Middle school mathematics teacher preparation in a Chinese and an Australian university: Different starting mathematics knowledge

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There is increasing concern with respect to the quality of teacher education preparation processes in the West. This study used case study methods to examine learning opportunities and knowledge of mathematics in a sample of 192 Australian trainee middle school teachers and 94 Chinese Bachelor of Science trainee teachers. It was found that the Chinese teacher preparation program ensured the prospective teachers had mastery of basic facts and processes and extended opportunities to deepen this knowledge and connect this mathematics knowledge to pedagogy. Many of the Australian trainee teachers had struggled with the same material and had limited opportunity to remediate this situation prior to commencing classroom engagement. The implications are discussed with regard to program structure and academic governance within the study institutions.

Keywords: initial teacher education • mathematics knowledge • middle school

Introduction

It has been convincingly argued that quality education at all levels contributes to a nation's economic prosperity and social capital (e.g., Dinham, 2015; Keeling & Hersh, 2012; Marginson, 2002, 2006; Sahlberg, 2006, 2007; Sahlberg & Oldroyd, 2010). That is, the investment in human capital via increasing knowledge increases labour productivity as well as social well-being and stability. Questions are starting to be asked as to whether the political and institutional frameworks effectively promote this priority in the English-speaking West (e.g., Australia, Canada, England, New Zealand, USA). According to a growing literature base there is need for reform of tertiary institutions in general and education in particular (e.g., Chang, 2002; Keeling & Hersh, 2012; Kotzee, 2012; Marginson, 2002, 2006; Meyers, 2012; Sahlberg, 2006) since there is a perception of a drift away from expertise and fluency with professional knowledge associated with disciplines.

With respect to teaching and learning mathematics it has long been recognised that deep knowledge of mathematics is necessary for effective teaching (e.g., Australian Academy of Science, 2015; Australian Institute for Teaching and School Leadership [AITSL], 2012; Ball, Hill, & Bass, 2005; Burghes & Geach, 2011; Cai, Mok, Reddy, & Stacey, 2016; Goulding, Rowland, & Barber, 2002; Hattie, 2009; Hill, Rowan, & Ball, 2005; Krainer, Hsieh, Peck, & Tatto, 2015; Masters, 2009; Teacher Education Ministerial Advisory Group [TEMAG], 2014; U.S. Department of Education, 2008). This study examines the processes of middle school mathematics teacher preparation in two institutions, one in Wenzhou China and one in Brisbane Australia, in order to gain insights from two different teacher education systems. It is not intended to claim that the institutions are necessarily representative of each nation. It is up to the reader to consider the transferability of the data and findings to their circumstances. The review of literature and course and program structures provided in the results section assists in this endeavour.

The importance of content knowledge for teaching mathematics

The relationship between knowing mathematics and teaching mathematics has a long history and nuances underpinning the theoretical models have been refined. Shulman (1986) used the term mathematical content knowledge (MCK) which included mathematical concepts, fundamental assumptions, definitions, and procedures. Shulman distinguished MCK from pedagogical content knowledge (PCK). PCK included curriculum knowledge, knowledge of students, representations of content, analogies, models, and explanations of mathematics that make it understandable. Some authors have questioned whether or not it is possible to make a clear distinction between subject knowledge and PCK since all mathematical knowledge has pedagogical underpinnings (McEwan & Bull, 1991; Stones, 1992). Lannin et al. (2013) described PCK as "the most useful forms of representations, analogies… but as subject-specific knowledge of curriculum, learners, assessment, and instructional strategies…" (p. 423). The link between depth of relevant mathematical knowledge and teaching effectiveness has been well documented.

Ball et al. (2005) commented: "That the quality of mathematics teaching depends on teachers' knowledge of content should not be a surprise" (p. 14). These authors went on to claim of United States teachers that "the mathematical knowledge of teachers is dismayingly thin… we are failing to reach reasonable standards with most of our students, and most of those students become the next generation of adults, some of them teachers" (p. 14). The Teacher Education and Development Study in Mathematics (TEDS-M) noted that "knowledge of content to be taught is a crucial factor in influencing the quality of teaching" (Tatto et al., 2008, p. 19). In the UK, Burghes and Geach (2011) reported that "a prerequisite to be an effective teacher of mathematics, is that you are confident and competent in mathematics at a level significantly above that which you are teaching" (p. 17). More recently, Gess-Newsome (2013) suggested that "Teachers' content knowledge for teaching mathematics (CKT-M) significantly and positively predicted student achievement…. The only variable that approached CKT-M in explaining student achievement was students' socioeconomic status" (p. 258). The debate is not so much that teachers should have a deep understanding of mathematics, but just what level of mathematics is necessary for particular levels of teaching. Ball (1990), Hill et al. (2005), and more recently Speer, King, and Howell (2015) have noted mathematical majors or the equivalent do not necessarily bestow prospective teachers with depth of understanding of the concepts they will teach. The mastery of rules and procedures is not enough: deep understanding of the structures underpinning the mathematics to be taught is needed. As Ball et al. (2005) commented, there is a need for "teachers to have a specialised fluency with mathematical language, with what counts as a mathematical explanation, and with how to use symbols with care" (p. 21). Ball et al. (2005) stated that specialised fluency with mathematical language is a starting point for being able to construct mathematical explanations. The specialised language of middle years' mathematics is related to order convention, index convention, whole-number place-value structures, the detail of algebraic processes, and conventions associated with logarithms. The need for depth of knowledge in order to provide necessary scaffolding has been supported by other studies (e.g., Englemann, 2007; Kirschner, Sweller, & Clark, 2006; Owen &

Sweller, 1989). Needless to say, it is difficult to be effective in enacting explicit instruction if the teacher has a limited understanding of the material, something of which the father of social-cultural principles, Vygotsky, was well aware.

While the exact relationship between depth of content knowledge and pedagogical content knowledge remains an area of development, the emerging literature indicates that a strong subject-specific knowledge supports the development of PCK (Krauss et al., 2008). These authors used pen-and-paper mathematics tasks to assess MCK and probe PCK. Their test items were samples of the concepts they were expected to teach. For example: "How does the surface area of a square change when the side length is tripled? Show your reasoning" (MCK) and "Note down different ways of solving this problem" (PCK); "Is it true that 0.999999…=1?" (MCK) and "Please give detailed reasons for your answers" (PCK) (p. 720). With respect to the use of pencil-and-paper testing of teachers and prospective teachers, Ball et al. (2005) defended "testing teachers, studying teaching or teacher learning, at scale, using standardised student achievement measures" (p. 45).

Support for teachers to have fluency with MCK, including fluency with basic facts and procedures as the foundation of broader mathematical success, comes from educational theorists who claim that mathematics is a hierarchical body of knowledge built upon specialised facts, procedures, and language (Bernstein, 1999, 2000; Muller, 2000, 2009; Muller & Taylor, 1995). Bernstein used the term "esoteric discourse" while Muller and Taylor (1995) described the nature of mathematics as "sacred and profound" (p. 263). The hierarchical nature of mathematics means that students who are not fluent in whole-number numeration and computation will find it incredibly difficult to succeed with fraction numeration and computation and algebra computation. Similarly, students who are not fluent in linear algebra conventions will find calculus a mystery. Cognitive load theorists support this position. The example Sweller (2016) gives is to "solve for 'a' in (a+b)/c = d". This is a relatively trivial Year 9 standard question where the student could apply reverse order of operations by first multiplying both sides of the equal sign by c and then subtracting b from both sides. The point is that, if the student does not know order of operation convention, solving for "a" is very difficult, in fact near impossible. Recently, Hattie and Donoghue (2016) used a similar argument and described the retention of accurate detail (surface learning) or "lower level learning" as a necessary foundation to higher level problem solving and creative thought. This positon is supported by the Australian Academy of Science (2015), the Organization for Economic Co-operation and Development (OECD) (2014), and Klein (2005). Every modern mathematics curriculum, including the current Australian curriculum, is structured and sequenced in such a way that acknowledges the essentially vertical nature of mathematics. The Australian Curriculum: Mathematics (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2012) has a proficiency strand "Fluency" (p. 5). Fluency is defined thus:

> Students develop skills in choosing appropriate procedures, carrying out procedures flexibly, accurately, efficiently and appropriately, and recalling factual knowledge and concepts readily. Students are fluent when they calculate answers efficiently, when they recognise robust ways of answering questions, when they choose appropriate methods and approximations, when they recall definitions and regularly use facts, and when they can manipulate expressions and equations to find solutions. (ACARA, p. 6)

Cognitive load theory helps us gain insight as to why a teacher's fluency and depth of content knowledge has a positive impact on measures of PCK and student learning. Sweller (2016) classifies knowledge into two forms: biological primary and secondary. Sweller contends that primary knowledge includes learning to speak and listen, recognise faces, and engage in generic cognitive processes such as problem solving by using solution knowledge of related problems. Such biologically primary knowledge tends to be acquired "without explicit tuition from others" (Sweller, 2016, p. 360). In contrast, almost all of what is learned in educational institutions is classified as secondary knowledge. The acquisition of secondary knowledge is conscious, relatively difficult, and effortful (Kirschner, Verschaffel, Star, & Van Dooren, 2017). Sweller points out that attaining secondary knowledge is greatly assisted by explicit instruction; he uses cognitive load theory and the well-known capacity and duration of working memory to explain the importance of quality explicit instruction in the learning process. Chen, Kalyuga, and Sweller (2016, p. 28) reported that, "for the learning of complex materials, explicit instruction is essential for novice learners". A further insight is that secondary knowledge is domain specific (Tricot & Sweller, 2014) and includes conceptual and procedural information. Learning mathematics is very domain specific (Kirschner et al., 2017). It has long been recognised that deliberate practice is necessary for the acquisition of expert knowledge (Ericsson, Krampe, & Tesch-Romer, 1993).

Cognitive load theory offers an explanation for the call for mathematics teachers to have a relatively high MCK, a position that is broadly supported in the mathematics teacher education literature cited above. Teaching is a very complex and fluid working environment with very high secondary as well as primary knowledge demands. In a complex classroom environment a teacher who is taxing her or his short-term memory capacity to think through mathematical solutions is likely to be unable to take account of a myriad of other critical classroom responsibilities including classroom management or taking account of individual students' learning needs. One of the claimed outcomes of low levels of MCK is a lack of significant deviation from the pedagogy that they experienced during their own school days (Ball, 1988). That is, teachers with low MCK will tend to be less willing to take risks and explore new ways of scaffolding mathematics. The prevalence of shallow teaching has manifested itself in "the school mathematics tradition" (Gregg, 1995). Gregg describes this as a pattern of teaching that emphasises rote, procedures, and shallow explanations that do not readily assist students to develop deep conceptualization. Gregg reported that this tradition was "ubiquitous and robust" (p. 444). Further, to attempt to teach mathematics without personal fluency and depth of knowledge is likely to be stressful and to impact negatively on self-efficacy (Henson, 2001; Watt & Richardson, 2013) and, in this way, negatively impact on student learning. Put simply, taking equality of primary knowledge such as the ability to communicate to students, an expert in their discipline is likely to outperform a novice in their domain irrespective of differences in their working memories (Sweller, 2016). A teacher who is not fluent in basic facts, procedures, and language cannot be considered an expert.

There are different ways to ensure that teachers are well prepared to teach, and all of the top nations have systems that insist on depth of discipline knowledge, which includes fluency with facts and processes. In China, South Korea, Japan, and Singapore, the use of high-stakes testing is a part of the checks and balances to ensure quality (Burghes & Geach, 2011; Tatto, Rodriguez, & Lu, 2015). With respect to Chinese education, Wang, Cai, and Hwang (2004) reported "a high degree of instructional coherence as a distinguishing feature in Chinese classrooms… and other East Asian countries such as Japan" (p. 112). Finland has achieved very substantial improvements in children's mathematical outcomes as measured on international tests without the use of high-stakes testing of children or their teachers. Rather, a more holistic approach to education is enacted there with a cohesive and centrally organised emphasis on high standards of knowledge at all levels and a trust in individual teachers' professionalism (Sahlberg, 2011a). In East Asian and Finnish systems, coherence is founded on sustained and in-depth teacher preparation where specific discipline knowledge is central (Burghes & Geach, 2011; Fan, Miao, & Mok, 2004; Sahlberg, 2011a; Tatto et al., 2015). These authors also noted that this preparation included close collaboration between schools, tertiary providers, government financing, and bodies accrediting teacher training programs. Common descriptors of highfunctioning educational systems included that they had selective entry requirements, cohesive structures, and stringent graduation requirements. Since the two case study sites in this study are located in China and Australia, it is worth expanding on the literature related to mathematics learning therein.

Teacher preparation in China and East Asia

Over the past two decades, teacher training in China has been the subject of strong government intervention to ensure that teachers are suitably qualified (Burghes & Geach, 2011). There is considerable competition for teaching positions in China as teaching is seen as a respected occupation, and this leads to competition that enhances standards. Ma (1999) and Li, Zhao, Huang, and Ma (2008) suggest that strong basic content knowledge (MCK) has been the foundation of quality mathematics teaching in China in recent decades. Dai and Cheung (2004) described the wisdom of traditional mathematics teaching as based upon "concise explanation of mathematical concepts with abundant practice" (p. 3). Similarly, Fan et al. (2004) reported that Chinese teachers emphasised fluency and proficiency with basic mathematics as a prelude to deeper problem solving. Consistent with Ball et al. (2005) and Hill et al.'s, (2005) recommendations, understanding the subject matter is a first step in developing the specialised mathematics knowledge and skills used in teaching. The epistemology reflected by Chinese teacher education processes is consistent with cognitive load theorists (e.g., Chen et al., 2016; Sweller, 2016).The literature on Chinese mathematics teacher preparation indicates that the Chinese education processes start with the development of advanced mathematical knowledge (MCK) (e.g., Gu, Huang, & Marton, 2004; Fan et al., 2004; Lai & Murray, 2012; Li, 2004; Li et al., 2008; Zhang, Li, & Tang, 2004) and build upon this for the development of pedagogy (PCK). Further, these authors concluded that China has established a unified pre-service teacher education system with considerable consistency across the country as well as systematic ongoing professional development programs for in-service teachers. Similar findings have been reported in other East Asian studies that include nations such as Korea, Japan, Singapore, and Taipei (e.g., Tatto et al., 2015).

Teacher preparation in Australia and the West

From the Western perspective, a matter of concern for some considerable time has been the depth, or lack thereof, of MCK of some teachers (e.g., Ball, 1988, 1990; Ball et al., 2005) and much of this concern has been focused on primary teachers (e.g., Brown, McNamara, Hanley, & Jones, 1999; Burghes & Geach, 2011; Wragg, Bennett, & Carre, 1989). Those studies that have undertaken a review of Western middle school trainee teachers' mathematics knowledge had similar concerns (e.g., Burghes & Geach, 2011; Hine, 2015; Krainer et al., 2015; Ma, 1999). The Krainer et al. data are particularly concerning for U.S. middle school teachers. In Australia, Masters (2016) commented that a critical factor with respect to teacher preparation is that entry vetting is very lax, especially with regard to primary teaching. Part of the reason for this is that teaching in general in Australia is not seen as a high-status profession and thus does not attract top graduates. More specifically, with respect to secondary mathematics teaching within the state of Queensland, The Queensland Audit Office (QAO) (2013) reported that 49% of teachers teaching classes of mathematics were out of field. That is, they had no formal training in mathematics teaching. One of the implications of this finding is that almost all trainee teachers who qualify to teach mathematics are in a very good positon to acquire employment. This factor would seem to militate against the free market forces driving reform through skilled labour competition for employment.

Henderson and Rodrigues (2008), Hine (2015), and Kotzee (2012) reported that it has become a tradition in teacher education courses at Western universities to focus on big-picture curriculum issues including thinking skills, problem solving, and team work, and to largely assume that students have a basic knowledge of content. Poulson (2001) provides a rationale for not being too concerned with a teacher's capability to articulate the detail of mathematics content: "there seems to be little evidence of a clear relationship between a well-developed formal academic knowledge of particular subjects and effective teaching in the primary phase of schooling"(p. 47). Further, teachers could and do continue to learn in the classroom. Whether the concerns of Henderson and Rodrigues, Hine, and Kotzee are valid is contested, since there is limited transparency of practice within tertiary settings and in teacher preparation in particular (QAO, 2013; TEMAG, 2014). However, some insight with respect to teacher preparation orientation and practice can be gained by examining entrance demands, delivery modes, and assessment modes.

Entry processes are similar across the nation, as is the program structure. In some institutions an Australian Tertiary Admissions Rank (ATAR) is stipulated to enter a bachelors program; however, universities offer alternative pathways for entry such as bonus points for disadvantaged students, rural or local students, and Indigenous students. Other institutions stipulate the completion of particular subjects at either high school or at a tertiary level. The most common metric for entry into a graduate program is that four mathematics-rich tertiary courses have been completed. Some indication of the structure of mathematics teacher preparation processes can be gained via universities' published course profiles. These documents outline academic learning time and assessment formats, goals, and pre-requisite courses, and describe each course in the program. Table 1 below illustrates the diversity of assessment pathways as well as opportunities to learn MCK and PCK in a range of teacher preparation institutions within Australia.

Table 1
Sample of Course Assessment Modes in Sandstone Australian Mathematics Middle Years Curriculum Courses (The study institution is Griffith University-2016 structure)

University*	Course code	Assessment forms	Recommended contact face to face
Griffith University	EDN3024/7024	Closed book exam 60% Classroom-based research assignment 40%	32 hrs
University of Queensland	EDUC6725	Review of digital resources 33% Mathematical investigation inquiry 33% Resource, working with families 33%	24 hrs
Monash University (graduate)	EDF5017	Tasks exploring numeracy-related issues 50% Critical reflections on numeracy 50%	24 hrs
University of Adelaide	EDUC4533A	Workshop activities 20% Lesson Plan 30% Unit Plan 40% Attendance 10%	4hrs/week

		Prepare teaching materials 50%	
Queensland University of Technology	CRB204	Learning log 60% Teaching plan 40%	Not stated
University of Sydney	EDSE3046	4000 word essay 60% 2000 word assignment 40%	32 hrs
University of Melbourne	EDUC90457	Two reports 50% each	36 hrs
University of Newcastle	EDUC1090	Essays/written assignments Lesson plan	8 hrs

Key: *Sourced from online university web sites.

The condensed review of course profiles across Australia presented in Table 1 suggests that they rarely focus on specific pedagogy or MCK in either the middle school or senior pathway. Griffith University has a test that assesses depth of MCK and PCK as part of the assessment package. As noted above, entry to mathematics curriculum courses is on the basis of past subjects studied. For example, Sydney University provides an extensive list of mostly calculus subjects, and undergraduates are expected to study between four and six of these (University of Sydney, 2016). Entry to course enrolment on the basis of past studies allows the assumption that the trainee teachers have the necessary MCK. Acceptance of this assumption allows the mathematics curriculum courses to focus on the development of planning and aspects of PCK via preparing teaching materials, writing reports, reviewing digital resources, and writing essays. Some course profiles tend to emphasise generic graduate skills and niche issues, including at the University of Sydney where brain-based research into gender differences in adolescence is explored with implications for practice in mathematics classrooms (University of Sydney, 2016, p. 1). Curtin University (2016, p. 1) claims that its master's program provides candidates with a strong background in learning theory, curriculum development, and in providing supervision.

For assessment purposes a trainee teacher in most Australian teacher preparation institutions is likely to write essays, and complete planning tasks such as units of work. For example, they will be required to prepare a unit on fraction development, and in that process learn the content and pedagogy related to teaching fractions. Similarly, a trainee teacher might prepare an investigation of developing geometrical concepts and learn in depth the mathematics underpinning, say, Pythagoras. It is highly unlikely that the same unit of work would cover trigonometry. Report writing, preparing a teaching plan, writing essays, and completing a learning log are unlikely to assess fluency across a broad range of middle school mathematics including algebra conventions, surds, quadratics, trigonometry, basic fractions, percentages, index notation, and whole-number computation; nor do they explain how this material might be coherently taught without access to the internet. Thus, while essays, planning tasks, and critical reflections are useful assessment tools, by their very nature any development of MCK or PCK within the mathematics curriculum course will likely be narrowly focused because there is limited time to engage with the trainee teachers in order to explore niche domains, general pedagogical principles, as well as the detail of middle years' content. The validity of assuming that prior mathematics course completion is a reliable measure of depth of mathematical fluency and understanding has been questioned previously (e.g., Ball, 1990; Ball et al., 2005; Burghes & Geach, 2011; Hill et al., 2005; Qian & Youngs, 2016; Speer et al., 2015) but has had limited empirical examination in Australian contexts.

With respect to the face-to-face time allocated to learning to teach mathematics, the usual practiced in Australia is a blend of lectures and workshops. Courses such as those run by the University of Newcastle rely on delivery via digital media.

Given this background, the aims of this study are to:

1. Document the content knowledge of middle school trainee teachers in an Australia and a Chinese teacher education program.
2. Document entry pathways and opportunities to learn to teach mathematics in the two institutions.

These data are used to reflect upon teacher preparation pathways and practices in each study institution.

Method

The methodology is a multiple instrument case study where several cases provide insight into the issue (Cresswell, 2015; Gay, Mills, & Airasian, 2006). Holosko and Thyer (2011) have suggested that such case study methodology is useful in investigating a number of variables and the relationship between these in influencing behaviour. Consistent with the recommendations of Yin (2009), several data sources including quantitative and qualitative forms are used to help the readers appreciate the phenomena.

The data sources include documentation in the form of summaries of assessment protocols, time delivery, and program structures in Chinese and Australian teacher preparation programs. This detail enables the reader to compare the processes. Quantitative data describing Chinese and Australian trainee teachers' mathematics knowledge illustrate the different starting knowledge of the cohorts. The statistical differences (or otherwise) between the samples are supported by analysis of the concepts of mathematics with which trainee teachers struggled. This detail adds relevance to the descriptive statistics in terms of the stage of preparedness for teaching children similar concepts and processes. Supporting cultural commentary provides background content to help explain and analyse the data. SPSS was used to calculate the means and standard deviations of different cohorts. In terms of the differences between Chinese and Australian overall means, statistical tests are not needed since the differences are so profound. Further, what is important is not if the means are statistically significant, but rather, if they are educationally significant.

Participants

The subjects were trainee middle school mathematics teachers in an Australian and a Chinese university. The Australian university is situated in the capital of Queensland, Brisbane, and is ranked 16 out of 29 nationally and 382 on the world ranking system (4 International Colleges & Universities, 2015). The program and courses studied by these Australian students were accredited by the Queensland College of Teachers (QCT), a formal statutory body that accredits teacher training programs and registers teachers to teach. The Chinese university was located in Wenzhou, a middle-sized coastal city with a population of 8 million. It ranked 179 out of 741 Chinese universities and 1,254 on the world ranking system (4 International Colleges & Universities, 2015).

The sample of Chinese teacher education students was almost the entire cohort of a first-year mathematics curriculum course (n=94) studying a 4-year bachelor degree specialising in mathematics teaching at a normal university ("normal" originally applied to institutions that focused on preparing school teachers and has been retained subsequent to the offering of courses beyond education-related study). In China, mathematics teachers do not have a second subject: they specialise in teaching mathematics. Entry to the program includes mandatory high scores in closed-book examinations of mathematics. There was only one pathway into teacher training at this university and all the trainee teachers were expected to teach both middle and senior mathematics. Some of these trainee teachers would find themselves teaching primary mathematics. A portion of the cohort was likely to choose to undertake additional study in the form of a Master Degree in Education since schools were increasingly showing a preference to recruit those with such qualifications. The data were collected during a mathematics pedagogy lecture in early 2015 and there is no reason to assume they are not indicative of the general standard of mathematics proficiency among initial teacher education students enrolled in mathematics curriculum courses in that institution. Several authors cited in the literature review have noted that there is considerable uniformity with respect to teacher preparation across China as a result of highly interventionist government policy.

In Queensland, as in the rest of Australia, there are two pathways to middle school and subsequently senior high school teaching. The first is the undergraduate pathway. Here, students complete specialist mathematics courses as part of an undergraduate degree, and then curriculum courses specifically related to teacher education. As part of the curriculum courses there are two mathematics-orientated curriculum courses: one for middle school and one for senior mathematics. All of the undergraduate pathways prepare students to teach senior mathematics and that includes considerable calculus associated with mathematical methods and specialist mathematics. Unlike in China, all Queensland teachers need to have two teaching specialities, mathematics and one other.

The second pathway for mathematics teacher registration in Australia is the graduate diploma pathway. The intake consists of individuals who have a degree that is considered to contain sufficient mathematics to give them the background to teach mathematics to Year 10. About half of these trainee teachers go on to teach senior mathematics, while the remainder are qualified to teach to Year 10; content at this level includes surds and quadratics.

The Australian subjects in this study were all enrolled in a middle school mathematics curriculum course. They include virtually the entire enrolments of middle school trainee teachers across two campuses in 2016 and almost all the enrolments from one campus in 2014 and 2015. The total number of Australian trainee teachers sampled was 192. This sampling gives us confidence that the data are representative of the case study institution.

Instruments and data collection

The programs and course structures of each institution were summarised in tables. These were accessed via the published material online from each institution.

The trainee teachers' mathematical competency with basic facts and processes was assessed using a modified Burghes (2007) *International comparative study in mathematics teacher training: Trainee teacher primary mathematics audit part B* (15 questions). This instrument is also Part A of the *Trainee Teacher Secondary Mathematics Audit* (Burghes, 2007). This use gives readers the opportunity to cross reference with the results of Burghes and Geach (2011) *International comparative study in Mathematics teacher training*. Of these questions Burghes and Geach (2011, p. 7) commented:

> These are the responses to the relatively straightforward questions on concepts that were also taken by the primary participants. We would expect the secondary trainees to do well on this part of the audit…. The audits undoubtedly stress procedural rather than conceptual mathematics… we have gone for consistency and reliability rather than complexity.

The justification for assessing trainee teachers' MCK via fluency and depth of content knowledge has been outlined in the literature review. The Chinese trainee teachers were permitted 30 minutes and the Australian trainee teachers were permitted 1 hour. The reason for this discrepancy was that the Chinese author considered most of the questions relatively trivial given the level of mathematics previously studied and the expectation that teachers of mathematics would be very fluent in basic procedures and problem solving. Calculators were not permitted and the test was closed book and supervised by one of the authors. The Australian participants filled out additional background and attitudinal data that were not reported in this paper, an activity estimated to take about 10 minutes. The authors of this study did not use Part B of the Burghes (2007) audit as the first author considered it sufficient to test to the concepts associated with Secondary audit Part A, in part because about half the Australian sample would only be registered to teach to Year 10 mathematics and not senior mathematics.

Results

Opportunity to learn to teach mathematics: Chinese trainee teachers

Table 2 is a summary of courses as translated from "Cultivation plan" for the Bachelor of Science in Mathematics and Applied Mathematics (teaching degree in the first tier university).

Table 2
Mathematics Learning Opportunity for Chinese Secondary Trainee Teachers of Mathematics

Compulsory mathematics foundation subjects	*Mathematics electives (240 hrs to be selected)*
Advanced algebra 1 (64 hrs)	Advanced geometry (48 hrs)
Analytic geometry (32 hrs)	Ordinary differential equations (48 hrs)
Mathematical analysis (96 hrs)	Mathematical modelling and experiments 80 hrs)
Probability theory (48 hrs)	Abstract algebra (32 hrs)
C programming (80 hrs)	Abstract algebra (32 hrs)
Advanced algebra 2 (80 hrs)	Functions of complex variables (48 hrs)
Mathematical analysis 2 (96 hrs)	Calculation methods (64 hrs)
Mathematical analysis 3 (64 hrs)	Discrete mathematics (32 hrs)
Total (569 hrs)	Mathematical statistics (32 hrs)
	Elementary number theory (32 hrs)
	Function of complex variables (32 hrs)
	Operations research (48 hrs)
	Further studies in algebra (32 hrs)
	Calculation methods (32 hrs)
	Abstract algebra (32 hrs)
	Further studies in mathematical analysis (32 hrs)
	Function of a real variable (48 hrs)
	Further studies in mathematical analysis (32 hrs)
	Discrete mathematics (48 hrs)
	Differential geometry (32 hrs)

Compulsory subjects make up 70% of the course requirements. In addition to the list above there were compulsory subjects related to specific mathematics elements including pedagogy of mathematics (48 hours), analysis of mathematics curriculum standards and text books (16 hours), training of mathematical teaching skills (16 hours), and studies of trends of mathematics teaching (16 hours). Chinese trainee teachers at this institution received well in excess of 600 hours of explicit instruction related to mathematics and how to teach it. These data complement the Burghes and Geach (2011) description of a 4-year Bachelor Diploma Degree and Certificate of Teacher Training which included components in pedagogy, psychology, educational technology, mathematics and applied mathematics, computer science, teaching practice, and a dissertation. The teaching practice component included observed lessons, marking homework, and teaching under the observation of supervising school teachers and university academics. Of relevance to the study is that enrolment is dependent on an entrance examination including Chinese, English, Mathematics, and Physics.

The program structure of Australian trainee teachers follows two pathways as described above. The specific courses are listed below.

Table 3
Summary of Content Prerequisite University-based Courses for Undergraduate Pathway Students and Entry Criteria for Graduate Pathway (4 years Bachelor Program)

Summary of content of Undergraduate Pathway to middle school and senior mathematics teaching	
Mathematics 1A	Basic differential calculus of one variable, partial derivatives, basic vector algebra in two and three dimensions
Mathematics 1B	Probability, complex numbers, differential equations, and linear algebra
Mathematics 2A	Multiple integrals of scalar functions, Gauss, Green, and Stokes, Fourier series and integrals
Linear algebra	Multiples of integrals, of scalar functions, differential equations, theorems of Gauss, Green, and Stokes
Numerical methods and MATLAB	Maths that cannot be solved by hand including non-linear equations, linear systems, data fitting, integration and solutions of systems of differential equations (use of MATLAB)
Introduction to mathematical modelling	Mathematical models related to derivatives, rates, integrals, optimisation, and ordinary differential equations
Graduate pathway summary of entry requirements for middle school and senior mathematics course enrolment	
Middle school teaching only	Degree that contains at least four university-based subjects rich in mathematics
Middle and senior teaching	Degree that contains at least six university-based subjects rich in mathematics

With respect to the undergraduate content courses listed above, it is typical for the lectures to be derived online via lecture capture, and for students to attend 26 hours of tutorials. A reasonable estimate of recommended contact time is 180 to 200 hours for Bachelor of Secondary Education students with about 50 to 60 mathematics curriculum contact hours.

Typical of other Australian institutions, with respect to the graduate entry, the program convenor looks for a spread of mathematics in calculus, statistics, and modelling. Thus, some of the graduate students have majors in finance or accounting, while others are scientists, engineers, or draftspersons. A few have doctorates in research science and even in mathematics. All of the above Australian trainee mathematics teachers will do one 10-credit point course in middle school mathematics curriculum. Those who wish to teach senior mathematics will undertake one further mathematics curriculum subject. At the study site this ranges from 28 hours of contact to 24 hours depending on the lecturing academic. In 2019 it is proposed to offer 10 hours of workshops and 15 hours of online lectures over 5 weeks for each secondary school mathematics curriculum course. There is no entrance examination for enrolment in Australian teacher education programs (at the graduate level) that might test the level of mathematics associated with upper middle years' mathematics. Depth of knowledge of mathematics is inferred from the completion of prior courses.

Results on basic knowledge of content

Summary of results from Burghes's (2007) 20-mark (MCK) scale with standard deviation on fundamental mathematics is provided in Table 4.

Table 4
Results on Burghes's (2007) Test for Primary and Secondary Samples, Means and Standard Deviations

Sample	Mean/20 (Sd.)
China 2015 (n=94)	18.4 (1.5)
Australia 2014 (n=44)	8.3 (3.9)
Australia 2015 (n=30)	9.5 (4.2)
Australia 2016 (Undergraduates n=21)	12.8 (3.2)
Australia 2016 (Graduate entry n=97)	7.5 (3.8)

By way of comparison, Burghes and Geach (2011) reported their Chinese sample with a mean of 16.6 (SD 1.6). The Australian scores in this study are less than those reported by Burghes and Geach for England (14.1/3.5), Japan (16.3/1.8), Russia (17.3/2.0), Ukraine (15.5/2.6), and Singapore (15.3/2.3).

We can see in Table 4 that in general the undergraduate cohort achieved higher grades. It is hardly surprising that the undergraduates outperformed the graduate entry cohorts, since the undergraduates had recently completed six tertiary courses focused on mathematics while portions of the gradate pathway had not studied mathematics for extended time frames. Since all these prospective teachers will be qualified to teach middle school mathematics and the graduate pathway outnumber the undergraduate pathway in the order of 6:1, the overall average Australian success rate is reported henceforth. There is no virtue in conducting statistical significance tests on the Australian and Chinese data (clearly the significance is about p=.000); the real question is, what is the educational significance? It is worth exploring a few of the questions in detail to gain an appreciation of the mathematics involved.

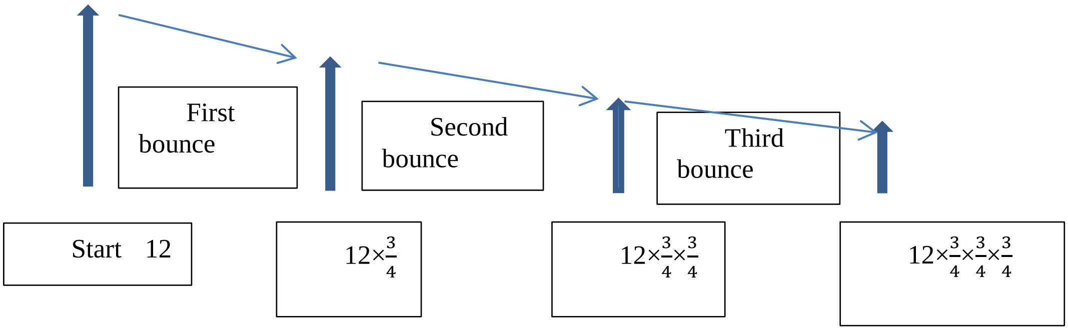
<u>Question 3</u>. Let a =2, b = -1. Calculate the value of H when $\frac{1}{H} = \frac{1}{a} + \frac{1}{b}$.

This problem is pure algebraic procedure and involves substitution, fractions, and integers. Fluency while working with algebraic fractions is a Year 10 expectation (ACARA, 2012). The Chinese sample had a 94% success rate and the Australian samples averaged 45% success.

<u>Question 5</u>: A ball is dropped from a height of 12 metres. It bounces on the ground and reaches $\frac{3}{4}$ of its height. It continues to bounce this way, each time rising to $\frac{3}{4}$ of the previous height. What height does the ball reach after three bounces?

The solution can be found with diagrammatic modelling.

Model the problem:



Thus the height after the third bounce is $12 \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = \frac{81}{16}$ or $5\frac{1}{16}$.

Figure 1: Possible solution to Question 5.

A trainee teacher who found the solution by modelling such as above would be demonstrating "mathematical knowledge for teaching" (Hill et al., 2005). A trainee teacher who could not solve the problem via any method would be demonstrating a low level of MCK.

The fraction computation in Question 5 is Year 7 level in Australian schools (ACARA, 2012). The Australian trainee success rate was 13% and the Chinese trainee teachers had a 90% success rate. This question is not pure procedure; the context requires a little thought unless the student readily recognised depreciation and index structure.

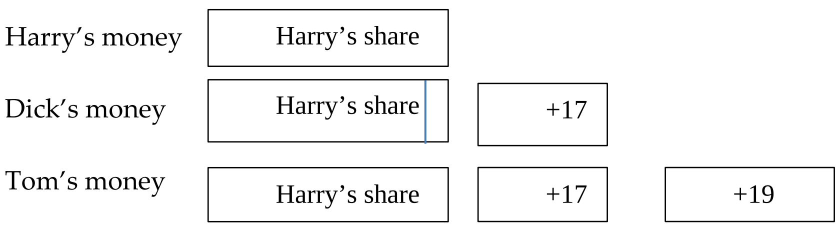
<u>Question 6</u> asked the trainee teachers to factorise x^2-7x + 12.

This question is pure procedure requiring knowledge of factorisation and integers and is content of Year 10 (ACARA, 2012). Since the "a" value in a ax^2+bx + c is 1, students need to consider two numbers that multiply to +12 and add to -7. Competent Year 10 students will recognise these to be -1 and -6, and express them in factorised form (x-1)(x-6) without using a pen to carry out any computations. The Chinese success rate was 99% and the overall Australian success rate was 18%.

<u>Question 7:</u> Tom, Dick, and Harry have a sum of $575 to be shared among them. They agree to divide it so that Tom gets $19 more than Dick, and Dick gets $17 more that Harry. How much does Tom get?

The solution is modelled in Figure 2 so the reader can get a sense of the problem solving in algebra contexts involved.

Modelling the problem:



Write the equation: Let H=Harry's share: 3H+17+17+19= 575
Simplify the equation: 3H+53=575
Solve for H (subtract from 53 both sides) 3H+53-53=575 -53
Solve for H (divide both sides by 3) $\frac{3H}{3} = \frac{522}{3}$
Do the division: H=174

Therefore Tom's share is 174+17+19=$210

Figure 2: Possible solution method for Question 7.

Problems of this structure are typically taught in Year 8 in Australia (ACARA, 2012), but the verbal context makes it more problem solving than pure procedure. The Chinese success rate was 86%; the Australian success rate was 33%.

In the main, the undergraduate students were considerably stronger in problems that were purely mechanical, such as finding and applying surd rules (Question 1), finding the cube root (Question 2), substitution (Question 3), and factorizing a quadratics (Question 6) and the margin of superiority was less for the problem-solving activities based on relatively simple arithmetic or basic algebra such as the questions presented in detail above. Consistent with cognitive load theory, the data suggest that while surface or procedural knowledge is a pre-requisite for more advanced problem solving, it alone does not necessarily guarantee higher level problem solving.

There was no post-test subsequent to the teaching of this material in China, but for the Australian students a number of questions were tested as part of the final assessment at the end of the course (2016). Needless to say the results were much better. The point of the paper is not to assess the effectiveness of an intervention, but to question whether basic content needs to be accounted for in this institution and potentially in other Australian middle school teacher preparation programs. Thus, the entry data are the critical data, rather than exit data after an intervention.

Discussion

With respect to enrolment selection in Australia, the use of proxy measures of mathematical knowledge such as those based on courses completed is typical of graduate entry to secondary mathematics teaching, and has been criticised by earlier authors (e.g., Ball, 1990; Ball et al, 2005; Burghes & Geach, 2011; Hill et al., 2005; Masters, 2016; Qian & Youngs, 2016; Speer et al., 2015). The data in Table 4 provide empirical evidence to support the assertions of these authors.

The sample of assessment formats across Australia, listed in Table 1, illustrates that learning opportunities and program structures are similar across Australia in that one or two mathematics curriculum courses are allocated to graduate entry trainee teachers and undergraduate trainee teachers after they complete specialist mathematics subjects. The normal modes of mathematics curriculum assessment across Australia use take-home assignments to assess and grade students; mostly, these assignments are associated with resource construction, essays, reflection, and lesson planning. While these forms of assessment are valuable, by their nature they tend to focus on one or at most a few mathematical concepts or pedagogical issues and frequently the topics are self-selected. In this way the data in Table 1 lend support to the concerns expressed by a range of authors (e.g., Chang, 2002; Keeling & Hersh, 2012; Kotzee, 2012; Marginson, 2002, 2006; Meyers, 2012; Sahlberg, 2006) that there may be a drift away from high demand with respect to expertise and fluency with discipline knowledge in some Western institutions. A point of difference between the course at the study institution and other Australian mathematics curriculum courses surveyed is that the case study subject course assesses mathematics knowledge upon entry and at the conclusion of the course. This final 3-hour closed-book test has components of MCK and PCK such as capacity to diagnose student thinking from student scripts and detailed teaching sequences for over a dozen key middle years' concepts. It would be difficult to pass such a test without a robust knowledge of middle years' mathematics content and specific pedagogy.

Descriptions of the Chinese trainee students' academic learning time indicate approximately 800 hours of pure mathematics learning (hours of lectures and workshops) prior to commencing curriculum and pedagogy courses. Specific mathematics curriculum courses amount to 100 hours of lectures and workshops. In addition, these beginning teachers receive ongoing professional development and are supported by experienced teachers in the workplace. The conditions of entry and general structure of bachelor programs conform to the Chinese processes described by Burghes and Geach (2011) and are consistent with other analyses of Chinese teacher training research (e.g., Fan et al., 2004; Lai & Murray, 2012; Li, 2004). Entrance to teacher preparation in China included formal examinations in mathematics to enable prospective teachers to demonstrate the currency of their mathematics knowledge. The academic learning in the undergraduate pathway in the Australian institution was about a quarter of the Chinese quota. The academic learning time for teaching of specific mathematics curriculum (pedagogy) in the Australian institution amounted to about a third of the Chinese quota of lectures and workshops. In Australia in general, including in the study institution, much greater use is made of self-study and online lecture capture.

The data on mathematical knowledge of the Chinese sample very closely mirror outcomes reported by Burghes and Geach for Chinese trainee teachers (2011). That is, for most of the Chinese cohort the questions based on the foundations of mathematical facts and processes were trivial. These data support the commentary of earlier researchers (e.g., Dai & Cheung, 2004; Fan et al., 2004; Li et al., 2008; Ma, 1999; Tatto et al., 2015; Wang et al., 2004) that Chinese teachers tend to be strong on MCK, including fluency with basic processes. According to cognitive load theorists, the ongoing testing of mathematical content encourages Chinese trainee teachers to commit key mathematical memory into long-term memory; that is, to progress towards meeting Sweller's (2016) definition of an expert. Further, the course structures illustrate that throughout the teacher education program there are considerable opportunities to link MCK to pedagogical practices both via the curriculum courses and in school-based internships. In other words, fluency with school mathematics concepts can be developed into mathematical knowledge for teaching. Cognitive load theorists (e.g., Kirschner et al., 2006; Chen et al., 2016), Asian educators (e.g., Gu, Huang, & Marton, 2004; Huang & Leung, 2004; Lai & Murray, 2012; Li 2004; Zhang et al., 2004) and general educational theorists (e.g., Ericsson et al., 1993; Hattie, 2009) consider deliberate practice over significant time to be necessary to develop expertise. The Chinese teacher education programs offer such opportunities to a greater degree than do the Australian programs.

From an accountability perspective, the data illustrated in Table 1 indicate that, in Australia, trust and flexibility of delivery and assessment are held in high regard. A range of authors (e.g., Ball, 2010; Ball et al., 2005; Dinham, 2015; Elton, 2000; Hill et al., 2005; Keeling & Hersh, 2012; Marginson, 2002, 2006; Meyers, 2012) have cited issues of governance and the role of universities in the market place. Marginson (2006) comments that universities in Australia, including top "sandstone" universities, have "focused more on numbers and revenues than positional value and student quality" (p. 29). Certainly, the international commentators (e.g., Ball et al., 2005; Burghes & Geach, 2011; Dinham, 2015; Fan et al., 2004; Sahlberg, 2006, 2007, 2011a, 2011b; Tatto et al., 2015) recommend that a system-wide, cohesive and comprehensive approach to teacher preparation is needed.

The results based on tests of the Australian middle school trainee teachers are little better than those for most European trainee primary teachers, and below those for Chinese, Japanese, and Russian trainee primary teachers (Burghes & Geach, 2011). This is concerning, not least because European primary school teachers are generalist teachers whereas the Australian secondary teachers will have two discipline areas. The low level of fluency with basic middle school mathematics confirms the earlier articulation by Australian authors that there might be reason for concern with respect to the levels of preparedness of Australian secondary teachers to enter classroom practice (Hine, 2015; Masters, 2009, 2016; QAO, 2013; TEMAG, 2014). The data do not mean the average graduate of this particular program will enter the classroom grossly unprepared: some of the trainee teachers of both the undergraduate and graduate pathways scored well on the basic entry tests, and many more learnt quite a lot of MCK as well as how to teach it over the following 7 weeks. Some trainee teachers had histories of self-directed learning, including the completion of doctoral studies in mathematics or science.

A number of authors in various fields (e.g., Kirchner et al., 2006; Meyers, 2012; Muller, 2009; Muller & Taylor, 1995) claim that Western education has become hostage to the view that teachers, as with other learners, can be assumed to be responsible to teach themselves or peer teach through shared learning and everyday experiences. Poulson (2001) argued that, for primary teachers, formal explicit discipline knowledge of mathematics might not be necessary for effective teaching of mathematics. Further, Poulson noted that much of what a teacher needs to know in order to teach effectively is learned on the job, including while teaching particular concepts. Indeed, this idea is implied in the widely cited descriptor "initial teacher education" rather than the more behaviourist "teacher training". If "pre-service teacher education students" are just embarking on a journey of lifelong learning, the shortcomings of not knowing the sorts of mathematics reported in this paper become less problematic.

The potentially negative interpretation of the Australian data is only valid if it is accepted that teachers ought to graduate with high levels of MCK, and that fluency with basic facts and processes is understood to be essential for effective teaching. Most educational theorists and report writers accept that this is a given (e.g., Australian Academy of Science, 2015; Ball, 1988; Ball et al., 2005; Burghes & Geach, 2011; Goulding, Rowland, & Barber, 2002; Hill et al., 2005; Krainer et al., 2015; Masters, 2009, 2016; TEMAG, 2014; U.S. Department of Education, 2008). Cognitive load theorists (e.g., Chen et al., 2016; Kirschner et al., 2006; Kirschner et al., 2017; Sweller, 2016) provide an explanation based on cognitive architecture as to why the MCK including fluency with middle school mathematics ought to reside in a teacher's long-term memory. They and others (e.g., Ball et al., 2005; Englemann, 2007; Hattie, 2009) argue that, without such knowledge, beginning teachers will be challenged to scaffold student learning, especially for those children most in need of explicit scaffolding, the most novice of high school students.

Conclusion

The second author was not surprised by the Chinese fluency in basic facts and processes. He was not familiar with the terminology of cognitive load theory but instantly recognised its manifestation in the Chinese educational process. The priority for him was to ensure that fluency in basic facts and processes was associated with deeper problem solving and was connected to domain-specific pedagogy. In this regard, MCK of the form tested can be regarded as what Hattie and Donoghue (2016) described as lower level learning, to be connected via the teacher education program to deeper mathematical understanding and creative thought associated with teaching. This is a significant task, but there was considerable opportunity to pursue these goals.

Across Australia there is a commonality of program structure for middle school mathematics teacher preparation in terms of the number of courses and the considerable flexibility of entry requirements. At the course level, there is considerable flexibility with regard to delivery format, allocated academic learning time, and assessment modes. This flexibility allows individual academics to treat mathematics curriculum courses as capstone courses that build upon the MCK that trainee teachers enter with. The Australian mathematics curriculum course modes of assessment indicate that MCK related to the specific mathematics to be taught is largely assumed. The data presented in this paper call into question the wisdom of this assumption. Testing data at entry to the mathematics curriculum course, especially with regard to the graduate entry pathway, indicated that most were not fluent in the mathematics they were being qualified to teach; according to most mathematic educational theorists and cognitive load theorists, to adjust program and course structure to account for this knowledge would seem necessary.

The challenge for the Australian academic was to attempt to build fluency and depth of understanding in content and connect this with effective domain-specific pedagogy within the constraints of a single mathematics curriculum course. The Australian program structures and reported condensed course delivery opportunities forced the academic to prioritise what knowledge forms are most important. In prioritizing, the authors consider the advice of cognitive load theorists that, in the absence of substantive discipline knowledge, general pedagogy is likely to be an empty vessel. Other English speaking Western teacher preparation nations might consider whether similar data are prevalent across their teacher preparation institutions.

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