Mathematics Teachers' First Engagement with Research Articles in Mathematics Education: Sketches of New Praxeologies

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This study is concerned with the first experiences of in-service mathematics teachers in consuming scholarly mathematics education literature. Growing from the meta-didactical transposition model, we were interested in the praxeologies that may develop from teachers' engagement with research articles. The data were collected from a cohort of 13 teachers who read, interpreted, critiqued, and designed a follow-up to an empirical study as part of a master's level course in mathematics education. The teachers reported that such a consumption of research contributed to the expansion of their mathematical knowledge, instigated changes in their teaching of particular concepts, and provided them with ideas for enacting this change. Furthermore, we explore how engagement with an article shaped the teachers' understandings about research in mathematics education.

Keywords in-service teachers · novice researchers · meta-didactical transposition · praxeology · mathematics education

Introduction

Engaging with research is a complicated endeavour that demands a complex set of understandings and skills (Shulman, 2010). However, what are these skills and how are they being developed and utilised? Research on professionalism and expertise repeatedly shows that experts excel in their core practices, but not necessarily in their analysis and communication to newcomers (e.g., van Someren, Barnard, & Sandberg, 1994). In some fields, such as medicine, research knowledge and its development among beginning researchers are studied systematically (Burke et al., 2005), whereas empirical research on this topic in mathematics education is nascent (Kontorovich & Liljedahl, 2018; Nardi, 2015).

In their reflection on the maturation of the mathematics education research community, Lester and Lambdin (2003) argued that "a mark of a profession is that it provides systematic mentoring to novices to ensure that these new members will be well prepared to contribute to the activities of the community" (p. 1663). Wilson (2006) notes that, for the education field in general, "we have done little to theorize about how people learn to do research" (p. 320). This reflects an earlier observation by Boaler, Ball, and Even (2003), who found that preparation of novice researchers, in mathematics education in particular, "has rarely been the object of systematic investigation" (p. 497). Over a decade later, Nardi (2015) argued that this is still the

case and she stressed the need to focus research on practices that "aim to foster skills and attitudes in postgraduate students, which are epistemologically specific" (p. 4).

We interpret these positions as an open call for explorations of newcomers' engagement with mathematics education research. Following Nardi (2015), we focus the study at hand on the epistemology of a specific cohort of novices — in-service teachers who study in graduate mathematics education programs. In particular, we are interested in their engagement with scholarly mathematics education literature, which Boaler et al. (2003) argue is a "core practice of disciplined inquiry" (p. 497). When suggesting that the ways in which teachers and researchers approach this "core practice" differ, Bartels (2003) noted the lack of research that focused specifically on how teachers read and react to research literature. Informed by Wilson (2006), we term this practice *critical consumption* to account for the myriad skills that it entails, including comprehending, synthesising, and evaluating. As such, our study explores the first experiences of practicing mathematics teachers in consuming research in mathematics education.

Teachers as novice researchers

Most of the current research on the development of novice mathematics education researchers focused on doctoral students transitioning into academia (e.g., Andrà & Brunetto, 2018; Boaler et al., 2003; Haser, 2018; Reys, 2018; Thanheiser, Ellis, & Herbel-Eisenmann, 2012). Nardi (2015) noted, however, there are different experiences and expectations for those student-researchers who come from, and intend to remain in, a school classroom. Research in this area has mainly focused on how mathematics teachers conduct research using methodologies, such as action research (Jasper & Taube, 2004), lesson study (Stigler & Hiebert, 1999), and communities of practice (McClain & Cobb, 2004; Wenger, 1998). At the graduate level, studies of practicing teachers conducting graduate studies in mathematics education are sparse. One study by Kontorovich and Rouleau (2018), examined how teachers cope with tensions that arise in interview settings. Another by Liljedahl (2018), described the preparation of graduate students, some of whom were schoolteachers, to become independent researchers. While both studies addressed the specificity of teachers' tensions and struggles, these investigations paid limited attention to the endeavour of critical consumption of research literature.

In general, studies of teachers reading research found that many of the teachers tended to value research that offers solutions to problems that were relevant to their practice, with Zeuli (1994) suggesting that teachers consider an idea useful when "it meshes with their experiences" (p. 52). MacDonald, Badger, and White (2001) found that teachers can view the language of research to be overly theoretical and difficult to use. Similar findings appeared in a study by Groth and Bergner (2007) that examined teachers' perspectives of mathematics education reports and positioned them according to Hammersley's (2002) models of the research to practice relationship. The teachers who valued practice-related research with immediate solutions were labelled as proponents of the engineering model, while those who found that research stated things the teachers already knew fell within the enlightenment model. Our aim was to sketch a more nuanced picture of how teachers consumed research literature.

Theoretical Framing

In noting the growing trend towards the involvement of practicing teachers in research, Reis-Jorge (2005) suggests that the goal is not to produce more academic scholars, but to prepare reflective practitioners, "who are more conversant with theoretical discourse and more competent users of research literature" (p. 303). Accordingly, we see teachers' consumption of research literature as an activity targeted at professional growth through sharing knowledge across communities in mathematics education, rather than a path towards teachers' full immersion into the research enterprise. This conceptualisation led us to frame our study with the theoretical model of meta-didactical transposition (Arzarello et al., 2014), which was introduced to capture the complex dynamic that takes place when teachers and researchers interact with each other.

Meta-Didactical Transposition Model

The meta-didactical transposition (MDT) model draws on the anthropological theory of Chevallard (1985, 1992, 1999). At the core of the theory are the notions of didactical transposition and praxeology. Didactical transposition highlights the way mathematical knowledge and activities transform within institutions, the particular configurations that appear, and the relationships among them (Bosch, 2014). Praxeology, in turn, encompasses *praxis* (know how), in which a family of similar problems can be solved with particular techniques and *logos* (knowledge), which pertain to a frame of discourses that describe, explain, and justify these techniques[1].

MDT extends Chevallard's anthropological theory to the context of teacher education, where it takes into consideration the meta-didactical praxeologies of researchers and those of teachers when both engage in professional development activities (Arzarello et al., 2014). These meta-level praxeologies "consist of the tasks, techniques, and justifying discourses that develop during the process of teacher education" (Arzarello et al., p. 353). Thus, a meta-didactical transposition is a dynamic process through which "both the didactic praxeologies of the community of researchers and of the teachers' community change within the institutional environment in which the two communities reside" (Arzarello et al., p. 355). The transposition is usually led by the researchers, who broker components of their praxeology to teachers, turning these components from external to internal, thereby, making the components accessible to the latter cohort. The dialectics of such a process ensures that praxeological components of both communities change their status over time. For example, a research finding in mathematics education may be an external component for teachers unfamiliar with the studies in which the finding emerged. By introducing and discussing studies in a professional development program or a graduate course, a mathematics research educator is brokering the shift of the finding to an internal component with the goal of developing new teacher praxeologies. This shift constitutes a meta-didactical transposition. Figure 1 summarises the model schematically, and it shows that as a result of MDT, each community may gain a new praxeology both emerging from a shared one "which consist[s] of a blending of the two initial praxeologies" (p. 354).

[1] This component could be further decomposed into Technologies and Theories, but the provided description is sufficient for our purposes.

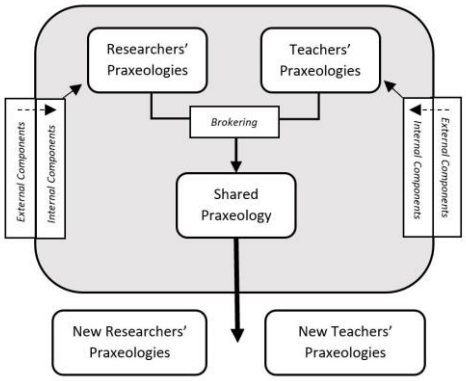

Figure 1. Visual representation of the MDT model

In MDT, the brokering of shared praxeologies is frequently accomplished through the use of boundary objects, which Star and Griesemer (1989) define as "objects that are both plastic enough to adapt to local needs and constraints of the several parties employing them, yet robust enough to maintain a common identity across sites" (p. 393). They suggest boundary objects function as a means for coordination of perspectives between communities and may result in "generative and productive tensions" (Cochran-Smith & Lytle, 2009, p. 123), which can be used by both teachers and mathematics research educators as a means for reflection, discussion, and change. In this paper we focus on Teachers' Change.

Framing the Study with MDT

In previous MDT studies (e.g., Prodromou, Robutti, & Panero, 2017; Wilkie, 2017), the brokering process was led by researchers, in which they were the ones to choose boundary objects, to instigate the delineation of initial components internal to both communities, and to promote the emergence of a shared praxeology. Yet, Arzarello et al. (2014) maintain that the flexibility of the MDT model makes it extendable to other contexts. We exploit this flexibility to frame the interaction between research literature and teachers who engage with it.

The literature, in our case, plays the role of a repository of boundary objects (Star, 2010). This is because by comprising articles on mathematics teaching and learning, the literature has the potential to appeal to the needs and interests of communities of researchers and teachers alike. We find Star's metaphor of repository useful as we do not assume that any one research article in mathematics education is a boundary object for any community of mathematics teachers by definition, that is, it satisfies their informational and work requirements (Bowker & Star, 1999) simply because it is concerned with mathematics teaching and learning. A repository is an encapsulation of modular units that can be selected and acted on individually. We associate such selections and acts with processes through which a research article can be placed at the boundary between the researchers' and teachers' communities.

Once an article is chosen by its readers it unfolds the voices of its authors brokering particular narratives. However, being written by mathematics education researchers and intended for such, research literature is aimed at transferring knowledge within the researchers' community. Thus,

teachers who engage with such literature are required to broker the praxes and logos that it narrates, some components of which might be more foreign to them than the other.

Through the MDT lens, the question instigating our study *was: What praxeologies may emerge when in-service mathematics teachers turn research articles in mathematics education into boundary objects?* Specifically, we were interested in new praxeologies of teachers and researchers that may develop from teachers' first engagement with scholarly articles.

It seems obvious that the emergence of new teachers' praxeologies would not be possible without teachers investing a considerable effort in the brokering process. In the next section, we describe the institutional environment that afforded the teachers' commitment to these processes.

The Study

We conducted an exploratory research study, which Stebbins (2001) suggests is the preferred methodological approach when the phenomenon of interest has received little or no systematic study. With exploratory research, the focus is on the inductive generation of empirical generalisations. The data for our exploratory study were collected from 13 participants, who were enrolled in their first course in a master's program in secondary mathematics education. All were practicing middle- and secondary-school mathematics teachers with teaching experience ranging from two to twelve years, with seven teachers having between 6 to 8 years of classroom experience (see Table 1 for details). This practical background was a prerequisite for their entrance into the master's program.

Table 1
Participating teachers

Pseudonym of the teacher	Years of teaching experience	Chosen Article	First experience reading general research literature?	First experience reading mathematics education research literature?
Alexis	12	Chernoff & Russell (2013)	No	No
Ali	2	Hiebert (2014)	Yes	Yes
Ava	5	Sirotic & Zazkis (2005)	Yes	Yes
Bonnie	4	Sirotic & Zazkis (2005)	No	Yes
Caleb	7	Hiebert (2014)	No	Yes
David	6	Sirotic & Zazkis (2005)	No	Yes
Eric	10	Hiebert (2014)	Yes	Yes
James	8	Hiebert (2014)	No	No
Julia	6	Hiebert (2014)	Yes	Yes
Levi	8	Hiebert (2014)	Yes	Yes
Mark	8	Sirotic & Zazkis (2005)	Yes	Yes
Naomi	9	Hiebert (2014)	No	No
Paul	7	Hiebert (2014)	Yes	Yes

As the entry point into graduate mathematics education, this first course offered a context for inquiry into the research enterprise and introduced practicing teachers to the research mathematics education community. This provided the teachers with the opportunity to gain a sense of how research in teaching and learning mathematics is designed and implemented, and to consider its potential impact on their own practice. Table 1 shows that for the majority of the participating teachers this was their first engagement with mathematics education as a research discipline.

The data for this study were collected from responses to a literature-based task, which was one of the written assignments in the course (see Figure 2). The structure of the task was informed by Nardi (2015), who offers three principles for supporting newcomers' transitions to post-graduate studies in mathematics education: *engaged pedagogy and participation*; *cultural sensitivity*; and, *independence, creativity and critical thinking*. The engaged pedagogy principle was used to overarch the task as a whole as it provided the newcomers with experiences of active agency and allowed them to develop an appreciation for possibilities that scholarly mathematics education provides. The first principle was evident in the request that each participating teacher choose one out of four pre-set articles for the task. The engaged pedagogy principle is also reflected in the final task, which requires the teachers to explain and justify decisions made when designing a follow-up study. The cultural sensitivity principle calls to provide newcomers with opportunities to enact their previous backgrounds, culture, and experiences in the new enterprise. This principle underpinned the first part of the task that asked teachers to summarise the ideas that they viewed as central to the article of their choice. It was also evident in the second part of the task where teachers were requested to design mathematical activities or problems based on ideas from the article that could be implemented in their school classrooms. The third principle is concerned with independence, creativity and critical thinking, which are positioned as "marks of the emerging membership to the scholarly community" (Nardi, 2015, p. 139). One feature of this principle is "the move from appropriating to creating knowledge" (Nardi, 2015, p. 140). Driven by this principle, the third part requested the teachers to critique the article's findings and the fourth part of the task, which invited them to design a follow up study to the one reported in the article. The last part contained a reflective questionnaire, in which the teachers were asked to comment on their experiences in completing the literature-based task. Specifically, they were asked to reflect on what part of it, if any, they found challenging or interesting, on the differences they experienced between reading a research article and other literature, and on what guided their choices of the article and designs of follow up studies. The submissions, along with the responses to the questionnaire constituted the data corpus for our study.

The purpose of this task is to expose you to a snapshot from research literature and invite you to consider how it can be used in your teaching and potentially in your research.
Choose one of the 4 articles provided and read it carefully.

IN YOUR SUBMISSION:

1. Summarise the central ideas of the article.
2. Design 2-3 activities/tasks/problems based on the ideas from the article. Briefly describe the particular setting for implementing your tasks.
3. Summarise and critique research finding. (What did the researchers want to find? What did they find?)
   What, in your view, is the innovation and limitations of the study? End this part with your perspective on the findings. What, if anything, surprised you?
4. Design a follow up study. (Participants? Tasks? Questions? Setting?)

FOLLOWING THE SUBMISSION AND CLASS DISCUSSION:

5. Respond to the reflective questionnaire (to be provided later).

Figure 2. Literature-based task.

The four articles that formed the basis for the assignment were selected from proceedings of the Psychology of Mathematics Education conferences, which allowed for uniformity of length and format. We also attempted to select articles that were compatible in their methodologies, in their accessibility to the teachers, in their relevance to their school teaching practices, and in their potential to enrich and enhance the teachers' mathematical content knowledge. These attempts were driven by the aims of the graduate course and master's program as a whole.

Table 2 provides a brief overview of the four articles with a focus on features that we considered important for making sense of the findings in the next section (see Appendix A for abstracts of the four articles). Generally speaking, all the articles explored mathematical concepts with which the teachers were expected to be familiar, either from their school practice or from course discussions.

Table 2
Features of the Articles

Article	Focus of the Study	Theoretical Underpinning	Data Corpus and Participants
Comparing the Relative Likelihood of Events: The Fallacy of Composition (Chernoff & Russell, 2013)	Probabilistic knowledge used to make relative likelihood comparisons of events	Fallacy of composition (Chernoff & Russell, 2012)	Written responses to a task were collected from 54 pre-service elementary teachers
Cardinality and Cardinal Number of an Infinite Set: A Nuanced Relationship (Mamolo, 2014)	Reasoning about, and with, concepts of infinity in set theory	APOS theory (Dubinsky & McDonald, 2001)	Individual task-based interviews were conducted with two post-secondary students
Students' Understanding of Square Numbers and Square Roots (Hiebert, 2014)	Obstacles encountered in problem solving with square numbers and square roots	Concept image and definition (Tall & Vinner, 1981); opaque and transparent representation (Lesh, Behr & Post, 1987)	Written responses to a questionnaire were collected from 51 secondary students; 9 of whom participated in follow up interviews
Locating Irrational Numbers on the Number Line (Sirotic & Zazkis, 2005)	Understanding of the geometric representation of an irrational number	Representation of prime and irrational numbers (e.g., (Zazkis, 2005)	Written responses to a questionnaire were collected from 46 pre-service secondary teachers; 16 of whom participated in follow up interviews

Our data consisted of assignments that the participants submitted as a response to a course task. Accordingly, these data did not provide us with a direct access to the praxeologies that developed as a result of teachers' engagement with research articles but to what may be conceived as *sketches* of these praxeologies. Cambridge dictionary (n.d.) defines a sketch as a simple drawing or a written or a spoken story that does not have many details. We used this notion to highlight that our findings came from written narratives in which the teachers reflected on their own learning and envisaged themselves in teaching and research situations. These self-portraits varied in length and amount of detail but still allowed us to outline the chief features of the praxeologies described.

We embarked on the data analysis with iterative analytical procedures (Glaser & Strauss, 1967). At the first stage, we coded teachers' responses to the first four parts of the task and to the reflective questionnaire using NVivo analysis software. The initial codes were derived from the

MDT model, but throughout the process the codes were gradually modified to account for the specificity of our context. For instance, when explaining their choices in a particular article for the literature-based assignment, many participants referred to their school teaching, which aligned with the construct of teacher praxeology in the MTD model. Yet, the participants, however, referred to different facets of teaching, making their explanations qualitatively different from each other. Accordingly, the original model's constructs required refinement to capture the differences among participants that we discerned in the data. In a similar fashion we engaged in multiple rounds of iterative alternations until the emergence of codes and findings that embodied an equilibrium between the theoretical MDT model and the data corpus.

Findings

We present our findings in four sections. The next section, *Considerations for turning (or not turning) an article in a boundary object*, is concerned with the teachers' choices of an article and considerations that underlaid these choices. The remaining sections report the praxeologies that emerged from the teachers' submissions: *Article consumption within teachers' and researchers' praxeologies*, *Sketches of new teachers praxeologies*, and *Sketches of researchers praxeologies*.

In the following section, we refer to each of the teachers by a pseudonym followed by the first letter of the author(s) whose article they chose. For example, Bonnie^{SZ} refers to a teacher who chose the article by Sirotic and Zazkis (2005) while Levi^{H} refers to a teacher who responded to Hiebert's (2014) article.

Considerations for Turning (or not Turning) an Article in a Boundary Object

Of the four articles that we offered in the literature-based task, eight teachers chose Hiebert (2014), four worked on Sirotic and Zazkis (2005), and the article by Chernoff and Russell (2013) was selected by one teacher. The adherence of the cohort to two articles and almost unanimous avoidance of the other two, attest to particular considerations that the teachers enacted via their choices. These considerations are the focus of this section.

Asked to explain their choices, the teachers referred to the connection between their teaching experiences and the mathematical focus of the article chosen. For example, Bonnie^{SZ} explained,

> I chose this particular article because I had just finished explaining what irrational numbers were to my grade 9s [sic] At the same time, I was teaching root simplification and was asked by many students why we had to simplify the root vs just rounding a decimal. They [the students] assumed that they were 'basically' the same thing.

Levi^{H}, in turn, wrote that he selected Hiebert (2014) because he was about to teach the same mathematical content to his students,

> I was, at the time, about to teach lessons on square numbers and square roots with my Math 8 class so this paper seemed appropriate.

Similar to Bonnie^{SZ}, Paul^{H} referred to common issues that he faces when teaching the concepts of square numbers and square roots to his students; in his words,

> The reason for me to pick Hiebert's article was because I could easily relate to it. I teach these concepts to my grade 9s and 10s and come across the same issues in my classes as well where students get confused [with] these topics and mix them up.

Overall, when substantiating their choices, ten participants described concrete episodes from their practice where they either taught the mathematics in the focus of the article in the past or were about to teach it soon. Such substantiations attest to the relevance that the teachers identified in the article of their choosing to their school teaching praxeologies.

Three other teachers, in turn, explained their choice of an article by highlighting their mathematical knowledge rather than a pedagogical experience. For example, Julia^{H} wrote,

> When reading Hiebert, perhaps because I had more knowledge of the concept, more things came readily to mind that I could comment on in the paper.

Similarly, Eric^{H} explained,

> While I read at least part of each paper, there were two that I read completely as I had a certain familiarity with the topic and felt more confident with it.

These two excerpts pertain to the teachers' fluency with mathematics in the focus of an article. Indeed, Julia^{H} and Eric^{H} referred to their knowledge of the concepts that Hiebert explored, which resulted in "more things" coming to mind and their feeling of confidence. These justifications can be associated with teachers having accessible mathematical knowledge that is necessary for consuming an article.

The above discussion is enriched by the reflections of three teachers, who referred to the articles that they did not choose for the task and specified some features that constituted a barrier for their engagement. One of them was Alexis^{CR}, who explained,

> I found the paper on infinities was a little too detailed for me regarding psychological or educational theories that I did not understand completely.

On a similar note, Levi^{H} wrote that "the language" in some articles was more accessible than in the others.

Theoretical issues and usage of specialised terminology are characteristic of researcher praxeology. These components were external to the teachers praxeologies at the time of data collection, which is apparent in the reflections that some of them wrote. Although present in all four articles offered, the teachers' reflections indicate that these characteristic components can be experienced differently by the teachers and in some cases, hindered their decision to engage with an article.

Article Consumption Within Teachers and Researchers Praxeologies

In their reflections, 11 teachers addressed the differences between their reading of the chosen article and their usual reading practices for teaching purposes. For instance, Julia^{H} mentioned that she read the article slowly and "with a more inquisitive lens." A resonating idea appeared in the reflection of Ali^{H}, who explained,

> Normally, I would have been reading with the more single-minded goal of finding classroom activities to use or getting a sense of what other teachers are doing these days. Reading the article was different in the sense that I read it several times. Also, it was different that I tried to understand the theoretical pieces that were unclear at first. Without an assignment of this kind, I might not have tried so hard to understand 'heuristics' and other specialized jargon like that. In this instance, I did

> seek out other academic papers to put context around some other research or specific terminology that was referred to in the article.

From the MDT perspective, these new ways of engaging with a chosen article for the participants can be ascribed to the emerging praxeology of the teachers. Indeed, the praxes of reading and rereading, attending to "the theoretical pieces", clarifying "specialized jargon" through seeking other articles are instances of researchers' praxeology. Furthermore, whereas the act of choosing the article was often substantiated by considerations of the immediate relevance to teaching and fluency with the mathematical content (see the Section, *Considerations for turning [or not turning] an article in a boundary object*), the new praxes that teachers described above are time- and effort-consuming. Accordingly, their reports on carrying out these praxes can be viewed as evidence of teachers' recognition that the act of consuming an article can look differently within the teachers' and researchers' praxeologies. Eric[H] addressed this recognition explicitly by writing,

> I realized that in a scholarly article we need to observe them both as an educator and a researcher. For me, it is the more technical explanations that need to be reread (and sometimes reread again) as a researcher. Only then can I begin to contemplate how my understanding and findings might apply to how I would engage my students.

The teachers also shared more general "logos" that they developed as a result of engaging with an article. For instance, Alexis[CR] reflected,

> I have learned that there is research going on that I did not know about, particularly regarding what prospective math teachers understand about some very specific topics. I have also learned that in my profession I am removed from this research because of the nature of my work and its isolation in the classroom.

We associate the words of Alexis[CR] with an emerging awareness of the differences between the two praxeologies and we maintain that this realisation cannot be taken for granted. As Alexis[CR] noticed, the nature of her teaching profession does not require (or at least allow) her to be aware of research that was "going on". Furthermore, we interpreted the teachers' recognition of the distinction as an acknowledgement that teachers' and researchers' praxeologies follow rules that may be incommensurable and even conflicting (see Kontorovich & Rouleau, 2018, for examples). We suggest such a recognition leaves room to explore researcher praxeology from within the domain of a teacher's own logos and praxis.

Sketches of New Teachers Praxeologies

When reflecting on what they had learned from their work on the literature-based task, five teachers indicated that reading an article provided them with an opportunity to expand their own mathematical knowledge. One of them was Bonnie[SZ], who wrote, "I learned more about my own understanding of irrational numbers". Ava[SZ] elaborated:

> As I continued to read the article [Sirotic & Zazkis, 2005], I realized that I was the one who made no connection between an irrational number and its geometrical representation. More shockingly, even though I have been learning and teaching the Pythagorean Theorem for many years, I have never given a chance to practice it as a tool to measure irrational length that is unique on the number line.

The expansion of mathematical knowledge was particularly evident among teachers who chose Sirotic and Zazkis' (2005) article on representing irrational numbers. Several of them spoke of their

knowledge as being "tested" and them being "pushed to think" about their own understanding. For instance, Mark[SZ] addressed the usage of a geometrical representation to placing irrationals on a number line and wrote, "I found the method of solving to be very interesting and engaging". He further suggested:

> Even as teachers, we often default to thinking about irrational numbers based on their decimal approximation simply because that's the way the textbooks force us to think.

Several of the participants expressed awareness that they experienced some of the same difficulties as the participants in the articles' studies. When reflecting on her engagement with Chernoff and Russell (2013), Alexis[CR] wrote:

> The irony is not lost on me that I did not immediately remember how to calculate the probability myself, as with the research subjects!

She went on to add that reading the article increased her knowledge of probability as "I did not completely understand the difference between fallacy of composition and equiprobability bias".

Along with addressing their expansion of mathematical knowledge, ten participants referred to the changes that they plan to make in their teaching on the level of praxis. The changes of some were concerned with the teaching of concepts and mathematical practices that were the focus of the chosen articles. As a result of engaging with Sirotic and Zazkis (2005), Ava[SZ] wrote that she plans to "change how I teach irrational numbers to the class". Levi[H] reported that the article made him aware of his students' struggles with definitions, and "based on this new awareness, I plan on introducing some new instructional strategies to attempt to address these student difficulties". Similarly, Eric[H] stated, "After reading Hiebert's article, I am very inclined to introduce square roots (and square numbers) to my students in a completely different way". For most, these changes revolved around providing students with more diverse mathematical experiences and time to explore mathematical concepts. Other participants accounted for more general changes. For instance, Mark[SZ] referred to "enhancing my lessons with activities that are more meaningful for the students" and Alexis[CR] indicated "the need for careful assessment of deeper conceptual understanding".

In terms of MTD, the described changes can be considered as sketches of new teachers' praxes that come into being alongside with participants' knowledge expansions. Bonnie[SZ] noted that these changes are inevitable since "It is a clear and direct line to see that we impart our new knowledge and understanding onto students". In line with this view, Paul[H] addressed the need to impart this new knowledge for finding ways to make concepts more transparent to the students,

> We have to understand the fact that our students who, in some cases, are seeing these concepts for the very first time, are not familiar with the basics related to that topic unless we take the time to explicitly state and reinforce them.

Levi[H] highlighted that the task is far from being easy since,

> One common challenge for teachers, no matter what the topic area, would be the challenge of identifying concepts that appear too simple to be confusing. As experienced math practitioners, it is sometimes hard to recognize such concepts and consequently, we often do not give them enough attention in our instruction.

Naomi's[H] reflection was in line with these views and she explicitly addressed the role that research literature can play in her teaching practice and her intention of further reading. She wrote,

> Up until now I did not rely on scholarly mathematics education articles to help design lessons on topics students struggle with. Now I see what an important tool it is to do these readings when creating lessons.

In MTD's terms, Naomi's^{H} new view on "scholarly mathematics education articles" can be ascribed to a new teacher praxeology that was enriched not just with new praxes, but with becoming aware of a new repository of boundary objects that may support her daily teaching.

Sketches of Researchers Praxeologies

The teachers participating in our study were asked to critique the article that they chose for the task since evaluating and reacting to the reading were a substantial part of our conceptualisation of research consumption (e.g., Bartels, 2003). Thus, the second and third parts in the literature-based task gave rise to teachers' sketches of researchers praxeologies with a focus on limitations and possibilities to extend the studies reported in the chosen articles. Again, we use *sketches* in the sense of a broad outline as our teachers had little to no prior experience with scholarly research in mathematics education, and thus, they could not be fully aware of current trends in our field in regard to critiquing and following up on research studies (e.g., Kilpatrick, 1992; Lester & Lambdin, 1998).

To start the talk on the sketches of researchers praxeologies, it seems worthwhile to note that out of twelve teachers who mentioned a limitation in their critique, ten attested to issues with methodology. For example, in expressing his criticism of Hiebert's (2014) selection of high-achieving students, Levi^{H} wrote,

> The only criticism I had, I based on my own sense of whether the method seemed reasonable. […] I do wonder why the interviews only consisted of 'A' or 'B' students. It seems to me that if the author was only interested in 'A' or 'B' students, then perhaps all 51 participants should have been such students.[2]

Levi^{H} was not the only one with this sort of criticism as eight of the teachers questioned the value of conducting studies with particular populations of participants. It appears that these teachers hold the view that research should cut across mathematical abilities and its findings should emerge from more than just one subset of participants. This image of research praxeology was reflected in the comments of Eric^{H} who, like Levi^{H}, was critical of Hiebert's (2014) focus on high-achieving students:

> It seems rather that this would limit our perspective on student responses. Would teachers not benefit to learn how 'C' students interpret square numbers and square roots?

Furthermore, for six teachers the main concern was the number of participants in the studies. This is exemplified by James^{H} who noted that "I wanted the participants to be chosen from a great pool of students." Rationales for this preference were similar across all six responses, they had concerns with students being from one institution: "I criticized the small sample of students taken from the same school, who all had the same teacher," stated Caleb^{H}, which led him to wonder "whether she [Hiebert] could state her conclusion for certain under the circumstances of her experiment."

[2] Fifty-one students of varying ability participated in Hiebert's study and answered a written questionnaire at the first stage. Then, nine high-achieving students were purposefully selected for follow up interviews.

Notably, the notion of experiment, or experimental design, was neither mentioned in any of the four articles nor in the course that the students were currently taking. However, the critique of several teachers seemed to be driven by experimental paradigm (e.g., Campbell & Stanley, 1963). These teachers suggested that the participants should be randomly selected, as exemplified by James^{H}, who stated, "Instead of taking an entire class to use for the study, I would like to take a random sampling of 10% of Pre-Calculus 11 students in 5 schools." James^{H} elaborated further upon a reason why this would be necessary:

> The reason for the percent instead of a fixed number is to make a better representation of the population. I would be taking a sample of over 25% of the schools. I would select one in central, east, west, north, and south Surrey. Given that [named school district] is subdivided into many social, economic and academic groups this should provide a good sampling of the district.

Similarly, Caleb^{H} suggested larger samples and random participant selection, and offered the following rationales:

> Not only would this increase our sample size and reinforce whatever our findings were to be generalized amongst students, but it would control and eliminate many biases that could affect smaller, more localized samplings (e.g. teacher's pedagogical differences, socioeconomic status amongst schools and students, a particularly bad day for students etc.).
>
> In the follow up study, I would divide the students into a control group and an experimental group within the classes studied.

This idea of conducting research by using control and experimental groups was also echoed by other teachers.

Another sort of critique came from five teachers, who expressed their disappointment with the articles not providing concrete methods for helping students overcome the identified misconceptions:

> The limitations of this paper, however, seems to be that there is no analysis of how these apparent incorrect concept images that the students had created could have either been avoided or amended. (Caleb^{H})
>
> I wish Hiebert had posed the question, what can be done to better assist students in their misunderstandings? She doesn't provide any answers to this question, which I believe is the more important issue at hand. (James^{H})

We see in these comments an explicit expectation for research to offer pedagogical solutions, describe how to correct or avoid students' mistakes, and not to constrain itself to raising readers' awareness to problematic phenomena. Having that said, the follow up studies, designed by the teachers in the fourth part of the task, did not tend to offer solutions either. Generally speaking, the suggestion for follow-up studies extended the scope of original studies by exploring larger groups of randomly selected participants.

Discussion

The study reported in this paper was concerned with the first experiences of 13 mathematics teachers as they engaged with scholarly articles in mathematics education. Given the particularity of their contexts, as students in a graduate mathematics education course but also as practicing school teachers, we expected their reading experiences to diverge from that of experienced

researchers (Bartels, 2003; Kontorovich, 2015, 2016; Nardi, 2015). This led us to frame our study with the MDT model, positioning teachers and researchers as members of distinct yet closely related communities, the transposition of knowledge between which might be mutually beneficial.

While MDT offers an opportunity to identify the transposition of knowledge for both teachers and researchers, the benefits for the teachers were in our focus. In alignment with MDT, we associated these benefits with new teachers' and researchers' praxeologies that our participants could develop as a result of consuming a research article. Accordingly, the findings pertain to considerations that the teachers applied for turning (or not turning) a particular article into a boundary object, their emergent awareness to the difference between research consumption within teachers' and researchers' praxeologies, and sketches of new praxeologies as these emerged from the analysis of their literature-based assignments and self-reflections. These findings are discussed next.

All the teachers in this study reported that the articles that they engaged with contributed to the expansion of their mathematical knowledge, instigated them to change their teaching of particular concepts, and even equipped them with ideas for enacting this change (see *Sketches of new teachers praxeologies*). While these praxeological developments emerged from self-reports, we do not take them for granted. Indeed, the disconnect between research and practice in mathematics education has been often discussed (e.g., Bartels, 2003; Even, 2003; Groth & Bergner, 2007), and thus capturing voices of a different nature is seen as a contribution of this study. Furthermore, Hammersley (2002) maintains that practitioners often treat educational research according to what he terms as an engineering model; a model, according to which research is expected to provide specific and immediately applicable solutions to practical problems in a manner that is similar to natural science and engineering. While some expectations of this kind were indicated in the current study as well, many comments that the participating teachers provided could be ascribed to a moderate enlightenment model positioning that research is just one among several sources of knowledge on which practice can draw. Moreover, Hammersley (2002) posits that the use made of research depends on practical judgments about what is appropriate and useful. In other words, the teachers adhering to this moderate model realise that research is not a provider of absolute pedagogical truths but something that should be filtered through their praxeologies before it could (and should) be acted upon in their classrooms.

Notable to our study was the offering of choice of a research article for teachers to engage with. The findings on these choices pertained to teachers' inclinations towards articles on teaching and learning of mathematics that were either relevant to their teaching or easily accessible in their knowledge base (see *Consideration for turning [or not turning] an article in a boundary object*, for details). At first glance, these teaching-informed considerations are hardly surprising in light of the praxeology that has been dominant for our participants at the time of data collection. Indeed, Zeuli (1994) maintains that reading research is one important way teachers learn about teaching, so why should they miss an opportunity to do so? But it is also possible that something additional lies behind these choices. According to Kontorovich (2015, 2016), carrying out a typical research practice in mathematics education (e.g., engaging with research literature) requires a confluence of three kinds of knowledge—mathematical, pedagogical, and methodological. The latter kind is characteristic of researcher praxeology since it is associated with everything related to conducting a study: from philosophical and epistemological conceptions, through research paradigms, to research designs and methodological techniques. Coming from a praxeologically different community, the teachers may have sensed that it would be necessary for them to invest in the enhancement of this kind of knowledge for coping with the assigned task. Indeed, several

of them reflected on the difficulties to make sense of the theories and terminology that the articles used (see MacDonald et al., 2001, for similar findings). Thus, if teachers have developed relevant mathematical and pedagogical knowledge in the past as part of their own praxeology, leveraging it for the benefit of the task would allow them to concentrate on the kind of knowledge that probably needed the most substantial enhancement. From this perspective, making use of their mathematical and pedagogical knowledge could be viewed as an appreciation of the challenge that critical consumption of a research article entails.

The appreciation discussed above is not likely possible without teachers recognising the distinction between teachers' and researchers' praxeologies. On this note, Arzarello et al. (2014) propose that teachers' interaction with the community of researchers may evolve into a shared praxeology, which is "a blending of the two initial praxeologies" (p. 354), those of teachers and those of researchers. Our study proposes that for the desired blending to take place, it is essential for teachers to recognise that both praxeologies do not constitute counterparts of the same whole. This recognition was in the focus of the section, *Article consumption within teachers' and researchers' praxeologies*, and its findings could be classified into three types of recognition. The first type regards researcher praxeology as a whole and it pertains to teachers learning something about what researchers do and what kind of research "goes on there" (see Alexis'^{CR} comment on her realizing that mathematics education research can explore mathematical understanding of prospective teachers). In the second type, a teacher engages in praxes that are characteristic of the researcher praxeology but not the teacher one (see Ali's^{H} comment on engaging with theories and additional articles). In the third type, a teacher engages in praxes that are characteristic to both communities, but their engagement is intentionally different from the one they are used to in their own teacher community (see Ali^{H} and Eric^{H} comments on reading their articles "with a different hat"). All three types of recognition may be associated with MDT processes that our teachers went through.

The sketches of researchers praxeologies that were made in the section, *Sketches of researchers praxeologies*, allow us to conclude with two points that might be of practical interest to those who teach courses on research in mathematics education. The first point is that while the participants in our study had had little to no prior exposure to research literature in mathematics education, they seem to hold solid views regarding what research methodologies are appropriate. Specifically, the participating teachers valued randomisation and generalisability of research findings, were inclined towards studies with large numbers of participants representing a variety of mathematical abilities, and which adhered to experimental designs. Assuming that these observations are not unique to the current study, research educators might find it informative that an experimental paradigm may have privileged status in their students' eyes.

The second point is that some teachers expressed disappointment with articles not offering pedagogies for overcoming learners' mathematical mistakes that were identified in the reported studies. Whereas similar findings on teachers' criticism have been reported (e.g., Even, 2003; Hammersley, 2002), when designing their follow up studies our participants also abstained from proposing and exploring these much-needed pedagogies. Possibly, this abstention is a result of the teachers lacking appropriate methodological knowledge at the time of data collection. An alternative explanation could be that the teachers' unfamiliarity with the format and focus of Psychology of Mathematics Education (PME) papers led them to interpret the lack of appropriate pedagogies as a norm in the mathematics education community and simply enacted it in their follow up designs. This interpretation better draws the attention of research educators to the

special status of the first research articles that newcomers consume since these might be viewed as prototypes setting the bar for mathematics education as a research discipline.

Conclusion

We believe that the study reported in this paper will be of interest to teachers of young researchers in mathematics education. Critical consumption of disciplinary literature is a core scholarly practice (Boaler et al., 2003), and thus it is not surprising that postgraduate students are often engaged in it as part of their postgraduate coursework. Yet, the commonness of this practice to experienced researchers, course teachers or research supervisors for instance, can easily overshadow the challenge that this practice entails for newcomers. The contribution of this study is in exposing some of the untrivial decisions and entangled processes that are ingrained in this practice. These range from choosing a research article with which to engage; to turning it into an object that has the potential to transfer praxeologically foreign knowledge; and finally, to the development of reading praxes themselves. This is in addition to advanced tasks such as critiquing a study and designing follow-up research, both of which are typical in literature-based assignments. In our experience, while literature-based assignments that beginning graduate students submit are often extensively discussed and formatively assessed as part of the course instruction, the considerations, doubts, and tensions that the students experience in the process of this reading often endow an anecdotal status. Accordingly, we believe that the rich palette of issues of such an ilk that emerged in this study is useful for teachers and supervisors in graduate programs to re-experience the complexity of this practice. It might also instigate them to develop educational resources explicitly targeted at supporting students' growth in critical consumption of research literature, a growth which is necessary for gaining access to new praxeologies. After all, as Fusselman (1926) suggests, "Today a reader, tomorrow a leader" (p. 56) … may these be leaders in teaching or research.

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Appendix A

Abstracts of assigned articles

Comparing the Relative Likelihood of Events: The Fallacy of Composition

Chernoff and Russell (2013)

The objective of this article is to contribute to the documented dearth of research on teachers' probabilistic knowledge. Prospective teachers of elementary school mathematics were asked to identify which result from five flips of a fair coin was least likely to occur. Participants, instead of being presented with sequences, were presented with events, that is, sets of outcomes, for five flips of a fair coin. A particular logical fallacy, the fallacy of composition, was used to analyze the juxtaposition of responses and response justifications. As a result, the fallacy of composition was found in the response justifications for both normatively incorrect and correct relative likelihood comparisons. Combining the results of this and prior research utilizing the fallacy of composition demonstrates that logical fallacies are a burgeoning area of research for those investigating relative likelihood comparisons and teachers' probabilistic knowledge.

Cardinality and Cardinal Number of an Infinite Set: A Nuanced Relationship

Mamolo (2014)

This case study examines the salient features of two individuals' reasoning when confronted with a task concerning the cardinality and associated cardinal number of equinumerous infinite sets. The APOS Theory was used as a framework to interpret their efforts to resolve the "infinite balls paradox" and one of its variants. These cases shed new light on the nuances involved in encapsulating, and de-encapsulating, a set theoretic concept of infinity. Implications for further research are discussed.

Students' Understanding of Square Numbers and Square Roots

Hiebert (2014)

Despite their apparent simplicity, the concepts of square numbers and square roots are problematic for high school students. I inquired into students' understanding of these concepts, focusing on obstacles that students face while attempting to solve square number problems. The study followed a modified analytic induction methodology that included a written questionnaire administered to 51 grade 11 students and follow up clinical interviews with 9 students. The study revealed significant obstacles relating to the representation of square numbers and confusion of concepts including both weak distinction between the concepts of square numbers and square roots and inconsistent evoking of their concept images.

Locating Irrational Numbers on the Number Line

Sirotic and Zazkis (2005)

Can the exact location of 5 be found on the number line? In this report, we consider the answers of a group of preservice secondary school teachers to this question, in light of their general conceptions of irrational numbers and their representations. The results indicate strong reliance on decimal approximation of irrational numbers. Pedagogical implications are considered.

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