Action Research on the Application of Variation Theory in Mathematics Teacher Education

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This article reports on action research that was undertaken while teaching a mathematics education subject to a series of cohorts of final-year primary pre-service teachers at a university in Australia. My aim was to increase the understanding of an assessment task, as well as its intrinsic value. Each action research cycle involved the introduction of a variation in the assessment task's instructions, a process informed by variation theory. The variations were introduced through cycles of planning, action, data collection, and reflective analysis related to clarification of the assessment task. The two theories (variation theory and action research) proved to be mutually supportive. Cycles of action research served well to identify the critical features that needed to be the focus of small variations in my presentation of the assessment task. Variation theory provided guidance as to a most effective way of highlighting these features, especially with regard to foregrounding varied requirements of the task.

Keywords · pre-service teacher education . open-ended problems . assessment tasks . variation theory . action research

Introduction

In schools as a teacher, I had always designed assessment tasks that were not only for the assessment of learning but were also teaching and learning tasks, and this is a principle that I have carried into teacher education. Of course, assessment for learning is not a new idea. It was used by Doyle (1983), and later developed extensively by Black and Wiliam (1998a, 1998b).

The Australian Curriculum Corporation supports the notion of "Assessment for learning". Using the definition of the UK's Assessment Reform Group (2002), they defined it as "the process of seeking and interpreting evidence for use by learners and their teachers to decide where they are in their learning, where they need to go, and how best to get there". Likewise, the Education Department in my state, Victoria, explains to teachers that:

> Assessment FOR learning occurs when teachers use inferences about student progress to inform their teaching. It is frequent, formal or informal (e.g. quality questioning, anecdotal notes, written comments), embedded in teaching, and provides clear and timely feedback that helps students in their learning progression. It has a formative use providing evidence that informs, or shapes, short term planning for learning. (DEECD, 2013)

As a teacher educator, my desire to have pre-service teachers learn through completing assessment tasks is continually reinforced by positive student feedback. Many lecturers in the faculty talk with our pre-service teachers about formative assessment tasks, and I make a point of modelling this principle — and students appreciate this, according to the University's regular student evaluations of teaching. Also, the primary pre-service teachers enrolled in the subject reported on here are all off campus, completing the subject wholly online, and I know from experience that with their busy lives during the final trimester of their course, they tend to focus the majority of their attention on completion of assessment tasks more than on learning for its own sake. Therefore, it is important to provide them with rich assessment tasks that connect their prior experiences with their everyday environment (Lloyd, 2013) for their development as teachers in the coming months.

The focus of our study is based on the research question, "How can we use action research to improve the educational design of an assessment-for-learning task, as well as students' understanding of the task, in our mathematics education unit?" With this aim in mind, a series of changes have been introduced in the unit of study (subject). This paper focuses on the variations made in the presentation of the assessment task and the reasons for them.

Theoretical Framework

The variation theory of learning offers a way of analysing and describing how learning takes place. The theory grew out of phenomenology, with how learners engage with the "objects of learning" (Marton, Runesson, & Tsui, 2004, p. 37) being the phenomena that are studied. In particular, the focus is on "critical features" of the content:

> … when teaching, we should take as our point of departure what is to be learned (i.e. object of learning). For every object of learning and for every learner there are critical features that the learners must be able to discern; critical features are critical because the learners participating in the study have problems with them, and different learners may have different kinds of problems. (Ling & Marton, 2012, p. 9)

Through comparing and contrasting critical features of the objects of learning with prior understandings that have been developed through previous learning experiences, knowledge is gradually refined. Seeing phenomena in new ways is at the core of learning.

In this research, the variations were made in an assessment task, and my aim was to increase students' understanding of the task. Thus the "object of learning" was the assessment task itself. I was keen to have the students understand the task better and hence to improve the outcomes.

Squares (An Example of an Object of Learning)

The features of variation theory just described are most easily explained with an example, so I will describe my students' (pre-service primary teacher education students) learning about squares.

If I ask a lecture room full of my students what the shape in Figure 1 is called, about 80% of them generally say it is a diamond, while relatively few will say it is a square.

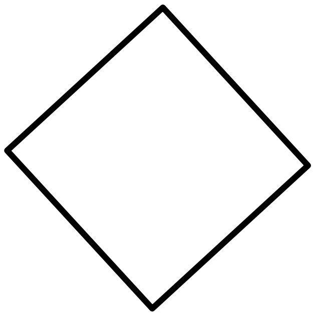

Figure 1. What is this shape?

The discussion that follows in the lecture room focuses on critical features of squares: four equal straight sides and four right angles. Then I ask the question again, and there is usually general agreement that the shape is a square. That is, our defining of a square has challenged the students' prior experience of traditionally-oriented squares, and they have realised that the orientation does not change either the shape or its name. The learning that has taken place has involved the pre-service teachers in "seeing" the phenomena in a new, more discerning way that is relevant to their future teaching.

The focus of the discussion is not about orientation of squares, per se, but this idea is "foregrounded" (Bowden & Marton, 1998; Marton & Booth 1997; Marton, Runesson, & Tsui, 2004). Until that discussion, many of the pre-service teachers have not had their understanding about the orientation of squares challenged since their early childhood experience where the traditional orientation would probably have been ubiquitous: "Reality and experience of the world [were] taken to be one" (Marton & Booth, 1997, p. 148).

With any such discussions and illustrations, the variations made in an object of learning not only focus on what is the same and what remains constant, but also what is different. Marton (2006) described the awareness brought about by experiencing such variation between values as "contrast", seeing this awareness as being a necessary condition of learning.

> Contrasts between values enable learners to discern values. One can never discern just one value. It is the difference that is discerned, and to experience a difference takes at least two values. Through contrast, the learner can discern values as well as the dimension of variation in which they are values. (Marton, 2006, p. 528)

In this comparing and contrasting, learners engage with "instances that we have encountered at different points in time, *at the same time*" (Marton, Runesson, & Tsui, 2004, p. 17; italics in original). This simultaneous processing of prior experience and new experience is a necessary condition for learning (Lo, 2012; Marton, 2006; Marton, Runesson, & Tsui, 2004).

After more experiences with squares oriented in many different ways, which now fit with the students' newly-developed understandings, so are not challenging learning experiences — just reinforcement of the concept, I ask if the same figure — the square — is a rectangle. Many students call out "No, it's a square!" while a few might say it is a square as well as a rectangle. After we look at the critical features of all rectangles, there is usually general agreement on one answer. (I will leave interested readers to explore that question with their own students.)

The teacher's role

Any teacher's role is to consider aspects of objects of learning that underpin common misunderstandings as well as the salient features of content. Thinking about them involves detailed consideration of students' lived worlds as well as the common understandings that usually have (or have not) arisen from experience.

Here, the teachers' experience of student misunderstandings is invaluable. With detailed consideration, it is possible to plan learning-focused assessment tasks that create the necessary "spaces of learning" (Marton et al., 2004, p. 20) that provide students with opportunities to understand both content and pedagogical processes well and to learn from the assessment tasks.

Designing learning-focused assessment tasks for pre-service teachers involves focusing pre-service teachers' attention on seeing curriculum content and tasks not only through the eyes of a learner but also through the eyes of a teacher; so providing them with a range of experiences with specific elements of relevant mathematics curriculum content as well as of associated pedagogical processes becomes vital. Marton et al. (2004) described the capacity of seeing situations that are critical in any profession as "professional seeing" (p. 11), and this is one avenue we use to try to develop better understanding of our assessment tasks.

The development of one particular assessment task is described below to illustrate how variation theory has been the focus of my action research. As the unit is offered only once per year, each cycle of action research takes one year. That is, variations of the task wording have been planned as a result of analysis of student assessment submissions, student feedback, analysis of online discussions, and staff discussions; all before changes have been made to the written assessment task instructions each year.

As the trimester progressed and assignments were marked, monitoring of the results has been possible. Notes taken by staff at this stage each year, along with consequent staff discussions, have helped to inform variations that needed to be made for the following year. Thus while students were not able to compare and contrast prior understandings of the task that had been developed through previous learning experiences (as in the "Squares" example above), the variations we made attended to misunderstandings and errors of previous students. In fact, you could say that that we (university staff) were the key learners about the assignment task that was the object of learning.

Methodology

To improve our presentation of the assessment task to each cohort of students, we used annual cycles of action research. Action research is an evolving paradigm (Burns, 2005; Masters, 2000). The action research cycle is most frequently traced back to the work of the sociologist Kurt Lewin (1946), although elements of like theory can be found in Dewey's writings. (For a discussion of pre-cursors see Masters, 2000; but the link with Dewey was later refuted by Kemmis, 1960). Lewin pointed out the importance of the participation of practitioners in all phases of social change. For Lewin, this new form of participatory research "consisted of analysis, fact-finding, conceptualisation, planning, execution, more fact-finding or evaluation; and then a repetition of this whole circle of activities; indeed a spiral of such circles" (Sanford, 1970, p. 4, drawing on a range of Lewin's publications).

In education in particular, action research is traced back to the teacher-researcher movement led by Stenhouse (1975), who envisioned teachers taking an active role in curriculum development. Stenhouse proposed that the collection and analysis of data formalises the inquiry processes used by all good teachers. He described action research as a cyclical process of planning, action, monitoring, and evaluation that underpins further planning, action, monitoring and evaluation, etc. Most importantly, though, he focused on the potential for action research to stimulate exploration of everyday practices to enable educators to explore historical, cultural, and ideological baggage; and hence to understand better what previously had been taken-for-granted. Hence, as an approach to participatory research, educational action research is aimed at increasing understanding the social context and limitations of educational practice as well as its improvement (Carr & Kemmis, 1986; Kemmis & McTaggart, 1988).

Action research in education, over the decades, has evolved through various conceptual and interpretive "generations" (McKernan, 1996; McTaggart & Garbutcheon-Singh, 1988). Typically, now, it involves self-reflective communities of professionals in the simultaneous development of educational theory and practice. Teachers and/or teacher educators engage in action research to examine their teaching and evidence of their students' learning as a basis for making changes, so it is more a means to professional development by an individual or group than a way of producing general and generalisable knowledge about teaching and learning.

Definitions of action research vary, but Sagor (2014) pointed out that however it is defined, it has seven steps:

1. Selecting a focus;
2. Clarifying theories;
3. Identifying research questions;
4. Collecting data;
5. Analysing data;
6. Reporting results; and
7. Taking informed action.

The second step involves identifying the values, beliefs, and theoretical perspectives that researchers hold relating to their focus. As outlined above, it was variation theory that I drew on to underpin each of the changes made to an assessment task for pre-service teachers to address the research question, "How can we use action research to improve the educational design of an assessment-for-learning task, as well as students' understanding of the task, in our mathematics education unit?"

I am not the first researcher to combine variation theory with action research in higher education. Åkerlind, McKenzie, and Lupton (2014) described a method of curriculum design that was based on combining phenomenographic research with the variation theory of learning in three stages: (a) identification of concepts needing intensive attention; (b) action research that collected data on variations in students' understandings and misunderstandings of those concepts; and (c) design of learning activities to address the poorer understandings; using variation theory as a guiding framework. This combined method was trialled during a two-year project, using physics and law disciplines as case studies. The researchers found that "phenomenographic action research served to identify the critical features that needed to be highlighted for students [while] Variation Theory provided guidance as to the most effective ways of highlighting these features during curriculum design" (p. 1). A key difference between the research of Åkerlind, McKenzie, and Lupton and mine was that their objects of learning were conceptual aspects of the curriculum content, while my research focused on students' understanding and performance of an assessment task.

The Pre-service Teachers and their Unit of Study

This report focuses on action research that was undertaken with a mathematics education subject that puts emphasis on designing an inclusive program, which is one subject studied in the last trimester of the final year of a 4-year Bachelor of Education (Primary). Its students have a concurrent extended practicum, so it suits them to study the unit online. Thus the students in my unit have no on-campus lectures or tutorials, but there is an online discussion area as well as access to substantial online library facilities. Earlier in their course, the pre-service teachers have all passed at least one unit of Mathematics that was taught by the Faculty of Science and Technology as well as two other units of Mathematics Education. A few have studied more Mathematics as elective units.

The "Problem Pictures" Task

In this unit, the pre-service teachers are assessed partly on their submission of a multimedia resource that focuses on mathematics curriculum. One task is to take a photograph of the everyday environment and to write three open-ended mathematics questions (often called "open questions") that are suitable for different grade levels (see Figure 3). It is this "Problem Pictures" task that has been the focus of my recent research. The image and questions below have been presented to pre-service teachers as an example of possible open-ended questions that can be designed based on an image.

Photograph with permission of Ben West

Question 1, for junior primary children: Describe and sort the shapes you see in the photo. (Teacher note: depending of the grade, the focus might be number of faces, edges, vertices or 2D and 3D shapes).

Question 2, for middle primary children: What time of day might this photo be taken? Explain and justify your reasoning.

Question 3, for upper primary children. Design a grid for the photo. Select an object in the photo and state its coordinates to describe the location of the object.

Figure 3. A Problem Picture with three open-ended questions.

In their prior mathematics education units, the pre-service teachers have met the idea of using open-ended questions (see, for example, Mousley, Sullivan & Zevenbergen, 2004; Sullivan & Lilburn, 1998) in mathematics classrooms, but they have had little practice in constructing these. The "Problem Pictures" learning and assessment task that is the focus of this paper reinforces and assesses pre-service teachers' abilities to construct suitable open-ended mathematics questions. We, the teaching staff in this unit, believe that learning about open-ended mathematics questions is an essential aspect of being able to design an inclusive program.

The most important characteristic of open-ended questions is that they have more than one possible correct solution. In mathematics classrooms, they are useful for fully engaging children with a range of abilities and hence being inclusive as well as using time productively. Take, for example, "A rectangle has an area of $24cm^2$. What might its dimensions be?" Those children who are generally less proficient in mathematics (perhaps using an enabling resource such as some squared paper) might find only one or a few answers to this question during the same time that others will find many correct solutions — perhaps using a logical pattern. With well-designed open-ended tasks, all children can make a start and all will have the opportunity to extend their knowledge. The whole class is working on a common learning task so all are able to make contributions to, and to learn further from, a class discussion about their responses and the mathematical concept being studied.

In Australia, where mathematics textbooks are generally not used heavily and curriculum guidelines can be relatively concise, primary teachers need to be able to create suitable learning tasks — and the use of open-ended questions is one aspect of designing an inclusive program. Hence learning to design suitable mathematical questions and tasks that allow for open entry (all students to start) and open exit (potential for extension and generalisation) is a standard practice in many primary teacher education programs. We aim to reinforce as well as to assess pre-service teachers' understanding of and ability to apply this skill in our unit of study.

For the assessment task, the pre-service teachers are asked to create a set of open-ended questions that are linked to state curriculum content, that arise from a photograph that they have taken. The steps involve taking and submitting a photograph, along with three open mathematics questions that use different mathematical concepts for three differing grade levels.

We do not talk with the students about variation theory; but its principles include that in any assessment task where we want to find out what has been learned, we must introduce variation between the way something is learned and the way it is assessed (Marton, 2006; Smedslund, 1953) to avoid rote learning — this time into technology use. Marton et al. (2004) advised that the main aim of any learning task must be for students to "see" and understand differently: "We try to achieve our aims, not in relation to the situation in an objective sense but in relation to how we see it" (p. 5).

For this task, over the past eight years, eight cohorts of pre-service teachers (n ≈ 2,800) have submitted about 9,800 photographs and posed a total of about 29,400 open-ended mathematical problems, questions, or tasks that were based on their problem pictures. The initial aims for this assessment task were (a) for the pre-service teachers to notice mathematics beyond the classroom walls; and (b) to assess students' ability to write open questions. However, cycles of action research over this time — consciously informed by variation theory — have resulted in important features of the task being better scaffolded and more productive. The progressive variations introduced in the different cycles of action research are described later in this paper.

Data Collection and Analysis

Over an eight year period, the photographs that the students have submitted each time the unit has been taught annually (which we called "Problem Pictures") as well as the attendant mathematical questions that they posed have been analysed. Over time, my idea has been to increase the variety of environmental contexts of the photographs (as evidencing growing awareness of mathematics in everyday contexts) as well as the types of problems posed and the range of mathematical foci and links between these (evidence of knowledge of the primary curriculum and understanding of linked mathematical concepts).

Data were also generated when the pre-service teachers were asked to write about what they thought they had learned from the task, and how. Specific questions that they were asked to answer in writing included "In what ways (if any) did this task help you learn more about teaching mathematics?" and "What, if any, were the benefits of completing the Problem Pictures task?"

While I am the Unit Chair, some lecturers from the other campuses of the university also teach the unit, and discuss potential developments of the unit via email; and a professional colleague in Canada has used adaptations of the task and engaged in discussions with me. Points made by the pre-service teachers, recorded in the unit's online discussion space, also became data — along with any relevant comments from the formal, periodic university Student Evaluation of Units and Teachers. We have also received feedback and suggestions from other academics when describing and sharing data via academic conferences and journal articles (Bragg & Nicol, 2008, 2011, 2013, 2015; Nicol & Bragg, 2009). Data analysis at each of the eight iterations of the assessment task took the form of document analysis (McKernan, 1996) using the pre-service teachers' work samples, online posts, emails, and survey responses; all supported by self-reflection on the verbal input of colleagues and notes taken by staff when marking.

Tests of pre-service teachers' learning outcomes were not possible within the project design, which placed a greater priority on the educational development and task improvement over different cohorts of students. The use of a control group of students was also not possible, as comparison of any current year student outcomes with the outcomes of the previous year's students would not be valid. Thus my aims in this paper are not about providing evidence that the variations in practice worked — at least not in a quantifiable and replicable way — but to describe the main foci for the action research with its application of variation theory.

Findings of the Research

Posing open-ended problems is challenging for many in-service and pre-service teachers. It was evident from my continued engagement with this task that support for creating, posing, and adapting open-ended questions was necessary to develop students' capabilities as mathematics instructors. Through an annual cycle of action research variations to the assessment task have been incorporated. Looking back over the series of changes made sequentially, there were three critical features of the task that were varied: (a) clarification of the task, including choice of pictures and posing of open-ended questions; (b) more focus on inclusive teaching, which brought in varied grade levels as well as within-class differences; and (c) new opportunities for seeing the task not just from the perspective of a student but also through the eyes of peers as well as a child. Details of these three themes are outlined below.

Clarification of the Task: Choosing Photos and Problems

At first students seemed to have trouble identifying aspects of the environment that would make a photograph "interactively" mathematical. Photographs in the initial submissions were largely decorative. For example, one student submitted a picture of a tram (a feature of Melbourne transport and a tourist attraction) and asked questions to do with time, distance, and speed, but the mathematics was not illustrated in the photograph as it would have been in, for instance, a photograph of the tram's timetable. Illustrative photographs did not meet one initial aim of the task: for the pre-service teachers to become more discerning about the mathematics embedded in the everyday environments of children — with potential for subsequent recognition of critical features of environmental factors for their mathematical potential.

Analysis of assignments showed that I needed to be more explicit about the mathematics "residing" in the photograph, meaning the pupils would be required to examine the content of the photograph in order to complete its accompanying problems. Such a problem picture was deemed "interactive". In the spirit of variation theory, some contrasting photographs were described as "illustrative" (Bragg & Nicol, 2011, p. 7). It was not long before "residing", "interactive", and "illustrative" became terms that the students started to use with confidence in the unit's online discussion area — and their choice of interactive pictures improved greatly.

Now a better description of the task is not only in the assignment description, but also in the unit's discussion space. The following is one message posted by me early in each trimester:

> Hi All. This is just a little reminder to start exploring your environment for potential mathematical opportunities for your Problem Pictures task. Take a minute right now to look around you. Can you see something that might be mathematically intriguing? Take a photo, take a few photos. You might not realise the potential right away but can come back to your shots later for inspiration. Remember that the mathematics needs to reside in the photos so that students will <u>use the images interactively</u> to answer the open questions. The images are not just pictures that illustrate what your problems are about. [Administrative notes including a reminder about the due date.] Have a wonderful time noticing the mathematics around you.

In the discussion space, students ask questions and make helpful suggestions about this requirement. A recent post was:

> A good way to test if your question is illustrative or interactive is to remove the photo and see if you can still answer the question. If you can, then the photo is illustrative. (Discussion post)

Also early in the trimester, I now model some suitable photographs and pertinent open-ended questions during several online non-compulsory activities that scaffold the students' understanding of the task. In the discussion space, I present some sample photographs and attached open questions, both to highlight the features of a problem picture and to show how problem solvers need to look closely at the picture to answer the questions. This is supported by a description of what makes the picture interactive and the problems appropriate. (The picture and problems are selected from a collection made over the years, and they are ideal for promoting noticing of quite a range of mathematical concepts.)

The next step in the current action research was to post further photographs without problems, asking the pre-service teachers to write suitable open-ended mathematical problems that require interaction with the pictures. Feedback from academic staff and peers is encouraged, and my online comments intentionally include how the problem pictures could be adapted for (a) different grade levels through varying the grade-level content of the problems whilst maintaining the same mathematical learning intention; and (b) varying the mathematical learning intention (concept and/or skill) while holding the grade level constant. As we discuss what has been changed compared with what has been held constant, and with what implications, the pre-service teachers are learning about how to adapt tasks to suit specific student groups via variance and invariance — although variation theory itself is not mentioned nor part of the course content.

The final scaffolding step related to choice of pictures and creation of open-ended mathematics problems has been to post an open-ended problem for one particular photograph and to invite students to share other photographs or descriptions of settings where this problem might also be asked.

We have found that this set of structured but non-compulsory activities based on the foregrounding of critical features of the assessment task has greatly enriched the learning, as evidenced by most students now submitting varied and suitable photographs together with a good range of appropriate open-ended mathematical problems.

Inclusive Teaching

It is important for pre-service teachers to learn to be inclusive in their teaching. In this unit of study, as well as other units, the students consider the need to cater for gender, social class, race, second-language, ethnic, and special education needs; but for this assignment I focus on use of open-ended questions as a way of catering for a range of ability and achievement levels, including children who are well above or well below national expectations. Here, I am trying to offer the pre-service teachers options other than the ubiquitous so-called "ability" grouping that risks disaffection, polarisation, and the construction of failure (Boaler, Wiliam & Brown, 1998; Dahllöf, 1971) as well as its reproduction of class-based inequities (Boaler, 1997). Turner Harrison (2003) found that teachers are able to give examples of open-ended questions and tasks that engage the full range of children successfully. She noted that it is not only the opportunity for all to have success but also the nature of that success that is important: "the positive sense of the students feeling in control" (p. 77).

In the first years that the Problem Pictures task was used, students showed little awareness of the need to consider inclusivity. Many of their questions tended to be closed or to have a very limited range of responses, and their written rationales for choosing their questions rarely included notions of catering for different achievement levels. To put more emphasis on this, I varied the assignment instructions by including more specific directions for the rationale, as follows:

> “The rationale (400 words) should address the following questions (included relevant citations):
>
> How are these problems inclusive of a range of children?
>
> How would you employ problem pictures in the classroom?
>
> What are the benefits of problem-pictures/open-ended questions in the classroom?”

The set readings that were added to support students’ writing about these points emphasise inclusive teaching of mathematics. Further, the unit already had a set of videoed lectures, including one titled *Teaching Mathematics to Classes of Diverse Interests and Backgrounds* (Sullivan, 2005), that were used in a different assessment task, so the "inclusive teaching" aspect of these is already foregrounded by lecturers in the discussion space.

These changes seem to have been effective variations. There is certainly more awareness shown of inclusive aspects in the unit discussion space as well as in the quality of open-ended questions and the pre-service teachers' written rationales.

Professional Seeing

Variation theory suggests that pre-service teachers need to develop a dual in situ perspective of any object of learning from their experience as (a) a learner; as well as (b) a teacher (Lingefjärd, 2011). We have now set up part of the set task to support pre-service teachers' looking at their problem pictures and questions with a critical, professional eye by looking through the eyes of others: first their peers and then by sample children.

Prior to submission of the problem pictures and questions for grading, the pre-service student teachers now undertake and report on two phases of trialling. First, they arrange — online or with a group of face-to-face peers (perhaps in their practicum schools) — for detailed feedback from fellow students, which might result in several iterations as they engage with several lots of critique. The peers gain insights from experiencing their peers' problems as learners, making further suggestions about the pictures' mathematical potential, and discussing the wording and potential solution processes of the open-ended questions.

It is often easier for other people to pick up poorly worded questions, typographical errors, and potential pedagogical problems than it is for authors of the tasks. The instructions now require that the peers give some answers (plural) for the open-ended questions, state what they perceived to be the learning intention, and suggest ways to enhance the effectiveness of the problem picture in achieving the intended learning. Overall, we have found that the pre-service teachers take this peer tutoring seriously, and student feedback has shown that this implementation of change has been highly appreciated.

A subsequent stage of "looking through the eyes of others" that was introduced was for students to trial their problem pictures face-to-face with at least three primary-school children from the grades for which their questions are designed. They collect the children's written responses as work samples, and complete a written reflection in response to four prompts:

1. Did the student answer my problem picture as expected? If not, why not? State your evidence.
2. What strengths or weaknesses of the students' mathematical understanding were highlighted through this question?
3. Did the problem address the mathematical intent of the question? If yes, how? If no, why not?
4. How might I modify/rewrite my problem picture in light of the students' responses and my reflective process, to address the mathematical intent of the question?

Trialling with children has alerted students to potential problems and has often resulted in rewording of questions, as shown in the following written reflection:

> Reflection on child's response:
>
> My original question was 'How many shapes can you see in the picture? Draw and name them, and state how many edges and corners they have.'
>
> "Lily" had a little bit of trouble understanding what the question was asking and was going to count all of the shapes she could see in the photo, until I explained that I was looking for the number of *different* shapes she could see and not every single shape in the photo. Once this was explained to her, she drew a triangle, rectangle, and square and correctly labelled them.

> The main issue with the question was not the question idea itself but the wording I had originally used. I have modified it so that it clearly states what is required in the answer. The rephrased question is:
>
> How many different shapes can you see in the picture? Illustrate your answer by drawing and naming them, and label how many edges and corners each one has.

Thus the pre-service teachers are now experiencing their problems through two sets of eyes (peers' and pupils') before adapting their assignment content accordingly ready for submission. Trialling and reporting on the results have increased their workload, but the submitted assignments are now of a very high standard and the open-ended questions are usually well worded, error free, and more appropriate for the grade levels nominated.

One Ongoing Variation of the Task

According to feedback from fellow academics (co-teachers) and the most recent round of analysis of research data collected, a small number of pre-service teachers still continue to find it difficult to assess whether their problems are open-ended or closed. One strategy to help such students has been trialled this trimester with a variation of the task description: getting the pre-service teachers to write, after each question, three possible responses. While this inclusion has assisted many, radically reducing the number of closed problems submitted, a few pre-service teachers in the current cohort presented a closed question with (say) one correct and two incorrect responses. Perhaps they understood the requirement as just writing three answers children might offer, whether correct or not. So the next iteration of the task description (for the present cohort undertaking the unit at the time of writing) will ask for "three possible correct responses". This is likely to bring about a better understanding of the nature of open-ended mathematics questions.

In summary, the variations made to the assessment task instructions are detailed in Table 1.

Table 1
Variations Implemented to the Assessment Task through each Action Research Cycle

Variation	Issue Identified	Major Variation Implemented in the Assessment Task Instructions
1st variation	Questions were predominately closed. Analysis of problem pictures indicated that approximately 39% were illustrative and 61% were interactive.	"The problems must be open-ended. After each question write 3 possible answers to your questions." "Ensure the mathematics in the questions resides in the photos. Questions should link directly to the content of the photo so the photo is interactive rather than illustrative."
2nd variation	The mathematical intent of the question was not easily understood and the phrasing was poor.	"Initially trial your questions with your peers. Ask your peers to provide you with some possible answers for the open-ended questions, state what they perceived to be the learning intention, and suggest ways to enhance the effectiveness of the problem picture in achieving the intended learning. Based on their feedback, you may wish to rephrase your problem questions"
	More focused support needed for creating	"Support material modelling Problem Pictures is located online. This material is a work in progress and

	open-ended questions.	will grow with more examples of Problem Pictures shared amongst academic staff and your cohort."
3rd variation	Little variation in the mathematical concepts represented in the questions.	"Each of the content strands of the National Curriculum document must be represented in the range of questions."
	Many of the questions were not engaging or the PSTs had difficulty forming suitable questions.	"We have created an online space for you to examine a preselected photo and invite you to pose or refine questions to share with your peers. Academic staff and your peers will provide feedback on your questions. This is a supportive environment created to assist you in honing your open-ended problem posing skills."
4th variation	The content of some photos was dull and easily found in the classroom – therefore negating the need for a photo.	"Do not create photos of objects you can find easily in a classroom, e.g. dice, unifix blocks, counters, number charts, standard clocks. In the real setting of the classroom you would not use a photo of unifix blocks when you have them next to you. This includes items that are easily portable such catalogues, flyers, portable menus which you would bring into the classroom rather than take a photo of it. Photos of a school or classroom are also not suitable for this task."
	Support for considering creative contexts for open-ended questions.	"Considering creative contexts for your problem pictures is a unique way to engage your students in mathematics. In the online discussion forum we have placed one open-ended problem and a matching photo. We invite you to share a photo or describe a context to match this problem in an interactive manner. Be imaginative!"
5th variation	The wording of many of the problems were clumsy or confusing.	"After trialing your questions with your peers, you must trial every question with at least one appropriately-aged child. Give the child the photo and open-ended question. Ask the child to respond to your question in written or oral form. Reflect on the children's responses to each question. Include relevant citations to readings you have sourced yourself outside the unit materials. You need to address the reflective prompts below. Ask yourself: • Did the child answer my problem picture as expected? If no, why not? • What strengths or weaknesses of the child's mathematical understanding were highlighted through this question? • Did the question address the mathematical intent of the question? If yes, how? If no, why not? • How might I modify my question in light of the child's response?"
6th variation	Questions were not inclusive of a range of	Write an enabling prompt and an extending prompt for each question for your grade. These prompts

	children and some children found the original questions difficult to access or too simple.	address the same mathematical concept as the original problem. The rationale (400 words) should address the following questions (included relevant citations): • How are these problems inclusive of a range of children? • How would you employ problem pictures in the classroom? • What are the benefits of problem-pictures/open-ended questions in the classroom?

Conclusion

In this study, the systematic annual cycles of analysis, planning, implementation, and further data collection of action research served well to identify the critical features that needed to be the focus of small changes in the assignment. Variation theory provided guidance as to ways of highlighting critical features of the assessment task features, especially with regard to choosing pictures and posing appropriate problems, thinking about inclusiveness, and seeing mathematics questions through the eyes of a teacher, of peers, and of potential pupils.

Through using cycles of action research where different aspects of the assessment task have been varied over time, my understanding of this task as well as my grasp of its potential for enabling stronger "professional seeing" by the pre-service teachers have been developed. So not only are students coming to see open-ended questions in mathematics differently, but I am seeing this task and the process of assessment through new eyes. Thus, I believe that employing action research, with a focus on variations to the task, in the manner described above offers teacher educators greater potential to gain insights into their practice as well as to recognise the benefits that an iterative process of variation to assessment tasks has for their pre-service teachers. The changes applied to the task over the years have made it a more comprehensive assignment task that is also now better understood by the pre-service teachers. The teaching staff have noticed an improvement in the quality of problem picture submissions and the task is the subject of many positive comments in the university's formal evaluation of the unit and its teaching.

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