

Mathematics Teachers' Perceptions of Teaching Practices Alignment with Ambitious Teaching

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Research on practice-based mathematics teacher education has identified core practices, principles and design features that lead to effective programs. Yet, some teachers do not perceive such practice-based development programs as relevant or useful. In response to this, the study reported in this article investigated three Norwegian teachers' current perceptions on mathematics teaching and student learning. Findings from the study indicate that teachers' perceptions on classroom practices can be described by using key concepts similar to those that are used to describe practices in ambitious teaching. However, the teachers' perceptions are guided by underlying purposes that differ from the view of teaching and student learning grounded in the characteristics of ambitious mathematics teaching. The implications of the findings are discussed in relation to the potential benefits of studying teachers' current understandings of concepts related to a professional development program before entry to or during the start-up phase of the program.

Keywords · practice-based · craft knowledge · ambitious teaching · teacher professional development

Introduction

Ever since Ball and Cohen (1999) called for a practice-based theory of teacher education the focus on practice-based pedagogy in research on mathematics teacher education has increased (e.g., Charalambous & Delaney, 2020). The need for a focus on practice-based teacher education is argued by McDonald et al. (2013), who described it as "a major shift—a turn away from a predominant focus on specifying the necessary knowledge for teaching toward specifying teaching practices that entail knowledge and doing" (p. 378). Hence, there is a need to bridge the gap between knowledge for teaching and knowledge of teaching, between theory and practice, and between university courses and field work (Ball & Forzani, 2009; Grossman et al., 2009; Zeichner, 2012). According to Forzani (2014), there is little consensus as to what practice-based teacher education means, as the term has been used to describe a wide range of programs that differ from the academic model of teacher education. Practice-based teacher education has nevertheless led to understanding that teaching is a key part of the process of learning to teach (Ball & Forzani, 2009; Grossman et al., 2009; Lampert, 2010; McDonald et al., 2014). Bailey and Taylor (2015) argued that practice-based teacher education can be identified as having two directions. The first focuses on core practices of ambitious teaching and the second deals with a range of pedagogical practices relating to novice teachers engaging with representations, decompositions, and approximations of practice. In this article, the focus is on the direction that centres around core practices and ambitious teaching.

Ambitious teaching is described as teaching that attends to the learning of all students and aims to deepen all students' understanding of complex mathematical ideas and performances (Lampert et al., 2010; Lampert et al., 2013; McDonald et al., 2013). A large body of research on practice-based teacher education aims to facilitate novice teachers', teacher candidates' and prospective teachers' development and enactment of core practices of ambitious teaching. For example, Bailey and Taylor (2015) focused on novice teachers' learning of core high-leverage teaching practices through engaging in a problem-solving approach to explore learning and teaching mathematics. Having reviewed key findings from research on teaching practices and practice-based pedagogy undertaken since 2000, Charalambous and

Delaney (2020) discussed the progress in making practice a key aspect of understanding and improving teaching and teacher education. Kazemi and Wæge (2015) investigated prospective teachers' learning experiences of participating in a practice-based-methods course focusing on a set of core practices of classroom teaching. Less research, however, has been dedicated to investigating in-service mathematics teachers' learning of ambitious mathematics teaching. Most of the research found in this area centred on coaching that supports teachers in their enactment of ambitious teaching practices, and in understanding what in-service teachers learn from participating in such professional development (PD) (e.g., Fauskanger & Bjuland, 2019; Gibbons & Cobb, 2016; Gibbons et al., 2017), or centred on organising schools to support teachers' PD (e.g., Gibbons et al., 2019; Kazemi & Resnick, 2020). In this article, the focus is on in-service mathematics teachers, referred to hereon as "teachers."

Research of teachers' PD of ambitious mathematics teaching through practice-based development programs has largely focused on the knowledge the teachers should develop and what they learned through participation, or the specific goals of practice-based development programs. Little attention has been devoted to teachers' existing knowledge and understandings and how these might influence their participation in a practice-based development program. In response to this, the research reported in this article investigated teachers' current perceptions on teaching and students' learning before they entered a practice-based development program in mathematics and explored how these perceptions might influence their participation. Researchers have identified several key characteristics that are of major importance in the work of teachers' PD (Borko, 2004; Desimone, 2009; Timperley et al., 2007). One of these is related to the extent to which the teachers' learning is consistent with their knowledge (Desimone, 2009), which implies that what the teachers experience as taught in PD is in accordance with their existing knowledge. Timperley et al. (2007) argued that the extent to which conceptual understandings and practical resources offered through the learning experience make sense to the recipients in terms of their existing understandings and practice contexts strongly influences the degree to which new information is used. According to Elmore (2002), teachers' PD must be of high quality and relevant to their needs if it is to be effective and successful. Teachers' PD is an ongoing process in which their continuous growth depends on their effort (Pokhrel & Behera, 2016). Research indicated that more attention should be devoted to the start-up phase of development work so the teachers are supported in developing an understanding of the goal and why they should act on it (Postholm, 2008; 2021). Thus, it may be helpful to examine teachers' perceptions on what they are to develop and what impact these perceptions might have on their own learning process when participating in a practice-based development program in mathematics.

One way of investigating teachers' existing knowledge and understandings about teaching and students' learning in mathematics is through identifying their craft knowledge. Craft knowledge is described as the professional qualities, formal knowledge and set of competencies developed through practice and experience (Ruthven & Goodchild, 2008). The purpose of this article, however, is not to determine teachers' craft knowledge, rather it is to attempt to understand teachers' current perceptions on teaching and students' learning before entering a practice-based development program in mathematics, here defined as a part of their craft knowledge. This stance is based on research on teachers' PD that claimed teachers' knowledge has an impact on their engagement in PD programs (e.g., Desimone, 2009; Timperley et al., 2007). In this study, three Norwegian lower secondary teachers' current perceptions on teaching and students' learning before they enter a practice-based development program in mathematics are investigated. The research aims to answer the research question: What perceptions do three lower secondary mathematics teachers have about classroom practice and students' learning?

Background of the Study

The aim of the study reported in this article was to investigate teachers' perceptions of teaching and students' learning in mathematics before they begin a practice-based development program: *Mastering Ambitious Mathematics Teaching* (MAM project) for in-service mathematics teachers in Norway (e.g., Fauskanger & Bjuland, 2019; Wæge & Fauskanger, 2021). The MAM project was developed and

contextualised to the Norwegian situation from the *Learning Teaching in, from, and for Practice* project (e.g., Ghouseini, 2017; Kazemi et al., 2016; Lampert et al., 2013), which aimed to promote opportunities for novice teachers to learn to enact ambitious teaching in practice (e.g., Kazemi & Wæge, 2015; Lampert et al., 2013). The MAM project adapted the pedagogy of ambitious teaching to mathematics in the Norwegian context. The work has led to the development of a model and related resources for school-based PD for teachers in Norway (Fauskanger & Bjuland, 2019), for which the aim was to support teachers in learning to enact the complex and demanding endeavour of ambitious teaching (e.g., Lampert et al., 2010; McDonald et al., 2013). The study reported in this article focused on teachers' perceptions of their classroom practice and students' learning before participating in a PD program based on promoting the use of ambitious mathematics teaching practices.

Theoretical Framework and Related Research

The study was grounded in social-constructivist theory, meaning that individuals and their social environment are dialectically related to each other (Postholm, 2010; Prawat, 1996). In school this means that the teaching context is decisive for the pupils' learning. A key element in the view of ambitious teaching is that the emerging ideas in the classroom are built on and extended directly from student thinking and reasoning. To both elicit and respond to students' thinking and reasoning, the teacher needs to create a discussion-based classroom community (Kazemi et al., 2009). The work of ambitious teaching is also about orienting the students to each other's ideas and the mathematical goal, which means that the teacher must attend to the way students make sense of mathematics and relate to one another, both socially and mathematically (Ghouseini et al., 2015; Gibbons et al., 2017). The view that ambitious teaching aims to enhance the learning outcome for all students requires the creation of an inclusive learning environment that takes the students' experiences into account and supports meaningful participation.

Ambitious mathematics teaching is defined by Lampert et al. (2010) as the work of teaching that entails the intellectually and socially ambitious goals of mathematical proficiency (Kilpatrick et al., 2001). The work attends to improve the learning of all students and aims to deepen their understanding of complex mathematical ideas and performances (e.g., Forzani, 2014; Lampert et al., 2010). Ambitious teaching is built upon a set of principles relating to student and teacher learning that is pivotal in the demanding endeavour of ambitious teaching (Kazemi, 2017). These principles guide teachers in the use of classroom practices and mathematical knowledge and aim to maximise students' ability to learn important mathematics with meaning (Lampert et al., 2013). The principles involve treating all students as sense-makers, knowing the students as individuals and learners, learning with and from students and designing instruction with clear instructional goals (Ghouseini et al., 2015; Gibbons et al., 2017). Ambitious teaching involves an approach to teaching that, together with similar approaches such as realistic mathematics education (Van den Heuvel–Panhuizen, 2003; Freudenthal, 1991), problem-based learning (Lampert, 2001), inquiry-based pedagogy (Artigue & Blomhøj, 2013) and thinking classrooms (Liljedahl, 2016), is considered *reform-based* (Boaler, 2002), where the learning process is student centred. In contrast to this reform-based teaching approach, a *traditional* teaching approach is teacher-centred.

Teachers need to make a large number of choices during their day-to-day work. Their decisions are based on their set of competencies that has been developed throughout their careers within the practice of teaching, which can be referred to as *craft knowledge* (Ruthven & Goodchild, 2008). According to Cooper and McIntyre (1996), "craft knowledge describes the knowledge that arises from and, in turn, informs what teachers do" (p. 76). They further maintained that teachers develop professional craft knowledge through their involvement in processes of reflection and practical problem solving. Cooper and McIntyre (1996) also stated that craft knowledge is not the knowledge the teachers draw on when explaining their thinking that underlies their teaching practice, and that it thus must be distinguished from the knowledge that is not linked directly to practice. Craft knowledge in this sense is more directly linked to practice than other forms of knowledge and is of a practical nature. Teachers' current classroom practices can in this way be considered as elements of craft knowledge which are to be

developed towards ambitious teaching through enacting core learning and teaching practices. Teachers' perceptions on teaching and students' learning in mathematics will therefore be informed by their craft knowledge and will come to light through their classroom practice. In this article, Cooper and McIntyre's (1996) description of the concept is adopted.

Inferring teachers' perceptions and understandings of teaching and students' thinking is far from straight forward, Ruthven and Goodchild (2015) argued that craft knowledge is action-oriented and not generally made explicit by teachers. It may be that teachers find the ideas difficult to articulate or may be unaware of using craft knowledge. This research takes the approach that a teacher's perceptions make sense to the individual who has the perception. The focus is therefore on what the teachers perceive, rather than what they do not perceive. Such an approach also means that teachers may not consider their perceptions to be contradictory, even if an external observer might see them as being so.

The Study

The study, which took place in a lower secondary school in Norway, aimed to investigate mathematics teachers' perceptions related to their classroom practices and students' learning before the start of a PD program. Hence, the research reported in this article was part of the MAM project outlined above. By acknowledging teachers' perceptions as parts of their craft knowledge, it was necessary to conduct a qualitative study to reach an in-depth understanding of what these perceptions are all about. The data were collected over a period of three weeks, which provided enough time to become acquainted with the school and get a grasp of its daily life.

Methodology

The research reported in this article was a qualitative interview study (Brinkmann & Kvale, 2015) of three lower secondary teachers at the same school. Qualitative research is a situated activity that localises the researcher in the real world, and qualitative researchers thus focus their research on natural settings and attempt to understand and interpret phenomena based on opinions ascribed to them by individuals in these settings (Denzin & Lincoln, 2011). The aim of qualitative studies is to bring forward the emic perspective, which centres around the participants' points of view and emphasises their specific interpretations within a context (Wolcott, 2008).

The epistemological stance in qualitative studies is that knowledge and understanding are constructed in the encounter between the researcher and the participants (Lincoln & Guba, 1985), following a social-constructivist paradigm (Postholm, 2010; Prawat, 1996). Based on analyses of the data, narrative texts were constructed to present the findings (Polkinghorne, 1989; Riessman, 2008).

Participants and Data Collection

The research participants were three teachers working at the same lower secondary school in Norway. They were selected through purposeful sampling (Creswell, 2013). Four lower secondary schools planned to participate in the MAM project, all of whom were asked to contribute to this study. As the four schools were similar in terms of the number of students and teachers, and because no other factors that potentially could affect the aim of the study were discovered, the first school that volunteered to participate was selected. It was important that the teachers worked at the same school as their perceptions were to form the basis of the discussion on how these perceptions might influence their participation as individuals and as a school in the MAM project. The teachers at the school were asked if they were interested in contributing to the study, and many volunteered. The aim was to select teachers at the school that could satisfy the purpose of the study (Postholm, 2010), and who were willing to share their knowledge and experiences. As their teaching experience and membership in a working team might be possible factors influencing their perceptions, three teachers were chosen who had different teaching experiences, were working with students at different year levels, and appeared to be the most interested. The first participant, with 10 years of teaching experience and formal teacher

education of 30 credits according to the European Credit Transfer and Accumulation System (ECTS) (one year of full-time study is 60 ECTS), was given the pseudonym, Sofie. The second participant, with five years of teaching experience and formal teacher education of 180 ECTS, was given the pseudonym, Harald. The third participant, with 13 years of teaching experience and a formal teacher education of 60 ECTS, was given the pseudonym, Stig. Of these three, Sofie expressed the most interest in contributing to the study and thus became the main participant.

The data in this study were collected from three individual semi-structured interviews (Brinkmann & Kvale, 2015) with the three participants, and from classroom observations of one interviewee. The individual semi-structured interviews were conducted as a conversation because Brinkmann and Kvale (2015) claimed that "Knowledge is constructed in the interaction between the interviewer and the interviewee" (p. 4), but with a clear focus on six prepared questions. This type of interview conversation provided the teachers with the opportunity to refer to interesting aspects or themes the researcher did not think of before the interview. The interviews were audio-recorded, and the same interview guide (see Table 1) was used in all three individual interviews. A follow-up interview with Sofie was conducted to clarify concepts and ideas identified. The initial interviews took approximately 40 minutes while the second interview with Sofie lasted 35 minutes.

The aim of the observations was to provide information that was used to construct the interview guide and to provide contextual information that was used in the dialogue during the interviews. Observations of Sofie's classroom were focused on her classroom teaching of one group of 19 students in Year 9 (14–15 years of age). In all, nine classroom observations were conducted in the natural classroom setting (Angrosino & Pérez, 2000), which means lessons or teaching episodes that were part of the regular classroom routine in line with the learning program ascribed by the school were observed.

Table 1

Structure of the individual semi-structured interview guide

	Questions. All questions start with: In your opinion...	Purpose
1	What kind of knowledge is needed by mathematics teachers to teach mathematics?	Knowledge about teaching
2	What type of requirements are there for teaching mathematics?	Framework (e.g., given by the school management or education authority)
3	Is there a shared perception among mathematics teachers at your school as to what type of knowledge mathematics teachers need for teaching mathematics in school?	Knowledge about teaching
4	Are the mathematics teachers at your school aware of each other's teaching practice? If yes, how?	Teaching practices
5	Can you briefly describe a teaching lesson where your classroom practice is visible?	Teaching practices
6	What are your expectations for the MAM project?	Prepared for the project

The immediate impressions from the observations were written down in a logbook after each lesson. The observations were also video-recorded and studied several times, together with the written impressions in the logbook, in search of teaching actions that could serve as starting points for discussions during the interviews. The teacher informed all the students about the observation activity. The researcher did not interfere in the teaching and did not take part in the discussions during the lessons. The researcher assumed the role of *complete observer* (Gold, 1958), meaning that the researcher was present but did not take any active part in the lessons.

Data Analysis and the Construction of Narratives

For this study, the constant comparative analysis method (Corbin & Strauss, 2008; Strauss & Corbin, 1998) was used to analyse the data. Corbin and Strauss (2008) argued that this method can be used to analyse data in all qualitative studies. The transcription of the interviews started shortly after they were conducted, and they were transcribed in their entirety. To acquire an overview of the data collected in the interviews, the transcriptions were organised into a matrix with columns where initial analysis, related research, and labels and questions were entered. An example of one sequence is shown in Table 2. The transcriptions were further divided into smaller sections, one to three statements, in an attempt to understand the essence of what was expressed in the raw data (Corbin & Strauss, 2008). Related research and previous experiences from working with ambitious mathematics teaching were used as reflective tools to understand what was said from the participants' point of view. This way of interacting between inductive and deductive approaches, from theory to data and vice versa, can be considered an *abductive* approach (Alvesson & Sköldberg, 2009).

Excerpts of labelled data were labelled, and colour coded. These were compared and given codes, each covering whole sentences and sometimes even whole paragraphs. In this way, the codes covered larger units and as such made working with the data easier (Postholm, 2019). Then, the codes that could be related to teaching and students' learning in mathematics were grouped into categories. For instance, codes involving a particular teaching practice, such as "talking with students", were grouped into one category, and codes involving a particular characteristic of teaching and students' learning, such as "students' thinking", were grouped into another. Throughout this process, it appeared the data were either about mathematical discussions or the teachers' interest in students and their thinking. Thus, "Mathematical discussion", and "Engage with students and their thinking" became the main categories for this study. Furthermore, the categories were structured and specified by asking questions, such as when, why and under which circumstances did the categories materialise, and how and what did this lead to? This process identified eleven sub-categories, which are presented together with their related main categories in Table 3.

Table 2
An example of how the transcriptions were organised

Transcription	Initial analysis	Related research	Labels and questions
<p>Researcher: Can you say more about how you collect information from your students?</p> <p>Sofie: Well, that's not easy...In my opinion, I collect the most valuable information when I wander around in the classroom while the students are working on tasks. Then I can talk with the students and observe how they solve the task. Because not everyone raises their hand and asks for help. And also, through small tests or small "checkouts" as we call them, where I give them just a few minutes to solve one or two tasks on a piece of paper at the end of a lesson, and then I collect them and see how it went. Then I get a very good overview, I think, as long as they don't peek at their neighbour's work. Eh, but we try not to have so many big tests, but I think small tasks like that work very well...and that's it. And walking around, trying to talk with everyone during a lesson when they're working to collect information. And of course, there are a lot, or not a lot, but some students also contribute verbally. And then you get information in that way from those who ask about things and say "Oh, I don't understand that" or "I don't get it from here." But often they just say, "I don't understand" or "I don't get it", so they don't really know what they don't understand.</p>	<p>Sofie says she talks with the students, but she does not elaborate on how this conversation takes place.</p> <p>Sofie says some students are asking questions if they do not understand something. This can be an invitation into a mathematical conversation.</p>	<p>Mathematical conversation. See principles of ambitious teaching (e.g., Kazemi et al., 2009; Lampert et al., 2013)</p>	<p>Ask Sofie if she can elaborate on what "talking with students" implies</p> <p>Ask Sofie what she does when a student says: "I don't understand"</p> <p>Observing and talking with students</p> <p>Checkouts</p> <p>Students inviting into a mathematical discussion.</p>

Table 3
Main categories and sub-categories

Main categories	Engage with students and their thinking	Mathematical discussion
Sub-categories	Students' expectations Different types of students Knowing the students Interest in students' thinking Students' asking for help	Share students' thinking Orchestrating students' thinking Using talk moves Classroom discussion Ability to get involved in students' thinking Asking questions

In the process of developing a core category a question such as, "What is this all about?" (Corbin & Strauss, 2008) was asked, and the data were analysed in light of the literature relating to teaching and students' learning in mathematics. The teachers' perceptions were approached with the notion that they were related to each other and made sense for the person who had the perceptions. This process revealed that the teachers' utterances within the categories did not always cohere, as their purpose for acting on similar perceptions within their classroom practices and students' learning in mathematics varied. In the search for a unifying concept (Corbin & Strauss, 2008), it was discovered that differences in the teachers' purposes for acting on similar perceptions were informed by an incoherence in their approach to teaching and student learning. "Approaches to teaching and student learning" thus became the core category and will be explored further in the analysis and discussion together with the main categories. A narrative format has been used to present the data. As the participants provided rich and complex statements throughout the interviews, according to Riessman (2008), the findings will be more accessible to the reader if they are presented as narratives. A narrative text (Polkinghorne, 1989; Riessman, 2008) for each teacher based on their responses and the developed categories was constructed. The categories form the structure of the narratives.

Ethical Considerations and Quality of the Study

The study, approved by the Norwegian Centre for Research Data (NSD), complies with the ethical principles laid down by the Norwegian Ethical Research Committee (NESH, 2021). Informed consent was obtained from all the interviewees in accordance with the NSD guidelines, and participants were given pseudonyms for reporting purposes to ensure their anonymity. The teachers were also informed that all information in the study was completely confidential and that they could withdraw at any time without needing to provide further explanation (NESH, 2021). No participants withdrew from the study.

The quality of this study was improved by using member-checking (Lincoln & Guba, 1985). The three teachers received their respective narratives by email and were asked to approve or comment if they thought they were misrepresented in the narrative. All three approved the narratives without any data being either added or excluded. The findings from this study may have importance beyond the immediate data collection context. Even though the presented descriptions and the analysis were connected to three teachers from one specific school, the findings can contribute knowledge to understanding similar situations and contexts. This means that the reader can use the presented descriptions as a thinking tool if they perceive processes as parallel experiences and adapt them to their own situation, thus conducting naturalistic generalisations (Stake & Trumbull, 1982).

Findings

This section presents the three teacher narratives based on the responses from the interviews.

Sofie

Sofie had worked at the same school for ten years and described herself as well informed about her colleagues' teaching practice in mathematics. She did not consider the school's teaching practice in mathematics to be traditional. "The students are always allowed to collaborate with a partner or in a group ... but the teacher standing in front of the board and explaining, and things like that, is something we do. That's perhaps because we feel it's necessary." Sofie added that the students' work involved solving many tasks, which she considered to be a prerequisite for their mathematical learning.

Sofie believed that teachers should be good at explaining mathematical concepts and processes. "I want the students to understand what we're doing. And that idea is something that permeates everything in a way. If they are to understand the material, they need to have it explained in a way that makes them able to understand." She further expressed that teachers' explanations are mainly aimed at helping students understand how to solve tasks and why a strategy works. "If I'm going to explain how to solve a task by using an example on the board, then I have to be able to explain it in a way that helps them understand why we do what we do." Sofie further explained that she had tried to start the lessons

by giving the students a mathematical problem with the aim of motivating them, creating curiosity and encouraging them to develop their own strategies and share their thinking. However, she claimed that she had experienced this way of teaching as being too time-consuming. She said that it was not possible to do this in every lesson because it took time away from getting through the lesson plan.

Sofie maintained that the teacher must be interested in the students and curious about what they do not understand and why to determine which explanation to use in different situations. She said the explanations must be adapted to the students' level, and the teacher needs the ability to understand which explanation to choose and when. For example, she stated that "... the ability to explain is very much related to the ability to understand why students do not understand. You can't just learn a lot of different ways to explain." Sofie stated that she finds out what students do not understand when she walks around the classroom talking to them and observing how they solve tasks, and by asking her students to explain to her what they do not understand.

Sofie described that she must show and explain things to the students. For instance, she explained the standard algorithm for solving equations with different types of numbers, parentheses, and so on. Sofie further explained that the mathematical discussions that took place between her and her students where when she asks what to do next and the students suggest different approaches, such as moving all the unknowns to one side of the equal sign or solving fractions. Sofie maintained that group work provides opportunities for the students to ask each other for help if they need it while working on a set of tasks individually.

Stig

Contrary to Sofie, Stig maintained that the school's teaching practice in mathematics is traditional, where the teachers often stand in front of the board providing examples while the students solve the given tasks. "If you look into a mathematics classroom, then you either see a teacher in front of the board explaining and showing examples, or students sitting and solving tasks." Later, Stig added that he acknowledged this way of teaching can be beneficial, "But both the instruction and the tasks can be really good, so I don't claim it's wrong to do it this way."

Stig argued that the teacher must understand how students think and how they experience mathematics when it is taught, and that teachers not always considering the students' point of view might be the reason why the students fail to understand. "What do the students experience when we show them math? It's not always what we think." He argued that finding out how students work and think mathematically are important if teachers are to help the students at their level of understanding, and he believed that doing this makes him better equipped to explain the mathematical content to his students.

Stig stated that he would have liked to know more about misconceptions and why students do not understand mathematics. He also wanted to know more about different types of students, and which types have difficulty understanding various things in mathematics. Stig claimed that teachers need to know where and in what situations mathematics is needed so they can explain to the students why they need to learn mathematics.

Stig also claimed that facilitating conversations about mathematics with students and between students is something he does more and more often. The scope of the conversations may vary, he added, and this variation mostly depends on the teacher's preparation. "The better prepared you are able to be, the more beneficial questions you might have prepared," he said and added, "If the plan for the lesson is really good, more or less everything is possible." Stig maintained that the teacher also must consider along the way if the students are interested and eager, and then adapt accordingly, and identify students who do not like to talk as much so they can be put together in pairs or groups.

Harald

Harald suggested that "We math teachers need a 'toolbox' with suggestions about how to teach or change teaching to something more modern." He elaborated that the point was to determine how to get students to think differently than they do when the teacher is in front of the board and they are

solving tasks, as in traditional teaching. Harald argued that traditional teaching is out of date, but that some teachers, including himself, still persist with the practice to some extent, especially in relation to giving the students a lot of similar tasks. "I think math teachers would benefit from being challenged to teach the subject in other ways." Harald believed that mathematics teaching should be adjusted to all students, both the strongest and the weakest, which is important for student motivation.

Harald said he usually begins lessons with discussions, especially when introducing a new topic. Following such discussions, he said he often organises the students into predetermined groups based on how they normally work; he tried to allow the students to work in a way that suits them best. "I divide them into groups based on their personality and not based on what mathematical level they might be on. Based on what needs they have in a way. Some are quiet and some like to discuss, so I put them together." Harald explained that he often ends a lesson by letting students discuss in pairs what they think other students may perceive as difficult about the task, and how other students might have solved it. These conversations typically differ from one class to the next, he said, and they depend on the students' mathematical level.

One class I teach really likes to talk out loud. They're on a more equal level mathematically and are better at engaging in a plenary conversation. The students in the other class are on a very different level so if one speaks out loud the others may lose interest ... so it's different from class to class.

Analysis and Discussion

In this section, the teachers' perceptions relating to teaching and students' learning in mathematics, and how their perceptions make sense to them are analysed and discussed. The main categories, "engaged with students and their thinking" and "mathematical discussion", and the core category, "approach to teaching and student learning", are used to structure the discussion. The perceptions are discussed across the interviewees.

Engaging with Students and Their Thinking

The perception of engaging with students' thinking is particularly emphasised by Sofie and Stig, who both repeatedly pointed this out as an important practice in mathematics teaching. Their descriptions of engaging with students' thinking share many similarities with the principles of ambitious mathematics teaching, which focus on treating all students as sense-makers, knowing the students as individuals and learners, and learning with and from students (Ghousseini et al., 2015; Gibbons et al., 2017). For instance, Sofie and Stig argued that the teacher must show interest in their students, be curious about what they do and do not understand, and how they experience mathematics when it is taught. Both Sofie and Harald pointed out the importance of enabling students to develop their own strategies. Stig added that students' thinking must not be taken for granted, as it might not always be what one expects. With these perceptions, it can be assumed that both Sofie and Stig were acting on these principles of ambitious mathematics teaching in their classroom practice, or at least similar principles that focus on a student-centred teaching approach (Boaler, 2002).

Although Sofie's and Stig's perception of engaging with students' thinking seems in many ways to resemble some of the principles of ambitious teaching, their understanding of this concept is rooted in a different underlying purpose when it comes to acting on this perception. The aim of the principles of ambitious mathematics teaching is to deepen all students' understanding of complex mathematical ideas and performances by eliciting and responding to the students' thinking and reasoning as they emerge in discussion-based classroom communities (e.g., Forzani, 2014; Lampert et al., 2010). For Sofie and Stig, however, their purpose behind engaging with students' thinking was to obtain information that they could use when explaining mathematics to them. They claimed that engaging with students' thinking provides them with important information about how students learn, which in turn enables them to be on the students' wavelength and to determine their learning trajectory. Bearing this in mind, the teachers would then decide which explanation to use in each particular situation. Sofie and Stig

perceived that engaging with students' thinking can therefore be considered to be a method that better equips them to explain mathematics to students.

Mathematical Discussion

Although the three teachers emphasised different aspects of the mathematical discussion and the facilitation of it, they all agreed that it is particularly important in their teaching and the students' learning of mathematics. Again, their perceptions appear to be in accordance with the principles of ambitious teaching (Kazemi et al., 2009), and therefore associated with a student-centred teaching approach (Boaler, 2002). Sofie emphasised the importance of facilitating students to share their thinking by asking them questions. Stig pointed to the conditions for having classroom discussions and mathematics discussions between students, which he believed rely on the teacher's prior preparation of good questions. This way of preparing a lesson is an important aspect that enables teachers to enact key practices that ensure the principles of ambitious teaching (Lampert et al., 2013). Harald claimed that organising the students into groups enabled them to discuss each other's strategies and reflect on how other students might think. Orienting the students to each other's ideas is one of the key features in productive mathematics discussions (Ghousseini et al., 2015; Gibbons et al., 2017). However, Harald did not specify how the students' ideas were shared and oriented to other students' ideas. This makes it difficult to determine the extent of the similarities in his teaching practice with the principles of ambitious teaching. Nonetheless, it seems that some of the ideas were shared.

In the same way as for the perception "engaging with students' thinking", Sofie's and Stig's understanding of mathematics discussions also appeared to have some differences compared to ambitious teaching (e.g., Kazemi, 2017). Although Sofie and Stig maintained that they use mathematical discussions as a means for gaining access to the students' thinking, which partly aligns with the description of ambitious teaching (e.g., Kazemi, 2017), they also stated that the intention behind including mathematical discussions in teaching is to be better equipped to explain mathematics to students. This is an understanding that differs from the idea of creating a discussion-based classroom community where the discussions are based on the students' emerging ideas (Kazemi et al., 2009).

Approach to Teaching and Student Learning

The three teachers seemed to be in a developmental process in their approach to teaching and student learning. Although they did not agree on the extent to which traditional teaching characterises the school's teaching practice, the findings reveal traces of both a traditional and a reform-based teaching approach in their perceptions (Boaler, 2002). These traces are most evident in Sofie's and Stig's descriptions. On the one hand, they claimed that a traditional teaching approach is outdated. On the other hand, they believed that teacher explanation and solving many tasks can be a very good and necessary approach to helping students learn. Both Stig and Sofie seemed to be focused on teachers' explanations in their approach to teaching and maintained that this was very important for facilitating students' learning in mathematics. Such a view of teaching and students' learning in mathematics fits with how Boaler (2002) described traditional teacher-centred mathematics teaching, and also appears to imply a behaviouristic view on learning.

Teacher explanation was important in Sofie's perception on teaching and students' learning, and Stig also appeared to agree with such a position. As mentioned above, the perception on "engaging with students' thinking" sees it is a method that better equips teachers to explain mathematics to students, which also applied to the perception of "mathematical discussion". These two perceptions can therefore be considered as practices they act on to support their view on students' learning. Teacher explanation thus appeared to be a principle that guided Sofie and Stig in their classroom practice in the same way as the principles in ambitious teaching guide teachers in the use of classroom practices and mathematical knowledge (e.g., Forzani, 2014; Lampert et al., 2013; McDonald et al., 2013). Another principle that appears to guide Stig and Harald in their classroom practice is the perception that there are different types of students who can be categorised by the way they learn. This perception affected both the composition of student groups and the number of mathematical discussions. This perception

also seemed to be an important reason for engaging with students' thinking, as this could inform the teacher both about how students learn and their learning trajectories. The teachers' intention behind engaging with students' thinking and facilitating mathematical conversations may therefore appear to be of a different nature than described for ambitious mathematics teaching (e.g., Kazemi & Wæge, 2015; Lampert et al., 2013).

Conclusion and Potential Implications

This study aimed to explore teachers' perceptions of their classroom practice and students' learning before participating in a PD program based on promoting the use of ambitious mathematics teaching practices. Bearing the analysis and discussion of the findings in mind, this last section will conclude and point out possible implications that the teachers' perceptions can have when participating in a practice-based PD program that promotes the use of ambitious mathematics teaching.

A Need for Common Understanding of Key Concepts

The findings reported in this paper show that the teachers' perceptions on key concepts related to classroom practice and students' learning in mathematics may differ, even when they initially appear to correspond. When analysing the data, the researcher found that the teachers used concepts similar to ambitious teaching when describing their perceptions, which they also believed were important features of good classroom practice. These perceptions were grounded in their understanding of how to facilitate students' learning and appeared to work as principles that guided them in their work as mathematics teachers. Sofie's situation is a good example because her first description relating to engaging with students' thinking and facilitating for mathematical discussions aligned with the principles in ambitious teaching (Gibbons et al., 2017), while her underlying purpose for acting on practices supporting this principle was not. Therefore, the difference in the teachers' perceptions on these key concepts is not found in their description of the perceptions but in the underlying purpose for acting on these perceptions. This underlying purpose is based on the ideas embedded in the teachers' craft knowledge, which guided their classroom practice (Ruthven & Goodchild, 2008). Both Sofie and Stig maintained that mathematics must be explained to the students in the process of helping them to learn. The underlying purpose can therefore be considered as their view on how students learn mathematics and can in this sense be understood as an important part of their craft knowledge (e.g., Cooper & McIntyre, 1996; Ruthven & Goodchild, 2008; 2015).

This study has shown that the teachers used some of the same concepts as in ambitious mathematics teaching (Gibbons et al., 2017) in their description of their perceptions of teaching and students' learning. However, their understandings did not always align, which was evident when the underlying purposes behind these perceptions were revealed. Working with teachers who have perceptions on key concepts that differ from those described in the development program might lead to possible implications for the providers or others involved. For example, teacher educators or other actors contributing to the teachers' PD work might be left with the impression that teachers are talking about key principles for ambitious mathematics teaching (Gibbons et al., 2017) when they describe how they perceive teaching and students' learning, rather than what really reflects their intentions behind their perceptions and their actual classroom practice. Thus, the program providers might benefit from clarifying the conceptual understanding of important key concepts used in the development program at an early stage to ensure that the PD leaders and teachers involved are using the same language. As such, clarifications might also contribute to aligning the information and practical resources provided in the program with the teachers' existing understandings and practice, which strongly influences the extent to which they are willing to use them (Timperley et al., 2007).

A Need to Understand Teachers' Perceptions

Differences in how teachers understand concepts may also concern teacher educators when conducting a teacher PD program. Timperley et al. (2007) argued that the conceptual understandings and practical resources offered through the learning experience within teacher PD must make sense to the recipients in terms of their existing understandings and practice contexts. The findings in this study show that it is important to take the teachers' view on teaching and learning into account as this view appeared to affect their perception of the aspects to be developed in the PD programme. In this way, the teachers will be better able to experience the content as relevant to their needs (Elmore, 2002). Moreover, the findings in this study show that the teachers have a different set of perceptions than what is seen on the surface. Bearing in mind Cooper and McIntyre's (1996) finding that teachers develop their professional craft knowledge through their involvement in processes of reflection and practical problem solving, PD programs therefore need to challenge the teachers' underlying perceptions and encourage them to reflect on their view on students' learning so they can develop their classroom practices and further expand their vision of what is possible. It seems essential that a practice-based development program, like the MAM project, must therefore aim to develop teachers' existing perceptions on what they are to develop by first mapping the teachers' craft knowledge in relation to the topic. It is this knowledge that the teachers relate to when assessing whether the teacher PD makes sense in relation to their existing understandings and knowledge (e.g., Desimone, 2009; Timperley et al., 2007). In this particular case, it may be necessary to identify the teachers' understanding of key concepts, either before starting a practice-based development program or in the start-up phase. Identifying and challenging the teachers' understanding of key concepts could support their development of the purpose of the program and why they should act upon it (Postholm, 2008, 2021). Additionally, the differences in understanding may be addressed, challenged, or promoted according to the goals and aims of the PD program, thus avoiding unnecessary misunderstanding that might undermine the teachers' development.

Mapping this terrain of teachers' perceptions is nevertheless easier said than done. A person's perception may not have the same meaning as what an observer might think it means, as the teachers' perceptions do not necessarily reveal the underlying purpose for acting on them. One unfortunate drawback could be to assume that there is a one-to-one correspondence between what is stated and the concept that is used by the teacher, and how those statements and concepts are understood by another person. However, in this study, the underlying purpose determined which practices made sense to the teachers and, therefore, also which practices they were willing to use. Hence, teachers must be challenged cognitively to reflect on these underlying purposes (Avalos, 2011), for example through questions such as, "What kind of classroom practices do these perceptions on teaching lead to? What impact do they have on students' learning?" and more importantly, "How do the teachers' perceptions align with (or not) the coterminous nature of teacher development programs? Examining teachers' craft knowledge to ensure consistency between teachers' existing knowledge and the content of a practice-based development program might therefore not be enough if "craft knowledge describes the knowledge that arises from and, in turn, informs what teachers do" (Cooper & McIntyre, 1996, p. 76).

The findings in this study show that teachers' actual perceptions on teaching and students' learning in mathematics are the ones related to their underlying purposes for acting on them. These perceptions are based on the knowledge they draw on when explaining their thinking that underlies these purposes, and they first become visible through these explanations. Moreover, the findings show that this knowledge is not directly linked to their practice, but largely informs the choices they make in their practice. The teachers' underlying purposes for what they do is therefore not accessible by only observing their practice or discussing it superficially. Therefore, if the teachers' existing understandings and knowledge are to be taken into account, there must be an investigation into their perceptions of the underlying purpose of their teaching and classroom actions. As has been seen here, the underlying purposes for what they do might be hidden and remain hidden, not only to the project management but also to the teachers themselves. The purpose is what needs to be understood and further challenged if their classroom practices are to be developed. Such an investigation into the teachers' perceptions

can form an important starting point for their PD and further contribute to understanding how a PD program, in which they participate, can function as an aid to broaden each teacher's understanding of ambitious teaching and how it promotes student learning.

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