Linking practice and theory: A case study of mathematical discourse between a mathematics teacher educator and a primary classroom teacher

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In recent years there has been growing concern as to how to bridge the gap between the theory pre-service teachers engage with as part of their learning in their tertiary classrooms and the profession. To enable pre-service teachers to make stronger connections with the profession, a mathematics teacher educator worked collaboratively with a practicing teacher by co-teaching two cohorts of pre-service teachers studying primary mathematics education. As well as providing benefits to the pre-service teachers, the project allowed the mathematics teacher educator to reflect upon her own practice and the impact of the co-teaching experiences and learning. In this paper we present two snapshots of this experience and a framework that was used to identify how the co-teachers helped the pre-service teachers to make connections between theory and practice, when engaged in mathematical discourse.

**Keywords** **.** preservice teachers **.** co-teaching **.** mathematical discourse **.** primary **.** theory and practice

# Introduction

Most pre-service teachers’ (PSTs) experiences occur in university classrooms (tutorials) or during practicum in primary school classrooms. When considering teacher preparation and learning, courses should assist PSTs to develop a deep understand of the mathematics they will teach, be grounded in research and assist with the transition of learning during coursework to teaching in real classrooms (Kulm, 2008). These experiences should also provide PSTs with current images of teaching that determine their understanding of mathematics education including how they might approach their mathematical teaching by linking practice and theory. There is agreement that knowledge for teaching mathematics includes a combination of theoretical and practical knowledge developed at university and in schools (practicum experiences) (Allen, Ambrosetti, & Turner, 2013; Novotná, 2009).

Currently there is an Australian initiative to improve the quality of pre-service teacher education including the knowledge for teaching mathematics (Teacher Education Ministerial Advisory Group [TEMAG], 2015) and universities need to consider how they might respond to these recommendations. The university where this study took place chose to invite practicing teachers to participate in teaching courses at both undergraduate honours level and masters level. The purpose of this initiative was to assist with building relationships between universities and schools, ensuring graduating teachers are classroom ready (TEMAG, 2015) and equipped with a diverse range of skills for teaching, including the specialised knowledge for teaching primary mathematics. As little has been written about co-teacher experiences in teacher education within university settings, this study will contribute to the research literature in this field.

Co-teaching is when two professionals such as a teacher and special education teacher work together to deliver instruction (Friend, 2008). Graziano, and Navarrete (2012) identified several benefits of co-teaching including opportunities to vary content presentation, individualise instruction, scaffold learning experiences, and monitor pre-service teachers’ understanding. They also suggest that co-teaching can promote equitable learning opportunities for all PSTs.

While a number of previous studies have documented situations where mathematics teacher educators (MTEs) have worked with practicing teachers to reflect upon and enhance classroom practices (e.g., Geiger & Goos, 2006; Geiger, Muir, & Lamb, 2015; Muir & Beswick, 2007), less common are examples of practicing teachers working alongside MTEs in their tertiary classrooms.

In the study reported on in this paper, a co-teaching initiative was enacted to enable a cohort of PSTs to make stronger connections with the profession, as well as providing an opportunity for the MTE to reflect upon her own practice and for the teacher to gain insights into current teacher education practices. The following research questions underpins our study:

How does co-teaching facilitate meaningful mathematical discourse?

How can a co-teaching situation help PSTs to make connections between the theory and practice of mathematics teaching?

Other strategies to help PSTs make connections between theory and practice have included the provision of video footage of mathematics teaching for them to view and critically analyse (e.g., Beswick & Muir, 2013), implementing lessons or co-teaching in classrooms (e.g., Anthony, Hunter, Anderson et al., 2015; Cavenagh & Garvey, 2012; Perkins, 2015), role-playing and rehearsal (e.g., Grossman, Hammerness, & McDonald, 2009; Muir, Allen, Rayner, & Cleland, 2013), the use of representations of practice, such as children’s work samples (Livy, Muir, & Downton, 2017) or providing opportunities for PSTs to observe their MTE teaching in a primary classroom (Livy & Downton, 2017). We believe that a focus on the transition to teaching in a real classroom can be improved by collaborating with schools and their teachers, such as inviting them to assist with teacher preparation during tutorial experiences. For most PSTs, working with practicing teachers only occurs through their professional placement experiences, and there are no guarantees that these experiences provide PSTs with examples of ‘best practice’, current research or exposure to quality mathematics teaching. Research, however, has shown that connections between theory and practice of mathematics classroom teaching can be challenging for MTEs, but achieved by making university experiences relevant to classroom experiences (e.g., Beswick & Muir, 2013).

# Review of the Literature

## Making explicit the links between theory and practice

There is ongoing concern of a possible disconnect between how PSTs make sense of what they learn in their tertiary classrooms with school based practicum placements (TEMAG, 2015; Zeichner, 2010). As MTEs, our teaching and research is informed by frameworks and explanations of terms to guide our thinking about the knowledge an effective mathematics teacher might use. For example, Shulman’s (1987) seminal study has guided many researchers as they consider important categories of a teacher’s knowledge base such as *knowledge of content*, *pedagogical knowledge* and *knowledge of learners*. Others have elaborated by describing *specialised content knowledge* when referring to a unique kind of knowledge mathematics teachers demonstrate (Ball, Thames, & Phelps, 2008). Further, Rowland, Turner, Thwaites and Huckstep, (2009) provided four categories of the Knowledge Quartet (KQ): *foundation knowledge* (including knowledge of content and pedagogical knowledge); *transformation* (representing the mathematics); *connection* (e.g., coherence of planning, sequencing of instruction); and *contingency* (when the teacher responses to classroom events) that can be used to develop and deepen teachers’ (and PSTs’) mathematics knowledge.

These frameworks can be useful for guiding MTEs’ instruction and helping PSTs to become confident teachers of mathematics. In particular assisting PSTs to develop their foundation knowledge, including beliefs about how and why mathematics is learnt (Turner & Rowland, 2011), is likely to impact upon their ability to adopt appropriate pedagogical practices into their future classrooms. The mathematical discourse that occurs in a tertiary classroom setting, therefore, must include discussion that advances PSTs’ *breadth* and *depth* (Ma, 1999) of mathematical understanding. Discourse, in this context, is an approach to teaching, which encourages students to discuss the mathematics as they reveal their understanding of concepts and engage in reasoning and debate (Cobb, 2006). Mathematical discourse is more than spoken words, rather learning that supports meaningful discussion (Staples & King, 2017).

Having clarified the knowledge that is needed for primary teaching it is also important to consider the role of the MTE in facilitating purposeful learning. An important role of teachers is to use strategies that encourage thinking such as turn and talk, think pair share (Kazemi & Franke, 2014; Staples & King, 2017). These strategies can also support PSTs to develop their mathematical discourse as well as model approaches to teaching. Similarly, students might be asked to share their ideas with the class explaining their thinking and strategies. The MTE can also model appropriate pedagogical moves and approaches, such as incorporating Smith & Stein’s (2011) five practices for orchestrating productive mathematical discussion (anticipating, monitoring, selecting, sequencing and connecting) into their tutorials. Analysing primary school students work samples can be used to illustrate the practices of selecting, sequencing, and connecting which have been shown to help PSTs’ make the links between theory and practice (e.g., Livy et al., 2017).

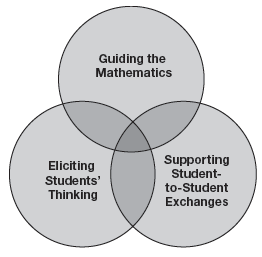
When conducting lessons, evidence of teachers’ Pedagogical Content Knowledge (PCK) can be enacted through the way in which they *transform* and *connect* the knowledge they are teaching and how they respond to *contingencies* (Rowland et al. 2009). When a teacher is using their *specialised mathematical content knowledge* (Ball et al., 2008) they are relying on their *mathematical knowledge* to consider ways to represent a mathematical idea. These mathematical ideas can be ‘transformed’ to students by using representations or materials that demonstrate and *transform* what the teacher knows when helping their students learn (Rowland, et al. 2009). The third category of the KQ, *connection,* is apparent when a teacher makes choices and decisions: making connections between procedures or concepts; as they consider the complexity of the learning; when making decisions about the sequence of the lesson; and recognising the conceptual appropriateness (Turner & Rowland, 2011). Within any given lesson, it is reasonable to expect that situations will occur when students might provide an incorrect response, or the lesson takes a different direction than that originally intended. Such instances are referred to in the KQ as *contingencies* because the teacher must consider how to respond with an occurrence that was unexpected (Rowland et al., 2009).

When supporting students to learn, Staples and King (2017) suggest providing tasks that make learning accessible to all as well as a student-centred approach to discussions where the teacher relies on a variety of strategies for guiding learning. When this occurs, the classroom can be transformed into a community of practice, where groups of people interact and contribute to a common interest, actively participating and sharing information, stories, and experiences to gain knowledge and skills (Wenger, 1998). Such interactions are important because a recent review of literature within Australia, recommended that collaboration and sharing of practice with other education communities is required if teacher educators are to improve their practice (Anthony, Cooke, & Muir, 2016). Others have also identified a community of practice as an effective perspective for examining co-teaching situations when teaching mathematics. For example, Enfield and Stasz, (2011) found that in their study, a community of practice helped their PSTs to develop as reflective professionals by encouraging reflection in action and discourse that forced explicit thinking or metacognition about an activity.

# Theoretical Frameworks

Facilitating mathematical discourse is an effective teaching practice (National Council of Teachers of Mathematics [NCTM], 2014) that also applies to MTEs’ practice. Next, we describe a framework for facilitating meaningful mathematical discourse with students. Then we present an adapted framework (Staples & King, 2017) for considering mathematical discourse and PSTs’ development of theory and practice of primary mathematics teaching.

The framework in Figure 1 shows three key functions teachers rely on when guiding students’ mathematical discourse include: eliciting student thinking; supporting student-to-student exchanges; and guiding and extending the mathematics (Staples & King, 2017). The key functions and inter relations between the functions supports students’ participation in mathematical discourse.



*Figure 1.* Three key functions of the teacher’s role in facilitating meaningful mathematical discourse (Staples & King, 2017 p. 39).

When teaching school mathematics, discourse may be used when engaging students through problem solving tasks, encouraging reasoning and understanding which are proficiency strands in the Australian Curriculum: Mathematics (ACARA, 2017). However, the focus of our study was to investigate the mathematical discourse that occurred when co-teaching in a tertiary classroom. After drawing on the literature we chose to adapt the three key functions (Staples and King, 2017) shown in Figure 1. Not only did we want to promote meaningful mathematical discourse, but mathematical discourse that supported PSTs development of theory and practice for teaching mathematics.

Staples and King’s, (2017) framework for facilitating meaningful mathematical discourse was useful for our study, along with other frameworks such as the KQ (Rowland et al., 2009). In addition, a community of practice was also a key function that promotes reflective discourse of PSTs (Enfield & Staz, 2011). As already supported within the review of literature, many categories of teacher knowledge are useful when helping PSTs to develop theory and practice and therefore became the centre of the revised framework (Figure 2).



*Figure 2.* Three key functions of the co-teachers’ role in facilitating meaningful mathematical discourse (adapted from Staples & King, 2017 p. 39).

Figure 2 shows three key functions:

1. Guiding and extending the mathematics, including pursuing common misconceptions and ensuring appropriate disciplinary norms to advance the learning of the class (Staples & King, 2017);
2. Supporting PSTs within a community of practice, establishing a supportive setting to help PSTs to learn as reflective professionals;
3. Eliciting student thinking, including providing opportunities for students (PSTs) to generate ideas with the class (Staples & King, 2017).

Within the classroom these three functions may overlap. For example, 1 and 3 overlap when the discourse includes making connections such as developing conceptual understanding of why a rule might work such as the formula for the area of a triangle. 1 and 2 overlap when the exchanging of ideas relates to guiding the mathematics understanding using one of the talk moves. Similarly, 2 and 3 overlap when the exchange of ideas relates to making connections and includes the talk moves. When all three functions overlap they assist PSTs to develop knowledge of theory and practice for primary mathematics teaching.

# Methods

Both case study research design and qualitative methods were used in this study. A case study may answer how or why questions (Yin, 2009) and describes specific instances or phenomenon (Merriam, 1988). The following case study research was designed to assist with reporting on the phenomenon of co-teaching experiences in a tertiary classroom when the co-teaching partnership assisted the MTE by helping the PSTs to make connections between theory and practice through meaningful mathematical discourse.

## Participants and co-teaching program

The participants in the study included two cohorts of PSTs (N=47), their mathematics MTE, Sally and a primary school teacher Sam (pseudonyms used throughout.) The PSTs were enrolled in a four-year teacher education Honours degree specialising in primary education. As part of the course structure all PSTs studied two units preparing them primary mathematics teaching. The first unit in second-year assisted PSTs to develop knowledge, skills and dispositions related to mathematics and numeracy education in the early years.

The second unit, reported in this study was in third year and designed to extend PSTs’ knowledge for teaching mathematics and numeracy by focusing on upper primary levels. Each week the PSTs attended 2 hours in their tertiary classroom during semester (10 weeks). The PSTs were expected to engage with independent study, complete two assessment tasks: a critical analysis related to an issue in mathematics education and the preparation and facilitation of a lesson to their peers, together with a reflection on their and other PSTs experiences. All PSTs completed 80 days of professional experience in primary school settings during their degree.

The MTE had applied for a co-teacher as part of an initiative to improve the quality of teacher education at the university and also in response to a call for teacher education providers to work together and assist pre-service teachers to develop a connected knowledge of theory into practice (TEMAG, 2015). Prior to the study, Sally and Sam had co-taught one tutorial in the previous year. After that experience Sally invited Sam to join her for the semester as a co-teacher in 2017 mainly because of his experience as a leading numeracy teacher in his school. Sam agreed to participate in the co-teaching program and was keen to share his expertise with future teachers. He was released from his school on a weekly basis to join Sally when teaching during 2017.

A goal of the weekly tutorials that the co-teachers planned was to engage the PSTs with significant mathematics and mathematical discourse that prepared the PSTs for teaching mathematics in the upper year levels. The discussion included making links to the Australian Curriculum for mathematics (ACARA, 2017), and discussion of pedagogical approaches when exploring tasks suitable for helping primary students to learn mathematics. Often a challenging task (e.g., Sullivan, 2017) was chosen and the PSTs solved the task for themselves, sharing and identifying different strategies and discussing their relative effectiveness. Many of these experiences also helped the PSTs to extend their mathematical content knowledge and to revisit the mathematics they were taught in schools. When eliciting, supporting and guiding mathematical learning as suggested by Staples and King (2017), discussion also “focused on concepts, procedures, problem-solving strategies, representations, or reasoning” (p. 37).

## Data collection

In Week 6 Amanda (researcher) observed the interactions between the co-teachers during both tutorials. Amanda took field notes and later wrote a vignette of her observations of the first lesson, reported in the results section of this paper. In addition, the PSTs provided written responses to a survey. They were asked, *‘What is something that Amanda will notice about today’s lesson* and *list three benefits from having a co-teacher in your mathematics tutorials’*. After the tutorials Amanda conducted two focus group interviews with PSTs (n=6) for 15 minutes. The PSTs were asked further questions related to the co-teaching program including benefits or difficulties they could recall when reflecting on their learning during the semester. Finally, Amanda interviewed Sam, then Sally, each for 15 minutes discussing their experiences of the co-teaching and responding to questions related to strengths and weakness of working together, what they were gaining from the experience and how they thought the PSTs’ knowledge of theory and practice was being extended because of the co-teaching experience.

In Week 7 Annabel (researcher) took field notes while observing two tutorials, and again, all PSTs provided written reflections of their lesson experiences, responding to the same questions they had answered the previous week. Anabel also interviewed Sam and Sally post lesson to gain their insights of the experience. There was no PSTs focus group interview following this lesson as few students were available to participate.

In summary, data included field notes of two lessons, transcripts of interviews with PSTs and both co-teachers, and written reflections from the PSTs.

## Data analysis and coding

Following an interpretative paradigm in qualitative data analysis, the authors coded the vignettes to identify evidence of meaningful mathematical discourse for supporting the PSTs to make connections between the theory and practice of mathematics teaching. Independently the authors each used open coding to find evidence of the three functions for facilitating discourse (Figure 2), evidence of the dimensions of the KQ, and evidence of supporting a community of practice. In collaboration, the authors conducted a further cycle of coding to derive the agreed categories. These are presented in Table 1 together with illustrative examples of the interplay between the co-teachers and PSTs as captured by the classroom observations.

Table 1

The categories derived from the analysis of the observation lessons and transcripts, with examples

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| Categories | Examples from the Lesson Observations |
| 1. Foundation knowledge (KQ) | How would you define perimeter, area and volume?  Articulating the meaning of volume verses capacity |
| 2. Making connections between concepts (KQ) | Considering the relationship between area and surface area of a cube. |
| 3. Eliciting PSTs’ thinking | Sam reframed the question to elicit more responses from the PSTs  Talk to your friend about your thinking |
| 4. Connecting to how children learn mathematics | Sharing insights of classroom experiences and student work samples |
| 5. Establishing a community of practice | Being co-constructors of the ideas with students  The interplay between co-teachers and PSTs |

These categories link to some of the dimensions of the KQ, the three functions for facilitating meaningful mathematic discourse (Figure 2), talk moves, the 5 Practices and community of practice.

# Results

The following vignettes report two episodes of Sally and Sam’s co-teaching to provide details of how they elicited PSTs’ thinking, and supported and guided mathematical discourse that helped PSTs’ to make connections between theory and practice. The discussion section that follows the results includes links back to the literature and three key functions for supporting meaningful mathematical discussion (Figure 2).

## Vignette 1: Algebraic Thinking tutorial observed by Amanda [12.00 pm class; 21 students; 5 table groups]

*Focus on the mathematics: Algebraic thinking*

The lesson began with Sally asking the PSTs to share what they thought algebraic thinking involved. When there were few responses, Sam reframed the question and elicited some more responses from the PSTs. Sam recorded the responses on the whiteboard, which included ‘solving problems’, to show how components are related, and to make predictions. At one point, the following exchange occurred:

PST: Substituting numbers for letters

Sam: Does anyone know what the numbers are called?

PST: Pronumerals

Sam: I haven’t heard that before

After some more contributions, Sally then directed the PSTs’ attention to the powerpoint slide which contained the three big ideas associated with algebraic thinking: Patterns, Equality, Function.

*Knowledge of resources*

Sally then provided a general overview of where to source teaching activities from, including websites and professional journals. At this point, Sam cautioned students to be critical when accessing websites and to “resist the temptation to use a resource without thinking about how it fits in with the conceptual understanding”.

*Links with the curriculum*

Sally then directed Sam to “go over the curriculum for us”. Sam spoke to a slide that contained an overview of algebraic thinking at different year levels. He then placed a line of tiny teddies on the desk and asked PSTs what they would do if students could not identify and then follow the pattern to place the next tiny teddy. When there was no response, Sam kept rephrasing until there were a few suggestions given. Photographs showing students from Sam’s class with examples of extending patterns were then shared with the PSTs.

*Modelling and representations*

Sam then modelled how a balance scale could be used to demonstrate equality to show that the concept of ‘balancing numbers actually works’. He emphasised the importance of students’ opportunity to ‘make, name and record’ and shared photographs and examples of students’ work. Sally then asked the PSTs to think about how they would use the balances at the next level (Grade 3) and then the PSTs were directed to use the balance scales at their tables to demonstrate the equality of number sentences. As the PSTs attempted to use the balance scales to represent number sentences, it became apparent that there were issues with using the balance scales provided. Some of the balance scales had weights that were marked in grams and sensitive to where the weights were placed. This meant that in practice, some of the PSTs could make their balance scale show that 20 grams were equal to 6 + 4 grams. This then prompted Sally to ask the PSTs to think about whether or not the balance scales were a good teaching tool or not and emphasised that “you need to think about whether or not the materials you are going to use help develop the concept”.

Further on in the lesson the PSTs were asked to construct patterns using yellow/red counters that Sally distributed. After PSTs spent about 5 minutes making their patterns, Sally directed the class to move to one group’s table and asked Mark to continue the pattern. She then asked the group to name the type of pattern [repeating] and then used the counters to show a growing pattern and asked PSTs to identify how the pattern was growing. The PSTs were then asked to use the counters to model triangular numbers as an example of a growing pattern. She then directed the PSTs back to the power point and provided an example and photograph of a ‘Year 5 lesson where ‘Jack’ wrote 8 + 4 = 12 + 5

PSTs were then asked to discuss why they might agree or disagree with that, and then looked at some similar work samples in their table groups. After sharing, Sam stated, “Here’s a different perspective on this – what do you think they would have come up with if there was no 12 in it?” After some prompting, Sally then suggested that “maybe equal means that’s where the answer goes”. She then explicitly explained to the PSTs that “we were trying to show you an example of students’ thinking which shows a misconception” and Sam responded with “What Sally’s struck on is fundamental – what do you think students are doing? I often get them (students in class) to teach me – I will deliberately make a mistake and get them to teach me how to do the maths correctly”.

*Focus on mathematics: Applying the ‘rule’*

Two examples were then provided to demonstrate how a ‘rule’ could be used to identify an algebraic pattern and then generalise for any number of steps. Sally made the point that it was “not about teaching the rule but about modelling and understanding.” Sam then suggested that in this context, ‘x’ could be replaced by ‘d’ to denote days, which was not commented on by Sally. The other example involved a variation on the ‘seating problem’ whereby the PSTs were required to work out how many people could be seated at six tables. They then shared their solutions with the class and Sally made the explicit teaching point of the importance of listening to students’ thinking and determining what it shows about their understanding.

*Focus on the mathematics: Functional thinking*

The final activity focused on functional thinking. Sally shared an example of a function machine and then shared the story ‘Two of Everything’ (Hong, 1993) to talk about the ‘rule’ and “What would my machine show if I put in 10 and 30 came out?” Sam then provided examples from his students where they showed examples of early multiplicative thinking. The lesson concluded with a summary from Sally who explained that the purpose of the tutorial was to show them what algebraic thinking involved and how it could be developed.

## Post-lesson feedback from the PTS

Following the lesson, three focus group interviews were conducted with eight PSTs. The PSTs were asked what they learnt as a result of the lesson, both in terms of mathematics and how to teach it, and how each of the co-teachers contributed to their learning. PSTs consistently made reference to the practical nature of the class and the focus on understanding as illustrated by the following comments:

It’s always a practical approach … we often get presented with a problem we have to solve and then discuss [Missy]

[Sally] gets us to do the activities ourselves so I guess that we can see how they work, how the children might think and then we compare how everybody else has worked out a solution [Sui]

Interestingly, when asked to comment on the contributions of the co-teachers in terms of impacting upon their learning, PSTs tended to focus on Sally’s teaching strategies in a whole group situation and Sam’s contribution to them individually or in small groups:

[Sally] breaks down misconceptions in most classes and we have to think about getting rid of those now so that when we go into a classroom, we don’t have those misconceptions ourselves [Jamie]

[Sally] also encourages us to share different answers because in a classroom when we ask a question, we have to be able to understand all the students’ responses, so she gets us to all respond in different ways to the same question. [Nina]

Sam always comes around and helps us throughout, asking questions, what are we doing [Kelly]

Even in a small group he’ll come over and he’ll look at what we’ve done and he’ll sort of say, ‘And what would you do if you were to change the level?’ … so he tries to get us to expand on what we’ve done and questions us differently. [Sui]

The PSTs also commented on the relevance of Sam’s classroom experience:

It’s quite helpful that Sam is actually teaching in a primary classroom and his teaching is very current. [Kelly]

It’s not that I don’t have trust in what Sally tells us because she has quite a lot of knowledge, but because he’s been in the classroom and done this with kids, she’d be like, “Oh I can see how that would work in the classroom” and he has that practical application. [Missy]

The PSTs were also specifically asked about the usefulness of Jack’s work sample in terms of helping them to understand an aspect of algebraic thinking. The following comments illustrate that the PSTs found it useful and that they were able to identify the misconception:

Yeah I think it was a really good example. It was really interesting to see, especially what the kids did – all the different methods they used even when they didn’t get it right – just to see how it could be interpreted.

Actually I wasn’t all that surprised by that only because in placement over the last couple of weeks, they have been learning about equals and I could see … how they struggled with one side being even with the other so that when I saw the question, I thought ‘I’m pretty sure I know what their thinking is behind that’ [Sui]

Well the misconception that the students have was that I think a lot of them were reading from left to right, so they weren’t seeing that both sides of the equation could be equal … they were just reading 4 + 8 = 12 and then they were like, what’s the five sign doing there – or 12 + 5 = 17. [Nina]

I think it’s a misconception of how you teach equals as well because when you’re younger, it’s just like 1 + 1 = 2, it’s not 2 = 1 + 1. [Jamie]

## Commentary

Lesson observations showed that the lesson was focused, with a good mix of instruction, participation in activities and opportunities for discussion. As the lesson excerpt shows, a number of instances occurred which guided the PSTs in their development of mathematical ideas, which is a key feature of *Guiding and Extending the Mathematics* (Staples & King, 2017). Further evidence of this aspect was also highlighted during the discussion about the equals sign and the misconceptions that PSTs might experience in the classroom. Providing the PSTs with work samples helped to situate the misconceptions into practice, and Sally was *explicit* about explaining this teaching strategy to her students. As evidenced by the PSTs’ feedback, it seemed this was an effective strategy in stimulating the PSTs to think about misconceptions and their causes. Sam provided a good example of *eliciting students’ thinking* when he asked the PSTs to speculate how students would respond if there was no ‘12’ in the answer. Their post-lesson interviews also showed there were plenty of opportunities and even expectations to contribute to discussions and share one’s thinking. Both the lesson excerpt and the interview data show that both co-teachers *supported class exchanges about mathematic ideas,* with Sam being particularly proactive in this area with small groups and individuals. The PSTs comments also indicated that Sally’s attempts to *make the pedagogy explicit* resonated with them. Both Nina and Jamie’s comments indicate that they could see the purpose behind the sharing of work samples and the expectation that they share and explain their strategies.

## Vignette 2: Measurement tutorial observed by Annabel [12.00 pm class; 19 students; 5 table groups]

The tutorial focus was on perimeter, area and volume and PSTs’ MCK and PCK of these topics as they engaged in activities that explored the mathematics underpinning these attributes of measurement and possible misconceptions students might develop.

*Focusing on correct mathematical terminology*

The session began with Sam asking the PSTs to individually brainstorm everything they know about measurement and he recorded their ideas on the board. Sally asked questions to probe PSTs’ thinking and to go deeper, “What are some attributes that we use but cannot see?” (e.g., time, temperature). “Can we measure all the things listed on the board? Is shape a measurement?”

Sally suggested that angles are aspects of shapes that can be measured, and that shapes are part of geometry. “In terms of definitions how would you define perimeter, area and volume?” Sam suggested that having a definition of these terms is helpful for Year 5/6 students. Sally asked for some examples and Sam recorded them on the board. Sally asked Sam what he would say to Year 1 /2 students when teaching this topic.

*Interplay between the co-teachers*

There was an engaging interplay between Sally and Sam as they explored the key ideas with the PSTs and how they might explore and model mathematical concepts with students of different year levels. When helping children learn correct mathematical language, Sally mentioned the importance of having word walls in the classroom. Sam agreed, he also pointed out the need for these to be co-constructed with the students and that displaying students’ work is much more effective. The PSTs were then directed to look at some definitions given in different mathematics dictionaries to clarify their own understandings of volume and capacity and identify definitions they might use with students. Sally asked them to articulate the meaning of volume versus capacity using a drink bottle and highlighted the misconception that some PSTs and students hold - that both volume and capacity are the same - is also held by some teachers.

*Role of teacher and teacher educator*

The roles played by both Sam and Sally were clear – one was the teacher educator and one was the classroom teacher. There were frequent interplays whereby Sally could be seen as a ‘knowledgeable other’ with Sam providing practical examples to complement the ‘theory’ provided by Sally. Sally would often reinforce what Sam said by making links to the curriculum. Sam extended on this and asked the PSTs what they might do to introduce a unit on the topic of capacity. He modelled the strategy he uses in the classroom of ‘talk to a friend about your thinking’ to get all students discussing and then invite different PSTs to share their thinking with a partner. Sam shared some of his experiences including when things do not go as planned, and Sally contributed some of her own.

*Making connections and challenging PCK*

The discussion shifted to when and how to introduce formulas and what the steps are before this. Sally drew a rectangle on the board. She asked PSTs to record the area and perimeter and for some to share their thinking. Some suggested the step before is filling in a grid and to model it with understanding. Sally suggested linking to multiplicative thinking and the use of arrays.

Sam questioned the use of a square when giving the area of a rectangular shape. What are the squares in everyday life? Sally then extended this to ways to find the volume of a cube without using the formula. A PST (Kelly) made a model of a cube and Sally asked how many blocks on each layer and how many altogether and how why we need to use cubic measures. She extended this by asking if they could work out the surface area of the cube as well. Sam made the point that this is a typical NAPLAN question. Sally then asked Sam, “If we were to make a cubic metre how many cubes would we need?” Sam drew the PSTs back to the surface area problem of a rectangular prism. One PST proceeded to find the surface area for each of the external faces would be 2 × (4×6), 2× (2×8), 2× (6×2). Seeing that she was struggling Sally provided an enabling prompt. “Draw the net of rectangular prism.” Sam said it would be better to build the net. Sally invited another PST (James) to use the model to visualise the net and then to record it on the board. Sally then asked different students to explain their understanding of surface area. She then said that this experience was an example of a ‘teachable moment’. Sam then linked back to the initial task: ‘Blocks and Boxes’ and the need to assess where students are at.

Sam: Some students come with knowledge of formulas and need to unpack it so getting students to break it down helps other students to understand how the formula was developed and what Sally did was gold as she broke the concept down using the covering of the grid. In school we need to get students to understand the concepts not focus on learning the rule.

Sally reinforced the importance of having a lesson plan, “to ensure you are aware of the mathematics, the questions and the key mathematical language.” She then introduced the area of a triangle task and both she and Sam roved and challenged the students as they worked. Sam invited different students to share their learning on the board and Sally then challenged them, “How could you find the area of a trapezium?” Sam asked them if they noticed the pedagogical action Sally used. “Going from the known and applying it to a new structure. It comes back to knowing the mathematics and deepening the students’ experiences.”

*Making connections between perimeter and area*

The session concluded with the PSTs working in groups of four to make a rug for granny’s hallway using newspaper that has an area of one square metre but is not in the shape of a square (Downton, Knight, Clarke, & Lewis, 2006). Both Sally and Sam moved around the groups asking them to explain and justify the process they were using. Because of time only one group was selected to share. They used the language of perimeter and area. Sally them asked, “What is the same and different about the rugs? What do we want the children to understand? It is important that they have experiences such as this so that they develop conceptual understanding. You need to think about the big idea of the lesson and the enabling and extending prompt and to get children to explore all different possibilities.”

*Links to planning and assessment*

Sam shared how he gets ideas from the Rich Assessment Task Book (Downton et al., 2006) to support his planning and assessment. One PST raised concern about the lack of support as to how they can cater for all students. Sally suggested they should plan enabling and extending prompts and to ensure they know the mathematics and to do the task prior to teaching it. Sam went off on a tangent about planning and said, “Look at the topic and task then unpack the conceptual understanding underpinning it.” Sally said, “Hopefully you will consider building conceptual understanding, language and links when planning and consider students’ self esteem and disposition towards mathematics.”

## Post lesson reflection: Sally and Sam

Following the lesson Annabel met with Sally and Sam to debrief on the lesson and Annabel asked then to share their thoughts on the lesson in terms of key messages and what they might do differently.

Sally indicated how worthwhile it was to start with the brainstorm and the revision of key ideas relating to the middle years. Sam mentioned that Sally reflected more on what the teacher would do and say and that he then drew on things from his experiences and what didn’t work and to learn from that. Excerpts from their reflections indicate Sam’s surprise at some of the PSTs limited MCK and how ingrained the use and reliance on formulas. Sally reflected more on how the PSTs were beginning to think of the learners and the *importance of linking to the classroom and pedagogical practices*.

Sally: The PSTs are starting to think about things through the eyes of their own students, which is why the links to the classroom experiences is so important.

Sam: We are giving them breadth of skills. Seeing PSTs operating and working through problems, affirming their strategies and knowing when to hold back or step in. There are some similarities between their strategies and Year 5/6 students. Also teaching them how to teach. I was not expecting to identify their misconceptions related to area and perimeter or their reliance on the use of formulas.

Sally: Addressing a misconception by getting students to explain what they did and to draw a diagram was important, modelled PCK and the use of teachable moments.

Sam: Big ideas were identified in an inclusive way.

Sally: Making connections back to the lesson planning and then as teachers linking to the pedagogy and modelling the language.

Sam: Evidence of shared philosophy in the example about learning intentions.

Sally: Next week we need to use more wait time and hold back before sharing.

## Commentary

It was evident throughout the lesson observation of a synergy between Sally and Sam that assisted in developing a *community of practice* (Wenger, 1998), in which PSTs were willing to discuss and share their learning and be challenged, in order to gain knowledge and skills. In these sharing situations Sally and Sam *guided and extended the mathematics* and in some instances challenged the PSTs’ own MCK (e.g., misconception of surface area). There were several examples in the lesson of the co-teachers *eliciting students’ thinking* and then assisting the PSTs to make connections between mathematics concepts. One particular example related to finding the volume of a cube and then Sally extended this to finding the surface area. Throughout the lesson Sally saw opportunities to make the *pedagogy explicit* such as when Sam said, “turn and talk to a friend”, is an example of ‘talk moves’ (Kazemi & Hintz, 2014), and addressing the misconception as a ‘teachable moment’, which links to *dealing with contingencies* (Rowland et al., 2009). Sam often extended on Sally’s probing by making connections to the classroom and his experiences with Year1/2 students, then he challenged the PSTs to consider how they might respond. Both co-teachers highlighted the importance of lesson planning and the PSTs knowing the mathematics(links to *foundation knowledge*), the key questions to ask and the mathematical language in order to deepen the students’ experiences and build conceptual understanding. Sally also drew their attention to the importance of being aware of the students’ disposition when planning. The PSTs found Sam’s sharing of his lesson plans beneficial as they promoted lots of discussion and this was clearly an area in which the PSTs needed further support. Some of the PSTs’ post class reflections indicated that having co-teachers assisted their learning. For example, one PST reflected, “Sally and Sam helped develop my measurement understanding of the difference between volume and surface area by their questioning and explanations and I realised the importance of making models.” This was an example of how the co-teachers *guiding the mathematics* (Staples & King, 2017), *elicited student thinking* and assisted them to *make connections between concepts.* In their post lesson reflection both Sally and Sam reflected on how they built on each other’s ideas and highlighted the importance of making connections back to the classroom experiences and planning. Sam’s comment about giving the PSTs breadth and depth links to the *foundation dimension* of the KQ (Rowland et al., 2009).

After co-teaching with Sam, Sally provided the following summary of their experiences including detail of their planning and preparation for teaching the PSTs.

## Sally’s reflection

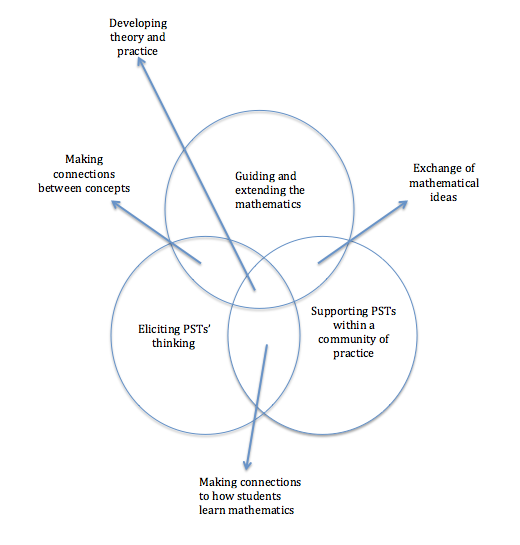
Many years ago, I was a primary school teacher and often taught alongside other teachers, team teaching a large class of students. I have also worked as a numeracy consultant mentoring teachers with their planning and observing or teaching with them in their classrooms. I also draw on these experiences and research knowledge when planning my teaching in the tertiary classroom with PSTs. These experiences also prepared me when considering my approach to the co-teaching partnership and the mathematical discourse for facilitating meaning learning for the PSTS learning to teach primary mathematics.

When teaching with Sam we chose many of the activities that I would normally use. However, Sam was able to contribute further to the lessons through justifying the year level students might complete the tasks they explored, and/or by sharing student artefacts from his primary classroom such as work samples or photos. The artefacts helped the PSTs to visualise how primary students learn in the classroom whilst also making connections to the discussion we had related to the curriculum content and pedagogical approaches for teaching mathematics.

Each week before teaching we would plan together. A power point was used to guide our teaching and as we selected which slides to use we were able to unpack the key mathematical ideas as we justified why we chose or did not choose different activities to share with the PSTs in their tertiary classroom. Sam was always eager to help and contribute. We also met each week after class to debrief about what we had done, always considering how we might improve the following lessons. Looking back, these interactions provide an opportunity to reflect and prepare for our co-teaching to ensure we maximised the experiences we offered to the PSTs. These experiences also provided an opportunity for both of us to develop new and shared understandings that were important when we co-taught.

## Refined model

From the analysis of the data using both the dimensions of the KQ (Rowland et al., 2009) and the adapted three-function model (Staples & King, 2017) we derived the relationship between the intersections of the three quadrants, which showed a relationship between all the frameworks and practices used. These are presented in Figure 3.



*Figure 3.* An interconnected model to assist PSTs to link theory and practice

1. *Guiding and extending the mathematics:* Links to Foundation knowledge (KQ), beliefs, PCK and MCK, transformation (KQ), and the 5 Practices
2. *Exchange of mathematical ideas:* Links to guiding the mathematics and supporting a community of practice (Staples & King, 2017).
3. *Supporting PSTs within a community of practice:* Links to talk moves, and supporting PSTs’ exchange in a community of practice.
4. *Making connections to how students learn mathematics*: Links to making connections to how children learn in the classroom, the affective domain and how we believe our PSTs should learn.
5. *Eliciting PSTs thinking:* Links to making connections (KQ), and dealing with contingencies (KQ)
6. *Making connections between concepts*: Links to talk moves, guiding and extending the mathematics, making connections between mathematical concepts.

# Discussion and Conclusions

Evidence presented in the vignettes, reflections and commentaries highlight the dynamics between the co-teachers and the rapport they developed with the PSTs to create a community of practice. Within such a community the PSTs were required to explain and justify their thinking, and communicate with and respond to the views put forward by others. As Enfield and Stasz (2011) indicated, a community of practice helps PSTs to reflect on their learning during the process of having to share their thinking. Others (e.g., Anthony et al., 2015) argue that “attending, interpreting, and responding appropriately to students’ mathematical thinking is a specialised pedagogical skill that needs to be explicitly taught within teacher education courses” (p. 8). For these reasons, the co-teachers made a conscious decision to focus on developing meaningful discourse within their tertiary classroom. In their planning and enactment each week they drew on the five practices for orchestrating productive mathematical discussion (Smith & Stein, 2011), the talk moves that support classroom discourse, the dimensions of the KQ (Rowland et al., 2009) and the three function model to facilitate meaningful mathematics discourse (Staples & King, 2017).

The co-teaching partnership also assisted the MTE to reflect on her practice because of the discourse generated by her co-teacher. Although her values for teaching and underpinning theoretical identity related to current research, her thinking about how PSTs learn was extended as part of the co-teaching partnership: “When Sam was talking, it gave me time to think and consider other ideas, sometimes deviating from the planned activities in a good way… endless sharing of ideas and examples of artefacts that guided the discussion … the PSTs really valued his input, his words were golden in their eyes … bringing the primary classroom to the tertiary classroom.” Similarly, Enfield and Stasz (2011) suggested that a community of practice provided unplanned experiences including opportunities for all to reflect on their practice and value critical discussion with PSTs.

Having a practicing primary school teacher as one of the co-teachers provided that direct link to the classroom and assisted the PSTs to make connections between the theory and practice of mathematics teaching. While other scholars have used videos (e.g., Beswick & Muir, 2013), co-taught in classroom (e.g., Anthony et al., 2015), used representations of practice (e.g., Livy et al., 2017), or had PSTs observe their MTE teach in a primary classroom (Livy & Downton, 2017), some were a one-off experience rather over a sustained over an extended period of time. Having a classroom teacher working with the MTE in the PSTs’ course on a weekly basis that involves them being engaged in discourse that at times challenges their MCK and PCK enables them to see that an essential part of being an effective teacher of mathematics is having breadth and depth of mathematical understanding (Ma, 1999) and specialised pedagogical skills.

Drawing on the dimensions of the KQ (Rowland et al., 2009) and the key functions model (Staples & King, 2017) provided a structure for the co-teachers to focus on both deepening the PSTs’ foundation knowledge and pedagogical practices when planning. An important element of the planning was Sam’s preparation prior to each week’s tutorial. He engaged his Year 1/2 students in tasks related to the topic being covered in the following week’s tutorial and brought artefacts to share with the PSTs (e.g., photos, work samples, reflections) and his weekly lesson plans. In doing so, he maintained a strong link to the classroom each week and assisted the PSTs build the connections between theory and practice.

In order to develop the links between theory and practice we argue that there needs to be a combination of the three functions of guiding and extending the mathematics, eliciting PSTs’ thinking, which includes assisting them to make connections and deal with contingencies, and supporting their exchanges within a community of practice. Within this framework and structure to develop meaningful mathematics discourse, are the use of talk moves, the five practices (Smith & Stein, 2011) and the dimensions of the KQ (Rowland et al., 2009).

In summary, the initiative of the co-teaching approach assisted with developing the teacher educator’s knowledge for teaching PSTs and assisted the PSTS to make deeper connections with theory and practice. It is hoped that the findings will deepen MTEs’ understanding of the need to make explicit connections between theory and practice in their courses and identify how a co-teaching model assisted to bridge this divide and better equip the PSTs with effective pedagogical practices for teaching mathematics. Using a co-teaching model can also encourage MTEs to critical reflect on their practice and to build strong relationships with those in the profession.

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