Messages Communicated through Mathematics Content for Elementary Teachers Course Syllabi: A Focus on Mathematical Disposition and Collaboration

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Professional organisations have provided recommendations for prospective elementary teachers (PTs) to engage in mathematics coursework designed for teachers. In this study, 37 syllabi obtained from instructors of these courses – termed Mathematics Content for Elementary Teachers (MCfET) courses in this paper – were analysed through the lens of messages communicated about the nature of mathematics teaching and learning. Findings indicate that the syllabi in this study communicated messages about mathematical content as well as messages related to mathematical disposition and the role of collaboration in the mathematics classroom. Syllabi presented mathematical disposition messages in two forms: mathematical (i.e., seeing mathematics in the world, mathematics as a sense-making activity) and personal (i.e., belief in oneself, self-assessment), while other syllabi presented potential to promote mathematical disposition. The role of collaboration was described in general and mathematical terms. Careful construction of these messages can help PTs develop a productive mathematical disposition and consider the role of collaboration as they prepare to teach mathematics.

Keywords Preservice teacher education • mathematics content courses for elementary teachers • syllabi analysis • mathematical disposition • collaboration

Introduction

An ongoing conversation in the mathematical preparation of elementary teachers involves the ways generalist prospective elementary teachers (PTs) engage with mathematics in their preparation programs. Many professional organisations have provided recommendations for PTs to engage in mathematics coursework designed for teachers, including recommendations for coursework that investigates elementary and advanced mathematics and connects the two from a teacher's perspective (e.g. Association of Mathematics Teacher Educators [AMTE], 2017; Conference Board of Mathematical Sciences [CBMS], 2001 and 2012). In this paper, courses that afford such opportunities are named Mathematics Content for Elementary Teachers (MCfET) courses and are presumed to have a greater focus on mathematics content than pedagogy.

MCfET courses are often required as part of PTs' preparation programs and, across countries, are typically housed in the mathematics department (e.g. Goos & Bennison, 2017; Masingila et al., 2012). Researchers have previously reported on the teaching of MCfET courses (e.g. Masingila, et al., 2012), instructors' knowledge of professional organisations and their recommendations (e.g. McCrory et al., 2008), and the mathematical content in MCfET courses as presented in the syllabi and textbooks (e.g. Greenberg & Walsh, 2008). Syllabi analyses in the mathematical preparation of elementary teachers have investigated mathematics methods courses (e.g. Taylor & Ronau, 2006). In Australia, findings have suggested such courses can have a positive impact on the mathematical beliefs of PTs (e.g. Beswick & Dole, 2001). To build on previous work investigating MCfET courses, this study explored course syllabi to unpack how the nature of these courses is communicated via these syllabi. The syllabi were investigated with these questions:

> In what ways do the syllabi of MCfET courses communicate messages about mathematical disposition to PTs?
>
> What is the role of mathematical collaboration in MCfET courses as indicated in their syllabi?"

Knowing more about the nature of these courses as represented in syllabi can inform mathematics teacher educators (MTEs) on current practices related to the mathematical preparation of elementary teachers and help them consider the messages communicated to PTs in their own syllabi.

Engaging with Mathematics: Recommendations and Research

Multiple countries have identified K–12 student collaboration in understanding mathematical concepts as a part of productive beliefs about teaching and learning mathematics (e.g., National Council of Teachers of Mathematics [NCTM], 2014), and recommendations for opportunities to collaborate extended to the post-secondary classroom for PTs in MCfET courses (AMTE, 2017). The Australian Association of Mathematics Teachers (AAMT) expects PTs to be "aware of a range of effective strategies and techniques for…promoting enjoyment of learning and positive attitudes towards mathematics" (2006, p. 2). Engaging with mathematics collaboratively in preparation programs affords PTs opportunities to experience mathematics in ways they are expected to use to facilitate mathematics learning in their future practices.

The National Research Council (NRC, 2001) described mathematical proficiency as "what we believe is necessary for anyone to learn mathematics successfully" (p. 116), where the authors emphasised children's learning of mathematics. However, the definition of mathematical proficiency applied to all people—including PTs. Five intertwined "strands" encompassed the ideas of mathematical proficiency: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. The first four strands addressed mathematical skills and abilities present in the international conversation of the teaching and learning of K–12 mathematics (e.g., AAMT, 2006; NCTM, 2000).

The fifth strand, productive disposition, addressed students' beliefs about mathematics and their own abilities. Mathematical skills and abilities alone may not afford PTs or K–12 students the ability to see mathematics as something worthwhile and attainable by all. This strand completed the holistic view of mathematics by incorporating beliefs in addition to the other four strands that involve concepts, procedures, and practices. However, productive disposition was noted as a part of mathematical proficiency often lacking in PTs because of unproductive mathematical experiences and beliefs (AMTE, 2017). While the intertwining of the strands makes it challenging to isolate any one strand (NRC, 2001), knowing more about individual strands can inform the others. In part because of its attention to aspects other than mathematical skills and abilities, this study further investigated productive disposition and the role of collaboration in the mathematics classroom.

Mathematical Disposition

In mathematics education, productive disposition has been identified as a component of mathematics teaching and learning (AMTE, 2017; Boaler, 2016; NCTM, 2014; NRC, 2001). Lowrie and Jorgensen (2016) reported findings from Australia that suggested teacher preparation programs have the potential to influence the mathematical disposition of PTs, regardless of age or level of mathematics completed prior to the coursework. Drawing from definitions of and studies related to productive disposition, mathematical disposition, and productive mathematical disposition in the mathematics education literature, the term "mathematical disposition" was conceptualised. Those definitions and studies are outlined below.

Disposition has been described as "a tendency to think and act in positive ways" (NCTM, 1989, p. 233), and, mathematically, students' dispositions are revealed in the ways they approach tasks and self-reflect. Productive disposition is a "habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy" (NRC, 2001, p. 5), highlighting the view that mathematics is a sense-making activity that students are able to do. This is part of the growth mindset that teachers can foster in students to encourage a positive disposition towards mathematics (AAMT, 2015). These statements have garnered support across countries (e.g., AMTE, 2017; AAMT, 2015), with assertions that well-prepared beginning teachers of mathematics exhibit "productive mathematical disposition," or that "teachers of mathematics expect mathematics to be sensible, useful, and worthwhile for themselves and others, and they believe that all people are capable of thinking mathematically" (AMTE, 2017, p. 9). Also stressed is the role played by teacher preparation programs in developing PTs' productive mathematical dispositions, as programs "represent mathematics as a useful, challenging, and interesting discipline" (AMTE, 2017 p. 33) so that teachers can provide "opportunities that encourage investigation and questioning over routine procedures" (AAMT, 2015, p 3) .

Having a productive mathematical disposition allows PTs to see mathematics as more than a performance opportunity, a finding Nolan and Dwyer (2002) discussed as they studied Canadian mathematics teachers and learners. This study highlighted that students "often embrace the view that you either get [mathematics] or you don't; that you are either good at it or you are not, as if mathematical ability is innate" (p. 5). Mathematical disposition pushes back against this notion, suggesting that mathematics is attainable by all, which professional organisations (e.g. AATE, 2015; AMTE, 2017; NCTM, 2014) and Boaler (2016) have suggested.

Bibby (2002) noted that mathematics evoked feelings of "shame" in practising elementary teachers in the United Kingdom as they felt pressure to get to the correct answer with an efficient and elegant strategy. She also noted that these feelings of shame emerged in teachers when mathematics was experienced as a procedural or algorithmic activity and highlighted two beliefs held by some PTs as sources of shame: that mathematics is a "private affair," and that the correct answer is the ultimate goal. Communicating about mathematics as more than a performance opportunity can help teachers, PTs, and students foster a relationship with mathematics and develop their mathematical dispositions.

Beattie (2002) and Black and colleagues (2004) referred to PTs and students taking ownership of their own learning as a productive practice, respectively. Beattie realised that affording PTs opportunities to "develop the abilities to understand and value others' ways of knowing and the capacities to work collaboratively with colleagues" supported PTs in developing their practice (2002, p. 27). Black and colleagues' work suggested that peer assessment and self-assessment were examples of ways teachers can help students take ownership of their own learning. Teachers noticed that students were more likely to ask clarifying questions of peers than of teachers, which led to the teachers being free to facilitate learning as students were engaged with one another.

The conceptualisation of mathematical disposition used in this study incorporated these recommendations and research findings. In developing their mathematical disposition, PTs would benefit from seeing mathematics as a useful sense-making activity where all are able to develop personal understanding and become active learners who take ownership and responsibility for their own learning.

Collaboration in the Mathematics Classroom

Researchers have asserted that intellectual development evolves from the social to the individual context (e.g. Confrey, 1995; Wertsch, 1985; Cobb, 1995) and that social interaction affords students opportunities to increase their learning potential (Vygotsky, 1978). Vygotsky (1979) theorised that "the social dimension in consciousness is primary in time and fact. The individual dimension is derivative and secondary" (p. 30). Researchers have suggested that collaboration in mathematics leads to learning opportunities beyond those of traditional instruction, allowing students to consider multiple perspectives rather than only that of the teacher (e.g., Confrey, 1995). To become well-prepared beginning teachers of mathematics and envision a classroom where students construct meaning together (AMTE, 2017; NCTM, 2014), PTs may need opportunities to engage in collaborative practices in their preparation programs.

Engaging with mathematical content through collaboration has been a practice in which MTEs have empirically found value. Harkness et al. (2006) reported that the majority of PTs in a problem-based course stated that collaboration was productive in helping them learn mathematical content. PTs increased in confidence when working together and explaining ideas to one another. In another study with similar findings, PTs themselves identified group work—or collaboration—as an effective pedagogical practice used in mathematics content coursework designed for a general audience (Hart & Swars, 2009). As mentioned earlier, Black and colleagues (2004) reported that collaboration in the form of peer assessment was a skill that teachers found productive because students were more likely to question or interrupt one another for clarification than they were to interrupt the teacher. Opportunities to collaborate allowed students to become more aware of their own learning, thereby developing their mathematical disposition through collaboration.

Prior Research on Mathematics Content for Elementary Teachers Courses

Researchers exploring MCfET courses investigated various aspects of the mathematical preparation of elementary teachers. Masingila and colleagues (2012) reported that the majority of MCfET courses offered by institutions were in mathematics departments. The majority of the institutions required PTs to take either two or three semester-long MCfET courses taught through a combination of lecture and activities. The use of activities suggested possible collaboration in the courses, but this question was not further explored in their study.

While previous researchers indicated that the textbook is a resource typically used by instructors of MCfET courses to organise the majority of class time (McCrory et al, 2008), McCrory (2006) noted that an entire textbook is not necessarily used in a course because many textbooks include more topics than can be adequately "covered" in a few semesters. This study builds upon the work of Masingila and colleagues (2012) and McCrory and colleagues (2006; 2008) through investigating the intended curriculum tailored to MCfET courses after MTEs have decided which topics, activities, and problems to include.

Syllabi as Intended Curriculum

Curriculum, whether "for ends or means" of education (Posner, 2004) can include (a) content, standards, or objectives (i.e. "ends") or (b) planned instructional strategies (i.e. "means"). Curriculum has been operationalised to include syllabi by researchers (Gorski, 2009; Posner, 2004; Stein et al., 2007); more specifically, researchers have often identified syllabi as intended curriculum (Billett, 2006; DeLuca & Bellara, 2013). This study drew on Stein and colleagues' (2007) operationalisation of intended curriculum as "the teachers' plans for instruction" (p. 321). Syllabi communicate intentionality to address various components in the course (DeLuca & Bellara, 2013), and these components could include but are not limited to course objectives, course outline, and course evaluation (Hess & Whittington, 2003).

MTEs communicate to PTs via syllabi prior to meeting them and as a course continues. Syllabi often provide an overview of the entire course and serve as an agreement between students and institutions about courses' purposes and directions (Lowther et al., 1989; Wankat, 2002). Doolittle and Siudzinski (2010) suggested that syllabi generally contain pertinent course information and communicate characteristics deemed important by the instructor. Through recommended components (e.g., course description, objectives, outline), instructors communicate to the students about the "nature of the course" and foster a "sense of belonging" (Hess & Whittington, 2003, p. 24). Other parts of the teaching and learning process exist in addition to syllabi (e.g., written curriculum, enacted curriculum). In this study, the decision was made to investigate syllabi separately from other types of curricula (e.g., standards, textbooks) in order to gain a sense of messages communicated to PTs in the intended curriculum.

Assessment structures.

The assessment structure is a syllabi component that communicates important characteristics of the course and may be revealed in the syllabi. Organisations have encouraged a broader conceptualisation of assessment than exams and quizzes (e.g. NCTM, 2014). Boaler (2016) referred to this as multi-dimensional grading, or assessing in various formats (e.g. self-assessment, student-written exam questions). Researchers have emphasised the need for instructors to provide feedback to learners in assessments, including exams, and to afford learners opportunities to demonstrate improvement in their learning after receiving feedback (Black & Wiliam, 1998; Boaler, 2016).

Peer assessment, self-assessment, and the formative use of summative exams are ways to promote a shared responsibility of learning between students and their teachers (Black, et al., 2004; Boaler, 2016). Having learners assess and reflect upon their own progress toward learning goals helped them take responsibility for their own learning (Black, et al., 2004), which is also part of developing mathematical disposition.

Syllabi Analyses in Teacher Education

Prior syllabi analyses in teacher education research informed this study. Gorski (2009, p. 309), in a syllabus analysis of multicultural teacher education courses, defined "official curriculum" as the course description, goals, objectives, and other contextual or descriptive text as he analysed the theories and philosophies "underlying" multicultural teacher education course designs. Gorski followed Strauss's (1987, p. 232) guidelines for open coding to identify more "subtle intricacies" (p. 312) than key words and phrases and limit bias through systemisation. Those guidelines were: (1) ask the data a set of questions, (2) analyse the data minutely, (3) frequently interrupt the coding to write a theoretical memo, and (4) never assume the analytic relevance of any traditional variable until the data show it to be relevant (Strauss, 1987).

Greenberg and Walsh (2008) evaluated the mathematics content of teacher preparation programs through an analysis of written and intended curriculum, including course syllabi, program requirements, and textbooks. Their findings suggested that the majority of teacher preparation programs were insufficient in breadth and depth of the mathematical topics recommended by CBMS.

Taylor and Ronau (2006) analysed syllabi for mathematics methods courses for PTs, concentrating on two syllabi components: common graded elements and common goals/objectives, finding that tests/quizzes and lessons and lesson planning made up nearly half of the course grade for PTs, being distributed almost evenly. Almost all of the syllabi stated goals addressing pedagogical content knowledge, where near one-third referenced disposition. Results suggested that MTEs have underlying goals "woven" throughout a course in the syllabus that are never directly stated; however, the report did not investigate these underlying goals.

The research exploring MCfET courses has identified who teaches the courses, the background of these instructors, and the mathematical content of textbooks designed for these courses. This study builds upon Taylor and Ronau's (2006) work by investigating MCfET courses, which tend to occur early on in the PTs' preparation programs, and Greenberg and Walsh's (2008) work by investigating the messages communicated throughout MCfET course syllabi. In this study, further examination of the syllabi was aimed to uncover messages communicated to PTs about mathematical disposition as well as the role of mathematical collaboration in MCfET courses.

Methods

This study drew from a larger mixed-methods study focused on the mathematical preparation of elementary teachers where MTEs in the United States were surveyed about mathematics teacher education program requirements, instructor backgrounds, and MCfET course details. Respondents described resources, content activities, and standards addressed, in addition to uploading syllabi and/or other course documents. Purposive snowball sampling (Devlin, 2018) was used in an attempt to reach all MTEs involved in the mathematical preparation of elementary teachers. There were 120 MTEs who completed the survey, describing self-identified MCfET courses, methods courses, general mathematics courses, and courses that equally combined content and pedagogy. Of those 120 MTEs, 44 MTEs described 82 MCfET courses. The survey presented, but did not require, a response to the following prompt: "If possible, please upload a syllabus and any other documents related to this course (e.g., course calendar, assignment list, topic list)." Twenty-four MTEs uploaded course syllabi for 37 MCfET courses. To ensure a complete set of data, MTEs were individually contacted for more information when the items in the syllabus were missing or vague.

Analysis

The syllabi were analysed using a six-step process. First, each syllabus was read, with the following curricular components previously identified by Posner (2004) and Gorski (2009): course description, goals, learning objectives, topic list, assessments, and assignments. Second, each syllabus was read again and written reflections were logged by the researcher. This included noting, among other elements, the mathematical focus, course/learning goals, PT expectations, and class structure. For example, one reflection noted that the syllabus expressed the expectation that PTs will persist, find alternate solutions, communicate, and respond to and connect mathematical work.

The collection of reflections was read to generate an emerging theme memo, as suggested by Strauss (1987), allowing the data to show the relevance of the themes. During this process, it was noted that many syllabi highlighted collaboration, participation, and attendance. The reflections and memos allowed access to the "subtle intricacies" communicated in the syllabi referenced by Gorski (2009, p. 312). In the reflections and memos, the perspectives of both the MTE writing the syllabus and the PTs reading the syllabus were considered. From this process, three components and two themes emerged because of their consistency or variety across syllabi and these are described in the findings.

The components of assessment structure, topics addressed, and attendance policies were traditional syllabus elements that could be quantified. Of these three components, the assessment structure provided the most insight into the courses by often providing percentages of the course grade assigned to each assessment and, at times, descriptions of assessments. The topics addressed gave a broad view of course content. It was clear that university policy had influenced the attendance statements, and, because these statements gave no insight into the messages communicated about mathematics, they were not investigated further.

The third step of analysis was to utilise protocol coding (Saldaña, 2016), in which the assessment structure and the themes of collaboration and connections to teaching were used as lenses to systematically analyse the syllabi. First, the assessment structure was detailed along with the breakdown of the course grade from each assessment in search of collaborative opportunities. Then each syllabus was examined, noting statements addressing collaboration of any kind (i.e., general, mathematical) and connections to the practice of teaching.

Fourth, the literature investigating mathematical collaboration was revisited, finding some of the aforementioned studies (e.g. Harkness et al., 2006) along with further exploration of the literature describing the five strands of mathematical proficiency: procedural fluency, conceptual understanding, strategic competence, adaptive reasoning, and productive disposition (NRC, 2001). Fifth, a summary for each of the strands was created and used to identify statements in the syllabi that addressed each of the five strands of mathematical proficiency.

Sixth, after reflecting on statements identified in steps three and five addressing collaboration, connections to the practice of teaching, and the five strands of mathematical proficiency, the focus was narrowed to the lenses of mathematical disposition, the role of collaboration in the mathematics classroom, and the assessment structure and/or assessment purpose. These areas were chosen because of their interconnectedness; mathematical disposition includes making sense of mathematics, which has been encouraged to happen collaboratively in recommendations and standards (e.g. NCTM, 1980, 2000, & 2014). The decision was made to include the assessment structure in an attempt to gain insight into the role of collaboration in course assessments, specifically analysing the assessment structure as it related to mathematical disposition and collaboration.

In addition to being interconnected, these themes also provoked further investigation because of the variety of attention the themes received across the syllabi. In this step, the syllabi statements were sorted by their theme. For example, the statement "[an] important goal of the course is to promote the development of the belief that mathematics is a sense-making activity and that learning mathematics involves figuring out how to solve problems in personally meaningful ways" presented mathematics as a sense-making activity. Similarly, statements indicating collaboration in the MCfET classroom were categorised by two themes: general and mathematical. The general collaboration statements were not discipline-specific, and the mathematical collaboration statements highlighted the mathematical activities and experiences in the classroom. For example, the following statement was classified as mathematical collaboration: "Class activities will involve engaging in mathematical discussions with peers, explaining one's mathematical thinking, questioning, and challenging the mathematical thinking of others." The language of "mathematical thinking" prompted this statement to be coded as mathematical collaboration.

Syllabi, Instructor, and Program Description

Syllabi

Thirty-seven syllabi from 35 unique courses were collected; two MTEs at the same institution provided syllabi for the same two courses, and these syllabi contained common components and identical statements about goals, philosophy, objectives, assessments, and more. Thus, these syllabi were included in the data set only once. Thirty-one (89%) of the 35 syllabi were identified as the first or second course in a sequence of MCfET courses. The mean page length was 5.6 pages (median five pages). Of the 24 MTEs who submitted syllabi, 11 provided syllabi for two separate courses. Hence, the sample size of syllabi is 35 (n = 35) from 24 MTEs (n = 24).

Participants

The 24 participants who provided syllabi for MCfET courses spanned 14 states at 23 institutions. All respondents held an instructor role involved in MCfET courses, and some had other responsibilities and experiences, like being an advisor or program coordinator (see Table 1).

Table 1
Roles and Teaching Experience of MTEs Providing MCfET Syllabi

Role	% of MTEs (n = 24)	Mean Experience (in years)
Instructor	100	9.2
Advisor	33	5.3
Program Coordinator	25	7.7
K–12 Teacher	71	6.2

Of those 17 MTEs reporting K–12 teaching experience, six (35%) reported teaching in a K–5 classroom for a mean of 1.3 years; eight (47%) reported teaching in a middle-grade classroom for a mean of 2.9 years; and 13 (76%) reported teaching in a high school classroom for a mean of 5.4 years. Seven MTEs had experience in more than one K–12 grade level.

All participants reported affiliation with the mathematics department at their institution, and five MTEs (21%) reported additional affiliation with the education department. Nineteen (79%) of the MTEs indicated conducting research in mathematics education, with two (8%) MTEs reporting research in mathematics or another field. One MTE reported no research area.

Programs

Twenty-three unique programs were described by participants, but because not all questions required a response before continuing the survey, 22 MTEs indicated the following characteristics of their programs: Showing a range of program sizes, 23% of the programs reported annually graduating each of the following ranges of PTs: 11–25 students, 26–50 students, 51–100 students, and more than 100 students. All programs prepared teachers for licensure in grades 1–4, with some programs preparing PTs to teach as early as pre-kindergarten or as late as eighth grade.

MTEs reported three variations of mathematics course requirements in their programs: MCfET, mathematics methods and general mathematics (e.g., mathematics courses not specifically designed for teachers). Table 2 presents the frequencies programs required each of these types of courses with the mean and median number of courses and semester hours required.

Table 2
Mathematics Course Requirements for PTs in Programs

	% of Programs Requiring	# of Required Courses		# of Semester Hour Credits	
		Mean	Median	Mean	Median
MCfET	96	2.1	2	7.0	6
Mathematics Methods	100	1.3	1	4.0	3
General Mathematics	48	1.1	1	3.7	3

Table 2 shows PTs took more MCfET courses than mathematics methods or general mathematics courses. MTEs indicated that the MCfET courses were designed primarily for first- or second-year PTs, with fewer than 6% of the PTs in these courses in their third or fourth year.

Findings

Findings of this study are separated by the themes of mathematical disposition and collaboration in the mathematics classroom. First, findings are presented from the syllabi through statements related to PTs' mathematical disposition in two ways: statements that clearly communicate aspects of mathematical disposition to PTs and statements that have potential to support development of mathematical disposition. Next, findings are reported from the syllabi related to the role of collaboration in mathematics—both by statements and in the assessment structure. These statements assisted in answering the research questions of "In what ways do the syllabi of MCfET courses communicate messages about mathematical disposition to PTs? What is the role of mathematical collaboration in MCfET courses as indicated in their syllabi?"

Mathematical Disposition

Seeing mathematics as a sense-making activity, having a growth mindset, and developing a personal relationship with mathematics are characteristics of mathematical disposition. Evidence of these characteristics, along with the belief that all people can learn mathematics, was present in many syllabi. In this section, statements considered to promote mathematical disposition and others that incorporated characteristics of mathematical disposition less clearly are identified.

Promoting mathematical disposition

"I know you can do this!". Statements in the syllabi related to promoting mathematical disposition were of two types: mathematical or personal. The mathematical statements addressed seeing mathematics in the world and perceiving mathematics as a sense-making activity. The personal statements communicated to PTs that they could be a source of mathematical knowledge through belief in themselves and that self-assessment of their mathematical understanding would happen throughout the course. Similar to the five strands of mathematical proficiency (NRC, 2001), these categories of mathematical and personal are not discrete and are intertwined. Table 3 presents examples of mathematical statements, and Table 4 presents examples of personal statements.

Table 3
Mathematical Disposition Statements from Syllabi by Theme: Mathematical

Seeing Mathematics in the World	Mathematics as a Sense-Making Activity
Acknowledge the relevance of mathematics in everyday life.	[To] construct personal meaning from all these experiences.
This course involves a careful examination of mathematical ideas behind the mathematics taught in grades K–6, and their history and applications to daily life.	Attention is given to connections with other areas of mathematics and to the need for developing the "habits of mind of a mathematical thinker."
Making the connections between the mathematics within the textbook to events that occur and are used on a day to day basis.	[An] important goal of the course is to promote the development of the belief that mathematics is a sense-making activity and that learning mathematics involves figuring out how to solve problems in personally meaningful ways.
[The mathematics] must be practical for students as they live as citizens in this world and as they live their personal lives.	[Develop] a disposition for understanding how others engage with mathematical concepts.
Knowledge of the relationship of mathematics to other subjects, its application in society, and relationships within mathematics itself.	Our work should support you in making sense of yours and others mathematical thinking.
To develop an appreciation of and an interest in mathematics and applications of mathematics [and]…to further develop positive dispositions toward mathematics.	Your job…is to carve out your personal "best route to understanding."

The 12 statements found in Table 3 came from 12 syllabi and communicated to PTs that mathematics is a sense-making activity that connects to everyday occurrences. The six statements related to *seeing mathematics in the world* all referenced PTs seeing mathematics in their daily lives. Viewing mathematics as sensible is productive for PTs as they build practical knowledge about the teaching of mathematics and the role of mathematics in everyday life. The six statements related to *mathematics as a sense-making* activity referenced the PTs individually and collectively having opportunities to make sense of mathematics. For example, the following statement highlighted the notion that mathematical thinking varies across people, as making sense of the mathematical thinking of others implies that the mathematics thinking is possible different from your own: "Our work should support you in making sense of yours and others mathematical thinking."

Table 4
Mathematical Disposition Statements from Syllabi by Theme: Personal

Belief in Oneself	Self-Assessment
Develop confidence in your abilities to understand and do mathematics.	[The] purpose for this structure [replacing a midterm grade with final grade if higher] is to promote your interest in continually improving your understanding, learning from your mistakes, and connecting what we learn in one chapter to topics studied in later chapters.
Students learn to depend on themselves and each other (rather than on the instructor) for problem solutions.	Letting students revise their homework is important to us as instructors, for in this process important learning can occur…a demonstration that the most important weaknesses have been understood and addressed.
I know you can understand this!	[PTs should have an] inclination to monitor and reflect on their own thinking and performance.
[Develop] confidence in their abilities to solve mathematical problems, and to determine whether a proposed solution to a problem is valid or invalid.	The honest assessment of your own understandings and misunderstandings will be significant.
You are expected to respect…yourself in this course. Negative and insulting language about your own abilities will not be tolerated.	To encourage you to become reflective doers of mathematics.

The 10 statements from nine syllabi presented via Table 4 emphasised that PTs are able to learn mathematics and gain mathematical authority. The five *belief in oneself* statements communicated the message to PTs that they are able to do the mathematics in these courses, a foundational component of mathematical disposition aligned with AMTE (2017), NCTM (2014), and NRC (2001). The repeated references to monitoring one's own work and improving understanding in *self-assessment* highlight what the standards and recommendations continue to ask K–12 students to do—reflect on the process and the product.

Communicating characteristics of mathematical disposition is productive for many reasons. It promotes the belief that mathematics is accessible, sensible, personal, and attainable for all learners. These statements explicitly promoted developing mathematical disposition in PTs.

Potential to promote mathematical disposition

"I am not a textbook for you to copy into your notebook". Some statements were less clear in operationalizing and communicating characteristics of mathematical disposition or the belief that mathematics is a useful sense-making activity in which PTs make personal meaning. The less

clear statements were in a minority of syllabi, and in order to illuminate these, sample statements from four syllabi are provided.

Certain statements may have led PTs to think that learning in mathematics is about performance on assessment tasks, such as "It is important that all activities be completed so that you will be able to do well on graded homework and on the three scheduled tests." This statement may have been an attempt to appeal to PTs who lack motivation to learn for understanding but are motivated by performance goals and grades. A statement from another syllabus may have resulted from considering similar motivating factors for performance: "Above all, for the sake of a good grade and the understanding of your future students, please stick with it till you get it." This statement seemed to appeal to two motivators: earning a "good" grade in the course and the mathematical thinking of future students. While making sense of the mathematics may be a part of the homework, tests, and grades in these particular courses, these statements alone did not clearly communicate that to PTs.

One syllabus communicated about mathematical disposition with a variety of statements in regards to course expectations by stating: "In keeping the theme that we are all responsible for each other's learning and success, we are going to be each other's resource for answering questions." With this statement, PTs are encouraged to take responsibility for their own learning and the learning of others and to collaborate to achieve that learning. This alone is part of developing a personal relationship as a part of mathematical disposition. However, the syllabus then stated, "PLEASE LIMIT your emails to me ... emails clogging my inbox will not earn you any brownie points." The syllabus continued with encouraging PTs to access other resources (e.g., classmates, university web platform) for help: "You WILL earn brownie points for answering each other's questions." The concluding statement was "Never hesitate to ask a question during class or meet with me before or after class if you cannot make office hours." At first, this syllabus referenced the PTs and instructor as a learning community. Then, the instructor seemed to remove herself from the community with a deterrent to emails. With the face-to-face encounters suggested at the end, the message appeared to be that the MTE was an available resource to help PTs develop their mathematical understanding. PTs reading these statements about the course may not know what type of communication to have with the MTE. The discussion of brownie points could lead PTs to question the possible actions that could lead to "brownie points." These statements may all be clarified in class sessions, but because PTs may read the syllabus before meeting the MTE, the interpretations made could lead PTs to question how this community functions in making personal meaning of mathematics.

Statements from another syllabus communicated to PTs the belief in their abilities to do mathematics. The syllabus described daily journals and personal textbooks that PTs will create, followed with

> Semester after semester, I get the question "How am I supposed to take notes when you don't write stuff on the board?" I am not a textbook for you to copy into your notebook. You and your classmates have a wealth of knowledge and are capable of solving complex problems.

These statements indicated that mathematics is not about taking notes and that PTs can access the needed knowledge to do mathematics. Framing the beginning of the paragraph about the textbook in a way that describes mathematics as a sense-making activity that is more than note taking could help clarify this message about PTs being able to do mathematics.

Collaboration in the Mathematics Classroom

To verify the statements about collaboration in the mathematics classroom, this topic was investigated by two means: through statements in the syllabi and the assessment structure indicated in the syllabi. The statements tell one story, and the assessment structure adds to the story, with some statements indicating collaboration as a part of the assessment structure.

Statements related to collaboration

Of the 35 syllabi, 33 included statements interpreted as encouraging collaboration—in or out of the classroom. In the syllabi that indicated collaborative language, there were two distinct categories. The first category included more general statements with language of "working together" that could apply across disciplines, and the second category included statements indicating the notion of mathematical collaboration. Findings are separated into these two categories.

Some statements were general with their notion of collaboration in the MCfET course, such as "A major part of this course will be spent working in small groups" and "Some of the activities [in class] will be completed in groups." A statement from another syllabus promoted variety in groups by encouraging PTs to "seek out opportunities to work with different members of your class to experience the breath of diversity our classroom setting offers." One syllabus explicitly justified the collaboration with

It is our conviction that problems are best solved in a cooperative learning situation, [as]:

- Group problem solving is often broader, more creative, and more insightful than individual efforts.
- Interaction with others may stimulate additional problems, insights, and discoveries.
- Students can motivate one another to excel and to accept more challenging problems
- Socialisation skills are developed and practiced.
- Motivation to persevere with a problem may be increased.

Communicating to PTs the perceived benefits of collaboration may help them think about its purpose and role in MCfET courses. This communication may also give PTs reasons to facilitate collaboration in their future classrooms.

Other syllabi indicated expectations for mathematical collaboration in the courses. One statement communicated to PTs about expectations for mathematical collaboration with "Students are encouraged to work collaboratively to explore, generate, and debate conjectures, build connections among concepts, [and] solve problems created." Another syllabus identified mathematical collaboration by saying, "Class activities will involve engaging in mathematical discussions with peers, explaining one's mathematical thinking, questioning, and challenging the mathematical thinking of others and developing a sense of what constitutes an acceptable mathematical justification." Again, this statement illuminated the expectation of collaborating mathematically. Another syllabus indicated the MTE's beliefs by saying, "I strongly believe there is value in conversation and collaboration in developing deep conceptual understanding of mathematics, so you are encouraged to find a study partner or study group."

Two syllabi implied that the purpose of working together was to perform well on practice problems, with "To get a good grade on homework, forming a study group is critical" and "Some parts [of homework] will be too hard to do alone." This statement implied that collaboration in some of the MCfET courses happens when PTs are working on homework.

Assessment structure through the lens of collaboration

Many of the syllabi included traditional items such as homework and exams in the assessment structure of the course. In fact, 34 (97%) of the syllabi included homework as a part of the assessment structure. Because of multiple suggestions for PTs to work together on homework outside of class, homework was considered to be an opportunity for mathematical collaboration. Class participation or attendance and group projects were also deemed as potential opportunities for collaboration.

Table 5 presents the findings from assessments affording the opportunity for PTs to collaborate. Thirty-four (97%) of the syllabi indicated in the assessment structure that PTs were graded on a traditional 90-80-70-60 scale for a grade of A-B-C-D, respectively. The "% of Course Grade" column in Table 5 represents these syllabi. One MTE used mastery grading with a "star" system that allowed PTs to earn stars in order to have multiple attempts on summative assignments. This syllabus included online homework, quizzes, a final exam, participation, and a portfolio with pencil-and-paper homework and reading assignments as a part of the course. However, there was no percentage breakdown in the assessments, unlike all the other syllabi. Therefore, this syllabus was included in the number of syllabi and percentage of syllabi but not in the average percentage of the grade.

Table 5
Assessments in MCfET Course Syllabi with Potential Collaboration

	Number of Syllabi	% of Syllabi (n = 35)	% of Course Grade (Mean)
Homework	34	97	21
Group Projects	6	17	14
Participation/Attendance	11	31	10

As seen in Table 5, homework was the most available opportunity for PTs to collaborate on an assessment. Group projects and class participation or attendance were much less common as an explicit component of the course grade, although one syllabus stated, "You will earn points for participating in a variety of in-class discussions and activities over the course of the semester."

Assessments like tests/exams, quizzes or papers, projects or presentations, and final exams are often individual assessments and were categorised them as such unless noted otherwise in the syllabus. Thirty-four syllabi (97%) indicated exams as a part of the course grade, and 30 syllabi (86%) indicated a final examination. Those results, along with other individual assessment breakdowns, are in Table 6. Again, one syllabus indicated these activities without a percentage breakdown and is only included in the number and percentage of syllabi.

Table 6
Individual Assessments in MCfET Course Syllabi

	Number of Syllabi	% of Syllabi (n = 35)	% of Course Grade (Mean)
Exams	34	97	43
Final Exam	30	86	23
Quizzes/Papers	16	46	17
Projects/Presentations	12	34	16

As seen in Table 6, exams constituted two-thirds of the grade for the majority of the MCfET courses. One syllabus identified a "group portion" of the exams, and more information was obtained from the MTE who wrote this syllabus. She indicated that PTs worked on a group problem for 20 minutes, turned in a group answer, and then received individual questions to assess PTs' understanding of the group problem. This allowed for PTs to collaborate and make sense of the problems together before continuing on their own.

Assessments with purpose indicated

Some syllabi included purpose statements for assessments, including opportunities for multi-dimensional grading. These findings are shared because of their potential to affect mathematical disposition. For example, one of the goals for a course was "To assess your learning in a variety of ways." Another syllabus indicated that assessments "will take many forms to accommodate different learning styles and providing a variety of ways to demonstrate your learning." Yet another example of multi-dimensional grading was presented with "There will be an oral exam attached to the second exam ... you will be given a list of potential questions for the exam and will be asked to present a single problem during the examination."

Attention to improving understanding was given in other syllabi. For example, one syllabus described a project tailored to the PTs' needs this way: "The purpose behind this project is for you to identify a mathematical area of weakness and intentionally learn more about the identified area over the course of the semester." Another syllabus addressed learning from mistakes with "You will have opportunities to revise your written work and your problem sets, and you are highly encouraged to do so." Another instructor referenced opportunities to clarify understanding with "We have designed opportunities for both problems and group homeworks to be revised...[so] that the most important weaknesses have been understood and addressed."

Discussion

MCfET course syllabi in this study communicated messages to PTs about the nature and learning of mathematics. In these findings, two particular messages were identified related to mathematical disposition and the role of collaboration in a mathematics classroom. Here, these results and their implications are discussed for MCfET course instructors and PTs in light of previous literature and provide recommendations for syllabi to describe and support development of mathematical dispositions and clarify the role of collaboration in a mathematics classroom in statements and assessment structure.

The syllabi communicated messages describing mathematical disposition in two categories: the role of mathematics and a personal relationship with mathematics. These categories are ultimately about PTs gaining mathematical awareness and authority. The role of mathematics involved seeing mathematics in the world and seeing mathematics as a sense-making activity. The personal relationship with mathematics involved believing in one's mathematical ability and engaging in self-assessment. Describing mathematics as a sense-making activity can support the view of mathematics as more than memorisation and symbol manipulation (AAMT, 2006, 2015; NCTM, 1980, 2000).

AMTE (2017) indicated that "well-prepared beginning teachers of mathematics expect mathematics to be sensible, useful, and worthwhile for themselves and others" (p. 9), and Lowrie and Jorgensen (2016) suggested that a positive view of mathematics is helpful for it to be useful across disciplines. Many syllabi in this study communicated this message to PTs. For example, the statement "[An] important goal of the course is to promote the development of the belief that mathematics is a sense-making activity and that learning mathematics involves figuring out how to solve problems in personally meaningful ways" was in alignment with AMTE (2017). This statement also tied the view of mathematics to having an individual relationship with mathematics. Beattie (2002) and Black and colleagues (2004) identified strengthening PTs' and K–12 students' individual relationships with content as a way to improve learning. Beattie (2002) studied helping PTs take ownership for their own learning and "find their voices in relation to the theory and practice of teaching" (p. 20). These findings illustrate that some syllabi statements encouraged PTs to find their voices and make sense of mathematics, therefore taking responsibility for their own learning and strengthening their personal relationship with mathematics. For example, one statement communicated to PTs that there may be more than one path to understanding with "Your job…is to carve out your personal 'best route to understanding'."

In some of these syllabi MTEs communicated attention to increasing one's own mathematical confidence and understanding. One way Black and colleagues (2004) indicated enhancing understanding was through peer assessment and self-assessment. The findings indicate that MTEs communicated this to PTs with statements such as "The honest assessment of your own understandings and misunderstandings will be significant" and at times described activities of written reflections or other means of self-assessment. Communicating about these aspects of mathematical disposition in the syllabi is a way for MTEs to assist PTs in their developing conceptualisation of mathematical disposition.

The syllabi presented two types of collaboration: general and mathematical. General collaboration included strategies or approaches that crossed over content boundaries, and mathematical collaboration was specific to mathematical language. Outlining the purpose for general collaboration showed support for learning in a social setting, aligning with Vygotsky's (1978) theory of learning and development as requiring a social component. For example, one syllabus included the following statement that indicated some of the benefits of collaboration, as viewed by the MTE: "Students can motivate one another to excel and to accept more challenging problems; socialisation skills are developed and practiced." These characteristics of motivating, combined with completing tasks that individually may be too challenging, highlighted the learning potential increase by social interaction as described by Vygotsky (1978) in the pathway to learning and development. PTs in Lowrie and Jorgensen's (2016) study viewed the constructivist approach like that of Vygotsky to be a positive way to experience mathematics. The attention to collaboration in classroom activities also supported the notion of the learning process as progressing from social to the individual context (e.g., Confrey, 1995; Wertsch, 1985; Cobb, 1995), as PTs have opportunities to make sense of mathematics collaboratively and then demonstrate understanding individually through exams and other individual course components.

MTEs can communicate characteristics of mathematical collaboration as they work toward the vision of well-prepared beginning teachers of mathematics set forth by international organizations (e.g., AMTE, 2017; AAMT, 2006). Statements with language that distinguished between general group work and mathematical collaboration afforded PTs the opportunity to see mathematics as something with which to engage from different viewpoints and not the "private affair" suggested by Bibby's (2002) results. For example, describing class activities as involving "explaining one's mathematical thinking, questioning, and challenging the mathematical thinking of others and developing a sense of what constitutes an acceptable mathematical justification" can help PTs envision collaboration in a mathematics classroom. The efforts of describing mathematical and general collaboration can assist PTs in envisioning a mathematics classroom that is not lecture-based. However, if the courses are rooted in collaboration, should students expect opportunities for collaboration on assessments? The instructor whose syllabus included a group portion in the midterm exams may have already considered this question and worked to incorporate Boaler's (2016) recommendation of multi-dimensional grading. This grading approach afforded PTs opportunities to demonstrate growth and learning through means other than traditional exams and homework.

Various syllabi indicated the purpose of the course activities and assessments, indicating that some syllabi were focused on performance goals and others on learning goals. For example, performance goals were communicated in the two syllabi that indicated that the purpose of homework was to perform well on exams. This confirmed a concern of Nolan and Dwyer's regarding many mathematics teachers and learners in their 2002 study, where teachers wanted students to "get it" without clarifying if it was a concept or a procedure they were "getting." However, the statements in the majority of these syllabi communicated messages pertaining to learning goals with some attention to performance. For example, one statement indicated the purpose of assessments was "to assess your learning in a variety of ways." However, with an average of 44% of courses based on exams and 23% based on a final exam, individual performance was still a major part of assessment in these MCfET courses.

MTEs likely have justification for this structure of individual exams. However, MTEs can still reflect upon the role of these exams and the ways syllabi communicate the purpose of these assessments. Two syllabi described using exams as a source of formative feedback, allowing for improvements to be made after an initial assessment—even an exam—which aligned with recommendations from Boaler (2016) and Black and colleagues (2004) to include opportunities for demonstrating improvement of understanding over time. Another statement encapsulated the idea of exhibiting growth this way: "the purpose of replacing midterm grades with a higher final exam grade" was explicitly described as to "promote your interest in continually improving your understanding, learning from your mistakes, and connecting what we learn in one chapter to topics studied in later chapters." Other statements regarding multiple attempts on homework or other assignments to demonstrate improvement in understanding also supported Black and colleagues' (2004) and Boaler's (2016) recommendations. Opportunities to demonstrate improvement on learning goals could also assist in the shift from a focus on performance to learning goals, thereby further decreasing questions of what students "get" as described by Nolan and Dwyer (2002).

The assessment structure can also influence the development of PTs' mathematical disposition. As Black and colleagues (2004) noted, assessment feedback may have a negative impact on students, particularly when numerical scores or grades are given, sending the message that students "lack ability and so are not able to learn" (p. 9). The assessment structure as presented in many of the syllabi here may have a similar effect. Even if the language in the syllabi encouraged collaboration, it could be disconcerting for PTs to see that exams are the majority of the assessment structure. This would especially be concerning for PTs if their prior experiences with exams were not positive or productive, which can often be the case (AMTE, 2017). As we know from Bibby (2002), this pressure associated with mathematical performance can cause anxiety and shame for elementary teachers around doing mathematics. When the syllabus conveys the purpose of each type of assessment, PTs may be inclined to see beyond the numbers. Testing appears to be the most important part of assessing in mathematics, considering that nearly two-thirds of the course assessments were through examinations. It is doubtful that instructors of MCfET courses want to invoke feelings of anxiety and shame from the syllabus, but the assessment structure presented to PTs before knowing them as learners and future teachers could very well invoke those feelings. The syllabus that included replacing lower exam scores with higher cumulative exam scores may help lessen the anxiety associated with mathematical performance.

Recommendations for MCfET Course Syllabi

Because the syllabus may be the first form of communication MCfET course instructors have with PTs, the document can set the tone for the course and communicate what the MTEs deem important. Therefore, intentional language use by MTEs can support development of a mathematical disposition and define the role of collaboration in a mathematics classroom.

Including statements that support a mathematical disposition in the syllabus can help PTs who have only considered mathematics as a performance opportunity to envision mathematics as something with which to engage instead. This includes clearly defining mathematical disposition, and one way to do that would be to reference the four themes found in this study: seeing mathematics in the world, mathematics as a sense-making activity, belief in oneself, and self-assessment. Tables 3 and 4 provided examples of statements that reinforce these themes, all of which support the language of professional organizations (e.g., AAMT, 2015; AMTE, 2017; NCTM, 2014) of productive mathematical disposition and productive beliefs teachers should hold about mathematics, respectively. Together, these themes can communicate important aspects of mathematical disposition.

Describing expectations of collaboration—whether mathematical or general—in the syllabus can help PTs envision a mathematics classroom that supports the vision where PTs are working together to make sense of mathematics. Communicating the advantages of collaboration in general (e.g., to motivate, to stimulate insights) and what constitutes mathematical collaboration (e.g., argumentation, justification) were two characteristics of collaboration found in these syllabi. These categories of collaboration afford PTs the opportunity to see a mathematics classroom as space to communicate mathematically with one another. Incorporating group assessments, like the syllabus that indicated that students work in groups for a part of exams, may also support this notion of collaboration. However, there are practical classroom considerations for MTEs that could present challenges for implementation (e.g., group dynamics, PT disability accommodations).

As with any study, this study has limitations. Because limited work exists that investigates the intended curriculum of MCfET courses, this study focused solely on the syllabi. Therefore, the experiences or entities that influenced the creation of these documents is unknown (e.g., past teaching experiences, university policy). Additionally, these syllabi indicate *intent*, which may not represent actual enactment. Future work investigating the development of MCfET course syllabi and oral communication about the syllabi to PTs could illuminate more aspects of the ways MTEs communicate to PTs about mathematical disposition and not only the role but the *use* of mathematical collaboration in MCfET courses not only in the United States but across countries.

Conclusion

Instructors of MCfET courses communicated messages about the nature of mathematics teaching and learning in syllabi. In addition to course content, the syllabi also conveyed messages related to mathematical disposition and the role of collaboration in the mathematics classroom. Careful construction of these messages can help PTs develop a productive mathematical disposition as they enter MCfET courses in their preparation. MTEs cultivate a desire to understand mathematics and see sense in it, all while promoting practices such as collaboration that PTs can implement in their future classroom to support students.

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