Reviewer A:

Comments and constructive feedback for the author(s) regarding its importance of the contribution to the field and suitability and readiness for publication.:
This paper looks at pre-service teachers' knowledge of and beliefs about pre-formal and formal proofs, particularly geometric proofs. The study is conducted using a focus on the triangle postulate (the interior angle sum in a triangle is 180°) [It should be noted that the triangle postulate is NOT being treated in this paper as a postulate but rather as a consequence (theorem) arising from the parallel postulate to which it is equivalent.]

The comment is well noted.

(In Discussion, p. 20, lines 1-8 after Figure 5).

It is our attempt in this investigation to testify the proficiency and affinity of using various pre-formal proofs about the Triangle Postulate. In the local mathematics curriculum, statement, deductive proof, and application of parallel postulate is a core content in lower secondary syllabuses. While the equivalence of the two postulates are not emphasized. In local curriculum, it does not specifically call the Triangle Postulate but the equivalence statement: “Sum of the interior angles of a tringle is 1800 “. Lower secondary students start to learn the geometric deductive reasoning in proofs through the justification of the statement (proposition).

The textbook’s topic of Hong Kong does not particularly demonstrate how to prove from the parallel postulate to Triangle postulate, but from the parallel postulate to the truth of “Sum of the interior angles of a tringle is 1800 “ (Treated as a Theorem)

The paper has the potential to be of interest to readers of MTED but there are a few aspects that could be improved or need to be addressed.

The Introduction and the Theoretical Framework sections are a bit muddled and there is some overlap that means that some of the same ideas seem to be revisited. It is not entirely clear to what extent the framework is actually a framework,….

We have revised the Frameworks, some previously done researches have been inserted etc.

Instead of using the term “Framework” we change to “Background” (Please see the sections Introduction and Theoretical Background on p. 2).

…. rather just the definition of some terms for different types of "proof" (and I am not sure that pre-formal proofs are necessarily "prematurely constructed", especially when they are advocated as a tool to assist less capable students). There is a brief discussion about teachers'

A rigorous description of proof is given by Hilbert (1928/1967) (see Theoretical Background on p. 2)

The whole study is diagnostic by nature – to understand how preservice teachers and novice teachers’ proficiency to use pre-formal proof and interconnect it with formal proof in deducing a geometric property or proposition.

Pre formal proofs are prematurely constructed – rough sketch, draft tabulation in connecting relationship between variables, etc., are not mature enough being a formal proof generally accepted by mathematicians. It is not example as well (Under *Preformal proofs versus formal proofs*,lines 10-15; p. 2)

…use of pre-formal and formal proofs (just before Figure 1)…., which needs some justification. Indeed, I wondered to what extent teachers will have met pre-formal proofs themselves anyway, or discussed their mathematical or pedagogical value.

(In Theoretical Background, pp. 4-5)

The use of pre-formal and formal proofs as well as their values are discussed in Theoretical Background, under the subheadings *Pre-formal proofs, visualization and formal proofs* and *Pre-formal proofs and formal proofs in mathematics teacher education*.

A competent teacher should possess the attributes stated by Vermunt (2007) (In Discussion on p. 20)

The methodology section is not particularly clear, particularly the development of the analysis, the coding and the rubrics. There are references to Appendix 2 but it would be better still to refer to the corresponding relevant tables in Appendix 2 (I hadn’t actually realised Appendix 2 was 13 tables long, which seems rather long for an appendix). In general, an appendix is for things the reader might LIKE to refer to but should not HAVE to refer to. In this case, I think the reader really does need to refer to some of the appendix material, in which case it should be in the main body of the paper.

Some explanation has been input, (extracted from Appendices in the previous version) and presented in Methodology, pp. 5-9.

It may make sense to include some of this coding information in the results section when you talk about the specific components of the questionnaire, rather than having it in the methodology because the reader will be more focused on the particular questions when looking at the results and so the coding will make more sense.

In the section Results and Findings (starts at p.9), amendments have been made accordingly.

Some of the coding tables could probably be omitted entirely, provided there is enough detail. (e.g., Table 1 in Appendix 2 about mathematical/didactical/other reasons for accepting or rejecting a particular pre-formal proof probably needs only a short description. For example, you probably only need to say that you looked at the reasons for accepting or rejecting each pre-formal proof to see if the justification was based on mathematical or didactical reasons or some other grounds.) If you do not bring the coding tables into the results section, then you must make clear reference to the relevant table in the Appendix 2 FROM the results section.

The revisions about coding are done according to reviewer’s advice.

I think there is a significant problem about the request for a formal proof of the triangle angle sum theorem. The standard proof relies on the construction of a line parallel to one of the sides through the vertex that is opposite that side. This clever little "trick" has to be either known, or, in the absence of having met the proof before, invented by the person constructing the proof.

I suspect that if you have not met the proof before, it is unlikely that you will think to construct such a parallel line.

(Last three paragraphs in the Discussion, pp. 20-21)

We believe that competent teachers’ idea of and approach of effective teaching can be stimulated. The participant teachers can obtain the insight when they are going through the questionnaires about the three methods of tiling the interior angles of the triangles. This tile will *prompt* their insight. Teachers need not get the idea of construction of parallel line solely by recalling of prior knowledge.

In fact, parallel lines are not unique construct to justify the proof. They can recall anything’s (eg the tiling of interior angles through the transformation of the triangle, as in the 2nd method in the questionnaire) or get the idea to create anything that is an essential construction for an aid for a formal proof.

Pre-formal proof helps teachers to build up the essential ingredients of a formal proof. (Results and Findings, p.11, after Figure 3; Discussion, p.20, paragraphs 3-4 after Figure 5).

This means, I feel, that the all of the proofs that are successfully completed are done by "recall" of an already seen proof, rather than constructing a proof. If a person cannot prove the triangle angle sum theorem, then it may only be because he/she cannot recall the construction, and NOT because he/she does not know about proofs. I strongly assert that saying "I don't know" or "I forget" (below Figure 4) is NOT necessarily an indication of low competence "with respect to understanding what constitutes a proof . . . and in formulating it". Similarly, I do not think that being able to recall a proof necessarily indicates high competence.

(In Discussion, pp. 20-21) Vermount (2007) discuss about competency. Including “insight”, the item questions shown in the three Methods in the questionnaires have already stimulated teachers’ “insight”. It is not simply a recall of constructing a parallel line for the formal proof.

Detailed results of unsuccessful attempts are presented (Results and Findings, p.12, 1st paragraph) and discussed (Discussion, pp. 20-21)

It would be good to see examples of responses that were coded in various ways (e.g., in reference to Q5 (Tables 8 and 9 seem to be rather odd ways to represent this data, and it would be good to see examples of some other levels of response).
Revised accordingly (Results and Findings).

I think you need to be careful about the conclusions drawn. For example, in the discussion section, there is a claim that the teachers appeared unprepared to teach the concept of the angle sum, but all you can claim is that they appear unprepared to teach it spontaneously (i.e., with no warning that you want them to teach it right now, and no chance to prepare or think about how to do it). Just because someone does not know the formal proof right at this moment, does not mean that they could not review the proof and plan a good lesson.

Yes, that is true. Someone can make effort to prepare for a good lesson of teaching proofs. We revise our arguments in various places in the paper.

It is the expectation of the authors that those participate teachers should have leant the formal proof of the Triangle Postulate indirectly by reading the three methods of pre formal proofs demonstrated. A competent teacher can interconnect the approaches of the two kinds of proofs (pre formal and formal) with some hints provided.

Contingency (Tim Rowland 2005, 2007, last paragraph of Discussion, see on p. 21) is one of the teachers’ knowledge competencies described in the *knowledge quartet* of mathematics teaching model. A competent teacher can create a formal proof (described in this paper) without prior preparation when students suddenly ask a formal, deductive formulation of the proof after the teacher has demonstrated the pre formal proofs in a lesson.

Also, it is obvious that it is important to know a formal proof of the triangle angle sum theorem in order to teach it, but your claim seems to suggest that this is the only thing that you need to know; …

We revise our tone for not to argue in an exclusive manner.

We argue it in the perspective of knowledge, beliefs, and practices of proofs (Theoretical Background and in the Discussion).

…you certainly have not investigated what good didactical approaches would be good to teach it well and whether or not the students know this too. These would be important factors to take into account when talking about preparedness to teach.

Knowledge of pre-formal proof is not sufficient, but necessary.

We do not intend to investigate the best (or better didactical approach), but to say that it there is a good approach, knowledge on pre formal proof and its transition to formal proof is necessary.

The second paragraph, and the unexplained quote, in the section on "Affinity for pre-formal proofs . . ." could use further clarification, and the quote needs to be introduced better.

Revised.

I am not sure how helpful Appendix 1 is.

Revised.

Clarity of expression, freedom from errors, presentation of references in APA/MTED style::
In general the paper is readable, but it would benefit significantly from the attention of someone with strong English language skills as there are a few unclear or unusual expressions.

References in APA/MTED style are adopted…

Language editing is done.

It would be good to have subsections in the Results section of the paper, aligned with the six questions asked of the participants. These questions should be repeated in this section so that the reader does not have to refer back to the list at the end of the Methodology.

Subsections are inserted.

Please specify more clearly that pre-service secondary teachers are involved, and discuss what sort of mathematical background they are likely to have (i.e., what proof experiences are they likely to have had as students; are they likely to have met pre-formal proofs; are they likely to have had any discussion about the nature and purpose of proof in mathematics AND the nature and purpose of proof in school mathematics teaching).

In popular introductory course of Linear Algebra, Calculus, Analysis etc they studied in their early university year, students learnt how to use Venn diagrams to justify the identities of sets’ operations. In which they learnt the characteristics of sets such as inclusive and exclusive properties, as well as the unions and intersections of sets. Venn diagram by nature is a kind of sketches or pictures to represent the sets’ operations. This illustration is less rigorous than a regular proof via logical arguments with sequential statements, in terms of various steps, of axiomatically deductive reasoning.

We have investigated PTs’ backgrounds and correlations with their views (for instance, Results and Findings, p.17, Table11)

The references have some APA errors (e.g., some references have missing or too many initial capitals; some references are missing a complete list of editor names; some references are missing place of publication; there are page references in the wrong place; the Ball and Bass reference is NOT from the Melbourne PME conference; missing commas before &, Kuchemann & Hoyles reference is incomplete (need more details about PME conference), Lakatos reference is odd; Mullis et al. should have TIMSS spelled out; Niss conference reference is incomplete; Reid 1993 reference seems incomplete; Rips reference should be MIT not Mit (or perhaps even spelled out); Weisstein is missing publisher details)

Correct references style has been fixed accordingly. Correction of each reference mentioned is made.

Required amendments (if recommend for acceptance subject to completion of
revisions):
See advice above

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Reviewer B:

Comments and constructive feedback for the author(s) regarding its importance of the contribution to the field and suitability and readiness for publication.:
This empirical paper is quite well written on the whole with a strong logical structure. However, it would benefit careful proofreading.

Revised.

The paper addresses an important area of mathematics that continues to be difficult to learn and teach effectively. Mathematical reasoning and proof in particular may appear very abstract especially to pre-service teachers who are focusing on what they need to learn to be effective in the classroom. This study considers their competency in pre-formal proof, so I'm not sure why there is some mention and analysis of formal proofs.

In the literature backgrounds (pp. 4-5), we state the importance of both proofs in teaching. There is connection between formal and pre-formal proofs, it is our intension to see how teachers well-prepare both types of proofs for a proposition (using Triangular postulate as an example).

I think the description of the analysis and results is unnecessarily dense and needs to focus more tightly on one particular aspect of the data. I believe readers of this journal would prefer a more focussed presentation of data and analysis. I think the paper tries to present too much analysis and too little discussion.

We modify the arguments/discussion in focusing PTs’ knowledge, practices, and beliefs about proofs (Theoretical Background).

Appropriate relevant literature is cited to support the development of theoretical  framework for the paper.
Note “Framework” has been changed to “Background”.

Clarity of expression, freedom from errors, presentation of references in APA/MTED style::
Quite a few errors in expression
Language editing is done, revised ambiguities.

Required amendments (if recommend for acceptance subject to completion of
revisions):
Narrowing of focus to fewer aspects of the data

We have paid more attention to the teachers; knowledge on both proofs

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Reviewer C:

Comments and constructive feedback for the author(s) regarding its importance of the contribution to the field and suitability and readiness for publication.:

The study on which the paper draws has the potential to make a modest contribution to our understanding of how we can improve teacher preparation for the teaching of proof in the classroom. Please see attached annotated manuscript for my recommendations on how the manuscript can be rewritten and improved towards achieving this potential.
Revision done according the comments given in the Word file of the paper.

PTs verses teachers with little classroom practices / experiences (we cannot just call them in-service, as they are not completely qualified-not acquiring Qualified Teacher Status yet, by the time of filing the questionnaires) (p.5)

Our definition: novice teacher.

Clarity of expression, freedom from errors, presentation of references in APA/MTED style::
The paper's writing is relatively clear but overall awkward. Please see attached annotated manuscript for my recommendations on how the writing can be improved.
Done

Required amendments (if recommend for acceptance subject to completion of revisions):
Please see attached annotated manuscript.

Revised accordingly

Research questions have been revised and trimmed down in a specific scope that echoed by the collected data (pp.5-6).