# A Case Study on Specialised Content Knowledge Development with Dynamic Geometry Software: The Analysis of Influential Factors and Technology Beliefs of Three Pre-Service Middle Grades Mathematics Teachers

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This study characterises the development of Specialised Content Knowledge (SCK) with dynamic geometry software (DGS) throughout a semester. The research employed a single-case study with the embedded units of three pre-service middle grades mathematics teachers. Qualitative data were collected, and factors affecting these three teachers' SCK development were compared and contrasted through their experiences during semi-structured clinical interviews. The cross-case comparison indicated aspects such as opportunity to justify ideas in geometry, the level of common content knowledge, and beliefs about technology as influential factors for these pre-service mathematics teachers' SCK development with DGS.

**Keywords** Specialised content knowledge • pre-service teachers • middle grades • instructional technology • geometry

## Introduction

One of the continually evolving questions within the last few decades of mathematics education research concerns the mathematical knowledge teachers should hold to effectively teach mathematics within K-12 schools (Petrou & Goulding, 2011). In this paper, we examine this question within the specific context of the geometry knowledge of middle grades (i.e., grades 6-8) teachers.

Recent reports (e.g., Conference Board of the Mathematical Sciences [CBMS], 2001; Advisory Committee on Mathematics Education, 2006; Stanley, 2008; National Mathematics Advisory Panel, 2008) recommend the design of programs that support middle grades teachers to develop solid knowledge of mathematics with deep conceptual understanding (CBMS, 2001). The Mathematical Education for Teachers (MET) II report (CBMS, 2012) recommends a similar focus on knowledge development along with conceptual understanding for middle grades teachers. The interpretation of deep conceptual understanding in these reports requires pre-service teachers to know more than simply how to "do" mathematics, but also to develop a special kind of mathematical knowledge germane to the work of teaching. Ball and her colleagues referred to this necessary knowledge (2008) as Specialised Content Knowledge (SCK) (Ball, Thames, & Phelps, 2008, p. 400). The construct of SCK, while helpful to understand the knowledge mathematics teachers should develop, poses a challenge to mathematics teacher education programs as these programs must determine how best to develop such content knowledge specific to mathematics teaching in teacher education.

One possible way to accelerate teachers' development of mathematical content knowledge, including conceptual understanding, is through the use of electronic technologies. Literature pertaining to the learning of geometry (e.g., Laborde, 2000; Laborde, Kynigos, Hollebrands, & Strasser, 2006; Forsythe, 2007; Guven & Kosa, 2008; González & Herbst, 2009) suggests the integration of electronic technologies such as dynamic geometry software (DGS) in K-12 classrooms can improve students' and teachers' ability to identify and justify geometrical relationships. While previous research has examined students' and pre-service teachers' learning of geometry with technology (Marrades & Gutiérrez, 2000; Laborde, 2000; Jones, 2000; Gerretson, 2004; Coffland & Strickland, 2004; Hollebrands, 2007; Baki, Kosa, & Guven, 2011), the literature specifically addressing SCK and its development through DGS is limited. In response to this limitation we examined the experiences of pre-service teachers (PSTs) in a technology-integrated geometry course and utilised a clinical interview technique with three middle grades PSTs, to observe how their experiences with DGS during the interviews impacted their SCK. The following research questions guided this study:

1. How does the use of DGS influence middle grades mathematics PSTs' SCK? What are the influential factors that can support the use of DGS to develop PSTs' SCK?
2. How do PSTs' beliefs about technology influence their use of DGS to develop SCK?

## *Background Literature*

Whereas paper and pencil allow for a static exploration of geometry, DGS provides a dynamic environment to learn geometry (Comiti & Moreira Baltar, 1997). It is argued that both of the mediums are needed to be able to thoroughly understand basic geometrical concepts and their properties (Noss, Hoyles, Healy, & Hoelzl, 1994; Healy, 2000). Compared to static representations of concrete alternatives with paper and pencil, DGS are considered more advantageous to learning due to their affordance of the generation and alteration of many geometrical objects when needed (Moyer, Bolyard, & Spikell, 2002). Through the use of the dragging feature, students can compare and contrast geometrical objects hierarchically (Laborde, 2000), construct geometrical objects based on transformations (Jahn, 2000; Hollebrands, 2007), empirically experiment with different geometrical properties, and perceive the need for justification and proving (Marrades & Gutierrez, 2000; Healy, 2000; Healy & Hoyles, 2002).

The study of geometrical knowledge and its development with DGS is fairly evident in the field of mathematics education (e.g., Edwards, 1990; Comiti & Moreira Baltar, 1997; Hazzan & Goldenberg, 1997; Jones, 2000). As the data analysed for this paper addresses geometrical constructions and factors influencing the geometry knowledge, however, we intentionally focus our review on studies contributing to these topics in the development of geometry knowledge with DGS.

*Geometrical constructions.* While Mariotti and Bussi (1998) define a geometrical construction as the "procedure resulting in a figure which will not be deformed by dragging (p. 260)", Healy (2000) refers to their definition as a special type of construction. According to Healy, a construction is called *robust* if dragging cannot deform it; otherwise it is called soft if it cannot satisfy this condition. Research (e.g., Noss et al., 1994; Hazzan & Goldenberg, 1997) demonstrated students have a tendency to use DGS to create soft constructions of geometrical objects "by eye", but also indicated robust constructions are vital in order for students to practice reasoning beyond visual orientation (Healy, 2000; Sinclair & Robutti, 2013). For example, properties of a parallelogram softly constructed by a student who practices reasoning based only on visual orientation cannot be compared with the properties of a rectangle because parallelogram and rectangle for such a student would be viewed as two distinct, but not related, figures. On the other hand, if the student robustly constructs the parallelogram, then the dragging feature of DGS would allow the student to observe that there is an instant where the parallelogram is identical to a rectangle. In other words, robust constructions with DGS clarify the link between different geometrical objects and the relationships between their properties.

Although dragging is an important part of the activity to test the quality of geometrical constructions, students cannot have a solid knowledge of geometry without explaining how the steps for their constructions theoretically result in geometrical figures they aimed to construct (Mariotti & Bussi, 1998; Arcavi & Hadas, 2000; Jones, 2000; Sinclair & Robutti, 2013). Sinclair and Robutti's study (2013, p.16-17) of students' constructions with DGS demonstrated the importance of having a robust construction followed by a theoretical explanation. In their study, two students created a soft construction of a square, located the middle point of each side, connected these points through line segments, and conjectured that the internal figure was to be a rhombus because of what they observed on the screen. Since students did not explore the theoretical relationship between the square and the internal figure they constructed robustly, they could not identify the internal figure as another square. The students' attention to the length of the sides of the internal figure, but not to the other geometrical properties (e.g., the congruency of its angles and perpendicularity of its vertices), led them to identify it as a rhombus, but not as a square.

*Influential factors.* Research has revealed several factors that mediate the effectiveness of DGS in students' learning of geometry: knowledge of technological affordances and the ways to utilise them for learning gains (Battista & Clements, 1992), the quality of the tasks a teacher uses (Jones, 2000; Noss et al., 1994), and the quality of mathematical reasoning practiced by students (Healy, 2000; Marrades & Gutierrez, 2000). More importantly, students are more likely to see the relationships among geometrical concepts when they are guided to explain why a geometrical phenomenon occurred within the DGS (e.g., Jones, 2000; Healy, 2000; Marrades & Gutierrez, 2000). Without such questioning, students have a tendency to rely on their visual-based perception, which creates a problem in their understanding of geometrical properties.

Since we approach teachers as the learners of content in this study, we posit the factors we identified as influential factors for students are also applicable for teachers and their learning of geometry with DGS (e.g., Arcavi & Hadas, 2000; Belfort & Guimaraes, 2004; Laborde, Kynigos, Hollebrands, & Strasser, 2006). However, we also address an additional influential factor: *teacher beliefs*. The influence of beliefs on the potential of DGS for teachers' geometry knowledge has a two-sided relationship: teachers' interaction with this type of software affects their attitudes towards instructional technology (Jahn, 2000; Kynigos & Argyris, 2004); and their attitudes towards technology can hypothetically influence their computer use. While teachers' beliefs about technology have been reported in the literature (e.g., Coffland and Strickland, 2004), its influence on the development of PSTs' geometry knowledge has not been examined. In this paper, we attempt to begin to address this gap in the literature. Regarding the main constructs for our study, we define knowledge and belief as cognitive constructs in the following section. Then, as the focus in our research questions, we narrow it down to SCK and *beliefs about technology*.

## Theoretical Framework

For the framing of this study, we defined knowledge as procedural and conceptual propositions that may be projected onto facts negotiated as valid by a group of professional people in the domain (Lemos, 2007). Conversely, we defined beliefs as cognitive entities that have formed and emerged from individuals' experiences, and interpretations of the happenings around them (Kagan, 1992; Pajares, 1992). To differentiate beliefs from knowledge, we examined the degree to which participants' statements or observed behaviours indicated opinions, which are more individualistic, rather than facts.

## *Mathematical Knowledge for Teaching*

To understand the mathematical knowledge required in the work of teaching mathematics, Ball and her colleagues built a knowledge framework called Mathematical Knowledge for Teaching (MKT). According to their framework, MKT was differentiated into subject-matter knowledge and pedagogical content knowledge. These two facets were then further subdivided into three knowledge components. Figure 1 presents each of these components of the MKT Framework (Ball, Thames, & Phelps, 2008). As this study has focused on Specialised Content Knowledge (SCK), we have carefully described SCK as a construct and how it could be differentiated from Common Content Knowledge (CCK).

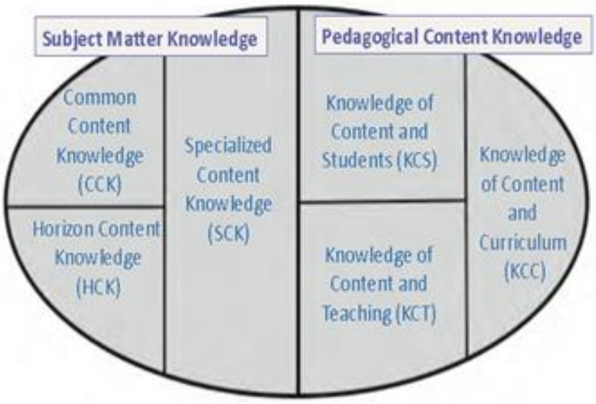


*Figure 1.* Representation of the MKT Framework (Ball, Thames, & Phelps, 2008, p.5)

*Common Content Knowledge* (CCK) is defined as "knowledge of mathematics that was common across professions and available in the public domain" (Hill, Sleep, Lewis, & Ball, 2007, p. 131). The availability of CCK enables teachers to make correct mathematical statements and differentiate an incorrect definition from a correct one to recognise students' incorrect answers (Hill, Schilling, & Ball, 2004; Ball, Thames, & Phelps, 2008). In considering these descriptions, we defined CCK for this study as the geometry knowledge an undergraduate student who is not necessarily majoring in education may develop through his/her tertiary study. For example, a mathematics teacher knows that $(x-h)^2+(y-k)^2=r^2$ is the algebraic representation for any circle where $r$ denotes the length of its radius and $(h, k)$ denotes its center. In addition, the teacher also recognises that a circle on the Cartesian coordinate plane is not a function by applying the vertical line test. Both of these examples are a part of the teacher's CCK because the fact about the circle's algebraic representation and the procedure to determine whether a given shape is a function or not can be acquired, developed, and conceived by people in other professions as well.

*Specialised Content Knowledge* (SCK) is the knowledge of mathematics needed specifically for the teaching profession (Ball, Hill, & Bass, 2005). Even though Ball and her colleagues in their studies did not specifically emphasise the content of geometry, we adapted the same definition of SCK pertaining to geometrical concepts. The following teacher skills would describe what constitutes as SCK and how it is different from CCK: providing reasons for common procedures in geometry, understanding the geometrical principles behind students' unusual procedures, recognising what is involved in a geometrical representation, and linking representations to underlying ideas in geometry (Ball, Thames, & Phelps, 2008; Herbst & Kosko, 2012). One of the main characteristics of SCK which differentiates it from CCK is it consists of knowledge indirectly used for students' learning, but is not taught *directly* to students (Markworth, Goodwin, & Glisson, 2009). That said, a teacher does need to have and develop SCK, a special kind of subject-matter knowledge, not in order to teach to students the same as CCK, but to utilise it when needed. In this respect, we consider that SCK is secondary to students' learning of mathematics during instruction. The teacher focuses on decomposing his/her CCK to achieve content-related learning goals, but utilises his/her SCK to be able to overcome possible problems with respect to students' learning when needed. For example, a student might ask why s/he is using the vertical line test to identify if a given graph of a circle is a function or not. The answer to this question is not in the scope of the instruction if the teacher has not included it within the learning goals. In other words, learning the reasoning behind the vertical line test might not be planned as a learning goal. However, a student might still ask such a question; and availability of SCK for the teacher about this concept helps him/her strengthen the student's understanding. Regarding this example, conceptual knowledge can be a part of the teacher's SCK as long as s/he does not aim or plan to teach it to students. One might wonder whether conceptual knowledge should be categorised as CCK or SCK. Ball and her colleagues raised the same concern (Ball, Thames, & Phelps, 2008):

> Some might wonder whether this decompressed knowledge is equivalent to conceptual understanding. They might ask whether we would not want all learners to understand content in such ways. Our answer is no. What we are describing is more than a solid grasp of the material. (p. 400)

Ball, Thames and Phelps (2008) considered that students might need to learn each detail for subject-matter knowledge that would enable them to understand the content conceptually. To expand the discussion about labelling conceptual knowledge as CCK or SCK, the authors gave an example:

> The mathematical demands of teaching require specialised mathematical knowledge not needed in other settings. Accountants have to calculate and reconcile numbers and engineers have to mathematically model properties of materials, but neither group needs to explain why, when you multiply by 10, you "add a zero." (p. 401)

Regarding our example, and the example given by Ball and her colleagues above, SCK includes conceptual understanding, but it is not equal to that. It is more than that. In Ball and her colleagues' example, teachers' knowledge of why they "add a zero" while multiplying a whole number by ten is labelled as SCK. It is a kind of trick for students' computational fluency. The vertical line test in our example is another trick that teachers would use for procedural fluency. However, a student might still ask why it does always work, and what mathematics is behind this trick. A correct response to this student's why question would be the result of the availability of the teacher's SCK. The teacher needs to decompose his/her conceptual understanding about functions to answer this question.

A narrowing of focus to literature addressing pre-service mathematics teachers' development of SCK produced limited results. One reason for this limitation could be the difficulty in differentiating SCK from other types of subject-matter knowledge or from aspects of *PCK* (Speer & Wagner, 2009). This difficulty could also explain decisions to orient studies on *MKT* in general rather than SCK, specifically (e.g. Hill, 2010). Our search did uncover two studies that examined pre-service teachers' SCK development (Morris, Hiebert, & Spitzer, 2009; Bair & Rich, 2011).

Morris and her colleagues (2009) investigated the development of SCK during teachers' pre-service education. In this study, SCK was defined as knowledge of mathematics unique to teaching mathematics, and considered as necessary knowledge for teachers to develop skill in specifying and unpacking learning goals into sub-concepts. In this respect, the authors examined how pre-service elementary teachers unpack learning goals into sub-concepts for planning, evaluation, teaching and learning. The participants responded to mathematical tasks which explored their ability to anticipate an ideal student response, to evaluate an incorrect student response, to evaluate a student's correct work, and to analyse a classroom lesson. Their written responses were coded according to pre-determined categories for each task, and they were scored zero, one or two according to the level of understanding apparent in the response. As a result, PSTs managed to identify sub-concepts for a learning goal in supportive contexts, but could not apply this knowledge for planning, evaluation, teaching and learning. Supportive contexts were the ones in which PSTs solved the problem by themselves, or examined students' incorrect responses. However, participants could not identify the sub-concepts for the learning goal when the context was non-supportive and the sub-concepts were hidden within the learning goal. The study demonstrated evidence for the development of pre-service teachers' SCK, but at the same time, indicated their development was limited as teachers had difficulty in using SCK for instructional purposes. Teachers might have SCK, but its enactment during instruction might come with more teaching experience.

While Morris and her colleagues (2009) studied pre-service teachers SCK through clinical interviews, Bair and Rich (2011) examined the same phenomenon over the span of two mathematics content courses. The authors questioned why some teachers are better in unpacking their SCK while teaching than others. Bair and Rich (2011) defined SCK as unique knowledge of teaching mathematics with understanding, where understanding was defined as having a sense of mathematical concepts as connected and related within the underlying structure of mathematics. The domain of number theory was the focus of the study. A grounded theory approach allowed authors to initially create a framework with five levels and indicators. When this framework was compared and contrasted with the Ball and Bass' (2000) eight descriptors of SCK, the number of levels was reduced to four components with five levels of indicators. According to Bair and Rich (2011), the four main components of their SCK progression are 1) explaining their reasoning, 2) using multiple standard representations, 3) relationships among conceptually similar problems, 4) problem posing. For each component, five levels indicated the progression of SCK from level zero, which only represents the CCK usage, to level four, which shows deep and connected SCK.

For this study, we also considered the geometric understanding behind a technological representation with DGS as an example of SCK, not CCK. When a teacher decides to use a technological tool for instruction, s/he is responsible to give meaning to the representation provided for the accuracy of the geometry knowledge embedded within the tool (Trgalova, Soury-Lavergne, & Jahn, 2011). The required geometry knowledge to give a meaning to the representation or to overcome possible lack of understanding is not something the teacher would transmit to students or shape the instruction around, but a specialised mathematical knowledge needed for teaching.

The analysis of participants' SCK for one of the tasks required us to consider the difference between soft and robust construction defined by Healy (2000). According to Healy (2000), robust construction is necessary for the verification to be a part of the construction. As such, we decided robust construction would be a requirement for evidence of participants' SCK. We also analysed if the participants utilised proactive or reactive strategies. Hollebrands (2007) described the distinction between these two strategies. Proactive strategies are users' actions with DGS, which are planned to achieve a certain result, while reactive strategies are not planned and performed for the sake of exploration without an expected result. Since proactive strategies demonstrate a better "mathematical understandings and nature of anticipatory and reflective activity" (p. 184), we evaluated participants using proactive strategies with DGS as having relatively higher SCK.

## *Beliefs about Technology*

We determined "beliefs about technology" as hypothetically influential factors for developing SCK. "Beliefs [are] psychologically held understandings, premises about the world that are thought to be true" (Philipp, 2007, p.259). The difference between knowledge and beliefs is not very straightforward. Knowledge is dynamic whereas beliefs are more static in terms of their capacity to change over time (Pajares, 1992). While beliefs are inflexible and mostly indisputable truths for believers, as each individual holds beliefs according to the interpretations of their experiences, knowledge is more open to change and dynamic as a consensus of a group of individuals after a possible discussion.

Pajares (1996) viewed beliefs as factors affecting one's potential for learning. In the context of technology integration to the instruction, beliefs about technology influences the potential to learn content with technology. Laborde (2002) studied the DGS integration of mathematics teachers, who have different level of teaching experience, to their instruction. It was found from this study that students' gains in geometry were more strongly conceptual that appreciates proving and generalisations, if the teacher used DGS as a medium to *mediate* students' learning. On the other hand, teachers who *instrumentally* integrated technology preferred to follow with paper and pencil in order to help students' learning of geometry.

Chen (2011) differentiated teachers' beliefs about technology in the same way. In this study, participants' beliefs were differentiated regarding the categorisation as applied by Chen (2011). In his theoretical framework, Chen (2011) describes *instrumental* beliefs about technology as those in which technology is an external device to the learner that can perform calculations or display graphs; and learners are in charge of making sense of mathematical relationships independent of the external device used. Chen (2011) also defines *substantive* beliefs about technology as those where the learner is aware of the potential of technology in the mediation of his/her learning. Rivera (2005) states that "when learners use tools to acquire knowledge without developing meaningful schemes, then the knowledge they produce may be superficial, and the operations they perform tend to be procedural" (p. 135). Regarding Rivera's (2005) point, we hypothesised participants holding instrumental beliefs about technology might have difficulty developing their SCK with DGS. With instrumental beliefs about technology, PSTs would use DGS for intermediate procedural operations (e.g., displaying a geometrical figure, or measuring a length or angle, when being asked) without interpreting or reasoning with what is being presented. Since the development of SCK requires deeper reasoning skills and conceptual understanding, we considered SCK development with GSP might be difficult to be achieved by PSTs who only hold instrumental beliefs about DGS.

# Methods

## Context of the Study

We examined middle grades PSTs enrolled in a technology-focused graduate-level geometry course, which took place at a South-eastern research university in the fall semester of 2013. There were 16 PSTs enrolled in the course and each was seeking a Master's of Arts in Teaching (MAT). The MAT program is an alternative certification program for recent graduates from a variety of majors who seek a career change towards teaching. We selected the *Geometry for the Middle Grades* course (hereafter identified as "the course") as our research site, as one of the expectations of the PSTs enrolled in this course was to use the dynamic geometric software, The *Geometer's SketchPad* (GSP) (Jackiw, 1995), flexibly and fluidly within the course's tasks.

## Participants and Methodology

The PSTs had varied educational backgrounds, having previously earned bachelor degrees in non-mathematics areas. At the beginning of the semester they had little to no instructional technology experience or knowledge nor had taken a geometry course in high school. All PSTs enrolled in the course were invited and agreed to participate in the study at the start of the fall semester. Of the 16 PSTs, we selected six to participate in three semi-structured interviews. To select these focal participants, we analysed survey data addressing participants' initial beliefs about the nature of mathematics, teaching, and technology. Our methodological design for this study employed a holistic multiple case study where each of three PSTs was identified as a case (Yin, 2008). We selected three cases to report in this space due to the richness of their data.

## Data Collection and Analysis

Three data sources were collected: 1) an entrance survey to measure PSTs' beliefs, 2) clinical interviews with focal participants, and 3) non-participant observations of the class meetings (Dewalt & Dewalt, 2002; Yin, 2008). The entrance survey was administered at the beginning of the study, consisting of ten open-ended questions (Table 1) examining participants' background, previous experiences, and their beliefs. We selected our six focal participants with respect to their beliefs. We utilised frameworks by Ernest (1989), Kuhl and Ball (1986), and Chen (2011) to categorise PSTs' beliefs about mathematics, teaching and technology. Three of the 16 PSTs' responses to belief-related questions were not sufficient to allow for categorisation and thus were excluded from selection. Results for Karl, Becky, and Sarah are presented in this paper.

We administered three semi-structured clinical interviews within the semester. While we used PSTs' responses to the geometry tasks during interviews to measure their CCK and SCK, we used their responses to belief-related questions as the means for us to measure their beliefs about technology. In this paper, we document findings from the first and the third interviews to answer our research questions. All interviews were audio and video recorded to capture the discussion between the participant and interviewer. Data collected during the two interviews for Karl, Becky, and Sarah were analysed using open coding strategy (Corbin & Strauss, 1998) to identify: possible examples of SCK; if the identified example of SCK was demonstrated by the PST; and what factors, including the use of GSP, may have influenced their SCK. After we identified the *emerging themes* from the open coding process, we separately assessed Karl, Becky, and Sarah's CCK, SCK and the emergence of these themes within their compiled data. We checked our interpretations with another researcher in mathematics education to assess the inter-rater reliability of our findings, which was found to be 0.86.

Table 1
*Open-ended questions in the entrance survey*

| | Questions |
|---|---|
| 1 | What was your major prior to the MAT program? |
| 2 | Can you briefly tell us about your work experiences so far? |
| 3 | Did you take any other undergraduate and/or graduate geometry course before that one? Describe these courses. How did you perform in these courses? What did you find most difficult and/or easy? |
| 4 | Which instructional technologies do you know how to use? What technology have you used for mathematics teaching/learning (high school or college)? Have you had any experience with dynamic geometry software? If so, describe that experience. |
| 5 | What do you see as the nature of mathematics? How do you think it is different from other subjects? |
| 6 | How should mathematics be taught? What supports do students need to learn mathematics? |
| 7 | What are your views on technology? What are the advantages or disadvantages of the technology use in mathematics instruction? |
| 8 | Have you learned or taught a mathematical concept with dynamic geometry software (for example, Geometer's Sketchpad)? If so, please describe such experiences. |
| 9 | How might the use of Geometer's Sketchpad contribute to your understanding of geometry as a learner? |
| 10 | How might the use of Geometer's Sketchpad hinder your understanding of geometry as a learner? |

Our clinical interviews included: 1) conversations around a geometry task that participants were asked to solve, and 2) questions designed to clarify and more deeply investigate participants' beliefs. Each geometry task was designed to provide insight into participants' geometric thinking and potential use of CCK and SCK. In addition, we looked at PSTs' strategies with DGS and determined if they are proactive or reactive strategies (Hollebrands, 2007) as a component of their SCK. If the PST used DGS to examine a geometrical principle or relationship with respect to a prior personal plan, we coded the PST's strategies as *proactive* and viewed it as an indication of higher SCK. If the PST, on the other hand, used DGS without a certain plan and did not predict the results of their action, we coded the PST's strategies as *reactive* and viewed it as an indication of lower SCK. The belief-related questions used in the entrance survey were included in the interviews. We mapped PSTs' responses to the belief-related questions in the interviews onto the theoretical frameworks for beliefs about mathematics, teaching and technology. For example, pre-determined descriptive codes in Table 2 were used to identify PSTs' beliefs about technology.

Table 2
*Pre-determined codes for PSTs' beliefs about technology*

| Codes | Description |
|---|---|
| Instrumental beliefs about technology | Participant considers technology is necessary for efficiency. It allows to save time during instruction and to deal with managerial tasks such as grading. Technology is advantageous to demonstrate the content precisely and visually appealing. It is a new way for tutoring through videos. Participant considers technology might be disadvantageous because it is a tool giving the correct answer. |
| Substantive beliefs about technology | Participant considers technology does not only help teacher for instructional efficiency, but also enhances students' mathematics ability, and aids students' learning, reasoning, sense making, and conceptual understanding. Technology should not be behaved as another means for old technologies (e.g., board), but better to be used to enhance mathematical ability. |

*Square construction task.* The first interview (conducted during the 4th week of the course) included a square construction task. Prior to the interview, PSTs had in-class experiences with GSP through individual GSP tutorials and prescriptive GSP tasks involving constructions. As PSTs worked on these tasks during the semester, they had opportunities to explore GSP affordances for different constructions. However, PSTs were not required to construct a square with or without GSP as a part of their course.

During the first interview, PSTs were first asked to construct a square by using compass and protractor on paper. Then, they were asked to complete a task a middle school student (i.e., a student who is 11-14 years old) had started, and predict what the student was thinking with her procedure (Figure 2).

**Square Construction**

Susan claims she can construct a square by using the properties of a circle and lines having parallel or orthogonal properties. She began by constructing a circle by its center and around a point, and then constructing the radius. Can you complete the construction? What do you think the student would do next? Predict their method based on what is described.

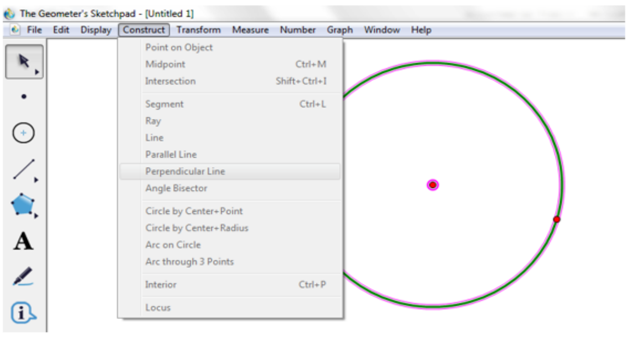


*Figure 2.* Square Construction Task

The identified *SCK* for this task was the prediction of the student's procedure to construct a square with GSP according to the given scenario. The required CCK in this task is knowledge of a square having four congruent sides, each interior angle to be 90°, and/or perpendicularly bisecting diagonals.

*Inscribed circle task.* During the third interview (during the 12th week of the course), each of the three participants were asked to examine the relationship among the area of a triangle, the perimeter of the triangle, and the radius of a circle inscribed in the triangle (i.e., $2 \times A_T = P_T \times r$, where $A_T$, $P_T$, r stands for the area of the triangle, the perimeter of the triangle, and the radius of the circle respectively). The interviewer first asked PSTs to explore the relationship on a piece of paper where they see the inscribed circle, its radius as the height of one of three intact triangles. The figure on the paper was the same figure they would see in GSP before they ran the animation (i.e., Figure 3a). They then explored the inscribed circle of the triangle by interacting with the animation built in GSP (Figure 3). The animation displayed the three triangles formed by the coincidence of the three angle bisectors. Two of these three triangles were folded out (i.e., Figure 3b) and rotated around their vertex so that the three bases of the triangle were collinear (Figure i.e., 3c). Finally, the top vertices of each were translated along a line, parallel to the base and until they were coincidental with the centre of the circle (i.e., Figure 3d).

**Inscribed Circle of a Triangle**

Are you aware of any relationships between the radius of the circle inscribed in a triangle, and the triangle's perimeter and area? Regarding the animation within the GSP, what could be the relationship? How did you understand that? Could you explain it to me?

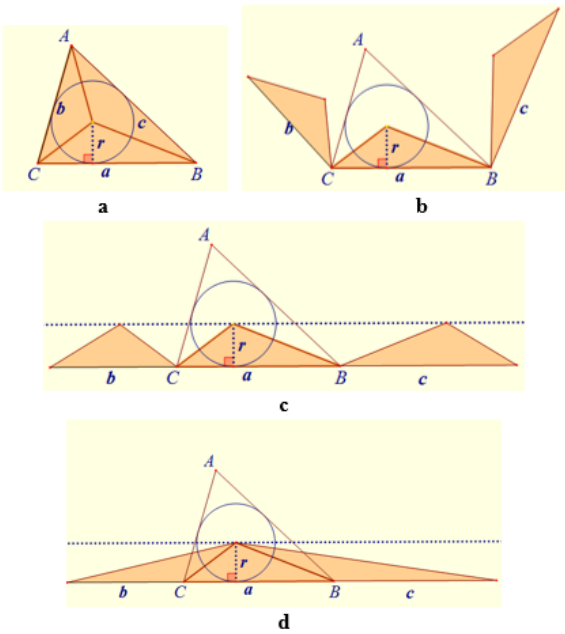


*Figure 3.* Inscribed Circle of a Triangle Task and Screenshots from the Animation

The *identified SCK* for this task was the geometry knowledge required to understand the procedure demonstrated through the animation. The animation represented that the triangle could be divided into three triangles, each one having a base length of one of the sides of the original triangle and a height equal to the length of the radius of the inscribed circle, as the radii are perpendicular to the sides of an inscribed triangle.

To understand the animation, participants needed to understand that the height of an obtuse triangle, in some cases, lies outside of the triangle (see Figure 4). We identified this knowledge as required *CCK* for this task, as the identification of the height of a triangle is factual knowledge that can be taught to students in class. On the other hand, teachers do not necessarily teach mathematical principles evident within the animation in order to represent the relationship between area, perimeter and the radius of an inscribed circle in a triangle. As such, the latter was categorised as SCK. Since this task was not a construction, we did not explore whether participants construct the figures robustly or not.

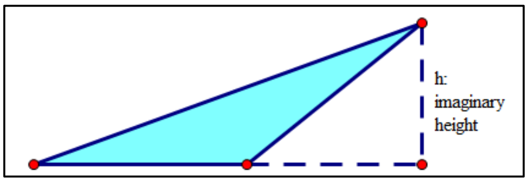


*Figure 4.* Imaginary Height of an Obtuse Triangle

In the following section, we present findings from our clinical interviews for the three cases separately. Each case starts with the description of each participant along with their belief profile at the beginning of the study. These preliminary descriptions are followed by each case's SCK development during each interview and how their beliefs appeared with respect to their experiences with DGS. The emerging themes out of these three contrasting cases are discussed in the last section of the paper.

# Findings

## *Karl*

*Karl* was 21 years of age and had completed his undergraduate major in economics. In the entrance survey, he viewed mathematics as a discipline independent of opinions, which requires critical and logical thinking. He referred to this notion during the first interview by saying that "mathematics is a way of thinking logically and analytically … to apply it to [real world] situations." He also stated that knowing all definitions, facts, procedures and related terms did not fully represent knowledge of mathematics; as understanding of mathematics required knowledge beyond compartmentalised mathematical entities. In the entrance survey, Karl considered technology a support for students' learning and instruction, but not a replacement for it. Karl's beliefs concerning technology were categorised as substantive at the beginning of the study.

*From rectangle to square.* During the first interview, Karl attempted to construct a square circumscribed around a circle using a compass and protractor. He began by using the construction of a perpendicular bisector. He constructed a perpendicular bisector on a line segment he drew, and kept the same procedure for each line that was perpendicular to the previous one. However, he did not manage to construct a square with compass and protractor, but a rectangle (see Figure 5).

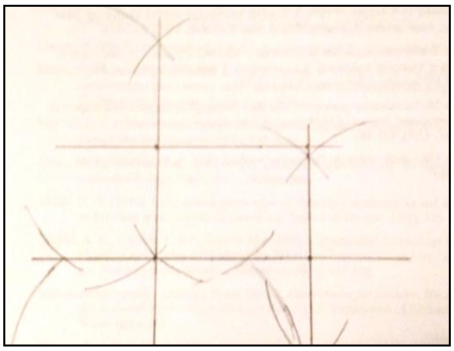

*Figure 5*. Karl's Rectangle Construction on Paper

Karl was unable to make his idea work using paper and pencil. The interviewer sought to support his idea by transitioning his work to GSP:

Karl: I know that we know how to draw perpendicular bisectors. Perpendicular bisectors will have a right angle. So, if you just do one on that one, and keep going all the way around…

Interviewer: But for the square?

Karl: Yeah, I forgot how to make sure all the sides are equal. Okay, it's a circle. And what I was going to do now was to draw tangent lines. That wouldn't work either… I was going to do an inscribed circle and then have a square around it.

Using GSP, Karl tried to construct a square circumscribed about a circle. He first constructed the diameter from the radius by constructing the line that contained the radius. He then constructed a parallel line to the given radius through the centre and precisely identified the diameter (see Figure 6a). He then constructed tangent lines to the circle by using the perpendicular line construction feature (see Figure 6b). Next he constructed parallel lines to the previously constructed perpendicular lines and completed his construction of a square circumscribed around the circle (see Figure 6c). His construction was robust, which did not require further attempts to correct it.

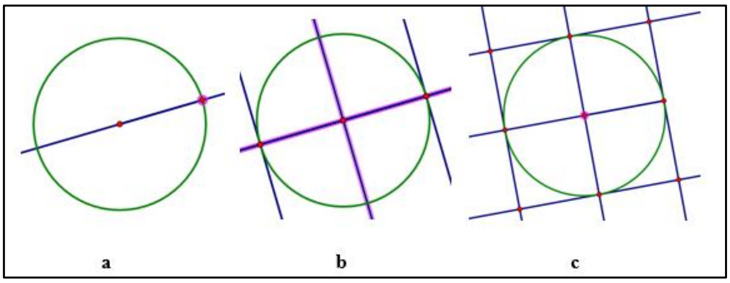


*Figure 6.* Karl's Square Construction on GSP

Karl's CCK for the definition of square was evident during his square construction with GSP. He was aware of both geometrical characteristics of a square: perpendicularity and congruency of sides. Since his work on paper allowed him to anticipate his construction on GSP, we labelled his strategies as *proactive* rather than reactive. When asked at the end of the interview, he justified himself by stating that "perpendicular lines provide right internal angles… and I set the diameter for the measure of congruency." Regarding his justification and the overall flow of his interview from the paper-pencil to the GSP parts, we considered his SCK evident for the task.

We categorised Karl's beliefs about technology as substantive at the end of the first interview. His approach indicated his flexibility to try his ideas to see if they would work with new mediums. His flexibility with mathematics allowed him to use GSP as a learning partner rather than just a tool for precision. During the interview, he suggested GSP was more conducive to his learning geometrical concepts than paper:

> Personally, I prefer the software only because I hate compasses. They [compasses] wobble everything. After you build it, I like being able to play around with it and seeing how it changes. I guess a lot of people in our class think you get more conceptual stuff by doing it [construction] on paper. But I think you can get just as much on the GSP.

Karl's statement indicated he viewed GSP more as a learning partner rather than as a tool to recreate paper-constructed models. His openness to exploration with geometry was enhanced by the software's affordance to dynamically represent how his personal assumptions about geometry could be generalised or refuted. Karl evaluated whether it was feasible to construct a square out of four tangent lines around a circle. His reasoning about what procedure to follow for square construction began to emerge on paper, but he recognised that he could not assess the validity of his reasoning in that medium. His preference to use GSP and beliefs about GSP also supported his conceptual development, which resulted in observable evidence of SCK for this task. However, he did not manage to construct a square with paper and pencil. Also, we want to emphasise the order we posed the tasks. If the GSP task had been posed first and the paper-pencil task second, Karl might have been more successful in constructing a square with compass and straightedge.

*The influence of CCK on SCK.* During the final interview, we first asked participants to examine a diagram of a circle inscribed in a triangle to find a relationship between the perimeter and area of the triangle. Karl's work using the static model was not productive in finding the relationship among the area and perimeter of the triangle, and the radius of the circle inscribed in the triangle. He did not do anything beyond writing down the general equation for the area of a triangle. We then asked him to examine the animation illustrating a circle inscribed in a triangle. We determined the concept of external height for an obtuse triangle (see Figure 4) as necessary CCK for this task. Karl presented evidence of the CCK for the task by identifying the external height of a triangle on the animation. He stated during the interview that the base and the altitude of each triangle stayed the same. As a result, Karl was able to make sense of the animation and construct the relationship between perimeter and area of the triangle:

| | |
|---|---|
| Interviewer: | The top points of these three triangles are moving on a parallel line, what's happening here? |
| Karl: | The areas aren't changing. It's just the shape of the triangle is changing. |
| Interviewer: | Why do you think that the areas are not changing? |
| Karl: | We didn't change the area because it didn't gain or lose any. We kind of just skewed the triangle back over. |
| Interviewer: | What are the parameters that we keep the same then? |
| Karl: | The base. |
| Interviewer: | What else do we keep the same? |
| Karl: | The altitude… So the base and the altitude of the triangles stay the same. So one-half base times height. So the area will still stay the same. The radius is the altitude of the three smaller triangles. So the radius could be the height… Oh, okay, I see it now. So if you add, cause of the new base, okay so if you want to find the area of this triangle, it would be b + a + c times one-half of the radius. So you could do b + a + c times one-half times r, the radius and your perimeter would also just be b + a + c. Area of the triangle would be equal to the perimeter times the radius divided by 2. |

The interaction demonstrated that Karl, supported by the questioning of the interviewer, was able to understand the geometrical principles behind the animation. He believed the reason behind the movement of the top points on a parallel line was to maintain the same area. He was able to explain the relationship between the area, perimeter of the triangle and the radius of the inscribed circle. However, his strategies on GSP were more reactive than proactive, as he did not use GSP in order to examine a pre-existing geometrical idea. Normally, the GSP environment would allow him to experiment and to explore a geometrical relationship. However, he preferred to make interpretations based on what he saw on the screen. Later we asked him to reflect on this experience with GSP and his beliefs about technology:

> If you would have just handed that [the inscribed-circle-of-a-triangle task], I would have just stared at it all day. But seeing animation definitely helps. Even thinking about rotating it out, I would have never thought about that… I would have never thought to put the three triangles as one big triangle. So, seeing that helped me think that the perimeter is your new base.

Karl's statement above again indicated his view of GSP as a learning partner. He was aware of how the program was advantageous visually and how it might have been hard for him to complete the task by himself using only paper and pencil. This experience along with his statement during the interview led us to categorise his beliefs about technology as substantive. We posit that the animation in GSP was the support for Karl's development of SCK, and he may have developed the same SCK if a similar demonstration was shown on paper.

## *Becky*

Becky was 35 years of age and came to the teaching profession from her career as a hair stylist. Before enrolling in the MAT program, she was a hairdresser for the last 12 years. At the beginning of the study, she stated in the entrance survey, "mathematics is a subject consisting of foundational laws, which is objective; but the application of laws is subjective." In addition, she viewed mathematics as more objective compared to other disciplines because of its accuracy and certainty. Her survey results also suggested she viewed technology as something to be used for saving time during instruction. Becky's beliefs concerning the nature of technology were categorised as instrumental at the beginning of the study.

*Inscribed square in a circle.* Becky first used a compass and straightedge in order to construct equal lengths for each side of a square. She began her square construction by drawing a line segment on the paper using the straight edge of a protractor. Next she opened the compass to copy the length of the segment at each end of the line segment and scribed an arc above each side of the line segment she drew. She scribed the arcs in such that they were approximately perpendicular to the line segment. However, she did not use the protractor to measure the angles to ensure they were 90°. She seemed to possess the required CCK, which would allow her to solve the given task correctly. For example, by setting the compass with the same width for both sides of the line segment, we concluded that she understood a square would have four equal sides and four right angles.

When Becky used GSP to construct the square, she began by creating the picture in the student's work in the presented task. She used the drawing tool for segments to extend the radius of the circle to make it a diameter. However, the diameter she constructed was not a straight line (see Figure 7a). Then, she constructed a perpendicular line to the original radius through the centre of the circle (see Figure 7b). After identifying the intersection points of the perpendicular line with the circle, she connected the four points that lay on the circle with line segments, which resulted in her "square" inscribed in the original circle (see Figure 7c).

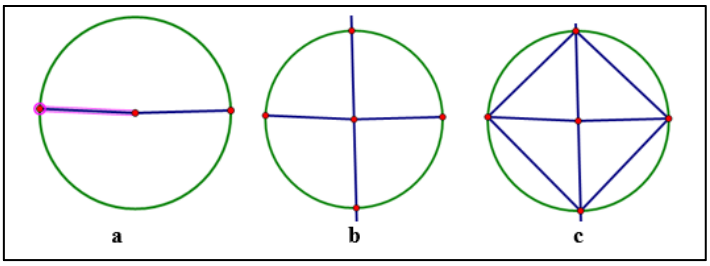


*Figure 7.* Becky's Square Construction

Becky's "square" was not constructed robustly because she did not originally construct the diameter as a straight line. Instead of using an affordance of GSP to create a precise diameter, she used *eyeballing* for the internal angles of the square; similar to her work on paper. We categorised her diagram as a soft construction. During the interview, the interviewer challenged her to justify her steps to construct a square on GSP:

Interviewer: How do you know that we can keep it as square if I play with points here?

Becky: I think it would be a square.

Interviewer: [The interviewer dragged the radius she constructed.] See, it is not a square anymore. What might be the reason?

Becky: Let me think… It might be off just a little bit because I didn't extend that line perfectly. But it's close… If it was the extension, then there's no problem with that one I believe.

Interviewer: How would you do that?

Becky: If I construct a straight line here [the centre]… I already have the radius… [She constructed a straight line overlapped with the radius given]. Now it is perfectly straight.

Becky's work with paper before GSP, where she emphasised perpendicularity and congruency of the edges, demonstrated she possessed the required CCK; that a square has four equal sides and four 90° angles. Through the interviewer's challenge of her ideas she recognised the mathematical and technological knowledge behind the error in her square construction and developed the identified SCK during the interview. However, her SCK was not as strong as that demonstrated by Karl. Becky did not try to justify herself by explaining why her construction would be a square. She constructed her square with the hypotenuses of isosceles right triangles within the unit circle as the sides of the square. Neither the perpendicularity of the sides of the square nor their congruency was explained clearly. As a result, we suspect Becky's conception of square was visually-oriented (Sinclair & Robutti, 2013), in which she identified a geometrical figure as a square with respect to its appearance, but not through the examination of its characteristics embedded within. We also considered her strategies with GSP as more *reactive* then proactive, as she did not follow the approach she started on paper, but applied a new train of thought using GSP.

We explored Becky's views of GSP as a factor influencing her SCK through the use of technology. During the first interview, Becky stated her intent to have students first create constructions on paper, and then to use GSP for "polishing":

> I would still guide students to use the archaic method to start and then probably go into something like GSP to polish it. It is better to represent geometrical figures, and you can do that with exact measures. But to explain parallel and perpendicular stuff, I'd rather let them conceptualise in their minds. Because, to me, this [GSP] gives too much.

Becky viewed GSP as a tool for precision and polishing. As a result, we categorised Becky's beliefs about technology as instrumental. Although Becky was able to develop SCK for this task as a result of having her ideas challenged by the interviewer, her statement indicated that she did not view GSP as a tool for exploration, but as a tool used to create precise measurements. At this stage of the course, Becky's explorative experiences with GSP had not changed her beliefs about the technology.

*A slow way to develop SCK.* As with Karl, Becky did not make significant progress toward on paper to observe the relationship among the area and perimeter of the triangle and the radius of the inscribed circle of the triangle. She understood from the image that the height of one of the internal triangles is equal to the length of the radius of the inscribed circle, but did not consider that her finding (i.e., radius as the height) was true for any internal triangle. Becky recognised the radius as height using the visual clues (i.e., right angle demonstration between the radius and the base of the triangle) available in the diagram. When it was clear that her discussion and work did not allow her to determine the relationship, the interviewer asked her to view the GSP animation.

Even after repeated viewings, Becky was unable to make sense of the animation. It appeared she struggled with the height of a triangle lying outside its interior. Upon recognising Becky was missing the required CCK, the interviewer used GSP to demonstrate that the area would not change when the top points of the triangles were translated along a line parallel to the base. Even with this support, Becky did not understand the logic behind the animation in terms of its related geometrical principles; her lack of conception of an external height hindered her SCK development:

Interviewer: On the parallel line, can you see why they've done this?

Becky: No. I don't know why they did that. It made sense when they were in points over here [before moving along the parallel line]. But once they elongated it, I have no idea why.

Interviewer: Okay. So how would you find the area of that triangle then? What is the area formula for that thing?

Becky: It's the same thing, but I don't know what this height is… When they move that point and they stretch the shape, the area would change.

Interviewer: So you're not convinced with that kind of animation?

Becky: No. I am not sure why I think that.

The interviewer proceeded to inform Becky about the concept of external height and showed area conservation when the top point of the height of a triangle moves on a line parallel to its base. During this time, she used reactive strategies with GSP in order to understand the relationship (e.g., constructing another parallel line with no purpose, or drawing an additional line segment on the radius of the inscribed circle). Although the relationship was explained to her, she was not convinced by the animation and the geometric principles behind it. She suggested that she would have better understood the idea without GSP:

Interviewer: Is this animation conducive to understand those things?

Becky: In a slow painful way. I might have seen that faster on a geo-board. Somehow [on GSP], it didn't seem like it was parallel. It seemed like the height changed as it slid over. I would have kept it [the triangle] together. For some reason the breaking them [internal triangles] apart just blew my little brain.

Becky's statement indicated not only did she not view GSP as a learning partner, but as a tool that actually impeded her learning. Again, we labelled her belief about technology as instrumental. She seemed to attribute the reason for her lack of SCK in this task to GSP, as opposed to her lack of understanding of the external height of a triangle. Such an interpretation seems consistent with the definition of the instrumental belief about technology as the participant perceives GSP external to her learning and views it as a cause for the lack of recognition of the targeted relationship. Although the interviewer explained the top vertices of the three triangles were moving on a line parallel to the base of the new triangle, Becky was not convinced the line was parallel and the height of the triangles was changing.

## Sarah

Sarah was 53 years of age and had worked as a nurse for many years before choosing teaching as a new career. She stated she was not very comfortable with technologies and preferred to do things "the old way" rather than with technology. At the beginning of the study, she viewed technology as a tool to make life easier. She provided examples from daily life to show how she used technology in easing tasks. Similar to Becky's beliefs, in the entrance survey, Sarah thought mathematics to be an objective discipline consisting of algorithms that can be applied in real life. Sarah's beliefs concerning the nature of technology were categorised as instrumental at the beginning of the study.

*Radii as a measure of congruency.* For her square construction on paper, Sarah did not use the straightedge as a ruler; she used the compass as a measuring tool to ascertain the congruency of the sides of the square. She also used the straight edge and eyeballed to approximately keep consecutive sides perpendicular to each other. At the end of her construction, she checked the perpendicularity of consecutive sides for a second time.

Sarah's procedure to complete the square construction using GSP was different from both Karl and Becky (Figure 8). She first constructed a point on the circle so that she could create a second radius that would be perpendicular to the radius given. Instead of using the construction of a perpendicular line affordance of GSP, however, she *estimated* the location of this point in a manner similar to Becky's approach (see Figure 8a). She then had to move her point onto the perpendicular line to make it visually perpendicular to the radius given (see Figure 8b). Next, she constructed two other perpendicular lines to create a "square" constructed from the radius given. When she finished her construction, the polygon approximated the appearance of a square (see Figure 8c). However, her construction did not hold its properties when the interviewer changed the size of the circle (see Figure 8d), which resulted in a *soft* construction of a square.

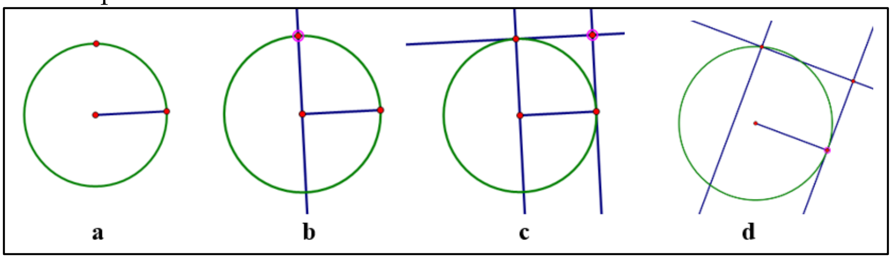


*Figure 8.* Sarah's Square Construction on GSP

Sarah was aware of the mathematical properties required of a square:

> I could measure the angles and make sure that they are 90°. But, I went from what I have and created with perpendiculars. So I am pretty confident that this is a square although I can verify by measuring each side if you want me to do that.

We coded her reasoning after her construction as evidence of CCK required for this task. When the randomly constructed point was moved, her construction would not hold its features in every case. Since this was an unexpected result for Sarah, we categorised her strategies with GSP as *reactive* more than proactive. Because Sarah was unaware of her mistake (see Figure 8d), the interviewer used it as an opportunity to ask her to justify her ideas and identify the error in her construction:

Interviewer: If it is a square construction, whenever I play with the points, it always has to be a square. But do you see what the problem is [see Figure 8d]?

Sarah: Well, it is not staying in the square when you are manipulating it. And ideally it should. Now why is it not?

As the interviewer continued to question Sarah's reasoning behind her construction, Sarah eventually recognised that, instead of "eyeballing", she could have utilised the intersection point of the first perpendicular line she constructed within the circle. This guidance enabled her to correct her error and construct the square robustly. Sarah's offering to use the measuring affordances of GSP to check her construction was indicative of a more robust SCK than Becky, as she explicitly referred to the properties of the figure rather than approaching it with an emphasis on visual orientation. However, her first construction was not as robust as Karl's. In addition, her justification of her construction was uncertain. Although she stated during her paper construction that she used the compass as a measuring tool for the congruency of each side, she did not reflect a similar mathematical insight for the justification of her square construction on GSP. Instead of referring to the radii of the circle to justify the congruency of sides, she preferred to measure each side on the software.

Sarah's beliefs about technology during the first interview underlined its potential to demonstrate concepts in different ways with more visual appeal. However, she did not fully view GSP as a learning partner. As a result, we considered her belief about technology as instrumental:

> I think it is a tool if used wisely. For example, our school lets students to go to computer lab. These computers have a mathematics software for students to use. Some teachers already use them during their instruction. I don't know the nature of that [software] and how my instruction is going to look like. But it is different than the teacher stands up there and talks from a smart board. It is much graphically and visually appealing now.

As opposed to Karl's viewpoint, Sarah did not explicitly view GSP as a medium to explore her geometrical conjectures. Even though she favoured its potential to visualise things in a better way, the data showed that Sarah had a reluctance to engage with technology to promote student learning. It seems that she had an interest to integrate technology for the sake of students' learning, but she had not learned enough about how to use it at this phase.

*CCK plus the views about GSP.* Sarah took a few minutes to work on the inscribed circle task with a piece of paper. She first assumed "the radius might be perpendicular bisector for the side that the radius lands on," but later reasoned her assumption was incorrect as, "it does not visually look like that the radius lands on the midpoint of the side."

Sarah's experience with GSP during the last interview revealed the important role CCK and her views about GSP play in her SCK. Sarah struggled to comprehend the required CCK for this task and her initial interactions with the GSP animation reinforced this misunderstanding. Instinctually, she believed the area of the three triangles had to stay constant after being rotated, but could not immediately understand why:

> I would say that the height is the perpendicular line at... or somewhere where it has a highest point with a perpendicular line. The height is not the highest point of the triangle, it is the height of the perpendicular line, I believe. I know it [r] is the height of the centre one because it is a perpendicular line… that is where I am getting hung up. I am used to height as a perpendicular line… If I go from here [the highest point of the obtuse triangle] to here [the base of the triangle], it would be the height… Yeah. So that would be the height, they [the areas of internal triangles] would be the same. Because you still have this base.

Sarah had to resolve her understanding about the concept of height for an obtuse triangle. Since her strategy was to find a perpendicular line within the area of the triangle, she was not confident that the height could lie outside the triangle. The animation, along with talking aloud through the process allowed her to refine her conception of height for an obtuse triangle: *the* *perpendicular distance between the highest point of a triangle and its base*. As a result of her reasoning and subsequent development of the CCK required for the task, Sarah constructed the relationship among the area and perimeter of the triangle. For the majority of her work with GSP, Sarah used *reactive* strategies without a specific plan to examine something specific in her mind. However, these strategies did not impact her SCK for this task. With the development of CCK, Sarah managed to recognise the relationship and linked it to the components of the animation.

When the interviewer asked whether the GSP animation was conducive to her understanding, Sarah explained she found it useful:

> I think it [GSP] helped being able to do it and undo it so that I could repeat. If I do it on paper, then I have to draw the whole thing again. But if I can see it multiple times, then I can watch the angles move and I can say even though the angles changed, the area did not change... One thing I liked about GSP is that they don't really skip steps.

Sarah used GSP as a learning tool and her experience during this task enabled her to learn a new concept. However, because of her limited experience with this technology, she did not recognise its potential to support her reasoning in multiple ways. She only viewed GSP as a tool, which allowed her to measure, draw and animate. Regarding her statement above and her experience with GSP during this task, we believe she began to see the potential of technology for learning, which, supported by additional positive learning experiences with technologies, could result in the future construction of a substantive belief about technology.

## Discussion and Conclusion

### *Research Question 1*

> How does the use of DGS influence middle grades mathematics PSTs' SCK? What are the influential factors that can support the use of DGS to develop PSTs' SCK?

Our examination of PSTs' development of SCK while using GSP, resulted in three potentially influential factors: 1) CCK availability, 2) opportunity to justify ideas, and 3) PSTs' beliefs about GSP.

Our analysis indicated the associated CCK for each task accompanied each development of SCK for the three focal participants. In some cases, PSTs constructed the required CCK while solving the task (e.g., Sarah during the inscribed-circle-of-a-triangle task), which enabled them to subsequently develop the associated SCK. Bair and Rich (2011) presented similar findings and posited CCK may be considered an important prerequisite for SCK, but also found that PSTs who have insufficient CCK could still develop some SCK. They also pointed out CCK is a requirement for demonstrating more complex or higher-levels of SCK. Although we did not initially differentiate the levels of SCK within particular tasks, we can classify the types of tasks utilised in terms of cognitive demand (Stein, Grover, & Henningsen, 1996). For example, our first task was an example of "doing mathematics" as it was an open-ended task that required PSTs to individually explore a geometrical construction with little to no direction or constraints. Conversely, the second task was more associated with "procedures with connections". This task focused on PSTs' understanding of a formula, and did not provide for individual exploration of the ideas. In order to classify the level of SCK it is also germane to consider the tasks' complexity associated. We would classify the mathematics associated with the inscribed circle of a triangle task as more complex than that involved with the square construction task. In addition, the CCK of the second task required knowledge of a common misconception in addition to the properties of a circle and of a circle inscribed in a triangle. Finally, understanding the relationship between the perimeter and area of the triangle required PSTs to develop a logical thought process, leading to a proof or a generalisation. Taking each of these influential factors into account, we concluded the SCK associated with the inscribed circle task was at a higher-level, which explains why the CCK in this case was more of a requirement.

Cross-case analysis for Karl, Becky, and Sarah revealed commonalities within themes as well as differences in the quality of their SCK. Evidence indicated the importance of challenging PSTs' mathematical ideas and providing them an *opportunity to justify* ideas. Bair and Rich (2011) included a similar component to our "opportunity to justify ideas" theme in their framework for SCK development: *explaining their reasoning*. In their study (Bair & Rich, 2011) to develop SCK, PSTs are expected to solve a problem first, explain their reasoning, and discuss possible students' errors. In our study, the interviewer's scaffolds afforded PSTs the opportunity to reconsider the conjectures they took for granted. A possible explanation for this result considers the definition of SCK and its differentiation from CCK. As defined in the theoretical framework, SCK is mathematical knowledge that a teacher does not seek to develop in students during the instruction, but uses to understand the mathematics evident within students' mathematical errors and unusual procedures. Whereas the teacher would use his/her CCK by recalling the facts, definitions and procedures to solve problems, SCK requires mathematical knowledge that is relational (Skemp, 1978) and allows PSTs to make sense of students' mathematics. Becky had an opportunity to justify her reasoning during the first interview. Although her construction indicated the characteristics of a square, her reasoning might have been challenged with more opportunities for justification. Without such opportunities, one cannot judge if her construction was the result of a coincidence or of a logical thought process. The use of Hollebrands' (2007) conception of proactive and reactive strategies became helpful at this point of analysis. According to the author's definition, we labelled Becky's construction with GSP as a reactive strategy rather than proactive, which would require a solid foundation of mathematical relationships. During the interviews, opportunities to justify reasoning were emphasised as a means for PSTs to employ more proactive strategies. However, such an emphasis on justification is not enough if PSTs do not use or develop the required mathematical relationships.

## *Research Question 2*

How do PSTs' beliefs about technology influence their use of DGS to develop SCK?

Close examination of the interviews with Karl, Becky and Sarah revealed conditions that can support the use of GSP in SCK. Data indicated that *beliefs about GSP* seemed to determine whether GSP could support PST's SCK. If a PST viewed GSP as a tool for precise measurements or demonstration (i.e., instrumental beliefs about technology) rather than a learning partner, then the role of GSP in SCK was often limited. During the third interview task, Becky claimed her learning experience with GSP was detrimental, which may explain why she struggled to develop SCK while using GSP. On the other hand, Karl and Sarah often considered the use of GSP advantageous for their learning (i.e., substantive beliefs about technology), and utilised its affordances to develop their SCK. Chen (2011) suggests at the early stage of learning content with an instructional technology, PSTs such as Becky tend to view technology as devices for efficiency and precise measurements. On the other hand, some PSTs, such as Sarah, can alter their view of technology towards substantive view and approach GSP as a learning partner. At first, Sarah was sceptical about the software's potential to impact student learning. With a learning experience during the last interview, her beliefs about technology *slightly* changed from instrumental to substantive one. We italicise "slightly" in the last sentence because a change in beliefs from a single experience is almost impossible. However, we consider such an experience to be an initiator in a process of possible belief change about technology in the long run.

## *Implications, Limitations and Future Research*

Our analyses provided insight on the process of SCK development and possible factors affecting this process. These factors can influence the design of mathematics courses offered for education majors. Although our study only documents findings from one-to-one interactions, its transferability as a viable teaching strategy to classroom context is not impossible. Exploration of mathematical ideas through instructional technologies should be provided within mathematics courses for PSTs in order to provide opportunity for conceptual understanding of the content. It is recommended for course instructors to focus on the simultaneous development of CCK along with instructional strategies emphasising PSTs' reasoning around possible errors and alternative procedures through mathematical tasks that require justification. These experiences can be potentially generative for PSTs, as successful collaboration with technologies can lead them to developing more substantive beliefs about technology. Secondly, it is necessary to find ways of changing PSTs' beliefs about technology in order to improve their SCK. Our findings indicated that PSTs had a potential to improve their SCK if they hold beliefs about technology underlying its integration as an influential factor for learning. Regarding that, teacher education programs should highlight the integration of technologies into their coursework emphasising its use as a means and an integral part for the learning of mathematical content.

One limitation of the study was the number of participants covered. The case study analysis allowed us to closely explore how beliefs of the pre-service teachers affected their SCK. However, the same approach would not allow us to make any strong claims about pre-service education programs, but might give some hints for further analysis with quantitative studies. There is a need for future research focusing on the quantitative analysis of SCK with a standardised test where the sample will be larger so that statistical inferences can be made.

Another limitation was the results' dependency on the tasks presented during interviews. The interview tasks were content dependent, where each PST's recalling abilities for the content might have affected his/her SCK they demonstrated. Moreover, the SCK they demonstrated only showed SCK for the specific content they dealt with the task. In a future qualitative study, we will focus on using tasks specialised for a unit in geometry rather than using tasks from different units. The complexity of these tasks should be similar for PSTs so that their difficulty would not be another factor influencing their SCK.

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