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How Deeply and how Well? How Ready to Teach Mathematics after a One-Year Program?

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The importance of teachers' content knowledge for effective teaching of mathematics has long been recognised. Recent national testing regimes in Australia (National Assessment Program–Literacy and Numeracy [NAPLAN]) have raised questions of the adequacy of teachers' content knowledge. This, in turn has focused attention upon the quality of teacher education programs. In this study 131 Graduate Diploma of Primary Education students undertook the 2008 Year 9 national tests of numeracy before and after a pre-service teacher education unit on mathematics education. The purpose was first to examine, upon entry to the course and then after training, their knowledge of the mathematics concepts they were likely to teach, and second to compare this knowledge with that of the students whom they would be teaching. While there was evidence of some inadequacy of entry-level mathematical knowledge, improvement over the life of the study was reported. The implications of these findings are examined in the context of a one-year graduate diploma pathway to teacher preparation.

While there is growing international acknowledgement that demand for professionals with mathematical and technological expertise is increasing rapidly, the supply of students willing to undertake study in the enabling sciences is diminishing (e.g., Barringtion, 2006; Hannover & Kessels, 2004; Yurtseven, 2002). The net effect is that the shortage in qualified workers will negatively impact economic and social potential in westernised countries (e.g., Department of Education, Science and Training [DEST], 2003; National Numeracy Review Panel and National Numeracy Review Secretariat, 2007; Tytler, Osborne, Williams, Tytler, & Clarke, 2008; Yurtseven, 2002). Recognition of the need for more students to have higher levels of mathematical understanding has been linked to a corresponding focus on curriculum reform and student assessment in recent decades across international boundaries (e.g., DEST, 2003; National Council for Teachers of Mathematics, 2009; Commonwealth of Australia, 2008).

In Australia, as in other countries, reform curriculum documents enshrine an expectation that students demonstrate 'deep understanding', 'mathematical fluency' and 'problem solving'. Concern that many students fail to reach acceptable mathematics understanding has been accompanied by increased use of high stakes testing nationally and internationally (Amrein & Berliner, 2002; Nichols, Glass, & Berliner, 2006; Phillips, 2007; Wilson, 2007). High stakes testing is typically defined as those tests, often standardised, that carry serious consequences for students and educators since schools and students may be judged, and affected by important decisions taken, on the basis of the results (Marchant, 2004). One effect of high stakes testing is that teacher quality has come under increasing scrutiny (e.g., Masters, 2009; U.S. Department of

Education, 2008) and this in turn has focused attention upon the quality of teaching education programs (e.g., Masters, 2009).

As Boyd, Grossman, Lankford, Loeb, and Wyckoff (2009) pointed out, there is a lack of understanding of how best to prepare teachers and accordingly further evidence in this area of research is required. There is ongoing debate about the most appropriate models of teacher education including the importance of content knowledge and how it might be best developed in teacher education programs (e.g., Ball, Hill, & Bass, 2005; Cavanagh, 2009; Osana, Lacroix, Tucker, & Desrosiers, 2006). Given that researchers almost uniformly agree that a knowledge of mathematics is central to its teaching, it is disturbing that a number of authors have noted that many elementary teachers have only limited understanding of mathematical structures (e.g., Brown & Benken, 2009; Ma, 1999; Osana et al., 2006). There has been little research on the level of mathematics understanding that graduate students typically bring to teacher preparation and the effect of teacher education courses upon that knowledge base. The research that has been conducted to date is inconclusive. For example, Boyd et al. (2009) found that the complexity of interactions between variables made assessment of the relationship between forms of teacher pre-service education and subsequent teacher performance very difficult to assess. Even describing teacher education programs courts the dangers of complexity and its interpretation.

Some of the problematic questions that challenge the researcher include deciding what aspects of knowledge are central to teaching and how the development of that knowledge can be promoted. The measurement of growth in teacher competency is also complex. What counts and is to be counted? Some of the unanswered questions include: Is the balance of credit points allocated to courses designed to promote learning to teach mathematics a reasonable indicator of "what goes on" in a teacher training program? Is the total time allocated to learning another important indicator? How does the mode of delivery affect student learning? How can the balance of content, pedagogical content, and curriculum knowledge be accounted for? Finally, how can a teacher's growth in knowledge be reasonably measured? One predictor that is almost universally recognised as critical to teaching mathematics is the teacher's content knowledge (e.g., Ball, et al., 2005; Ma, 1999; Osana et al., 2006; Shulman, 1987, 1999; Warren, 2009).

This study focuses particularly on pre-service teachers' knowledge of mathematical content, and the effectiveness of a pre-service mathematics curriculum subject in improving that knowledge. Harris and Jensz (2006) and Thomson, Wernert, Underwood, and Nicholas (2007) have already generated some findings in this respect. Harris and Jensz (2006) reported in depth on teacher education in Australia. However, their report focused upon specialist mathematics teachers who teach in secondary schools. The analysis of Trends in International Mathematics and Science Study (TIMSS) by Thomson et al. (2007) also provides background on the nature of primary school teachers and their confidence to teach mathematics. Thomson et al. (2007) reported that about 80%

of Year 4 teachers were female, highly experienced with on average, one and a half decades of teaching practice, and that 81% had tertiary teaching qualifications. However, the report by Thomson et al. (2007) did not contain any detail on teacher education specifically related to either mathematics teaching or levels of teachers' knowledge of mathematical concepts. Thus, little previous data have been published about the level of content knowledge of primary teachers or the nature of primary school mathematics teacher education. Ball et al. (2005, p. 45) noted that this state of affairs was in part due to "sharp criticism from some quarters… to testing teachers, studying teaching or teacher learning, at scale, using standardised student teacher measures." This paper investigates this issue through a comparison of mathematical knowledge before and after a pre-service mathematics curriculum course using a standardised test of numeracy. The following sections summarise existing research on the importance of content knowledge and describe various teacher preparation approaches. The paper then investigates postgraduate students' content knowledge upon entry and at exit of the major mathematics curriculum unit.

Teachers' Content Knowledge

The relationship between teachers' mathematical content knowledge and their ability to teach has been well researched and there is clear evidence on the relationship between them (e.g., Ball, Hill, & Bass, 2005; Darling-Hammond, 1997; Harris & Jensz, 2006; Ma, 1999; Shulman, 1987, 1999). But teaching knowledge is not a simple uni-dimensional variable. Rather, at the very least, teacher knowledge ought to include: content knowledge, pedagogical content knowledge, general pedagogical knowledge, curriculum knowledge, knowledge of learners and their characteristics, knowledge of educational contexts and knowledge of educational ends, purposes and values (Shulman, 1999). The concept of pedagogical content knowledge (PCK) is described as an intersection of subject knowledge and pedagogical knowledge (Shulman, 1987). That is, high levels of teacher subject content knowledge do not necessarily imply that individuals understand the material in a way that enables them to impart or teach it to students (Ma, 1999). Ma (1999) describes what is needed as profound understanding of fundamental mathematics. What is required is that teachers understand the material and ways of representing it to students. This has recently been described as mathematical knowledge for teaching (MKT) (Silverman & Thompson, 2008). However, both PCK and MKT pedagogical knowledge is dependent upon a fundamental understanding underlying mathematical structures (Silverman & Thompson, 2008). Banner and Cannon (1997) summed up the critical importance of teacher content knowledge as follows: "In order to teach they must know what they teach and know how to teach it; and in order to teach effectively, they must know deeply and well" (p. 7).

The importance of teachers' content knowledge was also more recently articulated by the U.S. Department of Education (2008, p. 36): "Teachers must know in detail the mathematical content they are responsible for teaching and its connections to other important mathematics, both prior and beyond the level they are assigned to teach." Masters (2009), in his report on the 2008 Queensland NAPLAN performance (Ministerial Council on Education, Employment and Youth Affairs [MCEETYA], 2008), similarly noted:

> Highly effective teachers have a deep understanding of the subjects they teach. These teachers have studied the content they teach in considerably greater depth than the level at which they currently teach and they have high levels of confidence in the subjects they teach. Their deep content knowledge allows them to focus on teaching underlying methods, concepts, principles and big ideas in a subject, rather than on factual and procedural knowledge alone. (p. 4)

It is widely acknowledged that teachers with more explicit and better connected knowledge are more likely to teach with a variety of representations and in a dynamic manner (Commonwealth of Australia, 2008; Sowder, 2007; Warren, 2009).

As important as they are, general pedagogical knowledge, curriculum knowledge, knowledge of learners and their characteristics, knowledge of educational contexts and knowledge of educational ends, purposes and values (Shulman, 1999), these cannot be effectively drawn upon unless the teacher has deep content knowledge. Ma (1999) noted that it was possible to pass advanced courses in mathematics without understanding how they might inform the teaching of primary mathematics, none the less, a deep conceptual knowledge of mathematics plays a vital role on mathematics teaching and learning. This observation has almost universal support among mathematics education researchers and so this study focuses on the depth of pre-service teachers' content knowledge.

Pathways in Primary Teacher Education

In some jurisdictions there are multiple pathways to primary teacher certification. For example, New York State has five (Boyd et al., 2009). Similarly there is a range of mathematics prerequisite requirements prior to teacher education entry. Most primary school teachers in Australia complete an undergraduate degree, usually full time over four years. This pathway is common across many countries including China (Li, Zhao, Huang, & Ma, 2008). The alternative pathway in Australia is a graduate diploma usually completed in one year subsequent to the completion of an undergraduate degree. This present study focuses on the Graduate Diploma of Education Primary pathway, a one-year pre-service teacher education program usually undertaken by students who have completed a Bachelors degree. Students throughout Australia increasingly favour the one-year graduate diploma pathway with proportional enrolments increasing at the expense of four-year undergraduate degrees. Unlike the New York State process of gaining registration as a teacher reported by Boyd et al. (2009), Australian primary teachers are not required to undertake registration examinations. Instead, state-based accrediting bodies review university course structures and students are accredited on the basis of their university assessments. The added criterion is that the student demonstrates 'reasonable classroom practice', a judgment made by the primary school in which the pre-service teacher gains classroom experience. There is thus no external assessment of the prospective teacher's knowledge of mathematics.

A major challenge in primary teacher education is, as Commonwealth of Australia (2008) noted, that primary school teachers are generalists who usually teach a cohort of students most of the subjects in the curriculum. For this reason pre-service teacher education courses look to ensure that graduate teachers have content and pedagogical content knowledge across a range of disciplines as well as curricular knowledge, knowledge of learners and their characteristics, and knowledge of educational contexts for teaching students from the early years of learning to middle school learning. The importance of teacher education programs providing a cohesive model that accommodates content, pedagogical content, and curriculum knowledge has been widely acknowledged (e.g., Ball, et al., 2005; Boyd, et al., 2009; Osana et al., 2006; Warren, 2009).

A survey of eight major universities across Australia carried out by the author reviewed common program structures by examining their online descriptions. Taking an academic learning time approach [ALT] (Berliner, 1990), it appears that in Australia, a typical graduate diploma program has two semesters with four units in each semester; generally one of these involves practicum or professional experiences, occupying 25% of all scheduled learning time. Learning how to teach mathematics (or literacy) might occupy one course, equivalent to 10 to 12 credit points (12.5% of scheduled learning time). In the case of the university presently being studied, there is one full course and a shared course (18.75% of scheduled learning time). It is difficult to establish exactly how many hours are allocated for lectures and workshops since this information is not generally published on university programs or course outlines. A reasonable estimate (obtained from informal discussions with mathematics educators across four universities) for specific mathematics related contact, for most graduate diploma primary programs, is between 24 and 40 hours of direct contact per course. At the study campus the teaching time was condensed into two, nine-week semesters, with lectures confined to six weeks, except for mathematics, which was permitted to extend the delivery of lectures over nine weeks. At an alternative Queensland based campus contact time was 24 hours over six weeks. Against this context, the inquiry question guiding the study asks: Is the time allocated sufficient to enable graduates to exit with high levels of content knowledge in the subjects they will teach?

The mathematics content knowledge of the cohort was tested pre- and post-course and the results are examined and discussed here. The related critical question posed by this paper is: Is it likely that such units provide adequate opportunities for students to develop the necessary content knowledge upon which to base effective teaching of mathematics? Thus the research questions were:

> Do entering pre-service teachers have adequate mathematical knowledge about the concepts they will be expected to teach?

After 39 contact hours in a mathematics curriculum unit do pre-service teachers achieve an adequate understanding of the content of mathematics that they might be expected to teach?

Method Overview

This section describes the courses and provides a description of their mathematics-related curriculum learning as well as the means of testing for content knowledge pre- and post- mathematics curriculum study. A quasi-experiment consisting of pre- and post-tests research was implemented. The intervention was the normal mathematics preparation that is provided in the Graduate Diploma of Education Primary. There was no control group in the study as it would not have been possible or ethical to omit mathematics education from the curriculum of a group of pre-service teachers or offer them an alternative program. The pre-tests were administered in the first week of the mathematics curriculum unit. The post-tests were administered in the last week of tutorials. In both cases, students completed the test under examination conditions and were not permitted the use of calculators. Changes in student content knowledge were analysed using a statistical test of proportional success based upon the mean data. This method of analysis is particularly robust with relatively large samples, as was the case in this study (Dimensions Research Inc, 2009). The pre- and post-test data were analysed for selected strands of mathematical knowledge. This enabled student change to be mapped against the time spent on various content strands during the learning process. The results were compared to those of the Year 9 Queensland school population (*n*=55952). This enabled a comparison of pre-service teacher knowledge mastery with that of the students they were being prepared to teach. The testing of content related to what the teachers would be expected to teach has the support of Ball et al. (2005).

Subjects

Almost the entire cohort (*n*=131, 89% female; 11% male for the pre-test; and *n*=105 for the post-test) of Graduate Diploma of Primary Education students participated in the study. The majority of students had completed high school since 2000 and, with few exceptions, had taken a degree before commencing teacher pre-service education. It is difficult to quantify how much mathematics study students had completed before commencing teacher pre-service education since they came from a variety of states, countries and age groups. Of the 131 enrolled in 2009, the University has intake records for 106. Of these, 67 students' records included their level of senior mathematics that could reasonably be converted to Queensland mathematics standards. The data on senior mathematics are tabled below:

Table 1
High school mathematics completed (n=67)

Mathematics A or Mathematics and Society	Mathematics B or equivalent
39 or 58%	28 or 42%

Mathematics A is the lowest-level senior mathematics subject with a focus on business mathematics and mathematics that might be of assistance to trades and general life preparation. Mathematics B (or its equivalent) is generally the minimum level of school mathematics needed to enter science-based courses at tertiary institutions, and is undertaken by about 20% of senior school students in Australia (Barrington, 2006). Of the students who had completed Mathematics B or equivalent, 32% received a high grade, 36% received a sound grade and 32% received a low grade or equivalent. These figures indicate that about 13% of the students had relatively high grades at intermediate or Mathematics B level mathematics. The remainder of the cohort had either the equivalent of pass or lower grades in intermediate mathematics, six had not studied mathematics to senior levels and the remainder had studied relatively low levels of senior mathematics (equivalent to Mathematics A).

The cohort was chosen on the basis of convenience: the researcher had the opportunity to collect data from its members. The numbers above represent virtually the entire course enrolment at the time of each test. There is no evidence that the sample is biased in any way. A description of the mathematics curriculum course is provided below and summarised in Table 2. The curriculum subject has been approved by the teacher registration body in the state (Queensland College of Teachers, 2006) as meeting the requirements for pre-service teacher education such that the graduating students are eligible to be registered as teachers in Queensland.

Mathematics Curriculum

The lectures in the mathematics curriculum unit were conducted over nine weeks and focused upon teaching principles for unpacking the mathematical structures. Two-hour lectures were followed by three-hour workshops in which students practised using mathematical materials and employing explicit language to make underlying mathematical structures explicit. Models of how specific mathematical concepts and operations might be taught were laid out in detail in a 360-page mathematics-teaching manual published by the author. Students demonstrated to each other how they would use the modelled strategies to solve and teach mathematical concepts and problems. The intention was to develop content and pedagogical content knowledge simultaneously. This approach is consistent with that recommended by researchers who emphasise the importance of understanding conceptual principles. This mastery is at the heart of mathematics and learning within a clinical setting (e.g., Ball et al. 2005; Cavanagh, 2009; Osana et al., 2006). In addition students were given past examination papers and were informed that they had to know both the content and pedagogy to pass a formal three-hour closed book examination (the data from this are not part of this study). Questions from the 2008, Years 3, 5, 7 and 9 NAPLAN numeracy tests formed the basis of three of 15 extended answer questions and students were asked to explain how they would teach the mathematics behind those questions. Most of the questions were of the form of diagnosis and remediation. For example:

"A Year 2 student carried out the following computation:

$$\begin{array}{rrr} & 4 & 9 \\ + & 0 & 8 \\ \hline & 0 & 17 \end{array}$$

(a) What are his or her main thinking errors? (1 mark)

(b) Set out clearly how you would remediate this misconception considering the use of materials, specific language and links to symbolism. (6 marks)."

"A Year 7 student completed the following computation.

$$\frac{3}{4} + \frac{2}{3} = \frac{5}{7}$$

(a) What is his or her fundamental misconception? (1 mark)

(b) Model the teaching you would use to remediate this error (illustrating the materials, language and symbolic representations you would use). (6 marks)."

The knowledge of proportional reasoning was of particular interest in the testing because it is central to mathematics and critical in understanding much science and technology (Nabors, 2002; National Council of Teachers of Mathematics, 2004). Proportional reasoning is used to describe the concepts and thinking required to understand fractions, rate, ratio, proportionality including scale in measurement, and geometry. A number of authors (e.g., Ilany, Keret, & Ben-Chaim, 2004; Lo & Watanabe, 1997) have noted that proportional reasoning is essentially multiplicative thinking. Ability in such thinking is needed for an understanding of fractions, rates, ratio and scale, percentages, gradient, trigonometry and algebra. Proportional reasoning is, thus, central to primary teaching beyond early years, and figures prominently in the NAPLAN tests at Years 5, 7 and 9 (Norton, 2009). The development of proportional reasoning in the mathematical contexts presented above is listed as required in middle and upper primary school learning in the recently released draft *Australian Curriculum: Mathematics* (Australian Curriculum, Assessment and Reporting Authority, 2010).

An overview of the course concepts is provided in Table 2, illustrating the emphasis on understanding and teaching proportional reasoning in its various contexts, with five of the eight weeks of tuition related to this concept area.

Table 2
Mathematics curriculum subject structure

Week 1 to 9	Critical concepts
1. Introduction to the number system (number strand). 5 hours contact.	Teaching early number, place value, renaming, rounding, index notation and scientific notation. Teaching problem solving will occur throughout the course. (Pre-test Year 9 NAPLAN numeracy test in first hour of workshops).
2. Teaching addition and subtraction (number strand). 5 hours contact.	Teaching the addition and subtraction concepts, algorithms and mental computations.
3. Teaching multiplication and division (number strand). 5 hours contact.	Teaching the concepts of multiplication and division, written and visual models and problem solving in multiplicative contexts.
4. Introduction to teaching fractions (number strand). 5 hours contact.	Teaching naming and renaming fractions, decimal concepts and operations. Introduction to fractions and fraction operations, addition and subtraction.
5. Teaching fractions (number strand). 5 hours contact.	Fraction computation including comparing fractions and fraction multiplication and division. Teaching rate, ratio, proportion and percent.
6. Teaching introductory algebra (algebra strand). 5 hours contact.	Teaching what primary students need to understand in order to make the transition to secondary algebra (coordinates, variables, relationships, expressions and solving in problem contexts).
7. Introduction to teaching measurement and geometry (measurement and space strands). 5 hours contact.	Teaching measurement and geometry for problem solving with emphasis on proportional reasoning associated with scale, measurement and geometry.
8. An introduction to teaching data and probability (measurement strand). 4 hours contact.	Teaching the collection, analysis and presentation of data and introducing probability, emphasising links to fractions, decimal and percent representations.
9. Revision and Assessment	Revision and consolidation in lectures, Post-test Year 9 NAPLAN numeracy test. Followed by a 3-hour closed book examination the following day.

Test of content knowledge

In order to gain a measure of students' content knowledge of mathematics at the beginning of the course, students completed the 2008 Year 9 NAPLAN non-calculator test (MYCEETA, 2008) under examination conditions. At the end of the course the students completed the second of the two Year 9 NAPLAN tests. (The full tests can be accessed from the MYCEETA website). In both instances the pre-service teachers were not allowed to use a calculating device. The tests are an accepted reflection of knowledge levels expected of upper primary and lower secondary school students and mirror the expectations of the emerging Australian curriculum. A test analysis of the NAPLAN items shows that, due to the structure of test items developed by MYCEETA (2008), students with a reasonable knowledge of primary computation ought not to have been disadvantaged by not having access to a calculating device (Norton, 2009). NAPLAN test papers are designed to assign students to particular band levels, and thus test a range of difficulty levels with questions that are of a standard lower than what is expected of a year level as well as some more challenging questions. There were, for example, 15 overlapping questions between the Year 7 and Year 9 tests (23% exact common questions). A sample of questions below illustrates the form and difficulty of items. The first such sample is of a number strand question.

9 There were only 14 students in Rina's class on Wednesday. The other 11 were absent.
What percentage of Rina's class was absent?

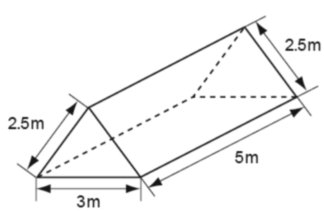
11% 44% 55% 56%

Figure 1: Sample of a number strand question

In this question students need knowledge of the concept of percent.
The question below is from the measurement strand.

30 This solid triangular prism needs all its faces paintewd.
The area of each triangular face is 3m²

Write your answer in the box.



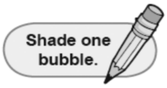
Not to scale

What is the **total** area to be painted? m²

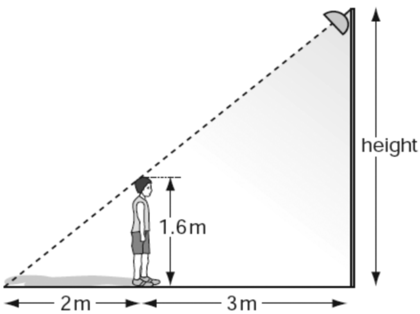
Figure 2: Sample of a measurement strand question

A sample of a space strand question is presented below. It is worth noting that while the context of the question is measurement, the solution depends upon proportional reasoning.

25 Joe is 1.6m tall. His shadow is 2m long when he stands 3m from the base of a floodlight.



Not to scale



2.4m ○ 2.6m ○ 4.0m ○ 4.2m ○

Figure 3: Sample of a space strand question

These items as well as the complete set of questions from both the calculator allowed and the non-calculator test papers are downloadable from http://www.naplan.edu.au/tests/naplan_2008_tests_page.html.

Limitations

A possible limitation of this study is that the tests were conducted at the end of the first semester; a further 36 hours of workshops and lectures in the second semester may have had an impact on pre-service teachers' content knowledge at the end of the year long program. However, the second course was "literacy and numeracy across the curriculum". It also needs to be noted that the focus of the second course was on curriculum issues including; assessment principles, planning principles, the use of technology in teaching, and analysis of investigations, integrated learning and planning to teach literacy and numeracy across the curriculum. In short, in the second semester there was limited time allocated to the specific teaching of content knowledge of mathematics.

A limitation of the use of the NAPLAN tests is that about 80% of questions are of multiple-choice formats. This means students can guess, or substitute the provided options to arrive at correct responses. These mechanisms serve to inflate estimates of student knowledge (Lange, 2007; Norton, 2009; Rindermann, 2007). It is thus likely that the multiple-choice format inflated the scores of students in this study (as it would have for the population of Year 9 students doing the tests in 2008) and this caveat should be taken into account when interpreting the findings. For less knowledgeable students, the guessing effect could possibly account for a significant proportion of their correct responses.

Results

Results of Students' Content Knowledge

The data on pre-service teachers' content knowledge upon entry to the unit and at the end of the unit are summarised in Table 3 below.

Table 3
Pre- and post-test results percentage of questions correctly answered by strand classification

Strand	Pre mean % correct (sd in brackets)	Post mean % correct (sd in brackets)	*z*	*p*
Number	45.13 (23.70)	69.88 (16.32)	3.72	<.01
Algebra	43.13 (19.56)	75.50 (17.24)	4.97	<.01
Space	58.13 (23.04)	71.00 (19.09)	1.93	<.05
Measurement	60.88 (18.98)	61.43 (31.62)	0.13	>.05

In relation to the first research question about entering pre-service teachers mathematical knowledge, the data indicated that many of the pre-service teachers struggled to understand the mathematics at levels normally taught in upper primary and lower secondary school. That is, the pre-service teachers' results were somewhat similar to those of average Year 9 students in the state of Queensland. The mean score on the 2008 NAPLAN non-calculator test (or pre-test) for Queensland Year 9 students was 47.19% while for the pre-service teachers the mean score was 54.06%. A test of proportional success percentage based upon the mean data indicates that there was not a significant difference between the two samples at the 5% level ($z= 1.516$, $p>.05$). Details of scores on the pre- and post-tests for Year 9 Queensland students and the pre-service teachers are contained in the Appendix (Tables 4 and 5).

For the second research question about adequate understanding of the content of mathematics after the 39-hour mathematics curriculum unit, there was evidence of improvement in mathematical understanding, but with wide variation among the pre-service teachers. The post-test data were compared to the pre-test data on an overall means basis and according to improvement in the four content strands. A test of proportional success percentage based upon the mean data indicates that there now was a high statistical difference between Queensland Year 9 students (overall mean = 49.53%) and the Graduate Diploma students (overall mean of 69.88% $z= 3.997$, $p<.01$). Further, the difference in means of pre- and post-tests of the Graduate Diploma students was statistically highly significant ($z=2.375$, $p< .01$). The data presented in Table 3 summarising pre- and post-test data according to strand performance helps to illustrate those areas in which the pre-service teachers most improved their content knowledge. (The items were classified in strands according to NAPLAN labelling (MCEETYA, 2008)).

The pre- and post-test data indicate that the greatest improvement was in the strands of number and algebra (p<.01) with significant improvement also in space. But there was very little improvement in the measurement strand. The lack of statistical difference in the measurement strand items pre- and post-trial results is explained by the higher starting score and almost no improvement upon this score. The spread of students remained relatively similar on pre- and post-tests across strands, with the exception of the increased spread on the measurement strand. This increased standard deviation on the post-test measurement questions can be explained by the low success rate on question 31 of the post-test where students had to calculate the surface area of a three-dimensional figure (Figure 2).

In summary the data indicate that, by the end of their nine weeks and 39 hours of contact, the pre-service teachers had demonstrated significantly greater content knowledge than the average Year 9 student and significantly more than they had at the beginning of their curriculum study. However, upon completion of the 39 hours of contact about 16% of students scored fewer than half marks (on the 32-item test) and about half the students had scores of less than 70% correct.

Discussion

Background data indicate that most of the pre-service teachers who were enrolled in the one-year program had studied high school mathematics at only relatively low levels. A small proportion had completed intermediate levels of mathematics with high grades. Testing during the first week of semester indicated that upon entry to the Graduate Diploma of Education Primary program the pre-service teachers in this sample had a grasp of content knowledge that was statistically similar to that of Queensland Year 9 students on the 2008 NAPLAN test. The finding is crucially important because, while it may not be stated in course descriptions, the rationale mounted for justifying a condensed one-year pre-service course is that, as graduates, students such as these can be readily transformed into teachers since they are presumed to have either solid backgrounds in mathematics or the capacity to quickly develop adequate levels of mathematical understanding.

The second major finding is that over the nine-week unit of 39 hours contact with an explicit focus on developing content and pedagogical knowledge simultaneously, many students made substantial progress in their mathematics content knowledge. Overall, the mean score on a similarly structured test with a similar level of difficulty increased from 54.06% of all questions answered correctly to almost 70% in the second test. However pre-service teachers' content knowledge at the end of the 39 hours of mathematics curriculum was variable. While 16% of the students gained fewer than half marks on the final tests, only about 39% scored over 24 correct responses out of 32. The gain in content knowledge was greatest in those strands that were a specific focus of the tertiary course – number and algebra, the strands that dominated contact or academic learning time. It needs to be noted that the teaching of algebra was closely linked to the teaching of number so that algebra was treated as a natural extension of number; in effect number and algebra dominated the learning time in this unit. In the strands where there were fewer lectures and less workshop time (e.g., space), the average gain in content knowledge was less and, in the case of the measurement strand, there was no increase in content knowledge. These results strongly support two conclusions: first, that it would be difficult to describe the majority of the students as having a deep understanding of the subjects they teach (Masters, 2009) or as meeting the criteria about knowledge and understanding required by the certifying body (Queensland College of Teachers, 2006); and second, that gains in student content knowledge were linked to face to face learning time, a not surprising finding.

Research from earlier studies has shown that, before teachers can apply pedagogical principles, they need to have a deep understanding of mathematical content knowledge (e.g., Banner & Cannon, 1997; Commonwealth of Australia, 2008; Darling-Hammond, 1997; Ma, 1999; Shulman, 1987, 1999). In particular, teachers' content knowledge of mathematics has been recognised as central to their capacity to teach mathematics in a connected and dynamic manner (e.g., Commonwealth of Australia, 2008; Masters, 2009; Sowder, 2007; U.S. Department of Education, 2008). This present study suggests that many pre-service teachers entered this teacher education program with very poor levels of mathematical content knowledge and that many struggled to remediate this deficiency in the condensed learning time. The subject structure in this unit attempted to teach primary mathematics content in parallel with how to teach the mathematics to primary school children. For many students this was a successful approach; for a significant proportion of students this was not the case. The data indicate that questions of intake content knowledge, learning time, balance of time spent, and duration of pre-service teacher education need to be examined. The testing data suggest that it takes considerable time for many pre-service teachers to develop reasonable levels of content knowledge. For example despite the unit emphasis on proportional reasoning post test results to question 25 (Application of proportional reasoning in the context of scale and triangles) was poorly done, with only 45% of students achieving success, almost identical to the Year 9 success rate. The importance of the findings potentially goes beyond informing curriculum design in just this institution. Deficiencies in pre-service teachers' content knowledge may be even more acute in other locations, particularly in those institutions where the focus in mathematics curriculum education is upon general pedagogy and theories of learning, with reduced learning time for mathematics understanding.

The one-year teacher preparation structure offered at the institute involved in the study is quite different from that offered in China, for example. Li et al. (2008) reported that elementary teachers were required to complete courses in: theories or elementary mathematics curriculum and instruction; mathematics analysis 1; advanced algebra; analytical spatial geometry; elementary number theory; mathematical analysis II; advanced algebra II; probability and statistics and mathematical thinking methods. There are also further electives available for those teachers who intend to specialise in elementary mathematics education. Li et al. (2008) concluded: "Based on the availability of different courses in mathematics, prospective elementary teachers are required to study mathematics systematically and in depth" (p. 442) and to have "strong training in subject content knowledge including advanced mathematical courses…" (p. 424). The programs described by Li et al. (2008) reflect the earlier findings of Ma (1999) that noted the depth of mathematical knowledge of the sample of Chinese teachers compared to her sample of U.S teachers. The structure of the studied program (and a number of other teacher preparation courses in Australia) does not reflect the same commitment to valuing mathematics content knowledge as the basis for teaching and learning, like the programs described by Li et al. (2008) and Ma (1999). Disproportionately, primary school teachers here are expected to teach mathematics for between 20% to 25% of student learning time, yet the mathematics teacher education curriculum is allocated between 12.5% and 18.75% of program time, which may amount to as little as 24 hours of contact. Most academics want more hours to engage with their students, the assumption being that more time-spent learning can be equated to improved quality, particularly when that learning is directly related to required performance, a stance strongly supported by Gladwell (2008). The finding that students improved the most in those strands, which were the focus of learning to teach mathematics and minimally in those that were not, supports the view.

Conclusions

This study is timely as it provides data that stimulates discussion on the total time allocated to mathematics units in pre-service teacher education as well as how that time is spent. There are growing calls within Australia and elsewhere for greater quality in teacher preparation in mathematics education (e.g., Commonwealth of Australia, 2008; Lange, 2007; Masters, 2009; McInerney, & McInerney, 2006; U.S. Department of Education, 2008). This study suggests that there is merit in exploring the content knowledge of teacher education students prior to, during, and at the end of their teacher education. The data add to the pool of knowledge requested by Ball et al. (2005) about what forms of teacher pre-service preparation are effective. The data suggest that there is merit in further exploring what time is needed and what other forms of learning are best suited to preparing students for professional service in primary teacher education. The models reported by Li et al. (2008) with a focus on mathematics and pedagogy within different courses are worth examination, as is the model used in this program of attempting to develop content and pedagogical knowledge simultaneously. As noted earlier (eg., Ball, et al., 2005; Banner, & Cannon, 1997; Cavanagh, 2009; Shulman, 1987; Silverman, & Thompson, 2008), simply having student teachers learn more content may not be the best option. It has been well documented that in order to teach effectively, teachers must first deeply understand mathematical structures and then know how to facilitate student learning.

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Appendix

Table 4

Pre-test results for the 2008 Year 9 NAPLAN Non-calculator test - % correct for Year 9 Queensland students (n= *55925) and pre-service teachers* (n=*131), and the identified content strand*

No	Description	S	Q	PT
1	Extending a pattern	n	83	87
2	Understanding a variable relationship	a	74	79
3	Calculating an angle in a triangle	s	60	67
4	Interpreting a stem and leaf plot	m	53	62
5	Geometric reasoning and visualisation	s	62	45
6	Co-ordinate geometry	s	79	81
7	Proportional reasoning	n	53	76
8	Scale and determining ratio	s	75	82
9	Spatial geometry and reasoning	m	68	81
10	Proportional reasoning in the context of data	m	53	70
11	Solving and equation variables on both sides	a	57	58
12	Logic in a measurement context	s	72	72
13	Applying the distributive law to expressions	a	41	58
14	Renaming a mixed number	n	66	74
15	Spatial reasoning	s	58	53
16	Reading an angle on a protractor	m	66	75
17	Geometric reasoning	s	42	53
18	Reading a scale and proportional reasoning	m	46	69
19	Percentage in the context of data	n	34	60
20	Using an understanding of rate	m	39	63
21	Dividing by a decimal	n	19	37
22	Proportional reasoning in a measurement context	n	22	47
23	Solving an equation with a variable on both sides	a	29	36
24	Equivalent fractions in a probability context	m	53	45
25	Number logic in an algebraic context	a	28	26
26	Coordinate geometry and algebra	a	45	34
27	Power notation	n	33	23
28	Substitution and power notation	a	27	28
29	Percentage and proportional reasoning	n	18	29
30	Inequalities and regions in algebra	a	16	26
31	Mean and mode understanding	m	24	22
32	Angle calculation within a hexagon	s	15	12
Mean percentage correct			**47.19**	**54.06**

Key to strands: n= number; a = algebra; m= measurement; s = space.
The questions in italics require a written response.

Table 5
Post-test results for the 2008 Year 9 NAPLAN – calculator allowed test - % correct for Year 9 Queensland students (n= 55925) and pre-service teachers (n=101), and the identified content strand

No	Description	S	Q	PT
1	Integer subtraction	n	80	94
2	Simple substitution	a	80	97
3	Relationship between variables	m	80	97
4	Concept of mean	m	75	98
5	Mixed number computation and number line understanding	n	70	87
6	Geometric reasoning with nets	s	83	90
7	Symmetry	s	87	90
8	Substitution in algebraic fraction	a	61	93
9	Percentage concept	n	67	86
10	Co-ordinate geometry	s	59	76
11	Geometric enlargement	s	42	83
12	Constructing a rule from a table	a	42	81
13	Using triangle properties	s	51	72
14	Percentage estimation	n	44	53
15	Circumference calculation	m	47	55
16	Properties of shapes	s	31	35
17	Calculating an area	m	63	55
18	Substitution	a	48	86
19	Calculating with time	m	48	72
20	Properties of triangles	s	61	75
21	Constructing rule from data	a	32	57
22	Money computations	n	26	64
23	Constructing a rule from a number pattern	n	49	57
24	Substituting into a formula with powers	a	30	75
25	Application of proportional reasoning in the context of scale and triangles	s	44	45
26	Compass directions	s	52	77
27	Algebra in an area context	a	37	67
28	Ratio	n	23	61
29	Logic, addition and subtraction	n	24	57
30	Using a net to find surface area	m	5	7
31	Solving given an equation	a	13	48
32	Mean, median and mode concepts	m	31	46
Mean percentage correct			**49.53**	**69.88**

Key to strands: n= number; a = algebra; m= measurement; s = space.
The questions in italics require a written response.