Surprises in the mathematics classroom: Some in-the-moment responses of a primary teacher

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The purpose of this paper is to describe some contingent moments during an in-service primary teacher's lesson on area, to analyse the teacher's responses, and to examine the relationship between the teacher's mathematical knowledge in teaching and her responses to these contingent moments. The teacher, in a public primary school located in Ankara, Turkey, taught five lessons on area measurement. During these lessons, the teacher was faced with contingent moments that she had not expected. Data for these moments were collected in field notes, video, and audio recordings of interviews with her. In this paper, five contingent moments are presented and discussed. In each case, after describing a contingent moment, the teacher's responses were analysed with reference to the codes of the Contingency dimension of the Knowledge Quartet model (Rowland et al., 2005). The analysis revealed that the teacher was able to acknowledge and incorporate her students' ideas into her instruction and respond to the unavailability of materials, and to turn them into opportunities. Although the teacher had difficulty in responding to the contingent moment that was initiated by herself, what was important here was that the teacher did not ignore her insight, instead; she tried to respond.

Keywords · Area Measurement · Contingency · In-service teacher · Knowledge Quartet

Introduction

There are undoubtedly a lot of interactions between the students and the teacher in classroom environments, some of which are unplanned, and these interactions are critical for the students' meaningful learning. Rowland, Turner, Thwaites, and Huckstep (2009) use the word 'contingent' to refer to these unplanned events. Teachers' responses to these contingent moments can influence students' mathematical learning (van Es, 2011). Similarly, Martin and Kasmer (2009) explain that teachers should take their students' ideas into consideration and encourage them to discuss and explore these ideas to build a strong foundation for their future learning. The present study describes some contingent moments in an in-service primary teacher's area measurement instruction and how the teacher's mathematical knowledge in teaching affected the ways in which she responded to these moments in her classroom. Describing these contingent moments is important not only to make other teachers aware of them, but also to help teachers be well-prepared for such moments in their own classrooms, and to gain insight into responding to such events during their own teaching. Furthermore, it is hoped that the findings of this study can inform researchers about the importance of contingent moments in mathematics education, thereby deepening our understanding about how contingent moments arise, and how to interpret and respond to them.

Theoretical Framework

Mathematical Knowledge

In the last three decades research has focused on finding a relationship between teaching and student learning, and increased attention has been given to the importance of teachers' knowledge. Many studies have described the characteristics and components of teacher knowledge (Shulman, 1986; Grossman, 1990; Ball, Thames, & Phelps, 2008; Rowland, Huckstep, & Thwaites, 2005). Shulman (1986) categorised teacher knowledge into subject matter knowledge (SMK), pedagogical content knowledge (PCK), and curriculum knowledge. Building on Shulman's model, Ball and her colleagues (2008) subdivide SMK and PCK into six distinct domains of Mathematical Knowledge for Teaching (MKT). Zazkis and Zazkis (2011) emphasise the interplay between SMK and PCK, since they interact and inform each other continuously throughout instruction. For example, teachers' knowledge about mathematical ideas underpinning a concept would inform teachers' pedagogical strategies and their choice of specific examples (Turner & Rowland, 2011). In the same way, McEwan and Bull (1991) emphasise that the SMK of teachers includes pedagogical underpinnings. Therefore, trying to categorise teacher knowledge as SMK or PCK "introduces an unnecessary and untenable complication" (McEwan & Bull, 1991, p. 318). Ball and her colleagues (2008) also acknowledge that the domains of their MKT model that they call 'common content knowledge' and 'specialised content knowledge' are not empirically distinguished. Ball et al. (2008) explain that "although the distinction may be compelling as a heuristic, it can be difficult to discern common from specialised knowledge in particular cases" (p. 403).

The Knowledge Quartet (KQ) model, developed by Rowland and his colleagues (2009), identifies how mathematics teacher knowledge is put into practice as the teacher goes about their work in the classroom. For this reason, Rowland et al. (2009) speak of mathematical knowledge *in* teaching. The KQ model has four interrelated dimensions: Foundation, Transformation, Connection, and Contingency; each has its own contributory codes (see Appendix). With reference to the Contingency dimension, Rowland and Zazkis (2013) observe that teachers' mathematical knowledge in teaching is "witnessed in teachers' responses to classroom events that were not anticipated or planned" (p. 138-139). Indeed, the inclusion of in-the-moment choices and responses of teachers to unexpected moments found in the Contingency dimension of the KQ distinguishes this model from the other models specific to mathematics education.

The first dimension of the model, Foundation, consists of teachers' content knowledge, theoretical knowledge of mathematics learning and teaching, and beliefs about how and why mathematics is learned. Teachers' *awareness of purpose*, *use of mathematical terminology*, and their ability to *identify students' difficulties and errors* characterise teachers' mathematical knowledge in teaching associated with the Foundation dimension. The second dimension, Transformation, which relates to Shulman's PCK (1987), focuses on teachers' ability to transform their mathematical knowledge in pedagogically helpful ways e.g., by their *choice of examples and representations*. The third dimension, Connection, includes teachers' *decisions about sequencing*, *making connections*, and *recognising conceptual appropriateness*. The last dimension, Contingency, is about unplanned events, such as students' questions and ideas, the (un)availability of materials, and the teachers' responses to them. While the contributory codes of this dimension were listed as *deviation from agenda*, *responding to a student's idea*, and *(un)availability of tools and resources* in Rowland and his colleagues' study (2009), the code of *teacher insight* was also included later (Rowland, 2013). Since the KQ model's Contingency dimension includes an additional demonstration of teacher knowledge – in-the-moment responses or decisions – we have drawn on the KQ model to focus on some contingent moments during an in-service primary teacher's lesson on the measurement of area, on her responses to these moments, and the relationship between these responses and the teacher's mathematical knowledge in teaching.

Contingency

The National Council of Teachers of Mathematics [NCTM] (2000) states that, "Effective teaching involves observing students [and] listening carefully to their ideas and explanations" (p. 19). Although using students' ideas and explanations may result in some changes in teachers' planned instruction, teachers may use these changes as potentially powerful learning opportunities to extend their students' learning (Davis, 1997). To be able to turn these moments into opportunities, they need to recognise them and to know how to respond to them effectively, which is closely related to teachers' mathematical knowledge in teaching (Rowland et al., 2009). To explain why the Contingency dimension was included in the KQ model, Rowland and Zazkis (2013) observe that teaching is not "attending to prescribed scenarios and delivering a predetermined curriculum;" otherwise, it would be enough to know the curriculum (p. 138). As may be surmised, no matter how teachers plan their instruction, they cannot plan or anticipate all the possible events that may occur while teaching. Therefore, there may be deviations from the lesson plan resulting from students' questions, comments, and ideas. Researchers state that if teachers know about the common difficulties, errors, and misconceptions that students may have, they can be ready for these events and can overcome them (Ryan & Williams, 2007; Lampert & Ball 1999). This same idea is implicit in some codes of the KQ, such as *anticipation of complexity* in the Connection dimension.

However, there are also researchers who state that knowing these possible difficulties does not guarantee that teachers will respond to them effectively, as having the necessary knowledge does not guarantee that it will always be applied in instruction (Cohen, Raudenbush, & Ball, 2003). Davis (1997) emphasises that "It is one thing to notice an absence of something from a learner but quite another thing to have a sensible pedagogical action come to mind when needed" (p. 183). This may be interpreted that although teachers may know the errors and misconceptions that are likely to occur and planned questions that they can ask when they observe them, they may need to ask new questions to understand and advance students' reasoning considering the objectives of the lesson (Smith & Stein, 2018). Even if teachers plan how to begin their questioning, their subsequent questions can be contingent on students' responses. Stein, Engle, Smith, and Hughes (2008) observe that teachers "are regularly surprised by what students say and do, and therefore often do not know how to respond to students in the midst of a discussion" (p. 321).

In their model including a set of five practices, Stein et al. (2008) explains how teachers can apply these practices to effectively use students' ideas – both contingent and noncontingent – to improve their instruction. The first practice, *anticipating*, is about how students solve or engage with the problem(s) that are planned to be used throughout instruction, and what kind of connections can be made among the solutions or between the solutions and the mathematical concepts. Anticipating includes considering students' probable difficulties and actions that will be taken to handle them. That is, this practice is more than just looking for correct or incorrect answers. Actually, the anticipating practice enables teachers to *monitor* students' solutions, *select* particular students who would present their solutions with their friends during discussion, *sequence* these solutions, and *connect* students' solutions presented throughout the instruction and the key mathematical ideas, which are the other four practices of the model (Smith & Stein, 2018). These practices in total allow teachers "to manage the content that will be discussed and how it will be discussed" and hence "the amount of improvisation required by the teacher "in the moment" is kept to a minimum" (p. 15). That is, although moments initiated by students may appear to be contingent to an observer, they may not actually be contingent because the teacher had anticipated them.

In addition to contingent moments initiated by students, teachers sometimes deviate from their lesson plan because they notice that they need to modify or correct something that they had planned; this is coded as *teacher insight* within the Contingency dimension of the KQ. The *teacher insight* code is also assigned when there is evidence that a teacher realises that something is particularly good or might be able to be taken in a useful direction, without being a reaction to some teaching shortcoming. Furthermore, the *(un)availability of materials* which the teacher had planned to use, another code of the Contingency dimension, may lead to deviation from the lesson plan. This code can include situations when technology does not 'work' as intended.

Rowland and his colleagues (2009) found that teachers can respond to these moments in different ways: by *ignoring what has happened*; by *acknowledging but sidelining the response from the child*; or by *acknowledging and incorporating the response into the lesson*. The response type of *ignoring what has happened* is observed when a teacher continues as if the contingent event was not noticed. The second response type, *acknowledging but sidelining the response from the child*, is observed when a teacher moves on with his/her instruction after accepting the idea or question as good. *Acknowledging and incorporating* is observed when a teacher elaborates the idea or question into the lesson in effective ways.

However, teachers' decisions and responses to these moments are affected both by what they know and by how they think about the content (Schoenfeld, 2010; Sherin & Star, 2011). That is, these moments are also related to teachers' knowledge in the Foundation, Transformation, and Connection dimensions. For example, a teacher may draw on his/her Foundation knowledge and respond to a student's question using his/her theoretical knowledge of pedagogy, or by considering the purpose of the topic. On the other hand, another teacher may decide to make a connection between the student's question and something that arose earlier in the lesson, or to previous topics that they had learned, using his/her Connection knowledge (Dogan Coskun, 2017). The purpose of the present study is to identify and describe some contingent moments in an in-service primary teacher's lessons about area measurement, to analyse the teacher's responses to these moments, and to consider what aspects of mathematical knowledge in teaching enabled (or limited) her responses.

Measurement of Area

Measurement is "the assignment of a numerical value to an attribute of an object" (NCTM, 2000, p. 44). Since measurement concepts are commonly encountered, needed, and used in our everyday life, measurement is one of the content strands of mathematics curricula in many countries (Australian Curriculum Assessment and Reporting Authority [ACARA], 2012; Ministry of Education, Culture, Sports, Science, and Technology [MEXT], 2008; Ministry of National Education [MoNE], 2005; NCTM, 2000). In order to emphasise the importance of the measurement content strand, Hart (1984) stated "If teachers of mathematics were asked to choose the five or six most important topics in the school mathematics curriculum, then measurement would be likely to appear on every list" (p. 16). Furthermore, measurement not only connects mathematics topics such as fractions, operations, geometric ideas, and statistics, but it also connects mathematics and science (Clements, Battista, & Sarama, 2001; Lehrer, 2003; NCTM, 2000). Despite its importance, Inskeep (1976) claims that "…the average citizen sometimes fails to appreciate the role of measurement" (p. 63). Similarly, studies on measurement show that teachers, as well as students, have difficulties with, and misconceptions about, measurement topics (Chappell & Thompson, 1999; Clements & Stephan, 2004; Murphy, 2009).

One of the concepts with which both students and teachers have difficulty, area measurement, is one of the most commonly taught and essential topics of the measurement content strand (ACARA, 2012; MEXT, 2008; MoNE, 2005; NCTM, 2000). It is an essential part of daily life and is related to other mathematical topics (Hiebert, 1981; Hirstein, Lamb, & Osborne, 1978). Area measurement involves assigning a positive number to a given two-dimensional surface (Kordaki & Potari, 2002). However, to fully understand what learners need to know to understand the area measurement concept, the measurement framework that discusses the stages through which understanding progresses, must be explained. The framework developed by Clements and Bright (2003) proposes five stages for the development of the area measurement. In the first of these stages, students are aware of the attribute and use its descriptive language. In the second stage, students start to compare, order, and match objects considering the related attribute. Students assign uniform units to the measurement result in the third stage, and move to assigning formal units to the measurement result in the fourth stage. In the fourth stage, students also start to estimate measurement results. Finally, in the last stage, students solve problems using computational knowledge and skills. In order to help students learn that rectangular area measurement is not simply a product of side lengths, teachers should help their students to pass through the above-mentioned stages (Piaget, Inhelder, & Sheminska, 1981, p. 262; as cited in Kordaki & Potari, 2002).

Teachers' explanations, demonstrations, examples, and responses to their students' questions and ideas are some of the factors that have the greatest influence on student learning (Ball, 1991; Brown & Borko, 1992). Primary teachers deserve special consideration, as they teach the basic concepts of mathematics – numbers, counting, arithmetic operations, fractions, length measurement, perimeter, area measurement, etc., – at an early stage, and students' knowledge of these concepts is foundational for their future learning. On the other hand, primary teachers do not necessarily enrol in specialist mathematics courses in their university education. Nevertheless, it is expected that primary teachers should possess a wide range of knowledge in different subjects. Area measurement is one topic which teachers may not feel comfortable with if they do not how to explain why the base×height or row×column formula works for rectangular figures (Jones, Mooney, & Harries, 2002). Rowland (2010) describes and analyses a lesson in which a primary teacher struggles to explain to his class why two particular non-congruent triangles have the same area. Stephan and Clements (2003) conclude that "Something is clearly wrong with measurement instruction" (p. 3). In order to take instructional decisions to improve the teaching of measurement and students' learning, we should start by analysing the nature of these instructions. Examining contingent moments in area measurement instruction and a teacher's responses to these moments provide a good context for teaching development. Therefore, the purpose of this study is to describe and analyse such contingent moments in one teacher's lessons, and her responses to them.

Rationale for the Study

There has been recent and extensive interest in the notion of contingency and in-the-moment responsiveness in teaching. However, these studies were mostly conducted with pre-service teachers and at middle and secondary school levels (Coles & Scott, 2015; Kula & Bukova-Guzel, 2014). Conducting a study at the primary school level is important, as a primary teacher is often the first person to teach basic mathematical concepts and to develop students' enthusiasm for mathematics. Therefore, the present study can be helpful in enabling researchers, teachers, and teacher educators to consider how the topic of area measurement is introduced to students at an early stage and how the participant primary teacher deals with her students' unexpected ideas and questions related to the measurement of area. This information could be used to assist pre-service and novice teachers to create a productive learning environment in which students' ideas and questions can be acknowledged and taken into account. The above-mentioned studies examined how teachers responded to contingent moments initiated by students. They showed that these contingent moments arise from both correct and incorrect students' ideas, and they show that when teachers do not know how to respond, they see these students' ideas as threatening and problematic, irrespective of their correctness. In this study, in addition to contingent events initiated by students, we also consider the teacher's responses to contingent moments resulting from the (un)availability of materials and fresh teacher insights. Presenting these contingent moments could help teachers to recognise their importance and to recognise the possibilities that they offer, both when planning and implementing their teaching. Furthermore, discussing the appropriateness of the responses in this study will raise teachers' awareness that their responses to students' contributions or the unavailability of materials can affect student learning.

A further aim of this study is to demonstrate how these unplanned events make demands on teachers' mathematical knowledge in teaching. Teachers can become more aware of, and take steps to improve, if necessary, the related dimension(s) of the KQ model. In short, this study could be a stimulus to develop teachers' mathematical knowledge in teaching, especially when considered in group discussions. Finally, when we reviewed studies of the triggers of contingent moments, we found very few such studies in Turkish classrooms: and those we did find were conducted with pre-service teachers at middle and secondary school levels (Kula & Bukova-Guzel, 2014; Koklu & Aslan-Tutak, 2015). Examining contingent moments in a Turkish classroom and a Turkish teacher's responses to them will be beneficial to understand contingent moments from a broader perspective. Some of the contingent moments in the above-mentioned studies were taken from the related literature to exemplify the triggers of contingent moments. In contrast, the contingent moments discussed in this study were from the primary teacher's own classroom. Therefore, examining the contingent moments of an in-service primary teacher's instruction will enhance the related literature regarding Turkish classrooms. This study examines some contingent moments in a Turkish in-service primary teacher's lessons on area measurement. It considers her responses to these moments, and the relationship between these responses and her mathematical knowledge in teaching. It is a case study of this teacher, seeking to answer the following research questions:

- What contingent moments can occur during an in-service primary teacher's teaching of area measurement?
- How did this in-service primary teacher respond to these contingent events?
- How was this in-service primary teacher's mathematical knowledge in teaching put into practice as she responded to these contingent events?

Methodology

Research Design

In order to show examples of the contingent moments in an in-service teacher's area measurement instruction, how she responded to these contingent events, and how her mathematical knowledge in teaching influenced her responses, a qualitative research methodology, specifically case study research, was used. Case studies are used to gain a deep understanding of some situation (Merriam, 1998) and are conducted in a real-life context "with a full variety of evidence-documents, artifacts, interviews, and observations" (Yin, 2003, p. 8). This paper presents research conducted to explore a teacher's responses to contingent moments related to area instruction, in the measurement strand of mathematics. It provides a detailed description of these moments as an exemplar of teaching practice, and is appropriately positioned as a case study.

The Context and Participant

Measurement of area, one of the important topics in the measurement content strand and the topic of this study, is defined as assigning a positive number to a two-dimensional space or enclosed region (Kordaki & Potari, 2002; Strutchens, Martin, & Kenney, 2003; Van de Walle, Karp, & Bay-Williams, 2012). The topic of area measurement is introduced in different grade levels in different countries. For example, while area measurement is firstly introduced to Turkish students in the third grade, it is introduced to students in Australia in the second grade. At these grades, students use nonstandard units to measure a region's area and to compare regions based on area. Then, Australian students in the third grade and Turkish students in the fourth grade start to use standard units to measure a region's area. Students in both countries start to learn the area measurement formula for rectangular figures and why the formula works for rectangular figures in the fourth grade (MoNE, 2005; ACARA, 2012). Specifically, the primary mathematics curriculum of Turkey gives the following objectives for area measurement for the fourth grade:

- Estimate the area using non-standard area measurement units and check it by counting the units.
- Determine that the area of plane region is equal to the number of square units covering it.
- Calculate the area of square and rectangular regions using square units (MoNE, 2005).

Prior to this study, the participant teacher's students were not taught any topics from the measurement content strand in the fourth grade. That is, the topic of area measurement is the first topic related to measurement in that grade. The present study was conducted at a public primary school located in a village outside the city of Ankara, Turkey during the 2014-2015 spring semester. The teacher and participant in this study, assigned the pseudonym 'Ebru', graduated from one of the four-year teacher education programs in the Department of Primary Teacher Education in Turkey. She had been teaching for four years, and she had taught students from grade levels one to four. After graduation, she started her teaching career as a first-grade teacher in this public school in 2010 and has continued in the same school. Because of the school's distance from the city centre, students at the school were residents in the village. There was one class for each grade between the first and fourth grades, and the number of students in each class was between 10 and 15. The class in which Ebru was teaching was a fourth-grade class with fourteen students at the time of data collection.

Data Collection and Analysis

The school curriculum assigned five lessons, 40 minutes each, to address the area measurement objectives listed above (MoNE, 2005). Each of these five lessons was observed and video-recorded by one of the researchers. The lesson texts were transcribed and referred to in interviews with Ebru, to bring her attention to the contingent moments in these lessons, and her responses to them. Before Ebru's lessons were video-recorded, the researcher-observer was present in Ebru's classroom for a week to help Ebru and the students get used to her being in the classroom. Including this week, the data collection took three weeks in total. A video camera on a tripod was located at the back of the classroom in order to not to disturb the classroom environment. The camera focused only on the teacher and the display board. Throughout this process, the researcher also took both descriptive and reflective field notes. The descriptive field notes included the date of the lesson, the duration of each contingent event in the lesson, and Ebru's gestures and facial expressions during the contingent events. The reflective field notes included the observer's thoughts about why it seemed that Ebru had not expected these events, and why she might have responded as she did.

A semi-structured interview was conducted following each teaching practice observation to assist the researchers' understanding and interpretation of her actions during the lesson. Before conducting these interviews, the researchers watched the video recordings of each lesson to identify contingent moments, and prepared a list of questions such as "Why did you respond to this moment in that way?" "What could you have done instead of what you did?" "How could you respond to this moment to help your students understand conceptually?" That is, they were mostly related to reasons for Ebru's responses to the contingent moments during her area measurement teaching. During the interviews, the video recordings of the contingent moments that had been identified were watched with Ebru, to help her remember them before the related questions were asked. All the interviews were also video-recorded and transcribed for analysis.

As can be understood from the above account, the analysis of this study began while the observer-researcher made the reflective field notes concerning the moments when it seemed that, for Ebru, what was happening in the lesson was unexpected. Before the other researchers watched the video-recordings, the observer-researcher watched them to see whether any contingent moments were missing in the field notes. Next, the other two researchers watched the video recordings of the observed lessons on their own and carefully judged whether there were other contingent moments in addition to those mentioned in the field notes. Although it can be problematic to decide whether an event in a lesson should be coded as contingent, the moments that seemed to be unexpected for Ebru were accepted as contingent moments in this study. Overall, five events in the area measurement lessons were identified as contingent. The researchers coded the data independently, and the codes were discussed until agreement was reached. The KQ contingency dimension codes assigned to them were: *responding to a student's question/idea*; *teacher insight*; or *(un)availability of tools and resources*. The researchers determined the codes which appeared the most relevant to each contingent event – more than one code, if appropriate. Watching the video recordings of the semi-structured interviews assisted our understanding of the nature of Ebru's responses and her reasons for responding as she did. Finally, examining the reasons underlying Ebru's responses informed our interpretation and understanding of how she applied her mathematical knowledge in teaching.

Findings and Discussion

According to the data, the contingent moments of this study resulted from a student's idea and an incorrect response, from unavailability of tools and resources, and from teacher insight. In order to focus on these moments, each contingent moment will be described, with the actions taken or responses given by Ebru to these moments. Description of these moments include transcripts from the conversations between Ebru and her students or among the students. When the actions or responses to contingent moments are described, they will also be coded as one of the three response-types identified in Rowland et al. (2009): *ignoring/disregarding*, *acknowledging but sidelining*, and *acknowledging and incorporating*. What could have happened in order to take advantage of these contingent moments will also be discussed. These five contingent moments are presented in the time-order in which they occurred in one of the five lessons.

Contingent Moment 1: Responding to a Student's Idea

As mentioned earlier in this paper, one of the objectives of area measurement in the curriculum is to learn that a plane region's area is equal to the number of square units covering it. With this objective, an example is provided in the textbook, as seen in Figure 1:

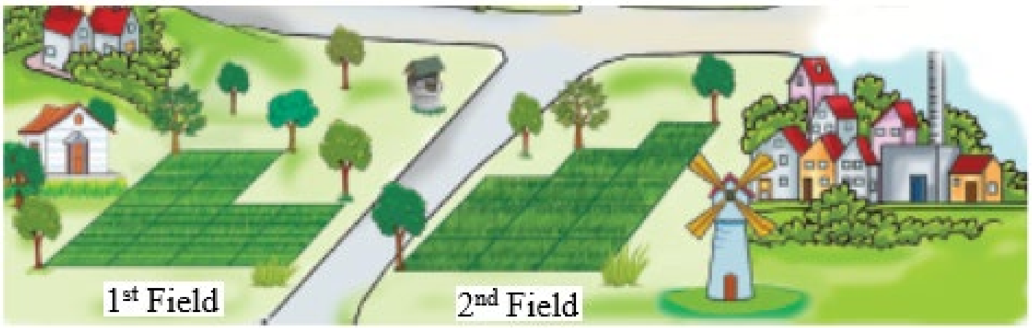

Figure 1. Example for area measurement (MoNE 4th Grade Student Lesson Textbook, 2005, p. 110)

Ebru asked her students to open this page of the textbook and to find a way to compare the area of the two fields. A few minutes later, Ebru asked them to say which one had a bigger area and to justify their suggestions. At this point, one of her students noticed the squares in the 2nd field and said it was bigger than the 1st field.

Student 1: I think the 2nd field's area is bigger.

Ebru: Why do you think so?

Student 1: Because it is taller.

Ebru: Hmmm, you say that since the 2nd field is taller than the 1st field, it must have a larger area. What are your opinions about your friend's assertion?

Student 2: Yes, it is tall; however, it is thinner. There are 4 squares in a row in the 1st field.

Ebru: He says that the 2nd field is taller but it is also thinner. Can we say?

Students: He is right. *(The students were agreeing with Student 2)*

Ebru: Then, we have to find another way to compare their areas. How can we compare?

Student 3: By counting squares.

Ebru: Is it possible? Can he be right?

Students: Yes.

When the above dialogue was discussed in the post-lesson interview, Ebru mentioned that she had not thought that her students would give such answers. That is, Ebru did not think that her students would only focus on the length or width of the figures or have difficulty in comparing the areas by counting on squares, and her students' ideas were unexpected. In her planning, she was sure that her students would count the squares and use those numbers to compare the fields' areas. Looking at Ebru's planning, it can be seen that Ebru anticipated that the students would be able to compare these two fields' areas by counting squares. Therefore, the thinking that Ebru anticipated before her instruction did not occur. The reason for this limited anticipation might be that Ebru taught this topic for the first time. To overcome this limitation, teachers can consult other teachers, as well as the related literature for possible student responses (Smith & Stein, 2018).

The above dialogues did show some evidence of recognising the related attribute, counting squares, but Ebru did not notice that there were some students who did not. Teachers need to identify and consider common errors and difficulties that students can have during their lesson planning drawing on their Foundation mathematical knowledge (Rowland et al., 2009). At this point, although Student 1's idea was incorrect, Ebru did not simply correct him. Instead, Ebru asked the student to explain why he answered as he did and repeated her student's idea to engage the other students. Since Ebru responded by *taking her student's response into account and incorporating it into her lesson* by directing the student's idea to his friends for further discussion, her introduction of area measurement was enhanced. This discussion is significant as it gave the students an opportunity to construct the meaning of area measurement – requiring a shift from one dimension to two dimensions. That is, Ebru changed the student's unexpected idea into an opportunity in line with the objectives of the lesson. In this way, Ebru used her mathematical knowledge in teaching effectively in this contingent moment.

Owens and Outhred (2006) mention that the foundational knowledge for developing the concept of area includes shape identification. It is helpful if teachers use appropriate and useful representations to introduce topics, especially new topics (Barwell, 2013). Therefore, if Ebru had not used only pictorial representations for the example given in Figure 1, but had used both pictorial representations and concrete objects, the students could have used the concrete ones to discover other configurations of the shapes in Figure 1. Discussing different configurations could have been helpful in developing their understanding of the concept of area. Furthermore, discussing these configurations could have assisted the students' understanding of area conservation, one of the important concepts in area measurement. Although Ebru *did take the student's answer into account and incorporated it into her lesson*, she did not use alternative representations to enhance student learning. This *choice of representations* is one of the codes in the Transformation dimension of the Knowledge Quartet.

Contingent Moment 2: Responding to the Unavailability of Resources

After the students realised that squares could be used to cover a region in to find its area, Ebru asked "Would you like to cover another area?" The students wanted to cover their tables as another covering activity. Since there were not enough paper squares to cover the table, a helpful response to the students' suggestion could have been to cover smaller areas. However, Ebru did not disregard her students' suggestion; instead, she asked the students to draw new squares to cover the table. As can be seen from the figure, the students drew new squares (Figure 2) and then covered the table.

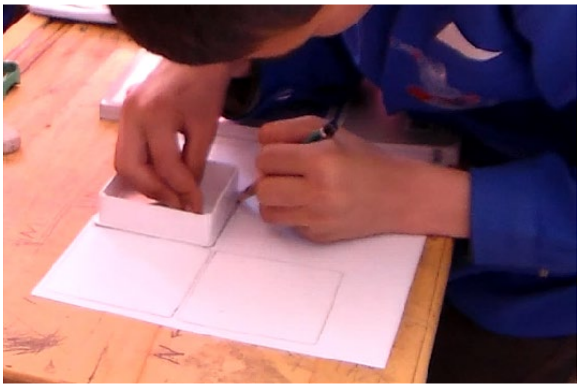

Figure 2. Drawing new squares for table covering activity

Although this process resulted in a deviation from her lesson plan, Ebru accepted her students' idea, allowed them to draw and cut new squares, and in doing so *responded to the unavailability of the unit squares,* related to *unavailability of tools*, which is another code of the Knowledge Quartet Contingency dimension. This covering process is important for the concept of structuring an array, as the students covered the surface of the table with these paper squares with equal rows and columns. Furthermore, although there were not enough paper squares to cover the whole surface of the table, the students did not continue to cut new squares. Instead, while some of them tried to cover by iterating one of these new squares, others preferred to iterate by visually estimating how many unit squares would fit the remaining part of the table. This process is also important in helping students acquire the concept of unit iteration, another important concept of area measurement instruction (Stephan & Clements, 2003). The table covering activity also provided Ebru with an opportunity to extend the conversation about the meaning of area and to invite her students to discover the relationship between the sides of a rectangle and its area. If the students had been given sufficient paper squares to cover the table surface, they might not have discovered the relationship between the length of the table's sides and its area. Therefore, it may be concluded that Ebru used her mathematical knowledge in teaching regarding the Contingency dimension to respond effectively to the students' need, stemming from *the unavailability of materials*.

Contingent Moment 3: Responding to a Student's Idea

After Ebru collected all the new paper squares drawn by her students, she asked them to estimate the number of squares needed to cover the table, and wrote each student's guess on the board. The students then came to the table and started to cover it. After they had covered the table, Ebru asked the students to find the number of squares. Although some of the students started to count squares one by one, one of them noticed that each column had five squares and started to count the squares in fives. Ebru noticed the student's counting strategy and decided that it would be good to share it with the whole class. As the students were about to conclude their counting, Ebru wanted her students to notice this student's counting strategy.

Figure 3. Table covering activity

Student 4: 5-10-15-20…

Ebru: She counted very well. What did she make? She counted squares in the column like 1-2-3-4-5, then she continued to count like 5-10-15-20-25-30-35-40-45-50-55-60-65.

Student 5: If we multiply 5 by 13…

Ebru: What did he say? Repeat it

Student 5: We multiply 5 and 13.

Ebru: Why? Which side corresponds to 5 and which side corresponds to 13?

Student 5: 13 is here (by showing the first row) and 5 is here (by showing the first column).

Ebru: Hmm. What do we do instead of repeated addition? I mean a short method for it.

Students: Multiplication.

Ebru: We multiply. 1-2-3-4-5 by pointing to the squares in a column and 1-2-3-4-5-6-7-8-9-10-11-12-13 by pointing to the squares in a row. We multiplied 5 and 13.

Students: 65.

Ebru: 65. Then, in order to calculate the area of a rectangle, we can multiply the length and width, can't we?

Students: Yes.

Rowland and his colleagues (2009) emphasise that some contingent moments result from the unexpected contributions of the students. What is important here is to notice these contributions (Mason, 2002) and know how to use them considering the objectives of the lesson (Smith & Stein, 2018). Ebru was pleasantly surprised by her student's counting strategy, which was significant in allowing the other students to discover the most efficient way. Although the students used different methods to count the squares, they got the same number as an answer. However, as can be imagined, the student who counted in 5s found the answer faster than her friends. Once the student had finished her counting, a short discussion of her counting strategy gave an opportunity to conceptualise how they could find the area of a rectangle more easily. That is, being able to monitor the potential available in the student's counting strategy and purposefully select this student to explain her strategy allowed Ebru to make a connection between this strategy and the objectives of the lesson. This discussion is important regarding the objectives of area measurement instruction, as one of the objectives stated in the curriculum is: "Calculate square and rectangular regions' areas using square units." Initially, the purpose of the table covering activity was to check the students' estimations. That is, Ebru had not previously planned to give the relationship between an area and the number of squares on the sides of a rectangle. Ebru helped her students discover this relationship by asking "What do we do instead of repeated addition? I mean the short method for it." *Being aware of the purpose*, one of the codes of the Foundation dimension, and *making connections between procedures*, one of the codes of the Connection dimension, allowed her to *take the student's response into account and incorporate it into the lesson*. In other words, Ebru changed this into an opportunity to help her students know that the area of a rectangle is equal to the product of the number of squares on adjacent sides of the rectangle. That is, the contingent moment relates to moving from unit counting to grouped counting in multiples, and then to multiplication as a way of working out the area of squares and rectangles. Therefore, it can be concluded that we see evidence of Ebru's mathematical knowledge in teaching in the KQ dimensions of Foundation, Connection, and Contingency, applied to the calculation of area using unit squares.

Contingent Moment 4: Teacher Insight

Another contingent moment in Ebru's area measurement instruction reflects the *teacher insight* code of the Contingency dimension. The teacher insight code is about moments in which the teacher realises that s/he is incorrect, or his/her example is not ideal for the topic, and tries to correct him/herself (Rowland et al., 2009). In addition to realising a teacher shortcoming, a teacher's realisation(s) about the usefulness and importance of some ideas are also taken to be teacher insight.

After the students discovered the 'row times column' (R×C) formula for area, Ebru wanted to write this as a rule on the board. Ebru gave the R×C formula by writing the following on the board:

> In order to find the area of a square region or a rectangular region, first of all, we have to find the number of squares in a row. Then, we have to multiply that number by the number of rows.

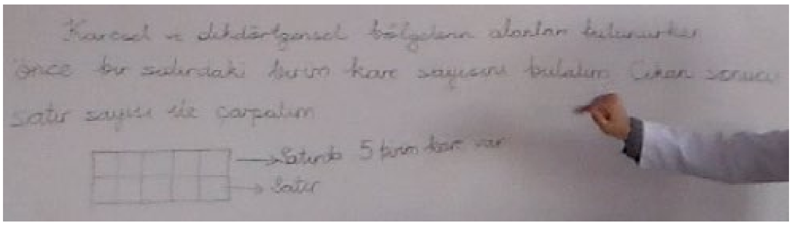

Figure 4. R×C formula definition and an example

Ebru drew the above 5×2 figure in order to help her explain the rule written on the board. For the first part of the rule, she counted the number of squares in a row, 5 in this case. When she continued to the rest of the formula, she thought that the statement "*multiply that number by the number of rows*" was wrong and erased it quickly, writing the following statement instead.

> Then, we have to multiply that number by the number of columns.

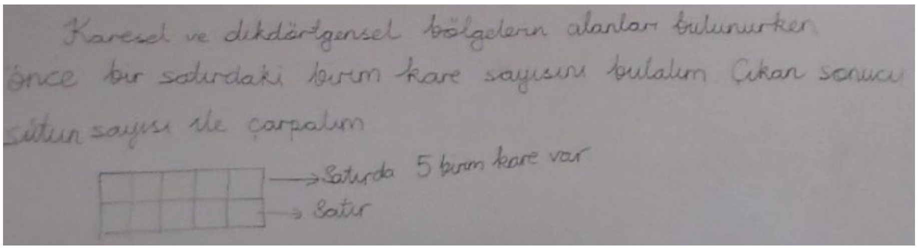

Figure 5. New R×C formula definition

Ebru was also unsure whether the new statement was correct or not; and she did not know how to handle the situation. Ebru's confusion over the statement of the formula for calculating area seems to have occurred unexpectedly – it was a contingent situation. The comment she made, "Let's check again in the textbook," shows that Ebru had experienced an insight. What is apparent here is that there is an effort to self-correct. However, her decision to check in the textbook and adhere rigidly to the textbook indicated some shortcoming in her Foundation knowledge (code: *adherence to textbook*) in the context of teaching. Instead, Ebru could have considered what was wrong with the statement and corrected it, realising that the number of squares in a row is the same as the number of columns.

In the post-lesson interview, we questioned this definition and the reason for the statement change. Ebru had repeatedly referred to the multiplication of the number of rows and the number of columns in relation to this rule. Although the definition included the word "row," it did not include "column," so, she concluded that it was wrong. However, during the post-interview, Ebru explained to us that the first definition had already been correct. Furthermore, she added that if she had changed the definition by writing "the number of squares in a column" instead of "number of rows," it would again be correct. That is, we see the sharing of a revised statement in the interview, pointing to a useful growth in her mathematical knowledge in teaching.

Contingent Moment 5: Responding to a Student's Incorrect Answer

The last contingent moment in Ebru's area measurement instruction was about drawing given areas on 'dotty' (square-dot) paper. During area measurement lessons, the students first discovered that squares could be used to cover a field, and learned how to find an area of a given figure by counting squares. Then, the students found that there was a relationship between area and the number of squares on adjacent sides of a rectangle and a square. Finally, they created figures with a given area on their dotty papers. During this activity, Ebru explained to her students that they would draw a figure with an area of 14 square units on dotty paper. Ebru moved around the classroom to monitor whether the students were drawing the figures correctly. The students drew different figures and rectangles containing 14 squares. However, one of the students drew a non-rectangular figure (Figure 6) on his dotty paper and wrote 7×2 at the top of the figure. To try to interpret what the student thinks or provide details about this student's thinking, Ebru began the following brief interchange with the student:

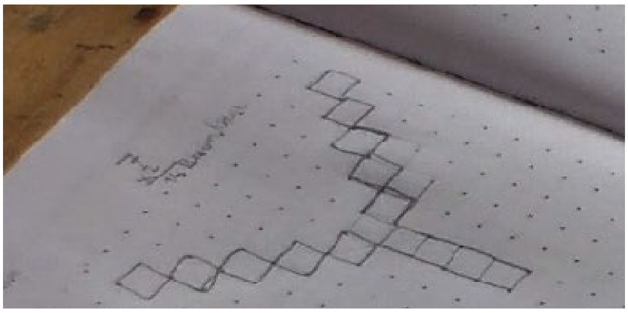

Figure 6. Student's drawing and answer

Ebru: Why did you multiply 7 by 2?

Student 6: To make 14.

Ebru: Just to make 14?

Student 6: Yes

Lehrer (2003) notes that some students prefer to use the area formula for all kinds of figures, including irregular figures. Here, it may be concluded that this student would also use the area formula for the non-rectangular figure drawn by himself. That is, although the student's drawing did have an area of 14 square units, his reasoning about the area was incorrect and might indicate misconceptions concerning the area formula. Ebru understood that her student was using the R×C formula incorrectly, and she wanted to take his solution into consideration in order to guide him to understand the difference between the R×C formula and the counting strategy. Smith and Stein (2018) emphasise that if teachers notice a common misconception, they "might want to have it addressed first so that the class can clear up that misunderstanding to be able to work on developing more successful ways of tackling the problem" (p. 13). Similar to this emphasis, after all the students had drawn their figures, Ebru showed his figure to the other students and asked them for their thoughts about how they could find the area of his figure. After the students had explained that the R×C formula above his figure was used for rectangular figures, she asked him to come to the board. Ebru drew two examples: one rectangular, the other non-rectangular. She asked the student to solve them and then explain his thinking.

Figure 7. Examples for a rectangular and a non-rectangular figure

Although Ebru realised that her student had applied the R×C formula to an inappropriate figure, she did not reject his incorrect answer. Instead, she *took her student's solution into account and incorporated it into her lesson* by spending some time discussing his solving strategies so that he might see his error and correct himself. The way Ebru responded to her student provides insight into her mathematical knowledge in teaching for the Contingency dimension. That is, having monitored the student's thinking helped Ebru to overcome the misconception. Furthermore, there may have been other students having difficulty in deciding which one of the strategies between the counting strategy and the R×C formula was more useful for finding the area of the figure, so Ebru's response was important in helping the students to understand when it is meaningful to use the RxC formula or for which shapes the RxC formula can be used. These discussions may help students comprehend how the row and column structure of a rectangular figure is related to the number of unit squares in the figure, and to the area formula for the figure (Sarama & Clements, 2009). Otherwise, the meaning of area for the students would be limited to a formula such as base×height, row×column, or length×width and they would try to apply this formula to all figures regardless of their shapes (Clements & Stephan, 2004; Outhred & Mitchelmore, 1996). Moreover, researchers state that if students do not have a good understanding of area, they want to use the counting strategy and the formula interchangeably (Dickson, 1989; Torbeyns, Verschaffel, & Ghesquiere, 2004). Providing students with examples of both rectangular and non-rectangular figures can be connected to Ebru's mathematical knowledge regarding representation (Transformation dimension of the KQ).

Conclusion and Suggestions

The purpose of this paper is to describe some contingent moments in an in-service primary teacher's area measurement instruction, to analyse the teacher's responses, and to examine how her mathematical knowledge in teaching influenced her responses to these contingent moments. For the first purpose, five contingent moments were identified in our data and those moments resulted from *the unavailability of materials*, from *teacher insight*, or from *a student's idea or incorrect answer*. The analysis of the contingent moments showed how a student's idea or response can change the direction of a lesson in a positive way.

For the second purpose of the paper, the ways in which Ebru responded to and dealt with these contingent moments were analysed and discussed. It was concluded that she did her best to respond to the contingent moments. Specifically, she did not ignore any of these moments but incorporated them into her instruction. However, there were some differences between the ways they were included and the depths of the given responses. The reason for these differences may be attributed to Ebru's mathematical knowledge in teaching, which was the final concern of this paper. If Ebru had been aware of the nature of such 'contingent moments' in mathematics classrooms and considered how a teacher can use these moments to assist the students' learning, she might have responded more effectively. In Contingent Moment 1, Ebru used the student's erroneous reasoning to help other students consider the suggested idea, so they would not make such an error. In the same way, sharing the student's incorrect answer in Contingent Moment 5 is important as the students would know when the area formula is applicable, and when it is not. Crespo (2000) emphasises that teachers need to change their ways of listening and responding to their students by just looking "for right or wrong answers" (p. 160). Instead, when students provide incorrect answers, teachers should encourage them to explain their thinking. If teachers effectively orchestrate this process and help students see what is wrong with the answer or why it is wrong, students can develop a deeper understanding of the related concept (Franke, Kazemi, & Battey, 2007; Smith & Stein, 2018). Although Ebru tried to orchestrate the discourse by pointing out the student's error to her other students, she did not consider the source of this error. Teachers with secure mathematical knowledge in teaching are better able to anticipate students' possible errors and reasons for them (Ball et al., 2008; Rowland et al., 2009).

Similarly, researchers emphasise the fact that teachers' mathematical knowledge in teaching affects the way they deal with students' errors (Bray, 2011; Son, 2013). Smith and Stein (2018) assert that teachers with secure mathematical knowledge in teaching are able to ask questions which direct their students towards the objectives of the lesson. In Contingent Moment 3, Ebru used another student's idea and asked questions to help the students discover the base×height or row×column formula, which was one of the objectives of the lesson. *Identifying students' error*s, *knowing reasons for these errors*, and *being aware of the purposes* of the lesson are codes of the Foundation dimension of the KQ. Ebru demonstrated some limitations regarding these codes; with attention to them, there would be opportunity to develop her mathematics teaching.

Likewise, limitations in Ebru's Transformation mathematical knowledge also becomes apparent, regarding her *use of representations*. Rowland and his colleagues (2009) state that appropriate representations help students to comprehend topics, especially when first introducing them. Ebru could have used concrete objects in addition to the pictorial ones displayed in the textbook. Contrary to the Foundation and Transformation dimensions, Ebru's mathematical knowledge in teaching for the Connection and Contingency dimensions seems more secure, as she made *appropriate connections between procedures* and quick decisions to respond to contingent moments, thinking on her feet. Teachers with secure mathematical knowledge in teaching are more successful in monitoring students' responses, noticing the potential of these responses, and making it visible to the whole class (Breen, 1999; Smith & Stein, 2018). Specifically, Ebru effectively took the student's counting strategy (explained in Contingent Moment 3) and the student's answer (explained in Contingent Moment 5) into consideration and successfully brought it to the attention of other students. Furthermore, throughout this process, Ebru asked questions to the students in order to hear their explanations "rather than assuming what the students did or how they were thinking" (Smith & Stein, 2018, p. 47), which is another characteristic of teachers with secure mathematical knowledge in teaching. To sum up, Ebru's way of responding to those moments was enabled by her mathematical knowledge in teaching.

In conclusion, the analysis of contingent moments of Ebru's lesson using the Contingency dimension of the Knowledge Quartet model has provided us with an insight not only into contingent moments of area measurement instruction, but also the importance of these contingent moments and the teacher's responses. Although this study was carried out in only one teacher's mathematics lessons, it demonstrates how contingent moments and teachers' responses to them can affect the course of instruction. Therefore, this raises further questions such as "What can be done in order to inform teachers about the importance of these contingent moments?" "How can teachers be trained to identify and to act during these moments?" and "How can teachers' mathematical knowledge in teaching be developed so that they can respond appropriately to these moments?"

Such contingent moments from 'real' mathematics classrooms can be used for discussion and reflection between teacher educators and pre-service teachers during the methods courses of teacher education programs. Presenting and discussing these contingent moments with pre-service teachers can guide their future teaching and help them take advantage of contingent opportunities as they occur. In this way, pre-service teachers give more thought into responding appropriately to contingent events, thereby creating a better learning environment. Teachers may tend to teach according to their lesson plan rather than deviating from it. However, since deviation from their plan because of students' ideas or responses may provide a more constructive learning environment, pre-service teachers should be encouraged to take these chances. In the same way, in-service teachers may take part in collaborative learning communities to generate and broaden a collective knowledge including importance of contingent moments and their responses to these moments.

Further explorations of larger samples of teachers' instruction would be beneficial to investigate which aspects of mathematical knowledge in teaching are related to their responses. It would be valuable to conduct a larger-scale empirical study into experienced primary teachers' responses to contingent moments in the mathematics classroom. In the same way, the effect of a professional development program informing teachers about the five practices on teachers' responses to contingent moments can also be explored.

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Appendix

The Dimensions and Contributory Codes of the KQ Model (Rowland, 2013, p. 25)

Dimension	Contributory codes
Foundation:	awareness of purpose adherence to textbook concentration on procedures identifying errors overt display of subject knowledge theoretical underpinning of pedagogy use of mathematical terminology
Transformation:	choice of examples choice of representation use of instructional materials teacher demonstration (to explain a procedure)
Connection:	anticipation of complexity decisions about sequencing recognition of conceptual appropriateness making connections between procedures making connections between concepts
Contingency:	deviation from agenda responding to students' ideas (use of opportunities) teacher insight during instruction responding to the (un)availability of tools and resources