on the assessments. The first category included a code for *conceptual questions*. To define a conceptual question, we focused on questions the teachers posed in their discussion scripts that encouraged the hypothetical students to think conceptually. We contrasted this with questions that encouraged them to think about procedures. Moreover, questions that asked how or why something is the way it is or asked students to analyse, create, or evaluate were often coded as conceptual. We examined each question asked by an IST and coded each as conceptual or not.

The second category included three codes used to measure ISTs' focus in the discussion script on *building connections*. To define building connections, we focused only on conceptual connections as specified by Gil et al. (2019). They identified three types of conceptual connections. The first type involves a connection where the teacher plays a central role in making explicit the big idea of the lesson. The second type of conceptual connection is related to knowing why a procedure worked. The third is when the teacher identifies or explicitly encourages students to identify multiple representations of the same idea found within or between solution methods.

Research Question 2

A qualitative, thematic analysis was conducted to investigate research question two to look for themes related to how ISTs learned during the co-creation (Braun & Clarke, 2006). The researchers used *Dedoose* (SocioCultural Research Consultants, 2020) to interpret and analyse the data. Initially, three researchers independently read through the interviews, generated emerging codes, and identified excerpts from each transcript representing each code. The team of coders then met to create a list of emerging codes. This list had eleven codes representing the different ways ISTs learned during the co-creation. Once the list of codes was agreed upon, the same three researchers independently coded the interviews and compared the results. All differences were discussed until consensus was reached. We choose to use more than one coder as a way of increasing coder reliability (Church et al., 2019; Kurasaki, 2000). Next, the original codes were further categorised and converted into themes. The themes were reviewed and refined until three themes emerged (Bruan & Clarke, 2006). The final themes were identified as (1) IST collaboration, (2) instructor-IST interactions, and (3) creation versus participation.

Results

Research Question 1: What do ISTs learn about facilitating effective discussions during the co-creation of a practice-based lesson?

To answer the first research question, we used data from the pre-and post-assessments to explore ISTs' learning about leading effective classroom discussions during the co-creation of the lesson. To do this, we looked at descriptive statistics for each sub-category: asking conceptual questions, building mathematical connections, and using talk moves. We found the average number of times each sub-category was used on the pre-assessment and the post-assessment and compared the difference in these two values. The results shown in Table 3 demonstrate an increase in the average number of discussion moves used from the pre- to the post-assessment in each of the three sub-categories among this sample of ISTs.

Table 3

Average Usage of Discussion Moves on the Pre– and Post–assessment

Discussion Move		Average Usage Pre-Assessment	Average Usage Post-Assessment
Conceptual Questions	Mean	2.86	6.43
	SD	(3.02)	(4.08)
Building Connections	Mean	2.57	5.43
	SD	(2.15)	(3.21)
Using Talk Moves	Mean	12.29	20.29
	SD	(10.53)	(14.34)

Notes: Pre-assessment n = 7; Post-assessment n = 7

The number of conceptual questions used increased, on average, from 2.86 on the pre-assessment to 6.43 on the post-assessment. This provides evidence that ISTs' questioning was more focused on conceptual understanding on the post-assessment. When examining the conceptual questions asked by ISTs, it became evident that many of the conceptual questions being asked prompted students to think more deeply about concepts and drew the focus away from primarily completing procedures. Here are two examples of such questions, "Why do you think he cut the sub into nine pieces?" and "Can someone explain why 2/3 and 6/9 are the same amount?" Notice in these examples that the teacher is focused on helping students understand why instead of focusing on how a procedure is completed.

Next, we looked further at ISTs' facilitation of *building connections*. The average number of building connections increased from 2.57 on the pre-assessment to 5.43 on the post-assessment. One reason ISTs made more attempts to build mathematical connections on the post-assessment was an increase in the number of times ISTs focused the discussion on multiple representations of mathematical ideas. As an example, on the post-assessment, one IST wrote, "Do we all see how each of Alfred's thirds has now been cut up into three pieces each? The subs are now cut into ninths." This quotation shows the IST linking the two different representations, first mentioning one-third of the sub as seen in Alfred's picture, and then connecting this with a second picture showing each one-third cut into three parts. After linking the two representations, the IST then explains, "So, each one-third is now three-ninths." In-service teachers also made more attempts to build connections that described the big idea of the lesson on the post-assessment. For example, Kelsey wrote, "That was a great way to tie multiplication to division since they are inverse operations." In another example, Marvin wrote, "So, what you're saying is that you saw a common factor and divided the numerator and denominator by the common factor." These examples show an IST working to draw students to one of the lesson's big ideas.

Lastly, we looked at the statements coded within the category of using *talk moves*. Overall, the average IST used eight more talk moves on the post-assessment than the pre-assessment. The difference was primarily due to increases in two of the five talk moves investigated, orienting students to each other's work and pressing students for explanations. When ISTs used the teacher move, orienting students to each other's work, they generally asked a student or students to (1) explain what another student said ("Mandy, can you explain in your own words what Alfred just said?"), (2) explain differences in students' work ("Can someone tell me why Mandy had extra pieces, but Alfred had too many?"), or (3) evaluate a student's work ("Do you agree with what D'Andre just said?"). One of the crucial goals of such statements is to encourage students to focus on the work of others and to create an environment where students see value in understanding one another's strategies and thinking.

In-service teachers also pressed students for explanations more frequently on the post-assessment. This commonly involved asking follow-up questions that probed for more information or gave students something additional to consider. For example, "What makes you say that?", "Why did you decide to divide each sub into three parts?" and "What are some ways their strategies are different?" As these examples show, ISTs were pressing students in ways that encouraged them to share details about their thinking. By making student thinking visible, both the student who is explaining their thinking and the other students in the class gain valuable insight into the mathematics being investigated.

Research Question 2: How do ISTs learn during the co-creation of a practice-based lesson?

The results described previously provide evidence about what ISTs learned during the co-creation. To investigate the second research question, we analysed the qualitative interview data to examine *how* ISTs learned during the co-creation process. Using thematic analysis, three themes emerged: *IST collaboration*, *instructor-IST interactions*, and *creating versus participating*.

Theme 1: IST collaboration

The first theme that emerged from the analysis dealt with how ISTs learned through IST collaboration. Six out of the seven ISTs indicated that their interactions with peers played a significant role in their learning. As we analysed the interviews, it became clear that the interactions between ISTs, whether within one's group or between groups, were significant in explaining how learning occurred. For example, Yolanda discussed how the feedback from her peers in class impacted her learning,

> I believe [feedback] opens the floor … for people to say, "Okay, let's reflect. What issues am I having? What can I improve on? What am I uncomfortable addressing?" because that was in my feedback. I was uncomfortable trying some mistakes … "Okay, I can own that. Why are you uncomfortable?" … Then once people get understanding that this is what we're going to work on, they know their truth. Their truth is out, everybody's truth is out. Now we can move forward and improve on that.

As relayed in the quotation, Yolanda pointed out that feedback from her peers not only enabled her to reflect and ultimately acknowledge her shortcomings, but also recognize that the group "can move forward and improve." This signifies the important role that peers outside her group played in providing feedback and the role that her group members played as they worked to improve their lesson, thus enabling them to build further knowledge.

Other ISTs also described the critical role peers played in their learning by helping them reflect on the implementation of certain concepts. For example, Roberta explained how the interaction with a group member helped with the redesign of the approximation, "That's when myself and Evelyn came up with the idea that we should make it an interactive skit and let the teachers themselves kind of chime in and say what they feel." The interaction between Evelyn and Roberta was central to the decision to change the approximation. Likewise, Marvin found the discussions with his groupmates helped him reflect on his own use of connections when he said, "I think that made the idea of connections a lot more, um, obvious, I guess. And then, it made me realise I'm not doing it as well as I should." The conversations with his peers were instrumental in prompting Marvin to reflect on his teaching and realise how he could improve.

The interaction among peers created through extensive discussion and feedback led to productive IST-to-IST interactions that promoted learning. The ISTs in this study took an active role in the learning process by co-creating and negotiating knowledge with their peers (Wenger, 1998). In the examples above, ISTs described their current knowledge structure and indicated the unique role conversation with peers had in helping them create new understandings. In addition to the essential role IST-to-IST collaboration played, we also noted the significance of instructor-IST interactions in promoting learning.

Theme 2: Instructor-IST interactions

The second theme that emerged from the data detailing how ISTs learned during the co-creation was instructor-IST interactions. Each of the seven ISTs in our sample pointed out that interactions with the instructor played a significant role in their learning. The instructor primarily supported ISTs' learning by providing resources, such as research articles or videos, and discussing these resources with them. For example, Amanda explains, "So we did more of our reading and emailing back and forth with [the instructor] … So, her insight and the reading selections really is what got us going." As shown in this quotation, the instructor and ISTs carried a conversation back and forth, building understanding from interaction to interaction.

Another IST, Kelsey, explained how the instructor offered ideas, and the group was able to use these ideas. "It wasn't easy to think about a way [to develop the approximation] … But in that sense, in the end, [the instructor], you know, took a look, and she gave us very good ideas, and we took different bits." The instructor played the role of an idea generator, and the ISTs were able to select and choose from these ideas to help them with the lesson creation. As these two examples demonstrate, the instructor supported the work being done, and the language provided by the ISTs indicates that they took from the ideas provided and built on them. Amanda commented this "really is what got us going," while Kelsey explained they "took different bits." This shows how the interactions between the instructor and ISTs played a role in moving forward ISTs' thinking.

In other instances, the language used by ISTs indicated that the instructor determined the path of the work instead of supporting the path ISTs were travelling. For example, Gretchen began by discussing how her group developed the lesson. At first, she described a collaborative process where the group worked together to select the best activity based on their goals, "So, what fits our goal the best, our end goal?" But, after this, Gretchen noted, "And, ultimately, [the instructor] picked the last [example], so you have that." The comment shows that while the team worked together to match the activity with their goals, the instructor directed the group's work by "picking" the last part of the activity. Then, when the interviewer asked Gretchen why the instructor picked the last one, she said, "I never really asked her why she selected it." In this example, the instructor dictated the content of the last part of the activity, and the IST did not understand why that choice was made; thus, she likely did not feel ownership over the decision.

The instructor-IST interactions in the co-creation process took many forms. In some cases, as we detailed with Amanda's and Kelsey's comments, the co-creation between the instructor and a group of ISTs led to deeper thinking and understanding. At other times, the instructor's role in the co-creation took on more of a traditional role where the instructor determined the path of learning by selecting the last activity used in Gretchen's lesson.

Theme 3: Creating versus participating

All seven participants discussed the final theme that emerged from the data analysis. In this theme, ISTs described how they believed the process of actively engaging in the co-creation of the lessons, instead of participating in the lesson, was central to their learning. It was evident that ISTs saw participation in the lesson differently than co-creating the lesson. As Evelyn explained, "I think just sitting in a class and learning about it is not as productive as actually doing it, creating it." For Evelyn, creation was central to her learning. She further explained how participating in lessons does not make her feel as connected to the content. It was not until she created and developed the lesson that she thought she would apply the concepts to her work as an elementary school teacher. Evelyn continued, "I didn't understand it personally, so, how would I be able to teach that [in my classroom] until we actually created a lesson and delivered it?" For Evelyn, the creation process was necessary to gain the understanding needed to implement this type of teaching in her elementary school classroom.

Kelsey described how being responsible for creating the lesson led to increased preparation and a deeper connection to the content. She explained,

> I am creating a lesson to teach so when I go to the lesson plan, I want to know everything. I want to make sure I know … you have a more emotional connection to it when you create opposed to when you are receiving it, once you learned it, mastered it, and got it, you will have that same connection, but the creator of it will ultimately have a deeper connection even understanding.

Kelsey's comment demonstrated her belief that the creator has a deeper connection with the material than the person who participates in the lesson. Yolanda described feelings similar to those expressed by Kelsey. In the quotation below, she explains how the creative process leads to increased understanding.

> Well, when you are creating something, number one, you are sensitive to it, because you created it, but you have an overall understanding like you really, I don't want to say you understand it more than the person that is receiving it but, you have planned it, you structured it, you've studied it, you've researched it, you kind of know it in and out.

From the comments provided in the interviews, it was clear the ISTs in this study saw a benefit to learning by being actively involved in the creative process. One noteworthy aspect of the quotations provided by Kelsey and Yolanda is that both showed uneasiness in saying that the creator knew more than the lesson participants but ultimately did conclude there was something about the creative process that led to deeper learning. For example, Kelsey said about the participants, they "will have that same connection" but followed it by saying the creator will ultimately "have a deeper connection." Likewise, Yolanda said, "I don't want to say you understand it more than the person that is receiving it," and she followed this up by arguing why the creator builds more understanding than the participant. These quotations demonstrate the powerful role creation played in their learning.

Discussion

Building on prior research in PBTE, our study examined the engagement of ISTs in the co-creation of practice-based lessons to determine what and how ISTs learned during the process. By engaging ISTs in the co-creation process, our study contributes new evidence about how PBTE promotes learning.

Using theories about the co-creation of learning and teaching (Bovill, 2020; Cook-Sather et al., 2014), we hypothesised that ISTs would develop their ability to facilitate mathematical discussions by actively creating practice-based lessons. Our analysis of pre-and post-assessment results offered support for this hypothesis, as we found evidence that ISTs' ability to facilitate an effective mathematics discussion improved in three areas: asking conceptual questions, building conceptual mathematical connections, and using talk moves to facilitate class discussions. These results provided evidence that learning about effective mathematical discussions occurred during the co-creation process, which led us to investigate how this learning happened. We interviewed ISTs in our sample and found three themes that helped explain how they learned. The themes were IST collaboration, instructor-IST interactions, and creating versus participating.

An important contribution emerging from this work was the essential role creation played in helping ISTs learn. While there is a large body of pre-existing research showing the value of participating in practice-based lessons, the ISTs in this study indicated that the decision to use co-creation was not only a significant factor in their learning but also led to deeper learning than would have resulted from merely participating in such lessons. For example, one IST commented, "the creator of [the lesson] will ultimately have a deeper connection even understanding." This result is not surprising given the extensive research showing the benefit of using student-centred, active learning experiences such as co-creation (Bovill, 2020; Cook-Sather et al., 2014). Our results provide evidence that engaging ISTs in the co-creation of practice-based lessons has considerable promise as a method for building their capacity to teach effectively.

Implications for Teacher Educators

As we reflect on the results from this study, we would like to highlight some areas that could be of value to mathematics teacher educators. First, we will discuss the *Framework for Teaching Practice*. During the co-creation, which involved design, presentation, feedback, and revision, ISTs identified representations of practice, decomposed the representations into manageable parts, and designed approximations to engage others in the teaching practices.

However, we note that while the ISTs were aware of the Framework suggesting implementing these practices in the following order: representation, decomposition, then approximation, the design process did not exclusively follow this order. For example, one group began the lesson development by creating an approximation of practice activity. Once this was complete, they wondered how they should present students with a representation of this practice and later decided how to decompose it. Another group started with decomposition (i.e., a list of talk moves, definitions, and examples), then developed an approximation, and later identified a way to represent this practice. Interestingly, the design phase did not exclusively follow the order of representing the practice, decomposing it, then developing the approximation.

In-service teachers were flexible during the design phase of the lesson and moved through the stages in the Framework in different ways. We highlight this because, like Janssen et al. (2015), we suggest teacher educators should be open to alternative ways to enact practice-based lessons. In our study, where ISTs took an active role in deciding how to incorporate these three practices into a lesson, we saw that ISTs were flexible. Given their improvement on the post-assessment, we hypothesise that this flexibility helped with their learning. While the Framework for Teaching Practice is undoubtedly useful, we suggest that teacher educators think flexibly about the order in which practice-based lessons are designed and presented in class.

The second implication from this work we would like to discuss stems from our findings about the complexity of the interactions between the instructor and the ISTs during the co-creation process. The co-created classroom is a social context where learning is linked to participation and the negotiation of meaning (Wenger, 1998). While there are many strengths to this, the very nature of co-creation challenges traditional teacher and student roles which can be troublesome for both students and teachers (Cook-Sather, 2014; Land et al., 2005).

For the instructor, engaging in co-creation is accompanied by, at least to some degree, a loss of control over the learning and teaching in the class (Cook-Sather et al., 2014). The loss of control over the content, goals, and activities in the classroom may provide a level of uncertainty for the instructor that can be difficult. On the other hand, for students who are used to sitting passively and taking in information, the new role as a co-creator may cause discomfort as they are asked to take on a greater level of responsibility and action. As a result, an instructor willing to engage in co-creation may still face resistance from students who are not used to this new approach to learning and teaching.

Even with students and instructors invested in and supportive of the co-creation process, there may still be struggles since effective engagement requires different skills than students and instructors typically need in traditional classrooms. The struggle exists partly because the instructor's role is not to give over the creation to students but to engage in a partnership with the students. As Bovill et al. (2014) explain, "It is not desirable to move from staff being completely in control to students being completely in control. Partnership implies a sharing of responsibility, a respect for others' views and a reciprocal relationship" (p. 4). So, managing the level of engagement among faculty and students can be difficult.

In this study, we saw evidence of the instructor's difficulty to engage in a partnership where they are neither in control nor without control. One of our main findings from this study identified the instructor's vital role in how ISTs learned. The instructor served as a source to provide valuable resources and ideas to springboard ISTs' thinking and decision-making. However, we also found evidence that the instructor occasionally took over the co-creation process and made decisions about the lesson plans without having buy-in from ISTs. This finding confirms the difficulty instructors may have in staying within the partnership and not falling back into the traditional role of one who controls the learning and teaching in the classroom.

Instructors need to strike a balance to continue to support and facilitate while actively minimising their control over the learning process. One approach might be to decide as a group how decisions will be made (i.e., through a majority vote) so that the power remains distributed across all who are involved (Bovill et al., 2014). Another approach that might be helpful is for the instructor to make their reasoning visible to students if they do decide to step in and direct the learning path. For instance, as explained in one of the examples presented in the Results, the instructor dictated to ISTs what the last example in their lesson should involve. The instructor should be aware of this choice and realise she placed her ideas above the partnership. This realisation can help the instructor reflect on why she made this choice, which might lead her to make a different choice or explain her reasoning to the students about why she is taking control of this decision. Being transparent is necessary if "we are to avoid students experiencing empty claims of partnership that can lead to disillusionment and a sense that we are manipulating students for our own purposes" (Bovill et al., 2014, p. 4). Consequently, for instructors considering co-creation, it is essential to realise it will likely take time to build the necessary skills needed to maximise this approach to instruction. Taking the time to engage in honest and open reflection about the instructor's role in the partnership and creating classroom procedures that will support a student-instructor partnership are essential steps toward making a productive co-created classroom.

Limitations and Future Research Directions

The results from this study established that the co-creation of practice-based lessons can be an effective way to help ISTs learn about facilitating mathematical discussions and provide insight into how this learning took place. Nevertheless, this study has several limitations. First, the sample size was small. Seven ISTs participated in the study, and all seven were from a single university class, making it impossible to generalise the results to a larger population. Second, the investigation into what ISTs learned during the co-creation was limited to three areas related to facilitating discussions asking conceptual questions, making connections, and using talk moves. This shows evidence that their discussion skills improved, but the question remains whether ISTs developed a deeper understanding of when and why to use these discussion strategies. Future studies should employ a larger sample across classes and universities to examine teachers' performance and conceptual understanding of facilitating discussions.

Another limitation of this study was the presence of three students in the class who were pursuing a doctoral degree and the role they may have played in the learning that took place among ISTs. Due to their status as doctoral students and their position as researchers, their presence may have impacted what and how ISTs learned during the class. That said, the doctoral students in the class were not experts in facilitating mathematical discussions or designing lessons and were learning about these concepts alongside their peers. Because of this, they were no more likely to direct the groups' decision-making than any of their peers. However, future research should consider this and investigate samples consisting exclusively of ISTs.

Lastly, the focus of this study was on ISTs' learning; however, future studies should also examine instructors' learning. The instructor has likely taken on a new role by engaging in co-creation. Understanding the instructor's role and learning more about how instructors and ISTs co-construct knowledge is essential to gain further insight into the effectiveness of using co-creation to develop practice-based lessons in a mathematics education course.

Past research established the potential PBTE has in supporting the development of teachers (Ball & Forzani, 2009; Grossman & McDonald, 2008; Grossman et al., 2009b). Our study contributes to the prior literature in this critical area by demonstrating that co-creating practice-based lessons helped develop ISTs' facilitation of mathematical discussions, and our results detailed how this learning took place. It is important to continue this research to investigate further how co-creation can build future and current teachers' understanding of essential mathematics teaching practices.

References

Ball, D. L., & Forzani, F. M. (2009). The work of teaching and the challenge for teacher education. *Journal of Teacher Education, 6*(5), 497–511. https://doi.org/10.1177/0022487109348479

Bergmark, U., & Westman, S. (2016). Co-creating curriculum in higher education: Promoting democratic values and a multidimensional view on learning. *International Journal for Academic Development, 21*(1), 28–40. https://doi.org/10.1080/1360144X.2015.1120734

Boerst, T., Sleep, L., Ball, D., & Hyman, B. (2011). Preparing teachers to lead mathematics discussions. *Teachers College Record, 113*, 2844–2877.

Bovill, C. (2020). Co-creation in learning and teaching: The case for a whole-class approach in higher education. *Higher Education, 79,* 1023–1037. https://doi.org/10.1007/s10734-019-00453-w

Bovill, C., Cook-Sather, A., Felten, P., Millard, L., & Moore-Cherry, N. (2016). Addressing potential challenges in co-creating learning and teaching: overcoming resistance, navigating institutional norms and ensuring inclusivity in student-staff partnerships. *Higher Education, 71*, 195–208. https://doi.org/10.1007/s10734-015-9896-4

Bovill, C., Felten, P., & Cook-Sather, A. (2014). Engaging students as partners in learning and teaching (2): Practical guidance for academic staff and academic developers. In *Educational Development in a Changing World*. Proceedings of the Annual Conference of International Consortium on Educational Development, Stockholm, Sweden.

Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology*, *3*(2), 77–101. https://doi.org/10.1191/1478088706qp063oa

Breen, M. P., & Littlejohn, A. (2000). The practicalities of negotiation. In M. P. Breen and A. Littlejohn (Eds.), *Classroom decision-making: Negotiation and process syllabuses in practice* (pp. 272–295). Cambridge University Press.

Carley, K. (1990). Content analysis. In R. E. Asher (Ed.), *The Encyclopedia of Language and Linguistics*. Pergamon.

Chapin, S. H., O'Connor, C., & Anderson, N. C. (2013). *Talk moves: A teacher's guide for using classroom discussions in Math, Grades K-6*. Math Solutions.

Church, S. P., Dunn, M., & Prokopy, L. S. (2019). Benefits to qualitative data quality with multiple coders: Two case studies in multi-coder data analysis. *Journal of Rural Social Sciences, 34*(1), 1–14.

Cook-Sather, A. (2014). Student-faculty partnership in explorations of pedagogical practice: A threshold concept in academic development. *International Journal for Academic Development, 19*(3), 186–198. https://doi.org/10.1080/1360144X.2013.805694

Cook-Sather, A., Bovill, C., & Felten, P. (2014). *Engaging students as partners in learning and teaching: A guide for faculty*. Jossey-Bass.

DiCicco-Bloom, B., & Crabtree, B. F. (2006). The qualitative research interview. *Medical Education, 40*(4), 314–321. https://doi.org/10.1111/j.1365-2929.2006.02418.x

Franke, M. L., Webb, N. M., Webb, A. G., Ing, M., Freund, D., & Battey, D. (2009). Teacher questioning to elicit students' mathematical thinking in elementary school classrooms. *Journal of Teacher Education*, *60*(40), 380–392. https://doi.org/10.1177/0022487109339906

Ghousseini, H. (2009). Designing opportunities to learn to lead classroom mathematics discussions in pre-service teacher education: Focusing on enactment. In D. S. Mewborn & H. S. Lee (Eds.), *Scholarly practices and inquiry in the preparation of mathematics teachers* (pp. 203–218). The AMTE Monograph Series.

Ghousseini, H., & Herbst, P. (2016). Pedagogies of practice and opportunities to learn about classroom mathematics discussions. *Journal of Mathematics Teacher Education*, *19*(1), 79–103. https://doi.org/10.1007/s10857-014-9296-1

Gil, I., Zamudio, L., & King, B. (2019). After presenting multiple strategies, what's next? Examining the mathematical connections made by pre-service teachers. *Journal of Mathematics Education at Teachers College, 10*(2), 9–20.

Grossman, P., Compton, C., Igra, D., Ronfeldt, M., Shanan, E., & Williamson, P. W. (2009a). Teaching practice: A cross-professional perspective. *Teachers College Record, 111*(9), 2055–2100.

Grossman, P., Hammerness, K., & McDonald, M. (2009b). Redefining teaching, re-imagining teacher education. *Teachers and Teaching: Theory and Practice, 15*(2), 273–289. https://doi.org/10.1080/13540600902875340

Grossman, P., & McDonald, M. (2008). Back to the future: Directions for research in teaching and teacher education. *American Educational Research Journal, 45*(1), 184–205. https://doi.org/10.3102/0002831207312906

Heron, J. (1999). *The complete facilitator's handbook*. Kogan Page.

Janssen, F. J. J. M., Grossman, P., & Westbroek, H. B. (2015). Facilitating decomposition and recomposition in practice-based teacher education: The power of modularity. *Teaching and Teacher Education, 51*, 137–146. https://doi.org/10.1016/j.tate.2015.06.009

Kavanagh, S. S., Metz, M., Hauser, M., Fogo, Brad., Taylor, M. W., & Carlson, J. (2020). Practicing responsiveness: Using approximations of teaching to develop teachers' responsiveness to students' ideas. *Journal of Teacher Education, 7*(10), 94–107. https://doi.org/10.1177/0022487119841884

Kazemi, E., Franke, M., & Lampert, M. (2009). Developing pedagogies in teacher education to support novice teachers' ability to enact ambitious instruction. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing Divides.* Proceedings of the 32nd Annual Conference of the Mathematics Education Research Group of Australasia, Palmerston North, NZ (Vol. 1, pp. 12–30).

Kazemi, E., Lampert, M., & Ghousseini, H. (2007). *Conceptualizing and using routines of practice in mathematics teaching to advance professional education: Report to the Spencer Foundation.* Spencer Foundation.

Kennedy, M. (2016). Parsing the practice of teaching. *Journal of Teacher Education*, *67*(1), 6–17. https://doi.org/10.1177/0022487115614617

Korthagen, F. A. J., & Wubbels, T. (2001). Learning from practice. In F. A. J. Korthagen (Ed.) *Linking practice and theory: The pedagogy of realistic teacher education* (pp. 32–50). Lawrence Erlbaum.

Krussel. L., Edwards, B., & Springer, G. (2004). The teacher's discourse moves: A framework for analysing discourse in mathematics classrooms. *School Science and Mathematics, 104*(7), 307–312. http://dx.doi.org/10.1111/j.1949-8594.2004.tb18249.x

Kurasaki, K. S. (2000). Intercoder reliability for validating conclusions drawn from open-ended interview data. *Field Methods*, *12*(3), 179–194. https://doi.org/10.1177/1525822X0001200301

Lampert, M. (2005, April 13). *Preparing teachers for ambitious instructional practice: Learning to listen and to construct an appropriate response.* Paper presented at *Demography and Democracy in the Era of Accountability*, the Annual Meeting of the American Educational Research Association, Montreal, Canada.

Land, R., Cousin, G., Meyer, J. H. F., & Davies, P. (2005). Threshold concepts and troublesome knowledge: Implications for course design and evaluation. In C. Rust (Ed.), *Improving student learning*: *Diversity and inclusivity.* The Oxford Centre for Staff Development.

Mann, S. J. (2008). *Study, power and the university.* Open University Press.

Martinez, M. E. (2006). What is metacognition? *Phi Delta Kappan, 87*(9), 696–699. https://doi.org/10.1177/003172170608700916

McDonald, M., Kazemi, E., & Kavanagh, S. S. (2013). Core practices and pedagogies of teacher education: A call for a common language and collective activity. *Journal of Teacher Education, 64*(5), 378–386. https://doi.org/10.1177/0022487113493807

Meyer, J. H. F., & Land, R. (2006). Threshold concepts and troublesome knowledge: An introduction. In J. H. F. Meyer & R. Land (Eds.), *Overcoming barriers to student learning: Threshold concepts and troublesome knowledge* (pp. 3–18). Routledge Falmer.

Michaels, S., & O'Connor, C. (2015). Conceptualizing talk moves as tools: Professional development approaches for academically productive discussion. In L. B. Resnick, C. A. Asterhan, & S. N. Clarke (Eds.), *Socializing intelligence through talk and dialogue* (pp. 347–362). American Educational Research Association.

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common Core State Standards for Mathematics*. Authors.

Philip, T., Souto-Manning, M., Anderson, L., Horn, I., Andrews, D. J. C., Stillman, J., & Varghese, M. (2019). Making justice peripheral by constructing practice as "core": How the increasing prominence of core practices challenges teacher education. *Journal of Teacher Education, 70*(3), 251–264. https://doi.org/10.1177/0022487118798324

Savin-Baden, M., & Howell-Major, C. (2013). *Qualitative research: The essential guide to theory and practice.* Routledge.

Schutz, K. M., Danielson, K. A., & Cohen, J. (2019). Approximations in English language arts: Scaffolding a shared teaching practice. *Teaching and Teacher Education, 81*, (10 –111). https://doi.org/10.1016/j.tate.2019.01.004

Schutz, K. M., Grossman, P., & Shaughnessy, M. (2018). Approximations of practice in teacher education. In P. Grossman (Ed.), *Teaching core practices in teacher education* (pp. 57–83). Harvard Education Press.

Shaughnessy, M., & Boerst, T. (2017). Uncovering the skills that pre-service teachers bring to teacher education: The practice of eliciting a student's thinking. *Journal of Teacher Education, 69*(1), 40–55. https://doi.org/10.1177/0022487117702574

Sleep, L. (2012). The work of steering instruction toward the mathematical point: A decomposition of teaching practice. *American Educational Research Journal*, *49*(5), 935–970. https://doi.org/10.3102/0002831212448095

Smith, M. S., & Stein, M. K. (2011). *Five practices for orchestrating productive mathematics discussions*. National Council of Teachers of Mathematics.

SocioCultural Research Consultants, LLC. (2020). *Dedoose* (Version 7.0.23). www.dedoose.com

Tyminski, A. M., Zambak, V. S., Drake, C., & Land, T. J. (2014). Using representations, decomposition, and approximations of practices to support prospective elementary mathematics teachers' practice of organizing discussion. *Journal of Mathematics Teacher Education, 17*(5), 463–487. https://doi.org/10.1007/s10857-013-9261-4

Wenger, E. (1998). *Communities of practice: Learning, meaning, and identity.* Cambridge University Press. https://doi.org/10.1017/CBO9780511803932

Zeichner, K. (2012). The turn once again toward practice-based teacher education. *Journal of Teacher Education*, *63*(5), 376–382. https://doi.org/10.1177/0022487112445789

Barbara King
Florida International University
11200 SW 8th St
Miami, FL 33199
Bking@fiu.edu

Carmen Petrick Smith
University of Vermont
85 South Prospect St.
Burlington, VT 05405
Carmen.Smith@uvm.edu

Jeehyun Park
Florida International University
11200 SW 8th St
Miami, FL 33199
Jepark@fiu.edu

Shemail Fatima
Florida International University
11200 SW 8th St
Miami, FL 33199
Sfatima@fiu.edu

Elizabeth Forde
State University of New York at New Paltz
1 Hawk Drive
New Paltz, NY 12561
Fordee@newpaltz.edu