Editorial

Scales, rubrics and frameworks: How we describe the complexity of mathematics teacher education

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This second edition of Mathematics Teacher Education and Development for 2017 presents eight articles from around the world addressing aspects of mathematics teacher education that are persistent concerns for researchers and practitioners in mathematics education. At their core are the beliefs, attitudes, knowledge and competencies of mathematics teachers and how these factors impact teachers' instructional behaviours and decision making. In particular, two themes emerge: the complexities of effectively teaching through non-routine problem solving and subsequent class dialogue, and the role that the knowledge and identity of the teacher plays in their instructional choices and orientation to teaching mathematics. These two themes, while tackled in separate papers, intersect in our teachers when they work with their students. How teachers see themselves, and their levels of knowledge and confidence, affect their planning, enactment, evaluation and modification of their mathematics teaching. The papers in this edition of Mathematics Teacher Education and Development consider the potential impacts of teachers' identity, attitudes, beliefs, knowledge of algebra, ability to conduct productive discussions, and ability to develop mathematical tasks on students' learning of mathematics.

Taken together, these papers give a strong impression of the complexity inherent in the teaching and learning of mathematics, and, in particular, the complexity of mathematics teacher education. Educating mathematics teachers adds an extra 'layer' or 'level' of complexity to understanding mathematics teaching and learning, as it concerns itself with how to improve teaching in ways that will impact learners. Teacher education stands at one remove from the everyday teacher-student dynamic, hoping to influence the choices and actions of teachers as they work with students. To deal with this complexity, researchers turn to scales, rubrics and frameworks to help them categorise and describe behaviours and phenomena of interest.

Describing the myriad of choices and actions of teachers observed in a classroom, or present in a piece of written work, presents a challenge for researchers. How can we coherently and concisely describe what we see in the data in a way that is rigorous, replicable and makes sense to a naïve reader? Journal article word lengths restrict our ability to explain all the nuances of our discussion as we coded the data or the subtleties of the choices we have made. Using frameworks from previous work helps, as readers can seek additional information elsewhere and we are then working together to develop common ways of seeing and more comparable data sets. This allows the aggregation of small studies over time and across contexts, leading to more robust claims about the efficacy of what we are advocating (for example, leading productive discussions in mathematics). There are several different frameworks and rubrics on offer in this edition's articles. They build on elements of previous work and might prove useful for researchers with aligned interests as they try to capture and code the complexity of mathematics teacher education.

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Stohlmann, Maiorca and Allen look at Model Eliciting Activities (MEAs) written by three elementary teachers on a summer intensive course. They use six principles for MEA development to analyse the problems written by the teachers. The article shows how these six principles can be used to understand the teachers' task development – both the adequacy of the task itself and the teachers' understanding of what is necessary in an MEA.

Magiera, van den Kieboom and Moyer gave 18 pre-service teachers 125 algebra problems and used their responses to investigate an algebraic 'habit of mind' - building rules to represent functions. This 'habit of mind' has seven features, which the authors use as a way to explore their pre-service teachers' algebraic thinking. Their article presents some strengths, weaknesses and relationships evident in the pre-service teachers' ability to build rules to represent functions, using the seven-factor framework as an analytic tool.

Leung and Lee investigated the acceptability and use of 'pre-formal' proofs with 79 preservice and novice secondary teachers. Three pre-formal proofs were presented to the teachers for their consideration. As well as providing a qualitative analysis, Leung and Lee also use the responses to construct a scale and to quantitatively explore the relationships among the participants' responses. They describe how they coded the participants' responses into three ordinal scales. Like the use of frameworks and rubrics, this scale construction aims to simplify the data in order to search for patterns that might not be visible when reading a set of written answers to questions.

Wasserman presents a case study of one elementary teacher, in order to tease out the impact of learning abstract algebra on the teacher's practice with elementary students. In this article, complexity is reduced by discussing one case in depth and by using the 'knowledge quartet' (Rowland, Huckstep, & Thwaites, 2005) as a framework. This is an example of reducing complexity in observing teaching: by focusing on particular dimensions (foundation, transformation, connection and contingency) and by seeing the teachers' utterances/actions as part of one idea – how knowledge impacts the teacher's choices. Wasserman further codes the teacher's utterances/actions as strong or weak, adding an additional layer of complexity to the coding. Presence or absence of a 'contributory code', such as 'responding to student ideas' is not enough to understand the changes in this teacher's practice – we need to know whether the responses were strong or weak in order to see the change brought about by the abstract algebra course. Wasserman is building on previous work here by extending the use of the knowledge quartet idea to the link between abstract algebra and elementary algebra teaching.

Crisan and Rodd identify a significant group of teachers who teach mathematics in secondary schools in the United Kingdom: non-specialist teachers without qualifications in mathematics. They report on the impacts of professional development work with these teachers which was designed to help the teachers see themselves as teachers of mathematics. These authors develop and present a framework of 'Modes of Belonging' which attempts to describe mathematics teacher identities. This analytic frame is used to present qualitative data from the non-specialist mathematics teachers to show emerging identities as teachers of mathematics.

Itter and Meyers underscore the link between beliefs and attitudes towards mathematics and teaching choices and behaviours. Their exploration of the attitudes towards mathematics of 152 pre-service teachers employed a narrative approach to data gathering. The pre-service teachers' written explanations were coded as to how positive or negative they were about mathematics. Three-quarters of their sample were neutral or negative about mathematics and the authors present an analysis of these negative responses, teasing out the effects of these beliefs on the pre-service teachers' engagement with mathematics teaching and learning.

The last two papers in this edition take different angles on the same concern: the use of rich problem solving and dialogue in mathematics classrooms. Xenofontos and Kyriakou consider the beliefs about problem solving and dialogue of 16 pre-service elementary teachers. These

pre-service teachers completed an open-ended questionnaire before and after attending a course in problem solving as part of their teacher preparation. They outline three themes in the pre-service teachers' responses which, like the Itter and Meyers article, suggest that pre-service teachers' beliefs and attitudes may be a barrier to problem solving and dialogue being used in schools.

Evans and Dawson present evidence that providing teachers with a set of worked solutions to a problem to use with their students results in deeper and more mathematically-focused discussion than when the solutions of students in the class are used. Using solutions that are 'depersonalised' appeared to result in a more in-depth discussion that could be more readily structured by the teacher than when the teacher called on students in the class to present their own ideas. To describe the differences in outcome they saw between these two approaches Evans and Dawson draw on several frameworks, both from the literature and of their own devising. The complexity of leading productive mathematical discussion based on solutions to rich problems is very evident in this article as the authors outline the key teacher actions and student responses using these frameworks.

References

Rowland, T., Huckstep, P., & Thwaites, A. (2005). Elementary teachers' mathematics subject knowledge: the knowledge quartet and the case of Naomi. *Journal of Mathematics Teacher Education, 8*(3), 255-281.