Facilitation Practices in Mathematics Teacher Education and the Mathematical Identities of Preservice Elementary Teachers¹

Lynsey Gibbons Boston University Ziv Feldman Boston University

Suzanne Chapin Boston University

Lisa Nguyen Batista Boston University Rachel Starks Boston University

Melissa Vazquez-Aguilar Boston University

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Supporting preservice teachers in their development of positive mathematical identities is important because of their future responsibility as teachers. In this self-study, we investigated a mathematics teacher educator's facilitation practices during discussions to examine opportunities for preservice teachers to develop productive mathematical identities. We analysed over 35 hours of video recordings of a mathematics content course for future elementary teachers and identified four key mathematics teacher educator facilitation practices that appeared to support preservice teachers to develop productive mathematical identities: emphasizing reasoning, promoting broader engagement, shifting responsibility for learning, and developing a supportive classroom community. We also analysed survey data of our preservice teachers' perceptions of their mathematical agency and authority. Implications for mathematics teacher educators are discussed.

Keywords Mathematics Teacher Educator, Preservice Teachers, Identity, Agency, Authority, Mathematics Content Courses

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Introduction

Research on mathematics teacher education indicates that well-prepared beginning teachers of mathematics should strongly believe that each and every student can learn mathematics with understanding, and they should take conscious and intentional action to build students' agency as mathematical learners (Association of Mathematics Teacher Educators (AMTE), 2017; Gutie rrez, 2009). Yet, it is difficult to support K-12 students' agency when, often, preservice teachers have not had the chance to develop productive mathematical identities themselves in their prior mathematics experiences. Therefore, recent recommendations about what should be emphasized in courses to prepare future mathematics teachers includes providing opportunities for them to develop their mathematical identities (AMTE, 2017).

Here, we report the analysis of a self-study research project in which we asked: What are the facilitation practices of one mathematics teacher educator who aimed to support preservice teachers' development of mathematical identity? By analysing classroom videotapes, we examined the practices of a mathematics teacher education professor (the third author) in a mathematics content course for preservice elementary teachers to understand how her practices may have provided opportunities for preservice teachers to develop productive mathematical identities. As a result of this analysis, we identified four practices that are used to facilitate mathematical discussions that also have the potential to support preservice teachers' mathematical identity. The facilitation practices are emphasizing reasoning, promoting broader engagement, shifting responsibility for learning, and developing a supportive classroom community. As we will portray, these four practices are not mutually exclusive and considerable overlap occurs during classroom interactions. These findings have implications for mathematics teacher educators who engage preservice teachers in doing mathematics within mathematics content courses, methods courses, or other professional learning settings.

Theoretical Framework

Mathematical Identity

An important goal of our work as mathematics teacher educators is to provide opportunities for preservice teachers to develop productive mathematical identities, which we believe will impact their future work as teachers. We aim to support preservice teachers' sense of themselves as prospective teachers of mathematics, starting with relocating mathematics in a lived activity (Brown & McNamara, 2011). We aim to support them to (re)position themselves in discourses about who they are in relation to school mathematics, and assist them in their emergent and evolving identities first as learners of mathematics and subsequently as teachers of mathematics.

We draw on research on identity in K-12 classrooms to describe what a productive mathematical identity consists of, in order to elaborate on our goal of supporting preservice teachers' development. As we will further elaborate in the methods section, this body of literature also helped us to conceptualize potential ways in which mathematics teacher educators could provide preservice teachers with opportunities to productively (re)position their mathematical identities.

Students, whether in a K-12 or a teacher education setting, enter mathematics courses with mathematical identities, which have been defined as one's knowledge, habits, beliefs, values and commitments as it relates to one's participation within a community of practice (Philipp, 2007). Students' mathematical identities can be impacted as they engage and operate within

mathematical contexts (Aguirre, Mayfield-Ingram, & Martin, 2013; Varelas, Martin, & Kane, 2012).

Two related aspects of one's mathematical identity are one's mathematical agency and authority (Braathe & Solomon, 2015; Childs, 2017; Engle, 2011; Schoenfeld, 2017; Sengupta-Irving, 2016). Mathematical agency involves students' capacity to think of themselves as doers of mathematics who can make progress on challenging issues, trust in the conclusions that they draw, create arguments and explanations, share their own ideas, and extend the ideas of others (Aguirre et al., 2013; Braathe & Solomon, 2015; Gresalfi, Martin, Hand, & Greeno, 2009; Hand, Kirtley, & Matassa, 2015; Oppland-Cordell & Martin, 2015; Schoenfeld, 2017; Turner, 2003). Students' mathematical agency is evident when they assert their power to ask questions, engage actively, make conjectures, support and extend peers' assertions, critique ideas, and author mathematics (Boaler, 2003). Mathematical authority relates to students authoring and producing mathematics through collaboration with their peers (Engle, 2011; Sengupta-Irving, 2016).

In our context, we support preservice teachers who are students within mathematics content courses. Our preservice teachers enter the course with existing mathematical identities that have been developed in their prior experiences with school mathematics. As teacher educators, we grapple with how to attend to the cultivation of preservice teachers' identities through supporting the development of a particular stance towards mathematics that will support their future work as teachers (Kazemi & Wæge, 2015). A big emphasis in our course is the importance of discussion in learning and supporting the development of "math talk learning communities" (Hufferd-Ackles, Fuson, & Sherin, 2004), where the teacher educator and preservice teachers use discourse to support the mathematical learning of all participants. Through facilitating dialogic communities, we aim to provide preservice teachers with opportunities to author and produce mathematics in collaboration with their peers, and develop productive dispositions toward mathematics.

Mathematics Teacher Educator Practices in Supporting Preservice Teachers

The research on effective facilitation of teachers' professional learning is in its infancy (Even, 2008). There is a small, but emerging set of literature that has unpacked facilitation practices of professional development facilitators, and, to a lesser extent, teacher educators who prepare future teachers. The existing research primarily examines how teacher educators in professional development settings support productive, substantive discussions among groups of teachers as they view videos of instruction (Borko, Koellner, & Jacobs, 2014; van Es, Tunney, Goldsmith, & Seago, 2014; Zhang, Lundeberg, Koehler, & Eberhardt, 2011), as they engage in mathematics tasks (Borko et al., 2011; Elliot et al., 2009), or as they engage in classroom-based learning experiences (Gibbons, Fox, Lewis, & Nieman, 2016). In mathematics teacher education, some studies have emerged over the last decade that have examined how teacher educators facilitate rehearsals (Anthony, Hunter, & Hunter, 2015; Averill, Anderson, & Drake, 2015; Ghousseini, 2017; Lampert, Ghousseini, & Beasley, 2015). These studies point to facilitator practices, such as establishing purpose, pressing for explanations and evidence, nurturing shared purpose among participants, stepping back and listening, maintaining focus on the instruction or the mathematics, and monitoring participation--as serving to deepen teacher conversations and promote teacher learning about teaching mathematics.

In our own context, we support future elementary teachers in a university setting. As preservice teachers progress through our program, we ultimately aim to impact their identity as a teacher, which has been defined as, "the constellation of interconnected beliefs and knowledge about subject matter, teaching, and learning as well as personal self-efficacy and orientation toward work and change" (Collopy, 2003, p. 289). However, in this analysis, we are concerned with impacting their identity around the subject matter of mathematics. Therefore, we examined our own practices as teacher educators to identify which practices appeared to support preservice teachers' willingness to share their ideas, ask questions, and build on their peers' thinking, which we conjecture provide preservice teachers with opportunities to (re)position their mathematical identities.

Context

Setting of the Episode

This research study was situated in a larger National Science Foundation funded project at a highly selective university in the northeast of the United States. The overarching goal of the project is to develop, test, and disseminate curriculum materials that strengthen elementary preservice teachers' understanding of mathematics for teaching. This project uses sociocultural theory to frame its initiatives. Represented in the works of Brown, Collins, and Duguid (1989) and Lave and Wenger (1991), sociocultural theories posit that learning occurs through coordinated social activity and is mediated by the use of language and the context in which the learning is situated.

The episode presented here took place in Fall 2017 in a mathematics content course for preservice elementary teachers that used the instructional lessons designed by the *Elementary Mathematics Project*. Preservice teachers engaged in cycles of learning where they worked together in small groups to solve problems and make sense of ideas. They then came together as a class to discuss these ideas, facilitated by the instructor. An analysis of the time allocation of this course showed that preservice teachers spent 54% of the class time during the semester in whole class discussion and 42% of the class time in small groups working with their peers on problems. Approximately 4% of the class time was used for other activities such as watching video clips or reviewing assignments.

This mathematics course is taken by undergraduate students (who are typically in their 2nd year of university coursework, approximate ages are 19-21 years old) who are training to become elementary school teachers. It is the first of a three-course sequence in mathematics education: two courses focus on mathematical content and the third focuses on methods of teaching mathematics to elementary students. The mathematics content of this episode involved whole number concepts, with a focus on number systems that use place value.

The course met twice per week, for one hour and 45 minutes each, for 14 weeks. There were 22 preservice teachers in the course: three males and nineteen females. Sixty-eight percent (68%) were White, 14% Hispanic, 14% Asian, and 4% African-American. Three students grew up speaking a language other than English in their home. Anecdotally, students overwhelmingly reported to us that they did not feel confident in their ability to explain the reasons behind elementary mathematical concepts and procedures when they started the course.

The instructor of this course (third author) has been a teacher educator for 30 years and has taught content courses for preservice elementary teachers throughout her career. She was the primary author of the curriculum materials used in the course and has had extensive experiences writing about and facilitating classroom discussions. The first and second authors have also taught this course during other semesters, and the fourth and fifth authors have served as teaching assistants.

Methodology

Self Study

As a team of mathematics teacher educators, we are constantly asking ourselves to what extent we are supporting our preservice teachers in their development as mathematical learners. Over the course of the *Elementary Mathematics Project*, we have quantitative evidence that our preservice teachers have developed richer mathematical understandings. We also had preliminary anecdotal evidence that the pre-service teachers were displaying more productive mathematical identities. As such, we decided to collectively engage in a self-study to examine the third author's facilitation practices that appeared to have the potential to productively impact our preservice teachers' mathematical identities. Our goal was to better understand aspects of our practice; in particular which practices seem to support our preservice teachers' willingness to share their ideas, ask questions, and build on their peers' thinking. By examining the third author's facilitation practices, we aim to contribute to the mathematics teacher education knowledge base about how to promote preservice teachers' productive mathematical identities (Kemmis & Grundy, 1997; Zeichner & Noffke, 2001). By supporting preservice teachers' own identity, we argue they will be better positioned to foster their future elementary students' mathematical identities.

Data Collection

Video of Instruction

As part of the larger project, the first twenty classes of the semester course were videotaped by a professional film crew, covering approximately 35 hours of class time. Two cameras were positioned to capture various small group interactions, as well as all whole class discussions. During small group discussions, operators moved the cameras to focus on different small groups, meaning that we do not have a record of every preservice teacher's participation in their small group discussion. However, since groups were randomly assigned in each class, the cameras captured a variety of group interactions.

Preservice Teacher Survey

Important to us as teacher educators was the opportunity to hear from our students about their experience in our course. Therefore, at the end of the semester, we electronically administered a 16-item survey to examine preservice teachers' perceptions of the course to better understand how the course may have influenced their identity. Preservice teachers responded to 14 Likert scale items and two open-response questions about their comfort, confidence, habits, and growth regarding the mathematical concepts studied throughout the semester. The open-response questions asked preservice teachers to share their perceptions of how they had, or had not, changed as learners of mathematics and how the course might have influenced their work as future teachers.

Data Analysis

Video of Instruction

Both individually and collectively, we viewed the whole corpus of video clips in order to examine the mathematics teacher educators' facilitation. We examined the moves the facilitator made (e.g., revoicing, restating, or pressing for reasoning) (Chapin, O'Connor & Anderson 2009) and considered to what extent those moves elicited behaviours of the preservice teachers that

prior research had identified as an exhibition of identity, agency, and authority (e.g., ask questions, engage actively, make conjectures, support and extend peers' assertions, critique ideas, and author mathematics). We then collectively classified the moves in order to identify facilitation practices that seemed to provide preservice teachers with opportunities to develop productive mathematical identities: emphasizing reasoning, promoting broader engagement, shifting responsibility for learning, and developing a supportive classroom community. Next, we selected particular video episodes that were highly representative of these four practices and chose one to examine for this self-study. We transcribed the video clip and analysed it to identify when and to what extent the instructor used the four facilitation practices.

Preservice Teacher Survey

In order to understand preservice teachers' perceptions of their experiences in the content course, we examined the survey data by looking at individual responses to understand whether, and to what extent, each preservice teacher felt he or she was comfortable, confident, and capable within the course context and to publically share their ideas, critique the reasoning of others, and build on others' ideas. Furthermore, we analysed pre-service teachers' ' reports of which aspects of the course structure (e.g., working with partners, receiving assistance from the math educator) had contributed to their growth. We looked across their responses to identify trends and notable responses.

Findings

We analysed a discussion that occured during the third class session of the semester. The content of the previous class focused on features of our base ten system using an analogy of packaging chocolate truffles in different size boxes. Preservice teachers had opportunities to talk about the differences between the face value and the place value of digits and how this work together to find the overall value of a digit in a number (e.g., the 7 in 571 has a face value of 7, a place value of ten and an overall value of 70). In the third lesson, the analogy of packaging was extended to the base four system. Students were asked to analyse the fictional B4 Company's rules for boxing and recording truffle orders. The point of this lesson was not to convert numbers from one base to another. Instead, through the analysis of another base, the goal was for preservice teachers to distil important base ten features. Furthermore, by asking them to draw parallels to base ten, the hope was that they would start to generalize key features, such as how the groupings used by a base affect the place values.

The transcript begins 30 minutes into a 105-minute class. During the first 30 minutes of class, preservice teachers worked in small groups with four people on problems such as how to count in base four, or how to package a certain number of truffles using this system. They had base four blocks at their tables, which were used to model the different types of boxes available (e.g., a carton holds 64 units, a flat holds 16 units, a long holds four units and a unit box holds one unit).

Prior to the start of this episode, the teacher educator circulated among the groups, listened to their conversations, and occasionally asked group members for clarification of statements overheard. She asked groups to talk about their conclusions to general questions such as "When does the B4 Company repackage?" or "How does the B4 Company record the number of truffles in an order?" with the expectation that everyone at the table would contribute and share their thoughts. She referred to each preservice teacher by name in order to learn them. Most groups had made a few errors in recording quantities or counting in base four where they use a "4."

She did not explain the error but pointed out to the group that they needed to go back and reexamine certain questions in light of packaging rules.

We begin the episode with the instructor bringing everyone together to discuss what they had done in their small groups. Analysis of this episode is presented in four parts. Each part consists of four sections: 1) a summary of preservice teachers' interactions; 2) the transcript; 3) the instructor's first-person reflections; and 4) the authors' collective analysis of instructional practices.

Part 1: "What are the major characteristics of this B4 packaging system?"

In a class discussion, I (the teacher educator, third author) pose the question, "What are the major characteristics of this B4 packaging system?" This question is designed to be open enough that preservice teachers can approach it in different ways, depending on what features of the system they wish to highlight. Because of where this question comes in the sequence of problems in the lesson, it also tends to elicit comparisons to base ten. I call on a preservice teacher who brings up the idea that this system uses four digits. A number of contributions follow about this idea where I ask each of the respondents to explain their statements. I also briefly restate each contribution to further reinforce the importance of their contributions in helping us understand the concept of regrouping. As expected, some contributions use base ten to help explain base four. I ask for others to add to the conversation to encourage participation.

1 Instructor:	What are the major characteristics of the B4 packaging system? Izzie, start us off.
2 Izzie:	One of the things we noticed is it only uses four digits: 0, 1, 2, and 3.
3 Instructor:	[Instructor writes four digits: 0, 1, 2, 3 on board] How come?
4 Izzie:	Because like if you were to use a four it wouldn't really make sense. Say if you had four units of something or four truffles you wouldn't use four separate boxes, you'd just jump to that one "long" box.
5 Instructor:	You'd trade it in or regroup it into one long box. Anyone want to add anything to this in terms of why we only have those digits being used? Sonja.
6 Sonja:	We compared it to the base ten system, and the base ten system uses ten digits and the base four system uses four digits so the number of digits is based on whatever the base is.
7 Instructor:	The number of digits is based on the base system. How come?
8 Sonja:	Because it is called the base of that number. So, when you are increasing in increments by the place value you have to go up by whatever that place value.
9 Instructor:	So, when I get four of something [such as four unit boxes, four long boxes] in base four what must I do? Tina.
10 Tina:	You have to move it up to the next place value since it would be kind of redundant to write it that way. Like we wouldn't write in our number system trying to fit a ten into the ones place, instead we'd move it into the tens place.
11 Instructor:	So, you have to move it into the next place since you have to repackage or regroup it. [points to the statement on board about 4 digits]. What else can you tell me about this base four system?

Instructor's reflections

As this is early in the semester, I do not yet know these students well. I want them to have positive experiences in grappling with the mathematics, but am aware that many are apprehensive about the course. I am trying to be encouraging without giving any indication of one contribution being "better" than another. At the same time, I want to make it clear that they must provide reasons and explanations. From my immediate prior interactions with the small groups, I think many preservice teachers are making sense of the role of digits in recording regrouping, which is one feature of a place value system. But I also suspect that many are confused. Furthermore, these preservice teachers may not have had mathematical experiences in which their role is to share their thinking as the class collectively works toward some generalizations. They may have never been involved in a discussion about mathematics that lasts longer than a minute or two. I do not want them to think the conversation is over or to assume that I will answer the question myself. So I rephrase my initial question and pose it again ("What else can you tell me about this base four system?").

Analysis of instructional practices

In this short interaction, the instructor leverages three instructional practices in order to support her preservice teachers' developing identity: emphasis on reasoning, promoting broader engagement, and developing a supportive classroom community. By asking 'why' or 'how come' (turns 3, 5, and 7), the instructor conveys the importance of sharing and making sense of one's mathematical ideas (e.g., the idea of regrouping). When she restates and/or builds on a preservice teachers' comment (turns 5, 7, and 11), the instructor validates the previous speaker's contribution and provides an additional opportunity for the class to collectively reflect on why the base 4 system only has four digits. In turn 5, the instructor also asks the class to add on to this discussion. The instructor is signalling that all members of the class are expected to participate in order for everyone to work together to make sense of ideas. In this short interaction, the instructor has supported three preservice teachers to share their ideas publicly about regrouping in a place value system by providing opportunities to reason and reflect on their peers' thinking.

Part 2: Emilia Presents her Idea

By restating the question, "What else can you tell me about this base four system?", I signal that we are not done talking about this topic and invite further participation from the class. Next, Emilia volunteers to share: "After a lot of discussion [in her group], the way you calculate the place value of each digit – do you mind if I come up to the board?" In the last class session, I requested that numerous students come to the board to present to their peers but this is the first time during this semester that a student initiates this. Emilia writes 432 _{B4} on the board (see Figure 1). It appears that she is going to explain how to determine what this base 4 numeral represents in terms of base ten (or number of truffles). Emilia begins by falteringly explaining how many truffles the four represents. She references the physical base four blocks that preservice teachers have been using ("cartons", "flats", "longs", and "units").

13 Emilia: And you're doing the B4 place value system, you would find the place value, by doing, four, hund-- no! no, no [Writes 4 then scribbles it, writes a 4 again under the 432].
14 Class: Laughter.

Laug

15 Emilia:	I was there! [signalling that she and her group had made sense of			
	notation moments ago]. I promise! Okay. [Erases her scribbles.] Oh! Oh!			
	Okay, four times [Laughs] I'm sorry, sixteen [Writes 4 × 16 under 432 with			
	an arrow pointing to the digit 4 in 432], because the face value of that would			
	be, it is the, I don't know how to describe it. [Turns to look at class]			
16 Class:	Flat [Referring to the base four blocks]			
17 Emilia:	The six, the flat, the sixteen.			
18 Alice:	No, it's the long.			
19 Emilia:	No, no, no. The flat.			
20 Class:	Laughter.			
21 Emilia:	The flat place, right? So flat. So, the flat is sixteen units [Above the 432 draws another arrow and writes, "flat" and "16 units"] which would be four times sixteen [Draws an arrow to 4 × 16], which is sixty [Writes 64], no, that's not it [Scribbles out 64]. Nope, I'm right, sixty-four. [Writes 64 below her			
	scribbles and circles it]			
	Flat			



Figure 1. Emilia's work, written on the classroom whiteboard.

22 Class:	Laughter
23 Emilia:	[She takes an audible deep breath] I did it.
24 Class:	Laughter

Instructor's reflections

Emilia's body language and rambling seem to be manifestations of her nervousness and her hesitation with the mathematics. She is not confident about what she is sharing. However, the class is generous in spirit toward her – while there is laughter, it is with her, not at her. During this presentation, I consciously decided not to intervene and correct Emilia's error in using a four in her notation, because, from past experience, I know that errors are often identified by others in the class. In fact, I purposefully move to the side of the room as a way to begin shifting the authority over the mathematics to Emilia and the class. If I were to take over, I would send the class a message that my job as the instructor is to correct their mistakes. By remaining quiet, I set an expectation that it is their responsibility to make sense of the mathematics with one another. Furthermore, 15 minutes earlier, during small group work, I overheard Emilia's group discussing the digits in base four. It appeared in that small group conversation that Emilia had

understood why 40_{B4} does not come after 33_{B4} when counting, so I considered that she might recognize the error of using a four in her notation herself.

In this moment, I recalled that during the first class, we discussed how we would work together during the semester to understand the mathematics, including when we make errors. I mentioned that some people find it unsettling or embarrassing to make mistakes publicly, so we will need to be kind and supportive of each other. This being the first public error, I was unsure of how the class would react.

Analysis of instructional practices

During this short segment, there is a *shift in responsibility for learning*, as the instructor does not facilitate the discussion and physically moves to the side of the room. First, she allows Emilia to take over the facilitation role as she comes to the board to present her thinking. Second, the instructor physically positions herself in a way that the class's focus is now on Emilia. Third, she does not interject when it is clear that Emilia has made an error and is struggling to articulate her thinking. All three decisions are made with the goal of having the class, not the instructor, make sense of how place value and face value interact to determine the value of a number in base four. The instructor sends the implicit message that it is the responsibility of the class to make sense of these ideas and the instructor will not necessarily intervene when they struggle.

Part 3: Addressing Emilia's Error

After Emilia's last remark, a number of students raise their hands. I call on Alvin but also indicate to Emilia (who is moving to her seat) to stay at the board. As shown below, four preservice teachers spontaneously join the conversation, while I only interject to encourage the class to help each other.

25 Alvin:	I mean, the math is right. But since we are using base four, you actually just		
	use the carton. You wouldn't, right?		
26 Olivia:	You wouldn't use, you couldn't use the four there [refers to 432B4 on the		
	board].		
27 Alvin:	Right, yeah.		
28 Olivia:	Yeah, there would be no four.		
29 Alvin:	It wouldn't be four flats.		
30 Emilia:	Oh man. It wouldn't?		
31 Class:	Laughter.		
32 Instructor:	So, help her out. Help her out here.		
33 Mary:	I have a question, I'm, I'm a little confused. How would, I don't get why		
	she's multiplying. Like I don't get, I don't, I'm not really understanding the		
	whole multiplication.		
24 Instructor	Emilia you're on Coll [on] noonlo		

34 Instructor: Emilia, you're on. Call [on] people.

Instructor's reflections

I tell Emilia to remain at the board because I want her to take ownership of the discussion, as it addresses her board work. However, when I notice Emilia's confusion growing, I push the class to help her (turn 32). I know this topic is challenging for preservice teachers, so I am patient and encourage the class to resolve the confusion. Mary's question in turn 33 confirms for me (and, I hope, for the class) that Emilia's confusion is shared by others. I notice several hands go up, but I want Emilia to maintain her role as the facilitator of this conversation, so I tell her to call on other people (turn 34). Had I started calling on people, I would resume my role as facilitator which would conflict with my goal of shifting responsibility to preservice teachers to take authority over the mathematical conversation.

Analysis of instructional practices

Two important facilitation practices are leveraged during this part of the exchange: *developing a supportive classroom community* and *shifting the responsibility for learning onto preservice teachers*. In turn 32, the instructor encourages the class to help Emilia with the mathematics. Emilia is at the board vocalizing her confusion (turn 30), and by saying, "Help her out," the instructor is indicating that there is a struggle that needs to be addressed by the entire class. Not only does this move support a sense of community that is beginning to develop in this class, but also shifts the responsibility for making sense of the underlying mathematics onto the preservice teachers. Interestingly, Mary quickly reveals her own confusion in turn 33, which might help Emilia recognize that this topic is complex and that she is not the only one who does not fully understand. By directing Emilia to call on others (turn 34), the instructor once again shifts the responsibility for helping resolve Mary's question onto the class. Both practices are meant to help strengthen Emilia's (and others') agency by signalling that everyone has a voice in the mathematical discussion, and equally importantly, that the instructor will not take over the mathematical thinking.

Part 4: Resolving Emilia's Error

Having shifted the facilitation to Emilia, I sit down to further emphasize that the class is in charge, and my role shifts to be a participating active listener. Emilia calls on Jose, who was part of her small group that day. Later on, another preservice teacher, Olivia, joins Emilia at the board, and the class is following their conversation closely. There is quiet chatter throughout as preservice teachers provide verbal support to their peers or agree with Olivia's explanation.

35 Emilia:	Jose.		
36 Jose: So, for the same reason that in the base-ten system, that when y			
	twent-, when you have nineteen and then you go up to twenty, you don't		
	have [pause]. I mean, it's difficult to explain. So when you get, when you get		
	to a nine in base ten, right? So you go one, two, three, four, five, six, seve		
	eight, nine, what do you do instead of just using ten in the ones place? You		
	use one in the tens place.		
37 Mary:	And zero in the ones?		
38 Jose:	Yeah.		
39 Mary:	Cause you have ten?		
40 Jose:	Yeah, exactly.		
41 Mary:	But, where, like, what does she do with the sixty-four? I don't, I don't really		
	understand the sixty-four. Sorry.		
42 Olivia:	Can I go to the board?		
43 Emilia:	Yeah.		
44 Olivia:	[Olivia goes to board] Like, I'm just going to do a complete example of what		
	you just did.		
45 Emilia:	Okay.		
46 Olivia:	So, like, if you did like, one two one, [Writes 121 _{B4} on board; see Figure 2];		
	this is an example from the packet.		
47 Emilia:	Okay.		

48 Olivia: So, the number of truffles was twenty-five. And how you would get that? [Writes = 25 truffles] So you would have to do, one times sixteen, [Draws arrow from the first 1 and writes 1 × 16 underneath it] cause this, [indicates the place value that the 1 is in by moving hand up and down over that place value] this represents the sixteen box.

49 Emilia: [Nods head]. Yeah.

50 Olivia: And then you would have to do two times four, [Draws an arrow from 2 and writes 2 × 4 underneath it] because this [indicates the place value by moving hand over the 2] represents the four box, and one times one [Draws an arrow from 1 and writes 1 × 1] cause it [Points to the place value she is referencing] represents the unit. So, you would get sixteen plus eight plus one which equals twenty-five. [Writes 16 + 8 + 1 = 25]



Figure 2. Olivia's work, written on the classroom whiteboard.

51 Pam:	Oh, I get what you are saying.
52 Emilia:	Wait, so, then what did I do wrong here? [points to her work] Because-
53 Olivia:	If you have a four here [points to the 4 in 432]
54 Emilia:	Yeah.
55 Olivia:	Then you would have to move it up to the next place.
56 Class:	To the carton.
57 Emilia:	Oh! There would be none. [Points to the 16 place value]
58 Olivia:	It would be zero and then you would put a one at the carton.
59 Emilia:	Okay, okay.
60 Olivia:	So, you had it right, you just didn't trade your fours. [pause]
61 Instructor:	Have we addressed Mary's question? Mary, have we answered, do you feel
	like your question has been answered?
62 Mary:	Relatively, yes. I understand how she gets the 25 truffles from there [points
,	at board]. I just was very confused why Emilia was multiplying before, but
	now I understand if you move it over to the other side, it makes more sense.
	If you switch it to the carton. Yeah, it makes more sense now.
63: Instructor:	Well done. [Claps with class joining her.]

Instructor's reflections

My decision to sit down and tell Emilia to call on people seems to have helped the class engage in making sense of Emilia's thinking. Although Jose, Emilia, and Olivia are the primary speakers in this interaction, the rest of the class is following along and reacting verbally. When Olivia provides an explanation to help Emilia understand why she needed to move to another size box in the 432 $_{\rm B4}$, Emilia confirms that she understands her error. I also go back to Mary and check

to see if her question was actually answered in order to further emphasize the need for all to understand. When I stand up, say, "Well done," and start clapping (as does the rest of the class), I want the class as a whole to acknowledge the good work that just occurred. I also hope that they will start to internalize that interactions with each other are normal occurrences in this class. Content-wise, it appears the class is beginning to see the need to repackage into different size boxes when particular values are reached, but I worry that the analogy of boxes is overshadowing the connection to the value of the places. Thus, I return to the front of the room to continue facilitating the discussion in order to highlight the place values and the relationships among these values in both base four and base ten. This subsequent discussion continues for approximately 13 minutes.

Analysis of instructional practices

The final part of the episode illustrates three instructional practices: *emphasizing reasoning*, *shifting responsibility for learning*, and *developing a supportive classroom community*. Although the instructor does not speak until the end of this conversation, the need to provide mathematical reasoning appears to have been internalized by her preservice teachers, as they do so without prompting from the instructor. For example, in turns 36-40, Jose and Mary volunteer reasoning to help explain why base four does not have the digit 4. And, in turns 48 and 50, Olivia provides Emilia with reasoning as to why 121 $_{B4}$ represents 25 truffles. Relatedly, by not speaking during most of the conversation, the instructor continues to shift the responsibility for learning onto her preservice teachers, as they work to make sense of Emilia's error and Jose's and Olivia's explanations. An example of this shift occurs at turn 42 when Olivia asks Emilia if she can come to the board. And when Emilia has a question in turn 52, she addresses the question to Olivia. In both instances, a preservice teacher, not the instructor, is seen as an authority figure.

The instructor also fosters a supportive classroom community by relinquishing her role of facilitator and through specific actions. At multiple points during this part of the episode, preservice teachers are supporting one another to make sense of the mathematics in respectful ways. In turn 36, Jose displays supportive behaviour by attempting to lead Emilia through his thinking about regrouping. Similarly, Olivia feels compelled to help Emilia make sense of her error by assisting her at the board (turn 42). Meanwhile, the rest of the class is attentive and supportive during this conversation (turns 51 and 56). In turn 61, the instructor follows up with Mary to make sure that her previous question has been addressed, signalling that the classroom community has a responsibility to help its members. And lastly, in turn 63, the instructor complements Olivia's and Emilia's work at the board to signify that their collective work is valuable.

Preservice Teachers' Perceptions of their Mathematical Identities

In our transcript analysis, we focused on the facilitation practices of an instructor and the interactions with her preservice teachers in an effort to better understand how the facilitation practices might provide opportunities for preservice teachers to develop productive mathematical identities. As seen in the transcript, many of the preservice teachers willingly engaged in the discipline; there were numerous examples of how they created mathematical arguments and explanations, offered their own interpretations or ideas, and built on others' ideas. (Braathe & Solomon, 2015; Gresalfi, Martin, Hand, & Greeno, 2009; Turner, 2003). They asserted their authority to ask questions and persisted in helping their peers make sense of an error.

We do not know if the identities of these preservice teachers were impacted by the instructor's facilitation practices by observing the classroom interactions. But, we conjecture that these types of interactions, which occurred throughout the semester, positively impacted preservice teachers' mathematical identities. However, we have no way of knowing their perceptions of themselves as doers of mathematics in those moments. We wanted to hear from our students about how their mathematical identity might have shifted or developed over the course. We therefore examined their responses to survey questions regarding mathematical identity administered at the end of the semester. Twenty-one out of 22 preservice teachers responded.

Table 1

	At the beginning of this semester, I had confidence in my ability to explain the reasons behind mathematical concepts and procedures.	Now at the end of the semester, I have confidence in my ability to explain the reasons behind mathematical concepts and procedures.
Always	0%	57%
Usually	14%	33%
Sometimes	47%	10%
Rarely	29%	0%
Never	10%	0%

Preservice Teachers' Change in Confidence from Beginning to End of Semester

The results, as a whole, showed a noticeable shift in preservice teachers' perceived confidence in their ability to engage in mathematical reasoning and sensemaking. As seen in Table 1, none of the respondents claimed to always have confidence in this ability at the beginning of the course, and only 14% felt that they usually were able to explain the reasoning behind mathematical concepts and procedures. Ninety-five percent of preservice teachers reported a shift in their confidence to explain the reasons behind mathematical concepts and procedures, with 90% of the preservice teachers selecting that they always or usually felt confident. Further, 71% of the preservice teachers indicated having grown in confidence by two or more points on the Likert scale (i.e., from rarely to usually, or sometimes to always). The following statement from one anonymous preservice teacher was similar to comments other preservice teachers made: "Overall [a] good class that was very demanding but also really changed my view of myself as a math learner, which makes me more confident and competent as a future teacher."

Emilia, a key participant in the episode described earlier, exhibited some struggle as she presented her thinking. This episode, however, did not stop her from continuing to share her thinking throughout the rest of the semester, even when she was challenged by particular ideas. Her survey results reinforced her fragile mathematical identity as she chose "rarely" to describe her confidence in her ability to reason about concepts and procedures at the beginning of the semester. At the end of the semester, she rated herself as "sometimes" having confidence in being able to explain the reasons behind concepts and procedures. This change indicates a moderate increase in her confidence regarding her ability to explain her thoughts. On a survey question that asked preservice teachers to rate the frequency with which they agreed with the following statement, "When I feel stuck on a concept I am able to use available resources to eventually grasp it," she chose "always." This indicates that while her confidence in her role as

a mathematical authority may not have changed dramatically, her confidence in her ability to make sense of challenging mathematics was high. Furthermore, two months after completing the course, Emilia wrote a blog on our university's website describing how important this course was to her and was "one of the best experiences" she has had as a preservice teacher. Her mathematical identity appeared to be evolving.

Discussion and Conclusions

The purpose of the study that we have reported was to identify mathematics teacher education facilitation practices that appeared to provide preservice teachers with opportunities to develop their mathematical identity and (re)position themselves as mathematical thinkers (Brown & McNamara, 2011). We identified four facilitation practices: emphasizing reasoning, promoting broader engagement, shifting responsibility for learning, and developing a supportive classroom community (see Table 2). We acknowledge that the use of specific instructional practices may be serving multiple goals, but our analysis focused primarily on those goals that seemed to support opportunities for preservice teachers to develop productive mathematical identities.

Emphasizing reasoning promoted preservice teachers to reason through their ideas and others' ideas, with the goal of supporting their willingness to engage with the mathematics and develop a belief that they can make progress on challenging issues and trust the conclusions they draw (Schoenfeld, 2017). By *promoting broader engagement*, the mathematics teacher educator invited preservice teachers to grapple with the mathematical ideas publicly. Through participating in a discussion, whether in small or whole group, preservice teachers have opportunities to reveal what they do or do not understand about a mathematical idea (Boaler, 2008; Sengupta-Irving, 2016). Through these interactions and attempts to vocalize their ideas, preservice teachers can better assess the quality of their mathematical understanding, and the mathematics. Public participation enables the mathematics teacher educator and others to provide that support. As Barron (2000) notes, collaboration on problems enables deep disciplinary engagement while also promoting individual agency.

The mathematics teacher educator aimed *to shift the responsibility for learning* to preservice teachers by pushing them to work through mathematical concepts with the shared understanding that it is their obligation to support one another's learning. The mathematics teacher educator did this in several ways, including encouraging them to facilitate their own mathematics discussion, refusing to provide the class with answers, and pressing them to provide evidence for their statements or reasoning related to their claims. Notice that through this practice of shifting the responsibility for learning, the mathematics teacher educator is also emphasizing the practice of highlighting reasoning in the discipline. The mathematics teacher educator developed a *supportive classroom community* where preservice teachers could share their ideas, even if partially formed, and critiqued the ideas of others. We conjecture that they were able to do this because they felt safe and respected by their peers and the mathematics educator.

Our findings have implications for mathematics teacher educators who prepare future teachers, and those who support mathematics teacher educators' learning. In our analysis, we decomposed the facilitation practices of a mathematics teacher educator in order to specify the work of changing preservice teachers' identities (Grossman et al., 2009). However, this work is complex, interactive, and interconnected (Lampert & Graziani, 2009), as was shown in the episode. We recognize the tension between how these practices interact in complex ways, with

the need to decompose these practices into moves in order to name and understand them. Decomposing and naming the facilitation practices is a step toward identifying instructional practices important in mathematics teacher education and can be learned by other teacher educators (Ball & Forzani, 2009).

Further research on the interplay between a mathematics teacher educator's facilitation practices and preservice teachers' development of productive mathematical identities as doers and teachers of mathematics is needed. We need to better understand the identities that preservice teachers enter their mathematics teaching preparation courses and *how* each of their identities shift over time as they participate in the course. Moreover, further research could examine how identity develops in different populations of preservice teachers, including examining the differences between generalists and specialists, as well as different ethnic and cultural backgrounds. Future research could longitudinally investigate the changes of preservice teachers' identities and the impact of that change on their work with young children as they become classroom teachers. Additional research can help us understand ways in which mathematics teacher educators can foster positive, productive interplay between individual and collective mathematical agency within preservice teacher preparation courses. In this analysis, we studied an expert mathematics teacher educator's practice. Future research can focus on how novice mathematics teacher educators develop such practices and understand how to combine practices in a variety of ways to productively impact preservice teachers' identities.

Table 2.

Summary of Mathematics Teacher Educator Facilitation Practices that Appear to Provide Preservice Teachers with Opportunities to Develop Productive Mathematical Identities

Facilitation Practices	Descriptions	Potential Changes in Preservice Teachers' Identity	Facilitation Moves or Examples
Emphasizing Reasoning	Promotes preservice teachers to reason about mathematical ideas	 Willingness to engage with mathematics Beliefs about the role of sense making Confidence in one's ability to grasp mathematical concepts and procedures 	 Instructor writes 4 digits: 0, 1, 2, 3 on board and asks, "How come?" (turn 3) "You'd trade it in or regroup it into 1 long box. Anyone want to add anything to this in terms of why we only have those digits being used? Sonja." (turn 5) "The number of digits is based on the base system. How come?" (turn 7)
Promoting Broader Engagement	Invites preservice teachers to engage with mathematical ideas publicly	 Contributions to class activities and discussions Work with others Abilities to extend peer's thinking and assertions 	 "So, you have to move it into the next place since you have to repackage or regroup it [looking out at the class, the mathematics educator asks,] What else can you tell me about this base four system?" (turn 11) Asking for additional voices or participation in the conversation.
Shifting Responsibility for Learning	Asks preservice teachers to figure out mathematical concepts and procedures	• Ability to explain one's ideas and understandings	 "Emilia, you're on. Call [on] people." (turn 34) Mathematics educator moves to the side of the room

		Willingness to collectively work on understanding mathematicsBecome a doer of mathematics	• Mathematics educator does not immediately correct preservice teachers' errors
		• Become an author of mathematics	
Developing a Supportive Classroom Community	Creates a community where preservice teachers are willing to share ideas	 Willingness to ask classmate for assistance Share confusion or errors 	• "Have we addressed Mary's question? Mary, have answered – do you feel like your question has been answered?" (turn 61)
		Encourage classmates	• Mathematics educator encourages the class to help Emilia with the explanation
			• Mathematics advice to real merula dass the second suffer

* * * * * * * * *

• Mathematics educator acknowledges the complexity of the content and when the class does good work (e.g., by clapping)

• Mathematics educator sets norms around the importance of making mistakes public and being supportive of one another

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Authors

Lynsey Gibbons 2 Silber Way, Boston, Massachusetts, 02215, USA email: lgibbons@bu.edu

Ziv Feldman 2 Silber Way, Boston, Massachusetts, 02215, USA email: zfeld@bu.edu

Suzanne Chapin 2 Silber Way, Boston, Massachusetts, 02215, USA email: schapin@bu.edu

Lisa Nguyen Batista 2 Silber Way, Boston, Massachusetts, 02215, USA email: Inguyen5@bu.edu

Rachel Starks 2 Silber Way, Boston, Massachusetts, 02215, USA email: rnstarks@bu.edu

Melissa Vazquez-Aguilar 2 Silber Way, Boston, Massachusetts, 02215, USA email: melvaz@bu.edu