Dear Editor,

The reviewers make some very useful contributions that have strengthened the paper. I have taken noted of all their detailed comments and outline my responses to each below. The changes have made substantial alterations of the paper. The main changes are summed up below:

1) Title chance to better reflect the purpose the paper.

2) I have included a much strong literature review in regard to relating the knowledge tested to potential to teach. This has involved greater commentary from East Asian educational academia, cognitive load theorists and those concerned with the special nature of mathematical discourse. The original paper had 70 citations, the current paper has 90 and this adds to the length of the overall paper in terms of total words. In the original draft I had assumed that since so much has already been written on the relationship between MCK and PCK that I could treat this area briefly. Comments with respect to the validity of the test items by the second reviewer particular encouraged me to be more specific in unpacking the relationships between mathematics teacher preparation; mathematics content knowledge; mathematical competency and “mathematical knowledge for teaching,” (MKT) The relationship between the 20 test items used and MKT is I believe quite strong now. For consistency I have used MCK throughout the paper, but the link to MKT has been well made.

3) I have removed of individual academic commentary. This was the source of much of the concern of the first reviewer.

4) There is increased clarity about the test instruments and what they measured and what the implications for teaching.

5) The paper has distanced any assertion that the data is necessarily relevant to other Australian teacher education programs.

6) The summary of course time and assessment of Australian middle school mathematics courses has been moved into the literature review. The purpose of this review is to show the commonality of program structure for mathematics teacher preparation across Australia and to also illustrate the flexibility that individual academic have with assessment format. This is a side issue, but given the wide spread Western concern with mathematics teaching and learning that is emerging from diverse sources, I think it is relevant. Similarly I have left in some comment on academic governance, abet in reduced from since reviewer 3 in particular found it interesting.

I think the paper is good to publish in the current from. It is a comparison between two institutions with respect to trainee teacher’s basic maths (MCK) and the opportunities to connect this to other aspects of teaching (PCK). In the current climate I think it adds to considerations with respect to mathematics teacher education course structure. I could not find any other Australian data that was similar. Without references (but including tables) the paper is 8800 words.

I hope that by attending to the insights of the three reviews the editors the paper is worthy of publication. We sincerely appreciate the time and effort the reviewers have dedicated to the paper and we hope we have done their efforts justice.

I hope you find the paper useful and I hope your readers find it provokes some consideration.

Kind regards,

Stephen Norton

**Detailed response to reviewers**

**The first reviewer A**

Reviewer A wished to have a more comprehensive treatment of “Mathematical knowledge for teaching.” The work of Hill and Ball and the relationship between knowing mathematics and teaching it has been greatly expanded. This has included about half a dozen new citations including some from cognitive load theory. In total almost 20 new articles are cited.

The first reviewer considered that the title was misleading and so it has been changed.

The first reviewer notes that the starting knowledge comes from only one Australian institution and only one Chinese institution. Thus, claims of any relevance beyond these institutions need to be treated with caution. To account for this concern the title has been changed and this aspect of the paper has been softened. If readers do not see any parallel between the study institutions and their own institutions they are free of any suggestion that the challenges facing the Australian institution are unique. These changes account for this reviewers points 5 and 6.

Additional literature with respect to consistently of practice in teacher preparation in China has been cited. It has been made clearer that in Australia similar selection mechanisms and program structures exist across institutions.

The first reviewer was concerned about the inclusion of academic commentary, and the way the raw data was added without coding. This was expressed in the reviewers paragraphs 3 and 4. This data has been removed. Consistent with this, the third aim has been removed from the paper. This action helps to balance the paper consistent with the recommendations in the review’s comments in paragraph 3.

Reviewer A notes that tests questions are fairly procedural. The justification of this has been made more explicit by adding several studies that have used similar metrics to assess MCK. Considerable literature review on cognitive load and the nature of mathematics has been added to justify the virtue of assessing basic knowledge. It has also been noted that the Australian mathematics syllabus expects children to be fluent in basic facts and processes, thus we might have similar aspirations of their teachers.

Reviewer A suggested that the Burghes data be presented in the literature review and this has been attended to.

**Reviewer B**

The first major point made by this reviewer is that the instruments (tests) did not seem to match the specific topic of the study. By shifting the paper away from “challenges” to starting “mathematics knowledge” in the title and in the paper I think this concern is accounted for.

This reviewer was also concerned about the detail of the relationships between “Mathematical knowledge for teaching” (MKT) PCK and connections between maths knowledge and pedagogy/teaching. This concern has been taken into account and the use of the terms explicitly defined and used consistently throughout the paper. Further references related to cognitive load have been added to help clarify the critical importance of teachers knowing the material they are about to teach. The argument becomes that is it is difficult to suggest a teacher can enact PCK or MKT without MCK. This addition, I believe, fulfils the reviewers belief that stronger theoretical framing increases the validity of using the items selected in the test.

Several additional test items were added to the main text and the mathematics underpinning them made explicit. This was a recommendation of the second reviewer. The detail shows that while some questions are pure procedure, some can be solved with insight and Year 7 understanding of procedure. Further, I think that since fluency with this mathematics is necessary for the children, it is difficult to argue that teachers need not be fluent in the material they will be teaching. The content validity is enhanced by linking the samples described in the main text to specific ACARA year levels. I do not think it is necessary to do test reliably analysis on the test. A mathematically informed reader can see the content is relevant. The full tests are available from Burghes (2007) citation in the references as is the breakdown of international performance on the items (Burghes & Geach, 2011). The full test including item success rates is reported in the appendix.

Reviewer B reviewer wanted some more detail on the mechanism of selection in order to enter teacher training. This data has been added. The addition of this data is not an attempt to make claims beyond the study institutions, but some readers might consider such comparisons have some merit.

I do not think I made the connection between teacher training and better TIMMS results, expect to say that due to the general slide in international ranking a bit more pressure was being applied to look at all variables including teacher training. Still, I take the point and have removed this section from the paper. This has the benefit of allowing other sections to be expanded in accordance with the three reviews suggestions. I have taken Reviewer B’s suggestion and named the cities and institutions in each study.

Reviewer B notes that Opportunities to Learn OTL was not part of the theoretical framework. Cognitive load theorists (about 6 new citations) as well as several more general educational theorists and certainly East Asian educational writers consider sustained effort over considerable time is need to develop a deep and connected understanding of mathematics. I have also added several citations unpacking the relationship between surface and deep knowledge or procedural fluency and deeper problem solving including teaching and the provision of scaffolding. I think the relationship between the opportunity to attain expert knowledge necessary to each and opportunity to learn such knowledge is clearer.

Most importantly the Reviewer B wanted more detail on the test instruments. Further questions have been added to the main text and the links to the Australian Syllabus have been made. This test is really just a simple test of lower middle school mathematics the teacher will be expected to teach. An informed mathematics teacher will recognise the importance of students and a teacher being fluent in the algebra and number concepts assessed in this test. As noted above the link between knowing this mathematics and teaching it has been made more explicit in the literature review.

Reviewer B makes the correct point that many of the questions in the test are very procedural. This is what makes the data so concerning to me. It is very procedural, very basic and very poorly understood by significant portions of Australian prospective teachers.

The reviewer asks what the pre-service teachers think about the mathematics. I have surveyed them extensively and they actually want to learn it because they recognise they need to know what they will be teaching. I have this data and I can add this data into the paper, but consider it is worthy of a paper in its own right and this paper is long and complicated enough as it is.

The Reviewer B has suggested I add and error pattern analysis into the paper. This could be done, but that is a paper in it’s own right and the data would mostly be Australian since most of the Chinese trainee teachers made few errors. So, I have compromised and added more detail on a few questions. A mathematics teacher will recognise that in questions such as those presented errors will come from not knowing what procedures to apply or making errors in the application of those procedures.

Reviewer B has stated the analysis is surface level in terms to relating to capacity to teach. I have tried to address this by strengthening the links between knowing mathematics to be taught and teaching it. I have taken on board the suggestion to confine the assertions to mathematics knowledge at the start of the program.

Reviewer B was concerned about the connections I am made between governance and standards. This material has been significantly reduced. This action is consistent with the suggestions of reviewer A and I appreciate their insights. I have not removed it entirely since I think it adds to the interest of a broader range of readers.

The minor revisions suggested by Reviewer B have been taken on board and attended to.

The doubling of time allowed by Australian trainee teacher to complete the test has been justified in terms of long and short term memory functioning.

Reviewer B wondered how the selection for Australian and Chinese teacher programs differed across each country. That is, were the processes relatively uniform across each the country. I have cited several authors who claim the consistency of process in China. Similarly, the review of entry requirements to Australian middle year’s courses is discussed in the paper. There are two mechanisms, ATAR scores and or number of university courses that contain mathematics completed at a tertiary level. This has been stated in the paper as a basic standard across Australia. Each state has relatively minor differences in terms of teacher accreditation processes. The paper is already long and I did not feel a thorough review of entry requirements was warranted, especially as the claim that the data might have relevance beyond the Australian study institution is left to the reader.

**Reviewer C**

Reviewer C commented that the tables were confusing. I have amended some details to facilitate clarity.

The reviewer considered that the overall purpose of the paper was unclear. I think the stronger linking of the knowledge assessed in this paper with the enactment of pedagogy makes the overall purpose clearer. I think the revamped literature review makes the significance of what was measured; content knowledge and opportunity to connect content with domain specific pedagogy much clearer for the reader.

The Reviewer C makes the point that lesser time given to does not necessarily mean that the students already possess the knowledge. The Review considers we might explain the Australian pre-service teacher’s poor mathematics achievement. I think the added literature helps the paper in this regard. The most logical assumption is that the material was never committed to long term memory, or the trainee teachers never fully understood the mathematics. I do not think I have the detail in data to unpack these ideas. What is clear is that at the commencement of the course a very significant portion of the trainee teachers could not do very fundamental middle school mathematics. This is like forgetting your multiplication facts or forgetting basic writing convention. A major question raised by the paper is, faced with some evidence do we; mathematics curriculum academics have a responsibility to remediate the situation while attempting to develop other pedagogical aspects? The review of course and program structure across top Australian institutions suggests that such a pathway has limited consideration. Entry requirements to middle school mathematics teacher education are pretty standard across the nation, there are one or at most two courses of limited duration and assessment is in the main take home lesson planning and essay writing. If ensuring knowledge of content is a key focus of mathematics curriculum courses, the assessment protocols do not reflect this. I have not made this claim as bluntly as that in the paper, but hope the reader might draw that conclusion.

Burghes et al (2011) results for other nations have been removed from the relevant table. If readers wish to cross reference the Australian 192 student knowledge with other trainee teachers studied by Burghes and his colleagues they can do so.

Reviewer C suggests that we attempted to point out the deficit in mathematics content knowledge by exploring their responses to two questions. Actually there were 20 questions and mean success rates are reported in Table 4. The success rates on individual questions were and are reported in Appendix 1 and I have made this data location clearer in the paper. The detail on the thinking underpinning four questions is reported in the main text. It is hoped that this detail will assist the reader in fully appreciating the implications of the short fall in basic knowledge of a portion of the Australian sample.

Consistent with the recommendations of the earlier reviewers we have not attempted to generalise beyond the sample, but encourage the readers in other institutions to determine if the findings have any relevance to their circumstances. The idea that this particular institution is unique in intake processes is hardly credible since academic selection, program accreditation and governance is similar across states. I have not made this claim in the paper, but leave it to individual readers to decide the relevance of the data to their circumstances. I have added four citations that question the virtue in assuming that such proxy measures such as course completion are wise.

Reviewer C has suggested we undertake a meta-analysis of Australian teachers (middle school) content knowledge. I have cited several studies, but the field is relatively scant on detail content knowledge. Burghes and his colleagues (2011) attempted to document Australian trainee teachers knowledge of basic mathematics but were unable to attain a valid sample. A number of studies (QAO, 2013; TEMAG, 2014) have commented on the limited detail data on teacher education practices. In terms of high school mathematics studied, each State has different assessment metrics and there is no common national examination. ATAR scores and specialist mathematics courses completed are estimates of what might have been studied. We have tried to add to the quantum.

Reviewer C stated “I find that many of the discussions in the authors’ literature review are very good and can be the argument in the position paper.” I hope that by attending to the insights of the three reviews the editors find this to be the case. We sincerely appreciate the time and effort the reviewers have dedicated to the paper and we hope we have done their efforts justice.