Differentiating Instruction for Students Who Fail to Thrive in Mathematics: The Impact of a Constructivist-Based Intervention Approach

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In this paper we explore how participation in a constructivist-oriented, classroom aligned mathematics intervention program advanced the learning and positive dispositions of Grade 1 students who were failing to thrive when learning mathematics. Intervention programs are an approach to differentiated instruction that some schools adopt, but there are questions about whether such interventions advance equity and inclusion for students. To provide insight about these issues, we draw on data from the Extending Mathematical Understanding for All study in NSW, Australia (2016-2020) to address two research aims. The first aim was to evaluate the effectiveness of the EMU intervention approach for progressing the mathematics learning of students who were failing to thrive mathematically, and for closing the performance gap with their peers. The second aim was to consider the impact of participating in EMU intervention on students' dispositions for learning mathematics. The findings confirm the potential of intensive constructivist-based intervention approaches for closing the performance gap for students who were not thriving in mathematics, and for improving the learning dispositions of these students. However, two years after the intervention, the performance gap was again apparent for many students. It is likely that a one-off intervention program is not sufficient for enabling all students to thrive mathematically, and that well-focused classroom-based differentiated instruction and further intervention support may be warranted for some students.

Keywords · inclusion in mathematics · pedagogy · differentiated instruction · mathematics difficulties · intervention programs · equity

Introduction

A strength of every classroom group is the diversity of learners who reflect the richness of the local community. This diversity provides a rich environment for facilitating mathematics learning and creativity, and for preparing all students to participate fully in a future global community that will be characterised by diversity and rapid change. Mathematics learners today need to prepare for this future by becoming creative, confident, collaborative and ethical problem solvers who can communicate their ideas using clear mathematical arguments and models, and who can persist when faced with new or complex problems. A challenge for mathematics teachers is to create classroom environments that represent and model this future while ensuring that all students flourish.

Amidst this endeavour for teachers, student diversity is also characterised by the wide range of students' mathematics knowledge, dispositions for learning mathematics, and engagement. A challenge for teachers can be how to successfully design instruction for those learners who currently, or persistently, do not to thrive when learning mathematics (Gervasoni & Peter-Koop, 2020). Differentiated instruction has been defined as a teaching approach that embraces student diversity, whilst acknowledging students' current strengths and limitations (Suprayogi et al., 2017). Tomlinson (2001) described differentiated instruction as: student-centred; proactive, in that teachers plan instruction to accommodate diverse learning needs; qualitative, in that the instruction reflects *quality* teaching rather than *quantity*; based on appropriate assessments; has multiple approaches to content, process and product; a blend of whole-class, group and individual instruction; and organic, in that the learner and the learning is monitored and adjustments are made as necessary. The core principle of differentiated instruction is providing the type of teaching and environment that enables every student to flourish.

The premise of this paper is to contribute new insight and deepen understanding about the potential of a constructivist-based intervention program for differentiating instruction, and contributing to ambitious and equitable mathematics education for all (Cobb et al., 2018). For this purpose, we draw on data collected during the *Extending Mathematical Understanding for All* study (2016-2020). Our focus on equity addresses the issue of how school communities can alleviate the disadvantages experienced by students who *fail to thrive* in mathematics due to their exclusion from "the opportunity to participate and achieve in mathematics, within our current practices and systems" (Atweh, 2011, p. 65). In so doing, we advance a social justice agenda, and pay tribute and attention to teachers and their work towards developing practices that include all learners in genuine opportunities to learn mathematics with deep conceptual understanding, and develop positive dispositions for learning mathematics. We begin by examining key literature that focuses attention on practices that support teachers to differentiate instruction and enable genuine inclusion.

Advancing Social Justice for All Through Mathematics Education

The fact that there are students who currently struggle with mathematics learning highlights the relevance of differentiated instruction for providing inclusive and equitable mathematics education for all. Before we discuss differentiated instruction in some detail, we first consider the phenomenon of students who *fail to thrive* when learning mathematics.

There are contested views for explaining the phenomena of students who initially do not thrive when learning school mathematics. Gervasoni and Lindenskov (2011) argued, from a social justice perspective, that this phenomenon occurs when students are excluded from, or do not have access to, high-quality mathematics programs and differentiated instruction that enables them to thrive. They argued that, historically, two particular groups of students have been excluded: students who are visually or hearing impaired, or who have physical or intellectual impairments such as Down syndrome; and those who underperform in mathematics. Many students in this first group have been directly excluded from opportunities and educational pathways in learning mathematics because mathematics was deemed an inappropriate field of study for them (e.g., Faragher et al., 2008; Faragher & Clarke, 2020; Feigenbaum, 2000). The second group of students may attend a school where mathematics is taught, but they do not receive the quality of instruction or experience that enables them to thrive mathematically (Gervasoni & Sullivan, 2007; Lindenskov & Weng, 2008). These students are indirectly excluded from mathematics education. Thus, underperformance in mathematics, or failure to thrive, can be explained through consideration of issues associated with exclusion.

Another contributing factor for explaining some students' failure to thrive in mathematics is the different theoretical orientations amongst the special education community and the mathematics education community regarding advice about appropriate instruction and curriculum for students who currently struggle with mathematics learning. For example, the mathematics education community has long argued that mathematics education be grounded in a constructivist approach (Cobb et al., 1992; Hiebert et al., 1997) that is characterised by active inquiry and problem-solving. Pedagogies that support all learners and align with constructivist, reform-oriented approaches to mathematics teaching have been identified by Jitendra (2013). These are:

1. maintaining the cognitive demand of the lesson throughout;
2. promoting the development of conceptual understanding;
3. providing opportunities for students to make conjectures about mathematical ideas;
4. attending to student thinking and mathematical reasoning by having students explain their responses or particular strategies or representations; and
5. using students' statements about mathematics to build class discussion (p. 6).

The special education community has been reluctant to shift towards a constructivist view of teaching and learning mathematics (Woodward & Montague, 2002), favouring practices characterised by direct instruction and repetitiveness.

A review of eleven studies that investigated instruction for students with learning difficulties found that constructivist aligned approaches, focusing on developing conceptual understanding, mathematical reasoning, problem solving, and visual representations, were seldom identified across the published research (McKenna et al., 2015). In a later review of 2477 research articles on the teaching and learning of mathematics, using a Disability Studies in Education framework, Lambert and Tan (2020) found that research on students with disabilities tended to draw on behavioural and medical theoretical orientations. In contrast, the authors found that research on students without disabilities tended to use constructivist and sociocultural theoretical foundations. Lambert and Tan (2020) argued that these differences in research orientations contributed to the segregation of students with disabilities, and those who are low-achieving, in ways that may lead to these students experiencing low quality mathematics instruction, low-level curriculum, and low expectations of their mathematical competence. Such potential outcomes place limits on students' opportunities to learn (Lambert & Tan, 2020; Scherer et al., 2016). New research is needed that focuses on the impact of reform-oriented teaching approaches for students who currently struggle when learning mathematics, and the contribution of these approaches for advancing inclusion.

Advancing a social justice agenda through mathematics education calls for policy and programs that promote equity through embracing student diversity, and ensuring that all students access quality mathematics education. Policy and programs that promote inclusion and equity provide students with the necessary supports in their local school that enable them to thrive, and ensure that teachers "recognise all students' potential and ensure that all students access opportunities to achieve this potential" (Gervasoni & Lindenskov, 2011, p. 310). We acknowledge that providing inclusive and equitable mathematics education is a moral imperative that reaches across international boundaries, but is an ongoing struggle for many teachers (Faragher & Clarke 2020; Gervasoni & Peter-Koop, 2020; Hunter et al., 2020; Lambert & Tan, 2020; Lindenskov & Lindhardt, 2020; Robertson & Graven, 2020; Rottman et al., 2020; Schenpel et al., 2020). Advancing this agenda begins with believing that all students can learn mathematics, given the necessary opportunities and supports.

Differentiated Instruction for Addressing Failure to Thrive in Mathematics

Differentiated instruction has been identified as an approach to cater for the wide range of student abilities, interests and learning profiles in the one classroom (Tomlinson et al., 2003), and to ensure that all student's learning opportunities are maximised. Differentiated instruction has potential for addressing the phenomenon of students who fail to thrive in mathematics. However, Cobb et al. (2018) argued that fewer than 20 percent of mathematics teachers effectively differentiated teaching for students. The literature related to the effectiveness of differentiation practices in education is ambiguous with respect to whether differentiation advances the agenda of achieving social justice for all students (Anthony et al., 2019). This has particular relevance for students who currently do not thrive in mathematics. For example, Bannister (2016) highlighted that there are four ways in which differentiation can further marginalise students who already experience educational disadvantages. First, inequity can occur in mathematics when differentiated instruction leads to practices that offer some students more direct instruction and routine practice as opposed to more meaningful, inquiry-based learning opportunities. In mathematics, this is more often the experience for students who struggle with mathematics learning (Anthony et al., 2019). Second, Bannister (2016) noted that some approaches to differentiated instruction can perpetuate the myth of learning styles, leading to students inappropriately being offered different types of learning opportunities based on unsupported assumptions about their ability to learn. Thus, it is essential that advice about differentiated instruction in mathematics does not perpetuate this. Third, Bannister (2016) noted that differentiated instruction can lead to inequity through with-in class ability grouping. Indeed, there is widespread recognition of the detrimental impact on students assigned to "low" level ability groups (Boaler, 2016; Boaler et al., 2000; Clarke & Clarke, 2008; Clarke, 2021). A major reason for this negative impact is that the instruction typically includes low level tasks, a lack of challenge, and "simple, dull material" (Pogrow, 1988, p. 84). This leaves these students with little opportunity to learn important mathematics, develop higher-level thinking and positive dispositions for learning mathematics, like their peers.

The fourth critique offered by Bannister (2016) was that differentiated instruction can lead to some students being viewed as more or less competent as learners of mathematics. The possibility of deficit framing or stereotyping of a student's capability has been shown to negatively impact students' opportunities for education (e.g., Faragher et al., 2008; Faragher & Clarke, 2020; Feigenbaum, 2000; Hunter et al., 2020). Furthermore, Cobb et al. (2018) reported that teachers' views about a student's capability can be difficult to shift, and the extent to which teachers developed ambitious and equitable teaching practices was shaped, in part, by their views of their students' capabilities.

Clearly, differentiated instruction in mathematics has to be carefully framed so that all students are positioned as competent and capable of learning, and are provided equitable opportunities to learn rich and interesting mathematics. This can be achieved through all students being offered tasks that are meaningful and inquiry-based and that align with constructivist reform-oriented approaches to mathematics education. Furthermore, Bannister (2016) and Anthony et al. (2019) highlighted the need for differentiated instruction in mathematics to be framed in terms of a social justice agenda so that student differences are embraced, and no limitations are placed on their capability for learning mathematics. This is the position that we take when considering effective approaches for differentiating instruction to support the learning of students who currently are not thriving in mathematics.

Mathematics Intervention Approaches

Many school communities offer mathematics intervention programs to provide supports that advance equity for students who do not thrive with learning school mathematics. Such approaches aim to accelerate or boost students' learning and achievement (e.g., Bryant et al., 2011; Gervasoni et al., 2019; Kalogeropoulos et al., 2020; Strand Cary et al., 2017; Thornton et al., 2010; Wright, 2003). The specific features of intervention approaches vary widely with respect to: whether students participate one-on-one or in small groups; how students are selected for participation; the age of the students; the duration of the intervention; whether assessment is aligned with a learning trajectory; and the mathematics focus of the intervention. Some interventions have a multi-level approach, with each level of intervention increasing the intensity of support. For example, *Response to Intervention* (RTI) (Berkeley et al., 2009; Gersten et al., 2009) and *Extending Mathematical Understanding* (EMU) (Gervasoni, 2004; 2015; Gervasoni et al., 2019) are three-tiered models. The first tier aims to address the needs of all students in their regular classroom through providing high quality mathematics teaching. Second tier instruction in small groups is provided for students who have not made meaningful progress in the regular classroom. This support aims to boost students' knowledge and skills so that they can be successful in the classroom once the more intense support is removed. Tier 3 support includes highly intensive instruction in small groups (EMU) or one-on-one (RTI). Other features of EMU and RTI are initial screening of all students, progress monitoring, and evidence-based instruction (Regan et al., 2015).

In an attempt to provide advice about the features of promising intervention approaches, the National Joint Committee on Learning Disabilities (2016) outlined five critical areas of attention to support currently struggling students to achieve high-quality education standards. These were:

1. high-quality, collaborative, professional development;
2. appropriate curriculum and instructional design;
3. appropriate assessments that reveal students' strengths, needs, and achievement levels;
4. a comprehensive understanding of the whole child; and
5. comprehensive and effective transition planning. (Gartland & Strosnider, 2017, p. 154)

This advice highlighted the need for teachers to have a vision of high-quality teaching, but it did not elaborate what this might be in the context of mathematics teaching.

In Australia, mathematics intervention supports for six-year-old students have included *Maths Recovery* (Wright, 2003) and *Extending Mathematical Understanding* (EMU) (Gervasoni, 2004; 2015). *Mathematics Recovery* (MR) is a one-on-one intervention and includes daily sessions for 12-15 weeks. A major element of MR is an interview-based assessment and an associated early number framework that is used to profile students' early number knowledge. Also included in the program is a bank of instructional activities from which the teacher chooses, according to the needs of the student. Teachers who administer the program undertake significant professional development about MR and the theory and practice behind the MR assessment and instructional activities.

EMU is a small group intervention for students who are failing to thrive with mathematics learning. Typically, students' performance is in the bottom quartile of the four *Mathematics Assessment Interview* (MAI) (Gervasoni et al., 2011) whole number growth-point scales for their cohort. Groups of three students are offered focused differentiated teaching by a specialist teacher for half of their mathematics lesson each day (30 minutes) for at least 10 weeks. There is also an EMU Middle Years form of the intervention for Grade 3 to Grade 8 students. EMU Specialist teachers initially undertake six days of professional learning, and complete at least 25 hours of field-based learning, and a programme of professional reading, in order to be accredited to teach the EMU Program. Guided by the EMU diagnostic assessment and the MAI growth point framework, EMU lessons and instruction are differentiated for the three students. Lessons focus on whole number learning, mathematical problem-solving, engagement with open tasks, and reflection on the mathematical focus of the lesson and each student's learning. There is an activity bank to guide teacher's task selection and instruction. The inherent challenge for students when engaging with the open tasks, and the subsequent discussion of solutions and strategies, builds alignment with classroom learning and teaching approaches. To engage families in the student's learning, a daily home task is provided at the conclusion of each EMU lesson.

Another intervention approach for older primary and secondary students is *Getting Ready in Numeracy* (GRIN) (Kalogeropoulos et al., 2020). In this case a specialist GRIN tutor or classroom teachers conduct GRIN sessions with one to three students for 15-25 minutes, typically three times a week for about 6 months. These sessions are timetabled to occur just prior to the students' classroom mathematics lesson with the aim of preparing the students for the key ideas, language, materials, and types of activities that they will experience in the classroom. Students are selected from those between the 20th and 40th percentiles in mathematics assessments for their cohort.

The positive contribution of intervention programs for providing equitable supports for students who struggle with mathematics is disputed. As previously described, there are concerns about whether the segregation of students during an intervention marginalises students and negatively impacts their confidence and engagement (Bannister, 2016). Other concerns include students being offered more direct instruction and routine practice as opposed to more meaningful, inquiry-based learning opportunities, or students being viewed as less capable as learners of mathematics. Another uncertainty is whether any positive effects of intervention programs are sustained for students. Several researchers have concluded that the academic gains from a mathematics intervention typically fade out over time (Clements et al., 2013; Protzko, 2015; Smith et al., 2013). The mechanisms underlying fadeout of the intervention effect on students' achievement are not well understood (Bailey et al., 2016). A commonly speculated reason for this fadeout is that the classroom environments that students experience alongside and post-intervention, do not sustain the effect. Further, Smith et al. (2013) concluded that intervention programs need to be well-aligned with the classroom approach to best ensure impact for students.

Bailey et al. (2016) offer two possible explanations for the fade out of a positive intervention effect. First is the *constraining content* hypothesis, positing that school environmental factors do not build on the skills students gained during the intervention and that classroom teachers offer material that students have already mastered. Therefore, students' lack of opportunity to engage with advanced content "impose a ceiling on higher achieving children's subsequent achievement trajectories" (p. 1458). The second explanation proposed by the authors is the *pre-existing differences* hypothesis, which suggests that relatively stable differences between students cause them to revert back to their previous individual achievement trajectories, after an effective intervention. Bailey et al. (2016) conducted a study to test the two hypotheses. They compared students from their control group with similar post-intervention achievement scores to students in the treatment group, testing whether their subsequent learning trajectories differed one year after the intervention. The authors found that pre-existing differences in students and their home environments explain more of the fadeout effect following an intervention than does school environmental factors such as low-quality teaching and curricula.

Some argue that believing that the effect of an intervention would be sustained, without further attention, is tantamount to believing in "magic" (Brooks-Gunn, 2003). However, Clements et al. (2013) showed that when on-going professional support, based on differentiated instruction and learning trajectories, was provided for the teachers who continued to support students after their participation in a mathematics intervention, learning gains were more likely to be maintained.

The Importance of Positive Dispositions for Learning Mathematics

A positive relationship has been identified between students' engagement and their academic performance and achievement (Fredricks et al., 2004; Marks, 2000; Renninger, 2000). Engagement is necessary for mathematics learning, develops over time, and can be modified by school practices to improve outcomes for students (Finn & Zimmer, 2012). Otherwise, students who are disengaged can be at risk of lower academic achievement (Hancock & Zubrick, 2015).

The construct of *interest* is also aligned with engagement. Interest implies a positive orientation toward an activity that has value for the person (Ainley, 2012). Interest is a critical motivational variable for determining: the amount of effort or persistence that a student might apply on challenging mathematical tasks; the utility or perceived value of a task in their present or in their future; and a student's self-efficacy, or their belief that they will be successful with a mathematics task (Middleton, 2013).

Despite the importance of student's interest and engagement in mathematics, studies of Australian students have identified that many do not enjoy learning mathematics (e.g., Attard, 2012; Howard & Perry, 2005; Markovits & Forgasz, 2017; Way et al., 2015), and dislike learning mathematics more as they progress through schooling (Russo et al., 2021; Thomson et al., 2017). These findings highlight that it is important for teachers to pay particular attention to facilitating student engagement when differentiating instruction for students who are currently not thriving in mathematics.

Background to the Study

The *Extending Mathematical Understanding for All* study (Gervasoni & Roche, 2018) is the context for our present research. It was conducted in NSW Australia from 2016-2020. The study aimed to improve mathematics learning for primary school students through advancing a whole-of-school approach (Fullan et al., 2006) that included developing the capacity of school leaders and mathematics teachers. The study was informed by findings from the *Early Numeracy Research Project* (Clarke et al., 2002), and the *Bridging the Numeracy Gap in Low SES and Aboriginal Communities* study (Gervasoni et al., 2011). Both studies incorporated a teaching model that included a diagnostic interview that was connected to a learning framework, and promoted problem-solving strategies.

As part of the *Extending Mathematical Understanding for All* study, three practices were implemented to advance differentiated instruction in mathematics. These practices included intervention support provided by a specialist teacher, which is a particular focus of this paper. It was anticipated that each practice would add equity-based supports for students who were not yet thriving in mathematics. The three practices were:

1. Using a diagnostic interview and associated learning framework (learning trajectory) to guide differentiated teaching to respond to student diversity. In this case, classroom teachers assessed all students using the Mathematics Assessment Interview (Gervasoni et al., 2011) and the associated growth point framework to find out about their students' current growth points in four whole number domains: Counting, Place Value, Addition and Subtraction Strategies, and Multiplication and Division Strategies. Teachers used these data to design appropriate curriculum and differentiated teaching and supports for students that were aligned with constructivist approaches.

2. Using a substantial core task in mathematics lessons, with enabling and extending prompts, to promote inquiry and problem-solving, challenge, persistence, creativity, and collaboration among students, and to respond to the diversity of learners in a class (Sullivan et al., 2005; Sullivan et al., 2009). Typically, teachers used open tasks for this purpose. The productive struggle and the challenge presented by difficult tasks are noted as important aspects of students' learning mathematics with understanding (Hiebert & Grouws, 2007). Using open tasks also aligns with constructivist approaches, and effective pedagogies such as those highlighted by Jitendra (2013). This practice supports what Scherer et al. (2016) termed natural differentiation, which they argued occurs when a rich core task is used for all students to investigate during mathematics lessons, thereby assisting the teacher to organise the learning processes more effectively. Additionally, Schukajlow and Krug (2014) found that supporting students to develop multiple solutions for mathematics tasks enhances their interest in mathematics; and
3. Employing a specialist mathematics teacher who provided a range of supports for teachers and students. These supports included acting as critical friend for teachers to support inclusive mathematics lesson planning, providing in-classroom support for students and teachers, and providing intensive differentiated teaching for students who were failing to thrive in mathematics. This included providing an EMU intervention program (Gervasoni, 2015) for Grade 1 students who were failing to thrive.

To illustrate the value of a learning trajectory for guiding differentiated instruction, Figure 1 shows the growth point framework for the Multiplication and Division Strategies domain. The growth points describe a pathway towards learning the multiplication concept. For example, research into young children's solution strategies for multiplicative problems indicate that they typically first solve problems by combining direct modelling with counting and grouping skills, and intuitive strategies based on addition (e.g., Anghileri, 1989; Clark & Kamii, 1996; Mulligan & Mitchelmore, 1997). This development is apparent in Growth Point 1 (GP1) and GP2 (see Figure 1). As their learning progresses, children move from relying on direct modelling (GP2) to partial modelling (GP3) and then to multiplicative thinking, at which point (GP4) they are operating on problems abstractly (Downton, 2008b).

Sullivan et al. (2001) suggested that abstracting, characterised by students moving beyond the need to create physical models, to forming mental images of a collection of objects as a composite unit, is a key stage in the learning of multiplicative concepts. This requires a shift in thinking from operating with single items to operating with composite units (Steffe, 1994), and a level of abstraction not required in additive thinking. Many students remain dependent on modelling or partial modelling in multiplicative situations into the later primary years (Clarke et al., 2002; De Corte & Verschaffel, 1996). Sullivan et al. (2001) argued that this is due to teachers' reluctance to engage students in problems that gradually remove physical prompts and encourage students to form mental images of multiplicative situations. This finding will be considered in the current study. Overall, the growth point framework assists teachers to identify a student's current stage in the learning of the multiplicative concept, and guides teachers to determine the type of instruction that best advances each student's learning.

The *Extending Mathematical Understanding (EMU) for All* study provides the context for our research that investigates how these three whole school practices support teachers to differentiate instruction for students who are currently failing to thrive in mathematics. Our paper has two specific aims. The first aim is to evaluate the effectiveness of the EMU intervention approach for progressing the mathematics learning of students who were failing to thrive mathematically, and for closing the performance gap with non-EMU students. To address this aim, we consider longitudinal growth point data derived from the Multiplication and Division domain of the MAI for the period 2016 to 2019. The second aim is to consider the impact that participating in EMU intervention has on students' dispositions for learning mathematics. To address this second aim, we analyse pre and post EMU program affective assessment data for Grade 1 EMU participants in 2020.

A. Strategies for Multiplication and Division

0. Not apparent
   Not yet able to find the answer in a situation involving multiple groups.
1. Counting group items by ones (all objects perceived)
   Counting one by one to find the solution in situations involving multiple groups when all objects are modelled or perceived.
2. Modelling multiplication and division (all objects perceived)
   Uses the multiplicative structure of the situation to find the answer when all objects are modelled or perceived.
3. Partial modelling multiplication and division (some objects perceived)
   Uses the multiplicative structure of the situation to find the answer when objects are partially modelled or perceived.
4. Abstracting multiplication and division (no objects perceived)
   Mentally solves multiplication and division problems (no objects perceived) using the multiplicative structure of the situation.
5. Basic, derived and intuitive strategies for multiplication
   Mentally solves a range of multiplication problems using strategies that reflect attention to the multiplicative structure such as commutativity and building up from known facts.
6. Basic, derived and intuitive strategies for division
   Mentally solves a range of division problems attending to the multiplicative structure using strategies such as fact families and building up from known facts.
7. Extending and applying multiplication and division
   Solves a range of multiplication and division problems (including multi-digit numbers) in practical contexts using multiplicative thinking.

Figure 1: Growth points for Multiplication and Division Strategies.

Method

The current study aimed to advance a social justice agenda through considering practices that enable teachers to differentiate mathematics instruction successfully for those who currently, or persistently, do not thrive when learning mathematics. In this section we outline the theoretical perspectives that underpin the study, the data collection methods, and approaches for analysing the data.

Two perspectives underpin our framework for considering differentiated instruction for the heterogeneous group of students who currently do not thrive when learning mathematics. First that mathematics teaching be based on a social constructivist view of learning (Cobb et al., 1992), and secondly the principle that all students can learn mathematics given access to the necessary resources, environment, and teaching (Cobb et al., 2018).

Our model for differentiating instruction includes:

- the use of diagnostic interview data and an associated learning trajectory to guide planning and instruction;
- the use of core tasks with enabling and extending prompts;
- advice for classroom teachers provided by a specialist teacher with deep pedagogical content knowledge; and
- an intervention program, facilitated by a specialist teacher, for those students who require additional support.

These perspectives and model form a framework for collecting data, examining the results, and discussing the findings.

Research Procedure, Data Collection and Analysis

In this research we draw on multiple data sources collected during the *Extending Mathematical Understanding for All* study. The research progressed according to approved ethical guidelines. Each school implemented the three whole school practices that were outlined earlier for differentiating instruction, with the aim of improving mathematics learning for all. The EMU intervention program was offered for Grade 1 students who were not thriving in mathematics, and who were ranked as the highest priority for receiving additional support, according to the EMU intervention guidelines described earlier (see also Gervasoni, 2015). The EMU program was taught by a specialist teacher who aimed to avoid any possible negative consequences of segregated interventions. For example, the teachers positioned students as highly capable mathematics learners, and aligned their teaching with the classroom approaches (Smith et al., 2013).

Data sources were selected for this study to provide insight about the effectiveness of the differentiated instruction and intervention supports provided for students who were not thriving in mathematics. These data sources included student affective surveys, and student growth point data derived from student clinical assessments conducted by classroom teachers. The data sources, participants and data analysis methods are summarised in Table 1.

Table 1: Summary of data sources, participants and data analysis methods.

Data	Year	Participants	Analysis
MAI growth-points for the beginning of the year	2016-2019	3272 Grade 1 students in 2016	Growth point distributions to measure longitudinal progress and calculation of median growth points for each group.
MAI growth-points for the beginning of the year	2016-2019	242 Grade 1 EMU participants in 2016	Growth point distribution to measure longitudinal progress and calculation of median growth points for each group
Pre- and post EMU Affective assessment	2020	307 Grade 1 EMU participants	Calculation of mean responses for each item and independent-samples t-test

Diagnostic Assessment and Learning Framework

In the current study, the *Mathematics Assessment Interview* (Clarke et al., 2002; Gervasoni et al., 2011; Victorian Curriculum and Assessment Authority, 2021) and an associated growth point framework were used for assessing all students at the start of each school year. In 2016, 21884 students from kindergarten to Grade 6, across 51 schools in a region in NSW were assessed in four whole number domains by their classroom teachers. These assessments also took place at the beginning of 2017-2019, which enabled students' post-intervention and longitudinal progress to be measured. These data were entered into the education system's database, and later analysed by our research team, according to the approved ethical guidelines for our research. This one-on-one diagnostic assessment and the growth point framework were first developed in Australia during the Early Numeracy Research Project (Clarke et al., 2002) and further refined during the *Bridging the Numeracy Gap in Low SES and Aboriginal Communities* pilot (Gervasoni et al., 2011). The processes for validating the growth points, the assessment interview items, and the comparative achievement of students are described in full in Clarke et al. (2002) and have been reported widely (e.g., Clarke, 2001; Clarke, 2013). The framework of multiplication and division growth points (see Figure 1) also enabled students' learning and longitudinal progress to be measured. The growth points do not represent an assessment score, but rather describe a learner's current knowledge in reference to the set of research-informed growth points or progressions. The idea is that the growth points guide teachers about how they might differentiate instruction in response to each learner's current knowledge, and provide the resources, supports, and teaching necessary to advance all students' learning. To achieve a growth point, students complete a series of tasks during the MAI. Teachers use their observation notes about students' solution strategies to determine whether a particular growth point has been achieved. For example, to determine whether Growth Point 3 (partial modelling multiplication and division) has been achieved, a student must demonstrate that they can solve three tasks using skip counting or known facts (Figure 2). In these tasks only partial models are provided to act as a visual stimulus. Students who are successful are able to make use of mental imagery and recognise the composite unit and structure of each problem (Downton, 2012).

It is important to note that each of these tasks (Figure 2) involves a different multiplicative structure: times as many (Unifix train task); allocation/rate (Tennis balls task); and rectangular array (Dots array task). As indicated in the literature, students need to experience the different multiplicative structures, as well as the different language they invoke, in order to gain a full understanding of these multiplicative (multiplication and division) concepts (Downton 2008a, 2012; Greer, 1988).

31) Unifix Train – (Partial Modelling - 'times as many' Multiplication)

a) *Show the bar of 5 unifix and say,*
I've made a train 5 unifix long.
Imagine that you make a train that is 3 times as long as mine.
How many unifix will you need?
Tell me how you worked that out.

b) *If the child appears to be counting by ones from 5, ask:*
Could you do it another way withoutcounting them one by one?
Tell me how you worked that out.

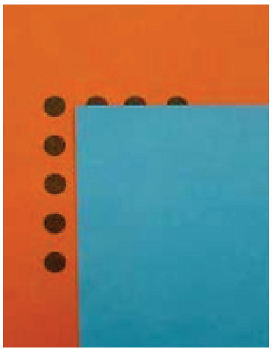
32) Tennis Balls Task – (Partial Modelling 'allocation/rate' Multiplication)

a) *Put out the packet of 3 tennis balls.*
Here is a packet of tennis balls.
How many balls would there be in four packets?
Tell me how you worked that out.

b) *If the child appears to be counting by ones, ask,*
Could you do that another way, without counting one by one?
Tell me how you worked that out.

33) Dots Array Task – (Partial Modelling Multiplication - Arrays)

a) Here is a page of dots.
Show red card - 5 × 4 dots - for an instant, in the orientation shown in the diagram.
I'm going to hide some.
Cover the bottom 4 × 3 section, and the bottom half of the 3 dots above it.
How many dots are there altogether on the whole page?
If the child answers 6½ lift up the screen momentarily and ask again,
How many dots on the whole page?



b) *If the child appears to be counting all, ask:*
Could you do that a faster way, without counting them one by one?
How did you work that out?

Figure 2. Multiplication and division items for growth point 3.

Selecting students for the EMU intervention

The research-based set of growth points (Clarke et al., 2002) were used to identify students for the EMU intervention program. Each student's current growth points were compared to a set of benchmarks known as the *On the Way* growth points (see Figure 3) according to the process outlined in the EMU Program Guidelines (Gervasoni, 2004; 2015).

Number Domains	February–June *On the Way* Growth Points for Each Grade Level								
	F	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Counting	[1]	2	3	4	5	5	5	6	6
Place Value	[1]	1	2	3	3	4	4	5	6
Add & Sub	-	1	2	3	4	5	5	5	6
Mult & Div	-	1	2	3	3	4	5	5	6

Figure 3. The *On the Way* growth points for Grades 1-8 students.

These growth points describe the mathematical understanding typically needed for students to fully engage with learning experiences in their classrooms, otherwise they might struggle to participate and learn. For example, Grade 1 students on Growth Point 0 (GP0) in Multiplication and Division Strategies were identified as vulnerable because it was anticipated that they would struggle to participate in classroom mathematics activities that assume that students can at least use a count-all strategy (GP1) to solve multiplicative problems. Similarly, Grade 2 students who had not reached GP 2 (using the multiplicative structure of the situation to find the answer when all objects are modelled or perceived) were identified as mathematically vulnerable in Multiplication and Division Strategies. The guidelines for identifying students as mathematically vulnerable were established during the *Early Numeracy Research Project* (Clarke et al., 2002) and extended during the *Bridging the Numeracy Gap Project* (Gervasoni et al., 2011) through analysing 3 sets of data: growth point distributions for large cohorts of students in the projects; mathematics curriculum statements; and teachers' recommendations (Clarke et al., 2002; Gervasoni, 2004; Gervasoni, 2015). Because resourcing may be insufficient to offer the EMU intervention program to all students who qualify, students are prioritised (Priority 1 to Priority 4) according to the number and combination of domains for which they were identified as vulnerable. For this purpose, students' growth point profiles (e.g., 2101) were examined which comprise a student's growth points for Counting, Place Value, Addition and Subtraction Strategies, and Multiplication and Division Strategies.

Of the Grade 1 children who were identified as mathematically vulnerable in any domain in 2016, 242 students, from 51 schools, participated in an EMU intervention program for at least 10 weeks (50 lessons). This was 15% of the Grade 1 cohort overall (N=3272). The EMU participants were the most vulnerable students in their class cohort, typically identified as Priority 1 or Priority 2 for EMU intervention. For any remaining students on the priority list, mathematics individual learning plans were co-developed by the specialist teacher and classroom teachers. Some students also received in-classroom support from the EMU specialist teacher.

Although the EMU program guidelines stipulate that students be provided with EMU intervention sessions on 5 days per week to achieve the intensity required for an acceleration effect, the mean number of days students actually participated in EMU lessons is typically fewer (Gervasoni et al., 2019). Teachers report that the major reason for missing lessons was a special school event or staffing interruptions, but not student absenteeism (Gervasoni et al., 2019).

Student Affective Assessment and Analysis

In 2020, Grade 1 students who participated in the EMU program completed pre-EMU and post-EMU affective assessments so that their teachers and the research team might gain insight about the students' engagement and interest in mathematics, and how this changed over time. The assessment items addressed key factors for mathematics engagement such as interest (e.g., Learning maths is interesting), effort (e.g., I like to think hard in maths), enjoyment (I enjoy learning maths at school), and preference for challenge (I like to be challenged when learning maths). These assessments were conducted using an online platform (Qualtrics) and included 19 Likert scale items and 3 open response prompts. For the Likert scale items, the students were invited to indicate how much they agreed or disagreed with each statement. Due to the young age of the students (six-years), the teachers read the statements, and an interactive approach was used to enable the students to record their responses. That is, the students used a mouse with their computer, or used a tablet, to maintain a neutral facial expression on an emoji (a straight mouth) or change the facial expression to either a sad or very sad expression to represent disagreement or to either a happy or very happy expression to represent agreement. This type of assessment has been shown to be a reliable measure of young students' agreement of Likert-type statements about attitudes to mathematics (Massey, 2021). For our study, deidentified responses for 307 students were extracted from the Qualtrics platform. For analysis, the responses were allocated scores from 1 to 5. A rating of strongly disagree was allocated a score of - 1, disagree - 2, neutral - 3, agree - 4, and strongly agree - 5, and mean scores were calculated.

The reliability of the scale was measured for the pre-EMU assessment, for 16 of the 19 statements. They were found to have good internal consistency, with a Cronbach alpha co-efficient of 0.85. As only pre-EMU items were analysed the post EMU item (Item 8, Table 2) was excluded. Two statements that measured the students' beliefs about the nature of mathematics (e.g. item 7, Table 2) had a low item-total correlation and these were also excluded from the reliability analysis.

To compare the extent of agreement between students' pre- and post- EMU program responses an independent-samples t-test was conducted. As these data were deidentified, we could not determine how many or which students from the pre-survey also completed the post-survey, and therefore we could not do a paired-samples t-test. The mean for each item, associated change in mean between Time 1 and Time 2 assessments, and P-values were calculated to indicate the statistical significance of any changes in mean responses.

Results

In this section, several data sets are examined to evaluate the effectiveness of the EMU intervention approach for progressing the mathematical learning of Grade 1 students who were failing to thrive mathematically, and for closing the performance gap with their peers. First, we consider longitudinal growth point data derived from the *Mathematics Assessment Interview* Multiplication and Division domain for the period 2016 to 2019. We then examine a further data set to determine the impact that participating in EMU intervention has on students' attitudes towards learning mathematics.

Longitudinal Progress of Students Participating in the EMU Intervention

Figure 4 compares the longitudinal progress (2016-2019) of 242 Grade 1 students in 2016 who were not thriving in mathematics learning with the progress of their class peers (n=3272) in the Multiplication and Division Strategies domain. This was one of four whole number domains assessed, and provides an illustrative example for discussion. It is important to note that the EMU group participated in an intervention program in Grade 1 (2016), but not in subsequent years. Hence, the 2016 data for the EMU group is pre-intervention data, and the 2017-2019 data is post-intervention and follow-up data. Data for the whole cohort of students is shown in the left column in each pair of columns in Figure 4, and data for the group of EMU participants is shown in the right-side column.

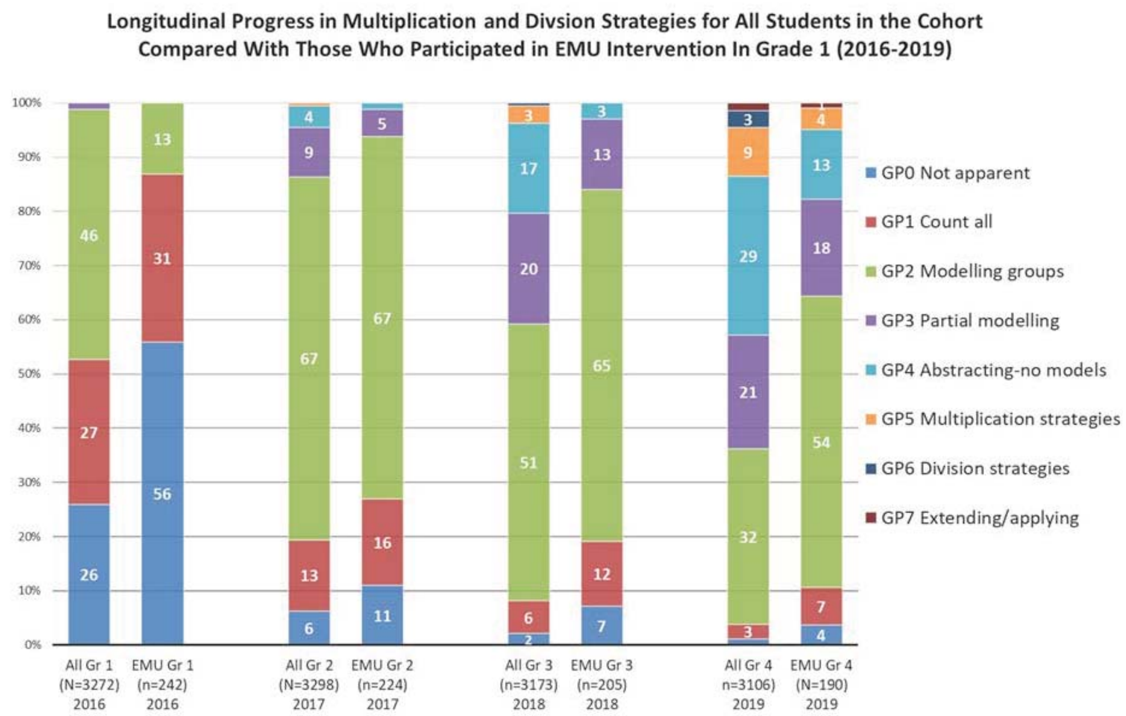


Figure 4. Multiplication and division strategies growth point distributions across 3 years for 242 students who accessed Level 3 EMU as a supplementary support in Grade 1.

Inspection of the results in Figure 4 shows that:

1. There is a wide-distribution of growth-points for each group in every grade level. The spread includes at least GP0 to GP4 from Grade 2 to Grade 4.
2. Prior to the EMU Intervention (2016), the growth point distributions for both groups were very different. The median growth point for the whole cohort is GP1 and for the EMU group is GP0. Fifty-six percent of students in the EMU group were on GP0 compared to 26% of the whole cohort, and only 13% of the EMU group had reached GP2.
3. Post the EMU intervention (2017), the distribution of growth points for both groups were very similar. The median growth point for each group was GP2. This represented a median increase of 2 growth points for the EMU group from 2016-2017, and one growth point increase for the whole cohort. A higher proportion of the EMU group was on GP0 and GP1 in 2017 (27%) compared with the whole cohort (19%). A lower proportion of the EMU group (6%) were on GP3-GP5 compared with the whole cohort (14%)
4. In 2018 (Grade 3), the median growth point for both groups remained GP2 (modelling). This represented minimal growth across one school year for most

students. The proportion of students on GP0 and GP1 decreased for both groups. The proportion of students reaching GP3 (partial modelling) or GP 4 (abstracting) increased to 37% for the whole cohort, and to 16% for the EMU group.

5. In 2019 (Grade 4) the median growth point for the EMU group remained GP2, and was one growth point less than the median for the whole cohort (GP3). The proportion of students reaching GP3 (partial modelling) or GP 4 (abstracting) increased to 64% for the whole cohort, and to 36% for the EMU group.
6. Progress for both groups was prolonged for the transition from GP2 to GP3, which required students to solve multiplicative problems in partially modelled contexts. This was apparent for the all student group from Grade 2 to Grade 3, and for the EMU group from Grade 2 to Grade 4.

The data indicate that overall, the EMU group's learning was boosted between Grade 1 and Grade 2, after the period of the EMU intervention, and that the performance gap between the two groups was minimal at the beginning of Grade 2 (2017) and after the long summer holiday. Although this learning was sustained for the EMU group from 2017-2019, two years after the end of the intervention, the performance gap had re-appeared.

Affective Progress of Students Participating in the EMU Intervention

Given the importance of interest, enjoyment, and effort for facilitating motivation and engagement in mathematics, in 2020, the *Extending Mathematical Understanding for All* study investigated the beliefs and attitudes of the 6-year-old students who participated in the EMU intervention program. The pre- and post EMU intervention assessments were administered by EMU specialist teachers as part of the intervention. The mean rating for each item, and the associated change in mean between Time 1 and Time 2 assessments were calculated. To compare the extent of agreement between students' pre- and post-EMU program responses, an independent-samples t-test was conducted. The results for the nineteen Likert scale items are shown in Table 2. Item 19 addressed students' beliefs about their experience during the EMU Program and hence was only asked in the post-EMU survey.

The results suggest that these students were positive about school (item 3) with a mean response of 4.3 out of 5 prior to their participation in the EMU program. Overall, the students' attitudes to learning mathematics were rated more highly after their participation in the EMU program. For example, the mean results after the EMU program for students' interest in mathematics (Item 2), was 4.4, and for their enjoyment of learning mathematics at school (Item 4) was 4.5. Notable is that the mean scores for these items were quite positive prior to the intervention, however, the improvement in mean ratings between the Time 1 and Time 2 assessments is apparent. Responses for Items 11 and 12 provide some confidence that the students were responding to each item thoughtfully as, for these two items, fewer students indicated agreement after the intervention program than before. Overall, we note that the mean item scores for the post-EMU assessment suggest that these students were more likely to identify that learning mathematics was interesting and enjoyable after they participated in the EMU program than before. The results also suggest that, after participating in the EMU program, students' liking of hard thinking and challenge when learning mathematics had increased (Items 7 and 18), and students were more confident about learning mathematics in their classroom and at home (Items 13 and 14).

The EMU students were also invited to respond to two further Likert scale items in the post EMU assessment that aimed to gain insight about whether they thought the EMU program assisted their mathematics learning. Students were asked to rate from one to five (1=not at all to 5=a lot) how much they agreed with the following two statements:

1. EMU helps me improve my mathematics learning.

2. The EMU Program helps me be better at learning mathematics in the classroom.

Table 2: Pre and post EMU affective assessment Likert scale item, frequency, mean, change of mean, and p value for Grade 1 EMU participants in 2020.

	Statements about Mathematics Learning and School	*Pre EMU n*	Pre EMU (Time 1) Mean out of 5	*Post EMU n*	Post EMU (Time 2) Mean out of 5	Change in mean	Sig. (2 tailed) *p<0.05
1.	Learning maths is easy	299	3.7	209	4.2	0.6	*0.000
2.	Learning maths is interesting	289	3.7	208	4.4	0.7	*0.000
3.	I like school	307	4.3	204	4.5	0.1	0.151
4.	I enjoy learning maths at school	296	4.0	207	4.5	0.5	*0.000
5.	I learn maths at home	295	3.4	199	3.9	0.5	*0.000
6.	I enjoy learning maths at home	293	3.6	192	4.1	0.5	*0.000
7.	I like to think hard in maths	296	3.8	202	4.5	0.7	*0.000
8.	I like to explain my thinking when doing maths	299	3.5	204	4.4	0.9	*0.000
9.	I like working with others when doing maths	301	4.2	211	4.6	0.4	*0.000
10.	Making mistakes in maths helps me to learn	293	3.6	209	4.4	0.8	*0.000
11.	Maths problems have one and only one correct answer	279	3.5	204	2.6	-0.9	*0.000
12.	There is only one way to solve a maths problem	286	3.6	208	2.4	-1.2	*0.000
13.	I feel confident learning maths in the classroom	292	3.7	204	4.4	0.7	*0.000
14.	I feel confident learning maths at home	294	3.6	199	4.2	0.6	*0.000
15.	I enjoy playing maths games at home	293	4.0	202	4.7	0.7	*0.000
16.	When I try hard I can learn most things in maths	290	4.1	211	4.7	0.5	*0.000
17.	Anyone can be better at maths if they keep trying	301	4.3	211	4.7	0.4	*0.000
18.	I like to be challenged when learning maths	293	3.6	203	4.3	0.7	*0.000
19.	I feel confident learning maths in EMU	n/a	n/a	210	4.8	n/a	n/a

The results shown in Table 3 indicate that most Grade 1 EMU students strongly agreed that the EMU Program helped to improve their mathematics learning, and that the EMU Program helped them to learn mathematics more successfully in the classroom.

Table 3: Frequency, rating and mean for students' perception of the EMU program's efficacy

Statement	n	Rating					Mean
		1	2	3	4	5	
#1 EMU helps me improve my maths learning	214	1 (0.5%)	0 (0%)	6 (2.8%)	30 (14%)	177 (83%)	4.8
#2 The EMU Program helps me be better at learning maths in the classroom	214	2 (0.9%)	0 (0%)	17 (8%)	50 (23%)	145 (68%)	4.6

Overall, these results suggest that the EMU intervention program, which was implemented to provide learning supports for students who were not yet thriving when learning mathematics, contributed to improving these students' interest in learning mathematics, and their resilience or persistence with challenging tasks. These findings suggest that the EMU support and the associated differentiated instruction, may have assisted in boosting the students' engagement and learning, so that they were better positioned for learning mathematics in their classrooms.

Discussion

One component of the *Extending Mathematical Understanding for All* study in 2016 was the provision of an intervention for the Grade 1 students who were not yet thriving in mathematics. This decision was intended to differentiate mathematics instruction more intensively for these students, given that they were failing to thrive after one year at school. A specialist mathematics teacher provided the classroom teacher with specific advice about how to differentiate teaching for these students during classroom mathematics lessons and provided small-group teaching within the EMU intervention program for a period of at least 10 weeks. The intent was to boost the students' mathematics learning and dispositions for learning mathematics so that they were more strongly positioned to flourish mathematically in their classrooms. A tension was whether the segregation effect of the intervention had any negative impacts on students' engagement in mathematics lessons and their dispositions for learning mathematics.

To provide insight about this issue, the longitudinal progress of 242 Grade 1 students who participated in the EMU intervention program was examined from Grade 1 through to Grade 4 and compared to the progress of their peers. Students' progress in Multiplication and Division Strategies was used as an illustrative example because this topic can be viewed as challenging for young children. The broader study similarly examined longitudinal progress for the domains of Counting, Place Value, and Addition and Subtraction Strategies. Also examined in this current study were the mathematical dispositions of a separate group of students in 2020 who were assessed before and after they completed the EMU intervention program. The key findings related to the longitudinal data analysis were:

1. The performance gap between the Grade 1 EMU group and their peers was reduced in 2017, after the EMU intervention in 2016. This was evident when comparing the distribution of growth points for both groups in 2017, which were remarkably similar. Likewise, the median growth point for both groups in 2017 was GP2 (modelling). Prior to the intervention, the growth point distributions and medians for both groups were different.

2. The median Multiplication and Division Strategies growth point for both the EMU group and their peers across 2017 and 2018 (post the EMU intervention) was GP2 (modelling). This finding suggested minimal growth for students between the Grade 2 and Grade 3 time points. In particular, progress for many students was prolonged for the transition from GP2 to GP3, which required students to solve multiplicative problems in partially modelled contexts. This prolonged transition was apparent for the all-student group from Grade 2 to Grade 3, and for the EMU group from Grade 2 to Grade 4.
3. By Grade 4, two years after the end of the intervention in Grade 1, the growth point distributions for the EMU group and the whole cohort were again quite different, and the median growth point in Multiplication and Division Strategies for the EMU group (GP2) was one growth point below the median for the whole cohort (GP3). This finding demonstrates that the performance gap between the EMU group and the whole cohort had re-emerged.

An issue that is highlighted by these findings is the pedagogical challenge of ensuring mathematics learning for all. In this research, the Grade 2, Grade 3 and Grade 4 teachers used the MAI and growth point framework to learn about each student's mathematical thinking and to inform their planning and differentiated instruction. All teachers were encouraged to use a core task with enabling and extending prompts to differentiate instruction. However, these instructional practices seemed insufficient to support progression from GP2 to GP3 for many students, and this was more apparent by Grade 4 for the EMU group. It is possible that the pedagogical content knowledge required by the classroom teachers to successfully differentiate instruction for students in the Multiplication and Division Strategies domain was insufficient. Alternatively, the teachers may not have implemented the recommended whole-school practices for differentiating instruction as rigorously as intended. This situation warrants further research to gain insight about how teachers differentiate their teaching in ways that support the transition to partial modelling (GP3) in Multiplication and Division.

It is likely that focused professional learning to support teachers to differentiate instruction for particular growth point transitions would be beneficial. Indeed, providing school based professional learning focused on multiplicative learning over an extended period of time can have a sustained impact on student learning (Downton et al., 2019), and on teachers' pedagogical content knowledge with respect to multiplication and division (Downton et al., 2018). For example, professional learning can support teachers to understand that to assist the shift in students' reliance on materials and counting based strategies in multiplicative situations (GP2), it is important to remove the materials (Sullivan et al., 2001) and encourage students to visualise or imagine the problem, and how they might solve it using drawings and representations (GP3). As noted by Mulligan et al. (2006) spatial structuring plays a key role in visualising and organising multiplicative structures such as unitising and partitioning. Frequently engaging students in a range of visualisation tasks using the array structure, such as "Look hide and make", "What do you see?" (Downton, 2008a; Jacob & Mulligan, 2014), and subitising tasks, can assist students to recognise a quantity as a composite unit and become less reliant on counting. Furthermore, progression from GP2 to GP3 and beyond requires students to explore the different multiplicative structures. Therefore, professional learning can deepen teachers' understanding of these structures, the different associated representations, and how these representations support students' development of multiplicative thinking.

Another key finding from our study is that two years after the end of the EMU intervention, the median growth point for the EMU group was again one growth point below the median for the whole cohort. This difference in means was also apparent prior to the intervention. Previous longitudinal research has shown that the academic gains from a mathematics intervention typically fadeout over time (Bailey et al., 2016; Clements et al., 2013; Protzko, 2015; Smith et al., 2013). A commonly speculated reason for this fadeout is that the classroom environments that the students experience, alongside and post-intervention, do not sustain the effect. It is critical that students are offered curriculum that is sufficiently challenging to advance their learning, and that teaching is informed by learning trajectories. For example, in our study, it seems that more attention was needed to support students' progress from GP2 (modelling) to GP3 (partial modelling). This was particularly relevant for the EMU group in the years following the intervention. However, it is worth noting that there is evidence that intervention fadeout effects are at least partially explained by persistent individual differences between learners, as well as home environment effects (Bailey et al., 2016).

The findings for the 307 EMU students in 2020 show that after their participation in the intervention program there were significant increases in the EMU group's mean responses related to their interest, effort, enjoyment, and preference for challenge when learning mathematics. This is encouraging given the importance of persistence and effort for achievement in mathematics (Blackwell et al., 2007). Moreover, the students were less likely to believe that there was only one way to solve a mathematics problem. These findings suggest that, overall, the students were more positive about learning mathematics following the EMU intervention.

Another important finding was that most of the group strongly agreed that the EMU Program helped to improve their mathematics learning, and that the EMU Program helped them to learn mathematics more successfully in the classroom. This is an encouraging outcome when considering the impact of an intervention program for providing equitable supports to assist students to thrive in their classroom learning environments.

Overall, it seems that access to more focused differentiated instruction provided through an EMU intervention program was a positive experience for the students who were not thriving with mathematics learning. However, it is important to note that the EMU intervention approach was deliberately aligned with the classroom approach (Smith et al., 2013); took place during only half of the daily mathematics lesson; instruction was differentiated and planned by a specialist teacher with deep pedagogical content knowledge; the approach was underpinned by constructivist approaches; and provided problem-solving opportunities for students in a small-group social situation. Strategically, the intervention approach was designed to avoid, as much as possible, the known negative consequences of ability grouping and streaming in mathematics, whilst providing focused differentiated instruction for students who were not thriving in mathematics. The overall goal remains for classroom teaching to be able to deliver this type of focused differentiated instruction for all students. It is likely that this requires teachers to have deep pedagogical content knowledge.

Conclusion

Mathematics teachers play a key role in preparing their students to participate fully in a future global community in which rapid change will be one certainty. This requires teachers to design equitable, inclusive curricula and teaching approaches that provide sufficient challenge and support to enable all students to thrive mathematically. The findings presented in this paper demonstrate that constructivist-oriented intervention programs can enhance mathematics learning for students who fail to thrive when learning school mathematics. However, it appears that one-off intervention programs may not be sufficient to maintain this enhancement over time (Bailey et al., 2016).

We propose a model for using differentiated instruction to advance the ongoing learning of students who are currently not thriving in mathematics: (1) using assessment instruments and associated frameworks that guide differentiated teaching to respond meaningfully to student diversity through building on their current strengths; (2) using a substantial core task in lessons with enabling and extending prompts that promotes challenge, persistence, creativity and collaboration for all through problem-solving; and (3) providing additional supports from a specialist mathematics teacher to amplify differentiated instruction for students who are not thriving. This includes providing advice and mentoring for classroom teachers, and classroom aligned small group teaching or intervention programs that advance equity and inclusion.

The findings of the current study showed that mathematics learning was accelerated for the group of students who participated in the EMU program in Grade 1 and that this effect was sustained through to the beginning of the following school year and across the long summer holiday break. Also, participation in the EMU program was associated with increases in students' positive dispositions for learning mathematics, and the students reported that the EMU program helped them to be better at learning mathematics in their classrooms. It appears that the differentiated teaching and the experience of the EMU program provided an equitable support for those students who were not thriving in mathematics, and boosted their learning and dispositions for learning mathematics in the short term. However, the positive intervention effect did fade over the two years following the intervention. As highlighted by Clements et al. (2013) and Bailey et al. (2016), we need to better understand the mechanisms that lead to the fade out of a positive intervention effect, and strengthen school practices to minimise this impact.

Several limitations for this current study are important to consider when reviewing the findings. First, according to the approved ethical conduct of this research, there was no control group of students; all students were provided with differentiated instruction through the whole school practices adopted by the schools. However, the data showing the progress of all students in a year level cohort was used as a point of comparison to interpret more clearly the comparative progress of students who accessed the EMU intervention. Second, there was not a comparative set of data that measured the dispositions of all students in a year level. The pre- and post- EMU affective assessment was used only for students who participated in the intensive small group teaching.

Another limitation of this study was that the research did not investigate level 2 EMU support whereby the EMU specialist teacher co-teaches with the regular classroom teacher to provide whole-of-class differentiated instruction to support students who are not thriving. This approach might reduce the potentially negative consequences of segregation and would be a profitable area for future research. Also, it was not known how well the Grade 2 - Grade 4 teachers used the growth-point framework to guide their differentiated teaching, and how often they used a substantial core task with enabling and extending prompts in mathematics lessons. Data that showed how these practices were enacted would be useful in future research.

The fact that there are students who currently struggle with mathematics learning highlights the need for differentiated instruction as a step towards ensuring that more inclusive equitable mathematics education is achieved. There is an ongoing need for further insights about the type of differentiated approaches that can best support mathematics learning for the students who fail to thrive. Also needed are deeper insights about how teacher education and ongoing professional learning can support teachers to provide more equitable, inclusive mathematics education. The findings presented in this paper provide hope that mathematics education can advance a social justice agenda to enable all students to thrive mathematically, through access to well-focused differentiated instruction that embraces diversity.

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