Making Learning Visible: Cases of Teacher Candidates Learning to Respond to Errors Through Multiple Approximations of Practice[1]

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Approximations of practice in teacher education provide learning opportunities for teacher candidates (TCs) in a space that is less authentic than classroom teaching, yet still maintains the complexity and interactive nature of core teaching practices. There exists a need for more research focused on documenting TCs' learning through such pedagogies. In this paper, we highlight two cases of secondary mathematics TC learning around the practice of responding to errors in whole-class discussion. We consider multiple components of resources TCs brought to the work of teaching that were supported by and made visible through two types of approximations—coached rehearsals and scripting tasks—at multiple moments in time over a semester-long methods course. This work contributes to the research on practice-based pedagogies in teacher education by offering detailed images that illustrate the nature of two TCs' learning over time and by offering approaches for understanding TCs' practice and development through multiple data sources.

Introduction

Practice-based approaches to teacher education, such as approximations of practice, offer promising learning opportunities for teacher candidates (TCs) by structuring TCs' investigation into and enactment of the work of teaching (Anthony et al., 2015a, 2015b; Kazemi & Wæge, 2015; Lampert et al., 2013). Approximations of practice provide a space that is less authentic than classroom teaching, yet still maintains the complexity and interactive nature of core teaching practices (Grossman et al., 2009). To support the growing body of work detailing the use of practice-based pedagogies in teacher education, there exists a need to characterise what TCs learn through approximations of practice with enough detail to evaluate the impacts of the

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pedagogical approach (Arbaugh et al., 2019, 2020; Janssen et al., 2015; Shaughnessy et al., 2018). Our previous work examined learning *opportunities* afforded by approximations of practice (Baldinger et al., 2021; Campbell & Baldinger, 2021). Here, we look across approximations to detail TCs' learning.

We use a case-based approach to unpack what two TCs learned through their engagement with approximations of practice—coached rehearsals and scripting tasks—over the course of a semester. The approximations centred the practice of responding to students' errors in whole-class discussions. This is a critical space in which teachers can value and leverage student reasoning, but may also potentially marginalise students and their ideas in problematic ways (Engle et al., 2014; Langer-Osuna, 2018). The cases illustrate what and how TCs learn through approximating practice, thus contributing to the scholarship on the use of approximations of practice and implications for their use in mathematics teacher education.

Perspective on Teacher Learning

Understanding how and what teachers learn and develop is a critical aspect of teacher education. We take the perspective that learning to teach entails teachers' changing participation in a community of practice (Lave & Wenger, 1991). Such changes in participation include teachers developing adaptive expertise to be responsive to the contexts in which they work (Hatano & Inagaki, 1984). We have adopted Hammerness and colleagues' (2005) "Framework for Teacher Learning" to delineate what teachers learn through their participation in communities of practice. This framework identifies *vision*, *dispositions*, *understandings of content and students*, *tools*, and *practices* as components of learning and within which teachers can develop as part of their changing participation. This conception of teacher learning enables us to avoid focusing solely on actions, such as instructional moves, disconnected from the complex, situational nature of teaching and the pedagogical reasoning drawn upon in that work (Horn & Kane, 2019; Kavanagh et al., 2020; Philip et al., 2019).

Vision represents teachers' sense of where they are going, how they will get there, and what is possible and desirable in teaching (Feiman-Nemser, 2001; Hammerness, 2001). A teacher's vision of teaching is an evolving, "dynamic view of the future" that guides practice and is shaped by opportunities to engage in the work of teaching (Munter, 2014, p. 587). *Dispositions* are "habits of thinking and action" related to teaching, students, and the teacher's role (Hammerness et al., 2005, p. 387). They inform a teacher's sense of what is valued in the teaching and learning of mathematics.

Understandings represent a teacher's deep knowledge of their subject and how to make it accessible to others. This component of teacher learning is consistent with the conception of mathematical knowledge for teaching (e.g., Ball et al., 2008), which includes knowledge of content in general and in ways unique to teaching, knowledge of how students develop and communicate mathematical ideas, and curricular knowledge.

Teachers need to develop a set of *tools and practices*, based on their understandings of content and students, that will enable them to enact their vision of and dispositions about mathematics teaching and learning. Tools, such as using talk moves to lead whole-class discussions (e.g., Chapin et al., 2013), mediate teachers' actions in the work of teaching. These

tools are integrated with practices—a sense of when, where, why, and how for using particular tools—that encompass one's "repertoire of classroom enactment" (Feiman-Nemser, 2001, p. 1018). Given their interrelation, we follow others in talking about "tools and practices" together (e.g., Arbaugh et al., 2019; Ghousseini & Herbst, 2016).

Approximations of Practice

Approximations of practice are opportunities for TCs to engage in the work of teaching and are a powerful support for TCs' learning (Arbaugh et al., 2020; Ghousseini & Herbst, 2016; Grossman et al., 2009; Kazemi & Wæge, 2015). In approximations, TCs experiment with practices that are critical for beginning teachers to enact with skill (Schutz et al., 2018). By their very nature, approximations of teaching are not "the real thing", with differences in terms of context, setting, and level of risk, designed to optimise opportunities for learning (Grossman et al., 2009, p. 2078).

We focus on two types of approximations, coached rehearsals, and scripting tasks, that differ from one another in their degree of authenticity, the resulting learning opportunities, and what they make visible about TCs' resources. In coached rehearsals, one TC acts as the teacher while other TCs act as students, and the teacher educator provides in-the-moment coaching through interjections (Kazemi et al., 2016; Wæge & Fauskanger, 2020). Rehearsals structure the relationship between teacher, students, and content so the teacher can engage in and with the contingent and interactive work of teaching and further develop their resources for that work.

Scripting tasks are approximations that require TCs to make sense of and respond to student reasoning (Crespo, 2018; Zazkis, 2017). TCs are presented with a classroom scenario and then demonstrate, through written dialogue, how they might continue the discussion. These scripts represent, in part, TCs' imagined response to a particular student contribution or classroom situation. They also represent TCs' sense of how students might contribute further, giving insight into TCs' view of what is reasonable or desirable in a classroom episode.

Coached rehearsals are more authentic than scripting tasks, in that they allow for interactions with others, and serve as approximations of the real classroom. Scripting tasks are less authentic, but still approximate practice by asking TCs to determine exactly what they might say during a discussion. By considering both approximations in concert, we have the potential to create a more nuanced view of TCs' learning.

Responding to Errors

Practice-based teacher education pedagogies are fundamentally grounded in a focus on the complex practices of teaching. In our work, we use coached rehearsals and scripting tasks to provide TCs with learning opportunities around responding to errors during whole-class discussions. We chose this focus because discussions are a critical space in which students can develop a broad set of mathematical proficiencies (e.g., Australian Association of Mathematics Teachers, 2006; National Council of Teachers of Mathematics, 2014). They are also a space in which students can be marginalised and in which inequities can be perpetuated (Engle et al., 2014; Langer-Osuna, 2018). This risk is most apparent when students contribute errors—ideas

that are not yet mathematically complete, precise, or correct (Brodie, 2014). Such contributions are often and unfortunately disregarded, quickly corrected by others, or used to position students at a deficit (e.g., Bray, 2011; Santagata, 2005; Shaughnessy et al., 2020). Furthermore, TCs tend to focus on procedural, rather than conceptual, aspects of student thinking (e.g., Casey et al., 2018; Runnalls & Hong, 2019). Such actions perpetuate harmful notions of what it means to be successful and valued in mathematics.

We aim for TCs to see errors as resources to the collective mathematical work of the classroom, even if those contributions are not yet consistent with canonical mathematics (Borasi, 1994; Kazemi & Stipek, 2001; Nesher, 1987). Teachers must learn to respond to student contributions in principled ways, negotiating dilemmas such as how students and their ideas are positioned alongside the needs of the rest of the class. Productive responses to errors support sensemaking and position students as capable doers of mathematics (e.g., Jacobs & Empson, 2016). Errors should be made public and can become objects for students to consider, question, and evaluate as part of building mathematical understandings (Bray, 2011; Stockero et al., 2020).

Practice-based pedagogies, and approximations in particular, are potentially powerful ways for TCs to learn about, and be better prepared for, responding to errors. Kazemi and colleagues (2009) saw pedagogies of practice centred around rehearsals as spaces to intentionally work on responding to errors, through instructional activity protocols as well as novel contributions from the teacher educator. Additionally, Lampert and colleagues (2013) found that addressing errors was a common topic of discussion in exchanges between coaches and TCs during rehearsals—what Wæge and Fauskanger (2020) call teacher time outs. Anthony and colleagues (2015a, 2015b) also found that rehearsals supported learning to respond to errors as TCs developed adaptive expertise in eliciting and responding to students and positioning them as mathematically competent. Rehearsals can be deliberately designed to foreground this challenging work through the strategic use of planted errors to provide opportunities for learning (Baldinger et al., 2021; Campbell et al., 2020).

Study Motivation

Approximations of practice can provide TCs with multi-faceted learning opportunities around responding to errors in whole-class discussions, but there exists a need for more research documenting the nature and form of TCs' learning through practice-based pedagogies (Arbaugh et al., 2019; Shaughnessy et al., 2018). To that end, we use the Framework for Teacher Learning to investigate what TCs learned about responding to errors in whole-class discussions through multiple approximations of practice. This framework enables us to richly document TCs' learning across components. The multiple approximations provide a nuanced perspective on learning, as each approximation makes visible different strengths and areas for growth in TCs' resources. Our focus on the practice of responding to errors represents a critically important space for TCs' learning and, thus, serves as a particularly illuminating lens for this study.

To uncover TCs' learning, we use a case-based approach (Merriam, 1998) to dive deeply into the experiences of two TCs—Greg and Travis[2]. We ask the following research question: What

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[2] All names are pseudonyms.

learning about responding to errors was made visible through TCs' engagement in multiple approximations of practice, with respect to each component? Understanding what Greg and Travis learned—and the learning these two cases represent—contributes to a more complete picture of how pedagogies of practice can document and support teacher development.

Methods

Our work is the product of a multi-year collaboration situated in secondary mathematics methods courses at two institutions in the United States. At the time of the study, Travis and Greg were both enrolled in yearlong post-baccalaureate initial teacher licensure programs. The methods courses were semester-long (i.e., 15 weeks) and occurred prior to full-time school experiences. Greg took the methods course taught by the first author, which met daily for 60-90 minutes at partner school sites. Travis took the methods course taught by the second author, which met in three-hour sessions one day per week in a university classroom.

Data were collected across the semester to capture numerous learning activities (see Figure 1). Our analyses focus on the two approximations of practice—scripting tasks and coached rehearsals—at the centre of this study, while acknowledging the presence of other coursework that provided learning opportunities both related and unrelated to responding to errors. Case selection and data sources are described in the following sections.
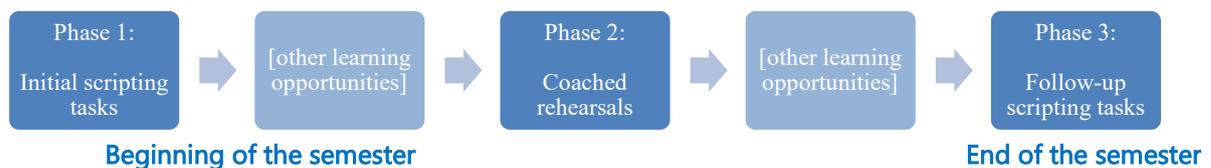
Phase 1: Initial scripting tasks		[other learning opportunities]		Phase 2: Coached rehearsals		[other learning opportunities]		Phase 3: Follow-up scripting tasks

Beginning of the semester   **End of the semester**

Figure 1. Data Collection Timeline

Case Selection

All TCs (13 at Greg's institution, eight at Travis's) completed two scripting tasks ("polygon," "graph") at the start of the course ("initial") and again at the end of the semester ("follow-up"). Six TCs (three at each institution) rehearsed leading a sorting activity (Campbell et al., 2020). We identified cases from among those TCs who rehearsed the sorting activity and completed both the initial and follow up scripting activities using theoretical sampling (Merriam, 1998). Based on earlier analyses of the scripting tasks (e.g., Campbell & Baldinger, 2021), we selected two TCs— Greg and Travis—who exhibited potentially interesting patterns of change from their initial scripts to their follow-up scripts, for more in-depth consideration. We selected Greg as a potential case of demonstrating multiple positive learning outcomes across components. Travis was a case where learning seemingly ran counter to our expectations, in that he did not resolve the errors in his initial scripts, but did resolve the errors in his follow-up scripts. We remained open to emergent themes as we embarked on our analysis, rather than restricting ourselves by trying to confirm our original hypotheses that motivated selecting these two cases. For each

case, we analysed data from responses to the initial scripting tasks, the coached rehearsal, and responses to the follow-up scripting tasks.

Scripting Tasks

Each scripting task presented TCs with a scenario in which a student contributed an error. The evaluation of the student contribution as an error left to the TC. TCs responded to the following prompts for each scenario:

1. Imagine that you are the teacher. Write the next 5-8 lines of script continuing this discussion.
2. Provide a rationale for why you decided to continue the discussion in this way.
3. What do you think [Student contributing the error] is thinking about [mathematical content]?
4. How, if at all, would you want [Student's] thinking to shift during this discussion?

After completing the follow-up scripting tasks, TCs responded to one additional prompt, asking them to reflect on how they felt their responses had changed over time.

The polygon and graph scripting tasks differed along two dimensions: mathematical content and classroom task situation. These differences were built in to diffuse the impact of TCs' specific mathematical knowledge and knowledge of particular tasks and classroom activity structures. The order in which TCs responded to the scripting tasks was randomly determined by Qualtrics, a survey management platform used to administer the tasks.

Polygon Scripting Task

This scenario centred on the use of a sorting activity designed to elicit and refine a definition of a polygon. The scenario was set at the start of the whole-class discussion after students sorted the cards in small groups, and is shown in Figure 2. The error included in this task occurs when Jessie identifies Shape J as a polygon, when it is not. TCs were asked to continue the discussion after Jessie shared reasoning for sorting Shape J as a polygon.
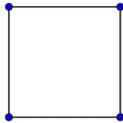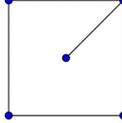

| Teacher: | Who can tell me a card that was easy to sort, and that you know for sure is a polygon? Rosalia? |
Rosalia:	In my group we had Shape Q as a polygon.
Teacher:	How do you know that this is a polygon?
Rosalia:	Because it is a square.
Teacher:	How does that tell you it's a polygon?
Rosalia:	Well, in a square, all the sides are straight lines.
	[*The teacher records this reasoning on the board.*]
Teacher:	Who can tell me another card that you know is a polygon? Jessie?
Jessie:	Shape J.
Teacher:	How do you know that Shape J is a polygon?
Jessie:	We noticed that it's just like Shape Q – the one Rosalia shared – it's a square.

Figure 2. Polygon Sort Scripting Task Scenario

Graph Scripting Task

This scenario focused on whole-class discussion about students' interpretation of a position-time graph. The discussion centres around interpretations of a graph that shows time in seconds on the *x*-axis and distance from home in meters on the *y*-axis. In the provided script (see Figure 3), Foster contributes an error by describing the shape of the graph (that it looked like hills) rather than interpreting the graph with respect to the axes.

A class of algebra students have been working on the task Journey to the Bus Stop.

Journey to the Bus Stop

Every morning Tom walks along a straight road from his home to a bus stop, a distance of 160 meters. The graph shows his journey on one particular day.
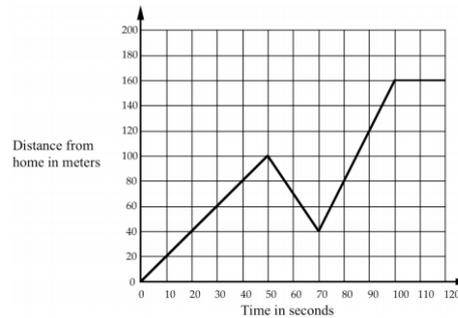
1. Describe what may have happened.
   You should include details like how fast he walked.

After students have had an opportunity to work in small groups, the teacher brings the class together for a discussion.

Teacher: Who would like to share their description of Tom's journey? Foster, thanks for volunteering.

Foster: Tom walked up a hill, down a hill, and up another hill to get to the bus stop.

Note. "Journey to the Bus Stop" image and text reprinted from *Interpreting distance-time graphs* (p. S-1), a Classroom Challenges lesson by the Mathematics Assessment Resource service (2015). CC BY-NC-ND 3.0.

Figure 3. Journey to the Bus Stop Scripting Task Scenario

Data from Coached Rehearsals with Planted Errors

All rehearsals were video recorded and transcribed. Following each rehearsal, TCs in each class participated in debrief discussions that were also video recorded and transcribed. After all the sorting activity rehearsals, TCs completed individual reflections and analysed and annotated the videos of their rehearsals.

The sorting activity rehearsals focused on defining *linear function*, with each rehearsal centred on a particular mathematical representation—graphs, tables, or equations (Campbell et al., 2020). Greg rehearsed the graph sorting activity; Travis rehearsed the table sorting activity.

For all rehearsing TCs, the mathematical focus and the sets of cards to be used were provided by the teacher educator. The provided cards included objects that would be easy to sort as examples or non-examples, and others that would elicit debate or confusion (boundary cases). To ensure that rehearsals included opportunities for responding to errors, a planted error was contributed by a non-rehearsing TC during each rehearsal.

Figure 4 shows samples of cards Greg used when rehearsing the graph sorting activity. The planted error involved a student sorting Graph D, a vertical line, as an example of a linear function. The student argued that "the graph is a straight line," making it a linear function.
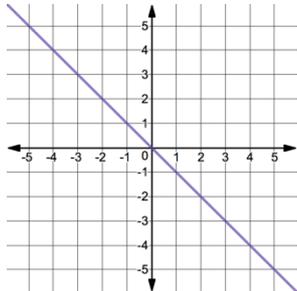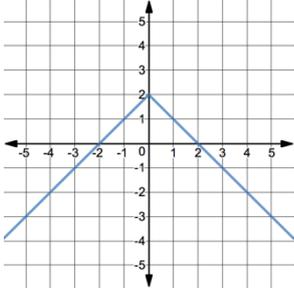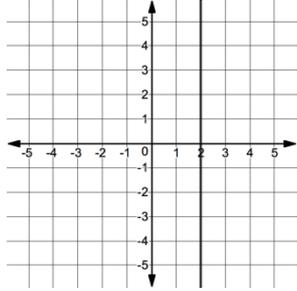
Provided Definition	Example (easy-to-sort)
A linear function is a function whose graph in the Cartesian plane is a straight line—with a constant slope everywhere—and does not have gaps.	**Graph E** Graph E shows a straight line through (0, 0) with negative slope.
Non-example (easy-to-sort) **Graph H** Students are likely to recognise Graph H as an absolute value function.	Boundary case (hard-to-sort) **Graph D** Graph D shows a vertical line, but it is not a function. Students can discuss the "linear" and the "function" parts of the definition.

Figure 4. Cards and definitions in the graphical sorting task.

Figure 5 shows samples of cards Travis used when rehearsing the table sort activity. The planted error for this task involved a student sorting Table D, which "skips" x values, as a non-example of a linear function. The student argued that "the values in the y column don't increase by the same amount," making it not a linear function.

Provided Definition	Example (easy-to-sort)
A linear function is a function where for any two ordered pairs in the table, the ratio of the change in y to the change in x is constant.	Table A <table><tr><td>x</td><td>y</td></tr><tr><td>0</td><td>2</td></tr><tr><td>1</td><td>6</td></tr><tr><td>2</td><td>10</td></tr><tr><td>3</td><td>14</td></tr></table> Table A shows x and y values increasing together at a constant rate.
Non-example (easy-to-sort) Table G <table><tr><td>x</td><td>y</td></tr><tr><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td></tr><tr><td>2</td><td>4</td></tr><tr><td>3</td><td>9</td></tr></table> Table G shows a pattern students are likely to recognise as quadratic.	Boundary case (hard-to-sort) Table D <table><tr><td>x</td><td>y</td></tr><tr><td>-2</td><td>-7</td></tr><tr><td>-1</td><td>-1</td></tr><tr><td>1</td><td>11</td></tr><tr><td>4</td><td>29</td></tr></table> Table D "skips" x values which may obscure the constant slope. Students can discuss how changes in x and y need to be related to one another.

Figure 3. Cards and Definition in the Tabular Sorting Task

Data Analysis

Our goal was to document a holistic picture of learning for each TC, with attention to all components in the framework for teacher learning, through "consolidating, reducing, and interpreting" our multiple data sources (Merriam, 1998, p. 178). We organised data for each case by timepoints in the semester: responses to the initial scripting tasks; video, transcribed, and written data from the coached rehearsal; and responses to the follow-up scripting tasks (see Figure 1). For each case, we first looked at data from a given timepoint (e.g., the initial scripting task responses) and crafted analytic memos (Miles et al., 2014) aligned to vision, disposition,

understanding of content and students, and tools and practices (see Table 1). Our memos characterised how each component was made visible in that set of data, leading to claims about the resources made visible by each TC at given moments in time. This resulted in 24 memos (one for each of four components of teacher learning, at three distinct timepoints, across two cases).

Table 1
Framework for Teacher Learning Definitions

Component	Definition
Tools & Practices	The use or mention of particular teaching moves or routines that support students' work in discussion
Vision	Indications of an aspect of practice presented as desirable
	Descriptions of typical classroom activity structures
Dispositions	Naming the commitments one has toward teaching and learning
	Perspectives on or preferences about mathematics
	Perspectives on students' mathematical thinking
Understandings	Indication of understanding of mathematical content
	Articulation of student approaches to tasks
	Descriptions of students' mathematical understandings and development

Note. Table adapted from Ghousseini and Herbst (2016, p. 86) and Arbaugh et al. (2019, p. 25).

In our second round of analysis, for each case, we looked across the memos written for each of the three timepoints for a given component of the framework (e.g., vision). We wrote additional memos to make claims about the TC's learning within the component, drawing on our observations from the first round of memos while also returning regularly to the data to check our hypotheses. In the third and final round of memo writing, we synthesised our claims about learning across components for each TC to identify overarching themes, with the aim of producing holistic and nuanced depictions of learning.

Findings

In this section we share depictions of Greg's and Travis's learning. These cases provide distinct images of learning and serve as examples of how that learning can be documented through coordinated analysis across two distinct approximations of practice.

Greg

Learning to Position Students to Contribute and Grapple with Mathematical Ideas

We make three claims about Greg's learning as revealed by multiple approximations of practice. First, Greg learned to focus more on orienting students to one another's ideas as a response to student contributions, through his actions and his sense of how a discussion should progress. Second, while Greg maintained attention to the mathematical ideas necessary to making progress toward an established goal, he exhibited shifts in how students were positioned as able

to offer those new ideas. Finally, Greg developed a more robust perspective of students' sensemaking, including valuing the mathematical reasoning behind students' errors.

Orienting Students to Others' Ideas as a Response to Student Contributions

A clear distinction in Greg's initial and follow-up scripts was the form of the teacher's response to the student who presented the error and in the inclusion of other student voices in the ensuing discussion. Table 2 shows Greg's initial and follow-up polygon scripts. The patterns visible in this task were similarly evident in Greg's response to the graph scripting task.

Table 2
Greg's Initial and Follow-Up Scripts

Initial Script		Follow-up Script	
Teacher:	Well what about this extra line here? Does it make a difference?	Teacher:	Who can tell me another reason why shape J is a polygon?
Jessie:	Idk, maybe	Student:	Like Rosalia said, all of the sides are straight.
Teacher:	What does it mean to be a square?		
Jessie:	All sides are equal length and opposite sides are parallel	Teacher:	Who can tell me why shape J might not be a polygon?
Teacher:	So does this fit the definition of a square?	Student:	There is that line in the middle so it is not really a square.
Jessie:	No, that line isn't parallel to anything	Teacher:	Can someone expand on what _____ just said?
Teacher:	So what can we say about it being a polygon?	Student:	Well that segment has one end not meeting any other sides.
Jessie:	that it is not because that line only connects to that one edge and the other lines have to [*sic*] edges		

In the initial script, Greg portrayed a conversation that involved only the teacher and Jessie, stemming from Jessie's contribution of an incorrectly sorted card. The teacher used a string of funneling questions (Wood, 1998). Through those questions, the teacher inserted new mathematical ideas that did not naturally build on Jessie's original or subsequent contributions (e.g., "What about this extra line here?"). The questions and new ideas were seemingly posed to evaluate the contribution as incorrect and to resolve the error within a few talk turns.

In contrast, in the follow-up script, Greg presented a teacher who oriented students to one another's ideas by asking students to provide additional reasoning in agreement or disagreement or to reason about and expand on peer's ideas (Chapin et al., 2013). In doing so, the teacher did not insert new mathematical ideas or imply a negative evaluation of Jessie's contribution. In fact, the first move elicited additional ideas about why Shape J could be a polygon. The follow-up script included multiple students (as many as three) and did not immediately resolve the error. This represents a clear shift in terms of the tools and practices used to respond to a student error.

Changes in using orienting moves to respond to a student error can also be seen in Greg's rehearsal. The first card elicited during the rehearsal (Graph D; see Figure 4) was incorrectly sorted as an example of a graph representing a linear function. The following discussion ensued:

Greg: Graph D, alright. Why was this one easy to sort? As a linear function [asking for clarification from Student G1[3]]?

SG1:   Yes. Because it is a straight line.

Greg: It's a straight line. Alright, you are correct, it is a straight line. Does anyone agree with this? Who agrees with this?

SG2:   [after a long pause] I agree.

Greg: You agree? Alright, does anyone disagree with this?

After eliciting ideas in disagreement, Greg tabled the discussion before any resolution or consensus, stating, "we can come back to it."

Greg improved his use of orienting moves as a response to student contributions during the rehearsal. After discussing a second card, the teacher educator interjected:

TE: Can I pause you for a second? I really like that you treated those two the same. You asked, "Is there any agreement?" and "Is there any disagreement?" I want you to try rephrasing those questions so it's not like putting it for a vote.

Greg: Okay.

TE: You asked the same both times, which was really good, um, maybe say something like "Who can give reasons that they agree?" or "Who can give reasons that they might disagree?" Because it's not democratic which things count [as examples or non-examples].

Greg: Yeah, yeah, for sure.

Greg enacted this refined version of the agree or disagree question during the rest of his rehearsal and in his follow-up scripts. Across the semester, not only did Greg begin to incorporate the tool of orienting moves but he also had the opportunity to further develop his practice of its use.

We also saw evidence of Greg's evolving vision related to orienting. When asked to consider how his responses to the scripting tasks had changed over time, Greg wrote that he tried to "expand more on the original ideas rather than going straight to find the 'right' solution." This demonstrates a different vision—a sense of what is desirable and possible—for how to respond to errors.

This vision is consistent with ideas that emerged following the rehearsal. During the rehearsal debrief, Greg talked about his satisfaction of having a "student-led" discussion where he did not evaluate student contributions in the moment. In the rehearsal video annotations, Greg and his group tagged many instances of orienting moves in which the teacher did not evaluate student contributions and supported students to lead the discussion. Overall, Greg's

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[3] Students are numbered sequentially; the letter refers to the rehearsing TC (e.g., Student G1 is the first student speaking in Greg's rehearsal).

scripting tasks and rehearsal experience highlight learning around the vision, tools, and practices related to orienting students to one another's ideas in discussion.

Shifts in Positioning of Students as Able to Offer Mathematically Valuable Contributions

Consistent across Greg's approximations of practice was his focus on the role of certain mathematical ideas in steering the discussion toward the established mathematical goal. For example, in his rationale for the initial polygon scripting task, Greg wrote:

> I believe it is important for them to re-think what a square is and modify what they are saying about Shape J. From there it might be easier for the students to recognise whether it is a polygon or not.

Greg did not focus on specific pedagogical approaches, instead emphasising the mathematical ideas that needed to be inserted in order to make progress toward the mathematical goal.

Identifying the relevant mathematical ideas of an activity reveals Greg's understanding of content. What changed over time was Greg's sense of who is able to contribute those key mathematical ideas, revealing learning across the components of vision and understanding of students. Greg's scripts for the initial and follow-up polygon scripting tasks provided in Table 2 demonstrate evidence of this shift.

In Greg's initial script, the teacher inserted many new mathematical ideas, in part through leading questions. For example, the teacher first asked, "well, what about this extra line here?", referring to the segment in the interior of Shape J. With that question, Greg, via the teacher, brought a mathematical idea into the discussion that was key to determining how Shape J should be sorted. In his follow-up script, Greg included the same mathematical idea, but did so through a contribution from a new student. The teacher's role was to elicit that contribution and further highlight that idea by asking another student to expand on it.

Through his focus on the need for certain mathematical ideas to surface in a discussion, Greg demonstrated a shifting vision for how that could be done. In his rationale for his follow-up polygon scripting task, Greg wrote:

> I would want other students to think about why it is possible for J to be a polygon, but then I would also want other students to explain why they think it is not a polygon. Although it is not a polygon, I want students to be thinking about both reasonings.

This rationale reveals attention to how students can be the ones to contribute ideas in a discussion, and that students need the opportunity to consider those varied ideas. Greg's representation of students as being able to offer these mathematically valuable ideas on their own, and to make sense of multiple, sometimes conflicting, ideas, demonstrates a changing understanding of students and a changing vision of mathematics teaching.

Valuing Mathematical Reasoning and Sensemaking in Student Errors

Finally, Greg learned to more fully value the complexity of students' mathematical reasoning, including in instances of errors. This aspect of learning connects to Greg's developing understanding of students and content, as well as his dispositions toward students' mathematical thinking. In his initial scripting tasks, Greg presented errors as something to be

quickly resolved. The teacher provided the ideas that would lead the student to come to a different conclusion.

In writing about Foster's thinking and how he would want to see it thinking shift in the initial graph scripting task, Greg wrote that Foster thinks the graph "looks like a hill" and, "I want him to take into account what the graph actually represents and *not just what it visually looks like* (emphasis added)," by attending to how the graph's axes are defined. This perspective reflected a theme in Greg's responses—that a student error is the product of how something looks on the surface and correcting the error would involve the student being directed to consider some additional information. We see this as positioning student thinking at a deficit, reflecting Greg's understanding and dispositions.

Greg's responses in the follow-up scripting tasks regarding Foster's thinking evolved in some notable ways. First, Greg provided more detail on what Foster might be thinking, stating:

He is thinking of the distance as maybe height, meaning since the graph goes up, then down, and up again that Tom is going up for a while then goes down and back up.

Greg provided additional information of what Foster might be thinking in coming to that conclusion. Instead of only focusing on what Foster was not attending to, Greg's follow-up response acknowledged that Foster was attending to something in order to come to his conclusion, thus validating Foster's idea, even though it was incorrect.

Additionally, Greg's responses to the follow-up scripting tasks acknowledge that the error is not necessarily a one-off mistake and that ideas, such as Foster's in the graph scripting task, "may be a common idea between students." Greg not only assigned mathematical value to errors, he also recognised that multiple students could be coming to similar conclusions. We see these shifts toward positioning student thinking as an asset as representing Greg's learning along the components of understanding of students and dispositions regarding student thinking.

Conclusion

Greg's scripting tasks and rehearsal revealed a transition from teacher-led discussions designed to correct errors toward student-led discussions designed to elicit multiple ideas and give students opportunities to engage with them. In that transition, Greg revealed learning across all the components of teacher learning in concert.

Travis

Developing Awareness of and Negotiating Competing Demands

When looking across all the initial and follow-up scripts, Travis's scripts stood out because he was the only TC to not resolve the errors in his initial scripts, but to resolve the errors in his follow-up scripts. For example, Travis's polygon scripts are shown in Table 3.

Table 3
Travis's Initial and Follow-Up Polygon Scripts

Initial Script		Follow-up Script	
Teacher:	What is your reasoning for determining that Shape J is a square?	Teacher:	Rosalia, can you re-state what Jessie was trying to say about Shape J?
Jessie:	Well it has 4 straight sides of equal length that are connected.	Rosalia:	I think Jessie said that Shape J is a polygon because it is a square.
Teacher:	Does anyone else agree with Jessie, that Shape J is a square, and therefore a polygon?	Teacher:	Alright, does anyone agree or disagree that Shapes Q and J are polygons because they are squares?
Melinda:	I disagree that it is a square and a polygon.	Student:	I don't think Shape J is a square. A square only has 4 sides that are straight lines. That shape has 5 straight lines.
Teacher:	And why do you believe that?		
Melinda:	Well it looks like a square with an extra line inside of it, and that line isn't connected to another line on both sides. So it isn't a square or a polygon.	Teacher:	What do you think about what student said about this shape not being a polygon because it isn't a square?
Teacher:	Could you say in different words why you think it is not a polygon?	Jessie:	If we are looking at the shape as a whole, then it would make sense that it isn't a square. If you ignore that diagonal line, we have a square, but I don't think we can do that after hearing that explanation.
Melinda:	The figure has a side that is not connected to two other sides. (Teacher records this reasoning on the board)		

In our initial analyses, we saw resolution of the error in a short dialogue as less-than-desirable, because it either involved disregarding the student and their idea or illustrated a student who can be convinced of a different idea in a way that seemed unrealistic. As such, this change in Travis's scripts raised questions for us in terms of what might have been learned. In this section, we describe three ways in which our deeper dive into Travis's learning revealed how a TC might learn to move beyond a generic sense of "good practice." First, while orienting students to one another was part of Travis's initial practice, over time he brought more intentionality to his orienting. Second, Travis developed a multi-faceted perspective on how to work with students to make progress toward mathematical goals. Finally, Travis began to be able to articulate the ways in which context matters for responding to errors.

Bringing Intentionality to the Work of Orienting Students to One Another's Ideas

From his initial scripts, Travis appeared to enter the semester with a repertoire of tools and practices that included orienting, accompanied by a vision for using such tools and understanding how students might respond in result. For instance, in both initial scripts, Travis used moves such as "Does anyone else agree or disagree with Foster?" with the intention, per

his rationale, "to give another student an opportunity to participate as well as to give a different opinion." In both initial scripts, a second student entered the conversation and disagreed with the student who made the error. In addition to a vision for how a discussion might unfold, this reveals Travis's understanding that students can and will disagree with one another.

Through his rehearsal, Travis continued to experiment with and consider the use of different types of orienting moves, such as turn-and-talk prompts, in part with the support of the coach. After a student shared their reasoning for sorting a card as an example of a linear function, Travis asked, "Did anyone have … a different reason they thought this could be an easy example to sort?" This orienting move, as opposed to generally asking for agreement or disagreement, is similar to the move Greg came to use through his rehearsal and different from what Travis demonstrated in his initial scripts. For Travis, according to his video annotation, this move was used to bring more students into the conversation and because he was "curious if they had another reason." During a coaching exchange, though, Travis highlighted his continued consideration of the intention and use of such a prompt, for instance whether he was "fishing" for a new idea or knew of other ideas from the group.

Another moment of coaching near the end of the rehearsal introduced new tools and practices to Travis's repertoire. As the discussion returned to the planted error, Table D (see Figure 5), the teacher educator advised Travis to check back in with the student who had originally argued (in error) that Table D was not an example of a linear function. Taking this advice, Travis brought the conversation back to Student T1:

Travis:	[Student T1], how would you use these change in y's versus these change in x's to show that this is an example?
ST1:	So, I get that the y's and x's are both changing at different rates, but I don't understand like how they're connected. I get that the x's aren't constantly changing by 1 and the y's aren't, but [...] I still don't understand how that would make it an example.
TE:	[Coaching interjection] You can ask someone else.
Travis:	[Student T2], could you give me a reason why you would think with these change-in-y's versus these change-in-x's, about what [Student T3] was talking about that the ratios are staying constant?
ST2:	So, what we're looking for, I think, with linear functions is a constant rate of change. The rate of change, as [Student T3] said, is the change in y divided by the change in x. We have the changes in x on the left and the changes in y on the right. So we would end up dividing 6 by 1, then 12 by 2 and 18 by 3. In each case, we end up with 6, so it stays the same through all the points [Teacher records ideas].
Travis:	[Student T1], does what [Student T2] was discussing there, does that make more sense about how we're relating the change in y's to the changes in x?
ST1:	Yeah, I think I get it now, because you have to divide the change in y by the change in x to find what the slope is. And when you actually do it, it gives you 6 every time. I think I get it now.

In returning to Student T1, Travis found that the student was not immediately persuaded by the arguments of other students. Travis used this opportunity to support students to help one

another make connections. After a more detailed explanation from Student T2, which Travis recorded, Travis again returned to Student T1, who was now convinced and able to articulate this new reasoning. Travis tagged returning to Student T1 in his video annotation, writing that the move, "gave [Student T1] the opportunity to respond to [Student T2]'s explanation (as well as other peoples' previous explanations that I didn't record) about there being a constant rate of change." This reveals Travis's developing vision related to this aspect of his tools and practices.

The evolution and intentionality that Travis was beginning to bring to orienting in the rehearsal remained evident in his responses to the follow-up scripting tasks. Most notable was Travis's uptake of returning to the student who made the error after ideas were shared by other students. In both follow-up scripts, Travis directed a question toward the student who initially contributed the error and specified the mathematics contributed by other students to which the original student ought to respond.

His vision around orienting was reflected in his follow-up rationales. For example, in his graph scripting task rationale, he reflected,

> It is also nice to give students opportunities to work with the reasoning of other students, an action they are not given to do very often. If that student gives a correct explanation of the situation, you can then go back to [the] original student who gave an incomplete conception about the graph given to check to see if they realised where they went wrong.

Across approximations, Travis shifted from a generic vision of students explaining their reasoning and multiple students contributing to a discussion to a more complex vision of giving students the opportunity to work with one another's reasoning. His evolving vision matched the evolution of his use of orienting moves. These changes in tools, practices, and vision were echoed by a change in Travis's understanding of how students participate in discussions. While Travis initially understood that students can disagree with one another, by the end of the semester he seemed to recognise that they can also reason about each other's ideas and convince one another of different ideas.

Identifying Students' Roles in Making Progress Toward a Mathematical Goal

When asked in the follow-up scripting task to reflect on how his responses changed over time, Travis wrote, "I think I tried to focus the discussion on the goal more than I did the first time." Given the way Travis's scripts changed over time, this reflection prompted further probing of what Travis might have meant. One interpretation might be that resolving the error could be equated with focusing on the mathematical goal. However, looking across approximations provides more nuanced possibilities.

Part of Travis's developing sense of how progress can be made toward a mathematical goal was linked to his identification of the role of recording student thinking. The rehearsal provided multiple opportunities for Travis to work on the practice of recording ideas and to develop a vision for its role. Several rehearsal video annotations focused on the challenges he felt. He wrote, "it was difficult at times for me to not only process what they were saying, but to also write it down in a concise manner than I thought stayed true to the point they were trying to say." Travis regularly referred to his board work to help motivate the conversation and connect student ideas. He came to see the benefit of these connections, writing that, "We finally got to the point of visually showing that the ratio of these y and x-value changes is constantly 6, which

was the biggest point to reach during this activity." While recording ideas was not made visible in his follow-up scripts, we saw ample evidence of how Travis connected recording and visualization more generally with the work of moving toward the mathematical goal.

In the follow-up scripting tasks, Travis placed increased emphasis on ensuring that students saw "where they went wrong." In his initial scripting task rationales, Travis wrote about wanting students to develop the correct conception, but did not specify having students understand the source of or reason behind the error, nor did the initial scripts illustrate any resolution. Travis's increased emphasis on students seeing the reasoning behind their answers and the answers of others suggests that Travis developed an understanding that students can make sense of the different ideas contributed during a discussion. Furthermore, Travis recognised that errors were conceptions on which the conversation could build. Alongside this developing understanding, Travis demonstrated a clear commitment toward helping the student with the error understand and learn from it, reflecting his evolving dispositions.

These shifts in Travis's understandings, dispositions, and practices are suggestive of learning that moving toward the mathematical goal requires students to understand the sources of an error. Travis learned that the teacher alone cannot move a discussion toward the mathematical goal; they must work interactively with students to do so. This provides a different lens through which to view Travis's inclusion of "error resolution" in his follow-up scripts. We are left wondering if Travis was focused on portraying a student—in particular, the student who originally contributed an error—who could make sense of others' ideas and revise their own thinking and that he was not focused on illustrating how an error might be quickly "dealt with."

Articulating Ways Context Matters for Responding to Errors

From the outset, Travis recognised that responding to errors was complex, and not something that could be accomplished in five-to-eight lines of dialogue. As he noted in his graph scripting rationale, "this discussion is definitely not finished." Travis's experience during the rehearsal heightened and deepened this understanding. He began to more clearly articulate how context featured in his decision-making processes. This was motivated, in part, by moments of coaching during his rehearsal. For example, the teacher educator pressed Travis on his use of orienting moves that asked for additional contributions but did not elicit responses from students. Reflecting on those coaching moments, Travis wrote that he wanted "to always ask if people had other things to say about each card being sorted because I did not want it to seem as if I single out the one person who makes an incorrect statement." He further considered how that goal weighed against the need to be intentional with his moves.

Travis's participation as a student in two other sorting rehearsals provided further opportunity to consider the importance of context. Travis observed differences in how other TCs responded to errors, noting that, "sometimes errors were discussed immediately, whereas in other rehearsals that discussion was held off until a later time." Attention to context accompanied a change in Travis's disposition. Even as he recognised that there was not one, generic form of "good practice," Travis developed a disposition toward accepting less control over the discussion and letting students take more of the lead. Travis made explicit his desire to continue learning about best practices for responding to errors and recognised that what might work in one instance might not work in another.

Knowing that Travis was inclined to consider the context of a situation in determining how to respond, we are left unsure what context he was considering when completing his follow-up scripting tasks. For Travis, the scripting tasks were a way to demonstrate particular tools and practices (such as "going back" to a student), vision (i.e., students engaging with one another's ideas toward a mathematical goal), and understandings of students (i.e., that students can reason about other ideas and revise their own thinking). However, the way those resources were made visible and put forward in the scripts could have been influenced by a read of the context that we are not able to fully determine. We can say, though, that Travis developed more appreciation for how context informs decisions about responding to student contributions.

Conclusion

These facets of Travis's learning, which extend across all components of the framework, help put into context his uniquely differing initial and follow-up scripting tasks. The generically "good" approaches that Travis exhibited at the beginning of the semester were refined and complicated through further approximations of practice. Travis developed an awareness of the competing demands teachers face in leading whole-class discussions and responding to errors.

Discussion and Implications

The cases of Greg and Travis reveal learning through multiple approximations of practice. This responds to the need to characterise what TCs' learn through practice-based pedagogies in enough detail to consider their impacts (e.g., Arbaugh et al., 2019). In this section, we discuss the affordance of considering two cases of learning through approximations of practice. We then suggest implications for research and practice related to the contingency of approximations of practice and moving from approximations to real teaching settings.

Cases of Learning to Respond to Errors

This paper provides a detailed, illustrative look at the nature of two TCs' learning over time, focusing on the work of responding to errors. The use of multiple approximations of practice—both pedagogically and in research—greatly enhanced what was revealed about both TCs. Each approximation, with differences in authenticity and complexity, made public and visible some components of learning, while others remained hidden. Together, the cases offer a "proof-of-concept" that systematic investigation of multiple approximations of practice can reveal learning across vision, dispositions, understandings of content and students, and tools and practices, contributing to a more holistic picture of each TC's development.

Greg and Travis acknowledged the challenge they felt responding to errors, echoing findings from earlier research (e.g., Bray, 2011; Santagata, 2005; Shaughnessy et al., 2020). Both named dilemmas they faced as they strove to embrace more productive ways of responding to errors. By the end of the semester, Greg and Travis's scripting task responses revealed efforts to create spaces where students are not marginalised for contributing errors and, instead, are supported as capable doers of mathematics (Jacobs & Empson, 2016; Stockero et al., 2020). This included representations of "imagined" students (Zazkis, 2018) who were positioned to do mathematical work and who contributed mathematically relevant (and often, complete and

precise) ideas, revealing increased understandings of students and a vision that students are capable of this kind of participation. We recognise that neither Greg nor Travis responded "perfectly" in their approximations. However, because productive responses to errors are necessarily context-dependent, no single "perfect response" exists. This further highlights the value of an investigation of TC learning that attends to multiple components of learning as revealed in various approximations and over time.

Contingency of Learning in Approximations of Practice

These two cases highlight just how contingent learning can be. Each case, by definition, depends on the particularities of experiences Greg and Travis brought to their learning and the contexts in which they learned. For instance, both Greg and Travis seemed to attend to particular moments of coaching that influenced what they included in their follow-up scripting tasks, which became one source for claims about learning. Greg took up how to ask an agree/disagree question after receiving coaching on his wording. That coaching, though, depended on Greg experimenting with asking that question in the first place. Similarly, Travis took up feedback to check in with the student who made the error after his rehearsal called for such a move. Had those situations not occurred, those coaching interjections likely would not have arisen. Though coaching can be planned at a level of goals or principles, the teacher educator must react to what happens in the moment, making each moment of coaching contingent on all that came before it.

These cases provide insights into the mechanisms by which approximations of practice may support learning, in part because TC learning is necessarily contingent. While we do not have data to support hypotheses about why Greg and Travis latched on to particular coaching moves, the cases emphasise how critical the coaching moves are as drivers of TCs' learning, thus illustrating the critical need to attend to the role of the coach in rehearsals (Kazemi et al., 2016; Lampert et al., 2013; Wæge & Fauskanger, 2020). The contingent nature of teaching is therefore well-captured by the contingent nature of the learning experiences provided by these approximations of practice.

The implication of this contingency is that teacher educators must take care to evaluate TCs' learning with respect to the contexts in which the learning occurred. The framework for teacher learning was a fundamental tool in allowing us to capture and depict learning. We also found that the intentional use of multiple approximations enabled more diverse data about TCs across components of the framework. Our analyses provided a way to capture contingencies as we made claims about learning.

From Approximations to Reality

We acknowledge the potential challenges of moving from approximations to teaching students (e.g., Campbell & Elliott, 2015; Kazemi & Wæge, 2015). While we make claims about learning from and in these approximations, we cannot necessarily predict what Greg or Travis might do in the field. For example, both demonstrated a vision of teaching that included students being able to describe robustly their mathematical ideas and respond to one another. This vision is an undeniably positive outcome. However, a potential pitfall is that TCs' vision of teaching, in terms of what teachers do and students do, hinges on that type of student participation. How does the

imagined setting of the approximation translate to the realities and uncertainties of classroom practice, and how does that influence a teacher's actions? With these questions, we simultaneously value TCs' learnings demonstrated across multiple approximations, and recognise the need for research that encompasses practice in schools.

Despite such limitations, we were able to depict robustly what two TCs learned through their engagement with coached rehearsals and scripting tasks over the course of a semester. TCs' learning was made visible within and across approximations used as part of a practice-based approach to teacher education. These examples of learning build on a body of scholarship around practice-based teacher education, responding to calls to more fully document learning and also adding to existing research on pedagogy, design, and opportunities to learn. Through a clearly articulated framework for teacher learning, we sought to overcome a potential pitfall of focusing only on teacher actions in isolation and instead represent a robust range of proficiencies that highlight the complexity of the work of teaching.

Acknowledgements

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