Exploring Connections between Content Knowledge, Pedagogical Content Knowledge, and the Opportunities to Learn Mathematics: Findings from the TEDS-M Dataset

Eileen Murray
Montclair State University

Kelley Durkin
Vanderbilt University

Theodore Chao
The Ohio State University

Jon R. Star
Harvard University

Rozy Vig
The University of California, Berkeley

Received: 3 April 2016 / Accepted: 6 March 2017

Past work on mathematics teachers' content knowledge (CK) and pedagogical content knowledge (PCK) has resulted in mixed findings about the strength of the relationship between and development of these constructs. The current study uses data from the Teacher Education and Development Study in Mathematics (TEDS-M) to examine the relationship between these knowledge types and the opportunity to learn mathematics within teacher preparation programs across 17 different countries. We consider the relationships between these constructs in various countries to further explore how teacher knowledge can be supported by their experiences in teacher education.

Keywords · international comparisons · mathematics education · pedagogical content knowledge · teacher knowledge · teacher education/development

Introduction

Sparked by recent economic and political changes, increasing globalisation, and results from international assessments in mathematics and literacy, many countries around the world have taken steps to re-evaluate and reform their educational systems. For example, the United States of America (U.S.) engaged in an accountability/standards movement (Conference Board of the Mathematical Sciences, 2012), Germany shifted to a "literacy" framework from an "individual humanism" paradigm of education (Neumann, Fischer, and Kauertz, 2010), Russia embraced a competency-based education model (Karp & Vogeli, 2010), and Chinese Taipei moved from centralised 5-year post-junior high Normal Schools to decentralised 4-year post-secondary Teacher Colleges to increase the professional and academic knowledge of mathematics teachers (Lo, Hung, & Liu, 2002; Weng, 2004). These reforms reflect a trend of heightened attention towards the preparation and evaluation of teacher education. In particular, many countries are focusing on the types of knowledge needed to support effective teaching. For mathematics teachers, this focus moves beyond just mathematics content, shining a light on what a mathematics teacher needs to know about the art of teaching in order to teach effectively.

While mathematics content and pedagogical content knowledge of mathematics teachers have been widely studied (e.g., Ball, Thames, & Phelps, 2008; Hauk, Jackson, & Noblet, 2010), questions remain about how these types of knowledge develop among future teachers. In particular, understanding the relationship between mathematical content and pedagogical content knowledge, as well as the kinds of courses or trainings that promote these knowledge types, are both important and open areas of research.

In this paper, we contribute to the international literature on mathematics teachers' knowledge and its development in teacher preparation programs. We explore issues regarding the relationship between teacher knowledge and the opportunities to study mathematics within the context of the Teacher Education and Development Study in Mathematics (TEDS-M). In particular, we use data from this large international comparative study to investigate the relationships between elementary and lower secondary future teachers' mathematics content knowledge (CK), pedagogical content knowledge (PCK), and opportunities to learn mathematics (OTL) in their teacher preparation programs. We use this data set because the TEDS-M data is (a) publicly available, (b) draw on a large and representative sample of participants ($n \approx 25{,}000$), and (c) include participants across 17 different countries, each of which offer a unique and powerful lens for exploring the constructs of CK, PCK, and OTL. We explore two research questions:

1. For future primary and secondary mathematics teachers, what is the relationship between content knowledge and pedagogical content knowledge?
2. For future primary and secondary mathematics teachers, what is the relationship between teacher knowledge and opportunities to learn mathematics?

In the following review of the literature, we discuss international educational reform efforts directed at K-16 education with links to teacher education. A common thread weaving through these global reform movements is the identification and exploration of the types of knowledge teachers need in order to be effective and how teachers gain that knowledge. We use the discussion of these reforms to lay the foundation for a deeper look at content and pedagogical content knowledge. We also explore current beliefs and assumptions in the field about teacher knowledge, including how such knowledge develops, the experiences that facilitate such knowledge and the opportunities to learn prospective teachers have in their preparation programs.

Educational Reforms

Countries around the world have undergone teacher education reform in different ways. In 1999, 29 European[1] countries signed the Bologna Declaration, a pledge to reform higher education structures and systems. Over the next 10 years, 18 more European countries[2] and Russia joined the Bologna Process, which subsequently expanded focus on curricula, the changing structure of higher education, and teacher education. Much of the Bologna Process's work on teacher education focused on understanding and developing a set of teacher competencies. Each Bologna signatory country crafted more specific policies regarding the subject-matter competence and pedagogical competence of novice teachers. For example, after the Bologna Process, the

[1] Signed by Austria, Belgium (French and Flemish communities), Bulgaria, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, Netherlands, Norway, Poland, Portugal, Romania, Slovak Republic, Slovenia, Spain, Sweden, Switzerland, and the United Kingdom.
[2] Additional countries include Albania, Croatia, Cyprus, Georgia, Serbia, Turkey, (see www.ehea.info)

requirements for teachers of mathematics in Russia are now based on two principles: (1) knowing mathematics well and (2) being able to teach mathematics to students (Stefimova, 2010). Similarly, Germany developed a core curriculum for subject, subject-didactics, and educational studies (e.g., educational psychology, philosophy of education) and connected the subject-centred and education-centred elements of teacher education (Moon, Vlăsceanu, & Barrows, 2003). In 2000, a group of European counties formed the European Network on Teacher Education Policies (ENTEP), which "exists to promote cooperation among European Union Member States regarding their role in initial, in-service and further teacher education policies" (Dimitropoulos, 2008, p.1). ENTEP advocates for teacher preparation programs to include subject-matter competence and professional competence in pedagogical theory and practice (ENTEP, 2010).

Over the past two decades, East Asia underwent a similar education reform movement, changing teacher certification and modernising teacher preparation institutions (Li, Zhao, Huang, & Ma, 2008). These reforms include changing the teacher preparation curricula, focusing specifically on professionalisation, and the quality of teacher education programs. Yet, throughout East Asian countries, questions about "What is good teaching?" and "How does one define a good teacher?" continue to linger (Shi & Englert, 2008).

Similar concerns are present in the U.S. Over the past 20+ years, the U.S. has worked to change student and teacher education to address deficits in international assessments and to remain globally competitive. In terms of teachers' pedagogical preparation, researchers and policy-makers focused on how teachers' can understand the mathematics they teach in order to present it to students in comprehensible ways. Because of the growing interest in this type of teacher knowledge and its relation to student learning (e.g., Bolyard & Moyer-Packenham, 2008; Hill, Rowan, & Ball, 2005a; Piccolo, 2008), teacher education programs in the U.S. are starting to include courses designed specifically to improve teachers' PCK.

As the examples above indicate, countries around the world have implemented reforms focused on improving the quality of teaching and thus the quality of teacher education. Implicit across these various reform efforts is a common interest in how to best develop teachers' content and pedagogical content knowledge, which are both necessary components of a teacher's knowledge base (Baumert et al., 2010a; Hill et al., 2005a). Yet questions about how to foster such knowledge among teachers are varied. Some argue that CK and PCK can be developed in content courses offered by teacher preparation programs (Baumert et al., 2010a; National Mathematics Advisory Panel, 2008; Schmidt et al., 2007). Others contend that CK and PCK are best developed in the mathematics classroom as teachers are actively engaged in the act of teaching (Seymour & Lehrer, 2006). To better understand how teachers develop content and pedagogical content knowledge, in the following sections, we further discuss the definitions of and relationship between these constructs.

Content and Pedagogical Content Knowledge

Content and pedagogical content knowledge were defined by Shulman (1986; 1987) as two of seven categories of important teacher knowledge. Shulman argued for examining "the distinction between teaching knowledge that is generic and teaching knowledge that is subject-specific" (Ferrini-Mundy, Floden, McCrory, Burrill, & Sandow, 2005, p. 9). Mathematics content knowledge, in Shulman's (1987) definition, refers to a teacher's knowledge of mathematics and its organising structures. Other researchers conceptualise mathematics CK as "a profound mathematical understanding of the mathematics taught in school" (Baumert et al., 2010). In contrast, pedagogical content knowledge refers to the mathematical knowledge necessary to teach mathematics and consists of two components: knowledge of instructional strategies/representations and knowledge of students' preconceptions and misconceptions in mathematics (Shulman, 1987).

While practical uses of PCK have been operationalised in many ways particular to mathematics (e.g., Krauss et al., 2008; Lim-Teo, Chua, Cheang, & Yeo, 2007), critiques of Shulman's CK and PCK framework have also emerged. Depaepe, Vershaffel, and Kelchtermans (2013) found a "lack of theoretical and empirical grounding for the existence of PCK as a distinct category in teachers' knowledge base" (p. 13), noting several researchers who have long doubted this distinction (e.g., Baumert et al., 2010; Bednarz & Proulx, 2009; Blömeke, Felbrich, Müller, Kaiser, & Lehmann, 2008; Huillet, 2009; Saderholm, Ronau, & Brown, 2010). In addition, Depaepe and colleagues noted that most of the research assumes that CK is "an important and necessary prerequisite for teachers' PCK" (p. 15).

The relationship between content and pedagogical content knowledge

Regardless of the various definitions and framings of the relationship between content and pedagogical content knowledge, many researchers agree that both knowledges affect instructional practice and students' mathematics learning [3](Baumert et al., 2010; Bukova-Güzel, Kula, Uğurel, & Özgür, 2010; Hill et al., 2005).

In the U.S., Buschang and her colleagues (2012) studied algebra teacher knowledge using four measures of teachers' knowledge, three for PCK and one for CK, finding moderate correlations between two PCK constructs and CK. In Germany, Krauss and colleagues (Krauss et al., 2008; Krauss, Baumert, & Blum, 2008) developed separate CK and PCK assessments for secondary mathematics teacher knowledge, finding that 10th grade German teachers with higher mathematical expertise had high correlations between CK and PCK (.96), whereas teachers with less expertise had lower correlations (.61).

In a further exploration of the relationship between teacher knowledge, instruction, and student progress, Baumert et al. (2010) analysed data with 9th grade German teachers. Similar to Krauss et al. (2008), their analysis indicated differences in both knowledge types for teachers trained for different types of schools – academic versus non-academic – and found a high correlation between CK and PCK for teachers trained for the academic track. Related, Kleickmann et al.'s (2013) examination of German future teachers showed strong correlations between CK and PCK. Through a factor analysis, these researchers found the latent correlations between content and pedagogical content knowledge as .64 for year one prospective teachers and .78 for year three prospective teachers. However, other results from the U.S. and Germany indicate that high CK does not always lead to the development of PCK, perhaps because the development of the latter requires CK, teaching experience, and extensive reflection on instructional practice (Hauk et al., 2010; Kleickmann et al., 2013). Furthermore, evidence indicates that German teachers with high CK may not always have high PCK and vice versa (Blömeke, Suhl, and Kaiser, 2011).

In a study using the same TEDS-M data we analyse here, Blömeke, Suhl, and Kaiser (2011) found interesting differences in the mean scores of both knowledge types among the highest scoring countries. For example, the U.S. ranked 7th overall for CK but 4th for PCK. Moreover, future teachers from the Russian Federation, Thailand, and Germany had much lower rankings in PCK than in CK. However, because this analysis focused on the mean scores of the tests, the relationship between CK and PCK can be obscured – a point acknowledged by the study's

[3] We acknowledge other frameworks for referring to teachers' knowledge, including mathematical knowledge for teaching (MKT) (Hill, Rowan, and Ball, 2005). However, we ground our work in the PCK framework for two reasons. First, the global literature on teachers' knowledge frames PCK as the dominant term used to discuss this issue. Second (and perhaps as a result), the TEDS-M assessment framework is organised around PCK.

authors. In particular, the apparent lack of correlation between content and pedagogical content knowledge (as found in this analysis of mean scores) challenges the assumption that CK is an important precondition for PCK. Based on these results, we find it insufficient to analyse mean scores of knowledge types to fully explore the relationship between these constructs.

In summary, content and pedagogical content knowledge are often positioned as related in the literature, even though past empirical work finds mixed results for the strength of this relationship. To further understand the link between CK and PCK, we must consider how the development of each knowledge type affects their relationship. For example, if knowing mathematics were a necessary component of knowing how to teach mathematics, then the development of CK would affect the development of PCK, and CK would be strongly related to PCK. The large international TEDS-M dataset provides an opportunity to further explore the relationship between CK and PCK across different countries with a large sample size. Thus, the current paper can uniquely contribute to existing literature on the relationship between CK and PCK using a large sample of future teachers in many different educational contexts.

The development of content and pedagogical content knowledge

Content knowledge. Despite disagreements about how to gauge teachers' mathematical knowledge, it is widely accepted that content knowledge grows as future teachers take more advanced and demanding mathematics content courses. For example, Kleickmann and colleagues (2013) found that future German teachers' CK developed most during their university studies. Similarly, the *Mathematics Teaching in the 21st century* (MT21) study found that future teachers in the U.S. who participated in programs requiring a larger number of advanced mathematics courses displayed higher CK than those who participated in other programs (Schmidt, et al., 2007).

This assumption that higher levels of mathematics courses positively affect teacher knowledge influences teacher preparation policy around the world. In the U.S., the National Council of Teachers of Mathematics' (NCTM) *Highly Qualified Teachers* statement (2005) and the *Mathematics Education of Teachers II* (MET II) report (2012) both outline specific coursework guidelines under the assumption that a strong relationship exists between the number of content courses taken and the quality of mathematics teachers' preparation. Similarly, in Europe, the Bologna Process prompted many countries to alter their teacher preparation programs by strengthening the academic orientation through a renewed focus on content (Bauer & Prenzel, 2012). Finally, Li and colleagues (2008) describe how a stronger concentration on mathematics in Chinese teacher preparation programs requires future teachers to take more content courses in advanced mathematics.

However, the empirical literature is mixed on exactly how the number and type of content courses taken is related to high quality teachers and teaching. Monk (1994) found that only secondary students in advanced courses benefited from teachers' mathematics course taking, and the positive impact of teacher course taking on secondary student learning diminished after a particular number of courses. Similarly, data from the 1992 and 2000 U.S. National Assessment of Educational Performance (NAEP) draws attention to how eighth-grade students with teachers who had a mathematics teaching certificate or a major or minor in mathematics or mathematics education scored significantly higher on the mathematics assessment (Ingersoll, 2007). Yet at the same time, the possession of subject-specific degrees by teachers had mixed or even negative effects on elementary students' achievement (Ingersoll, 2007). Similarly in Germany, a study analysing mathematics teachers' knowledge in relation to instruction concluded that simply requiring a mathematics major does not necessarily improve future teachers' knowledge (Baumert et al., 2010).

Pedagogical content knowledge. When considering how pedagogical content knowledge develops, the research literature gets even more complicated. Factors believed to support the development of PCK include coursework in both mathematics and mathematics education (Hauk et al., 2010; Lim-Teo, et al., 2007), as well as non-course experiences such as collaborative learning opportunities (Leikin, 2004), mentoring (Nilssen, 2010), and working in a professional community of teachers (Dalgarno and Colgan, 2007).

Wilson, Floden, and Ferrini-Mundy (2001) found little evidence of a correlation between PCK and some of the proxies for teacher knowledge, particularly the number of mathematics courses taken. Similarly, Youngs and Qian (2013) report that, "PCK related to mathematics teaching is associated with more practical experiences during teacher preparation" (p. 3). Buschang and colleagues (2012) found that pedagogical content experts and experienced teachers displayed higher correlations between PCK measures than novice teachers and subject matter experts, concluding that teaching experience influences the development of PCK. Similarly, Kleickmann et al. (2013) found that university studies and induction phases play an important role in the development of PCK; practicing academic-track teachers had higher PCK scores than future academic-track teachers tested at the conclusion of their initial teacher education. Thus, the practice of actual teaching seems to (weakly) contribute to additional PCK development.

Opportunities to Learn

The construct of OTL has been used in a variety of IEA studies, including the First and Second International Mathematics Studies (FIMS and SIMS) and the Trends in Mathematics and Science Study. However, the way that OTL has been addressed varies across these studies, including TEDS-M. The TEDS-M team used the construct to explore, in part, the mathematics topics future teachers reported having studied. To better understand this construct, its relation to CK and PCK, and to unpack the TEDS-M research teams' decision to use this as part of their work, we now briefly discuss the definition of OTL and its relation to teacher learning and preparation.

The first use of OTL for an IEA study was for the First International Mathematics Study (FIMS; Husen, 1967) where data was collected on both students' and teachers' views of students' opportunities to learn mathematics. In these studies, OTL was framed "as a content coverage variable without specific regards to allocated time" (Schmidt, Cogan, Houang, & McKnight, 2009, p. 5). This definition of OTL as content coverage allowed FIMS, SIMS, and TIMSS researchers to connect students' opportunity to study specific mathematical concepts and skills to their performance on assessments. In fact, results illustrated strong positive correlations between OTL and student achievement in all three studies (Schmidt, McKnight, & Raizen, 1997). Thus, the connection between prospective teachers' content and pedagogical content knowledge and their opportunities to learn mathematical concepts and skills seemed to be a natural extension of this work.

The TEDS-M research team used OTL to explore teacher preparation programs by using the construct to investigate curricular variations across programs. OTL allowed the researchers to represent "the diversity of content, both overall and for distinct groups of teachers" (Tatto, Schwille, Senk, Ingvarson, Peck, & Rowley, 2009, p. 44) in a coherent and consistent way. In reporting results from the study, the TEDS-M research team used OTL to describe teacher learning and to try to explain differences in the levels of CK and PCK of preservice teachers. That is to say, in order to better understand the impact of teacher preparation programs, using a construct such as OTL gave researchers the ability to "describe curricular variation among teacher preparation program types across countries and to investigate how such variation is related to differences in levels of knowledge of future teachers" (Tatto & Senk, 2011, p. 124). Results from

TEDS-M include high variability in teacher preparation programs with respect to mathematics pedagogy, tertiary mathematics, and some domains of school-level mathematics. In particular, primary teachers experienced fewer opportunities to learn mathematics content and pedagogy (Tatto et al., 2012). Additionally, the relationship between OTL and CK proved to be complex, with high/low OTL not necessarily corresponding to high/low CK scores (Tatto & Senk, 2011).

We know that the empirical literature is mixed on how courses taking is related to high quality teachers and teaching. As evidenced from the results described above, the TEDS-M work was unable to provide clarity on this issue, even with using OTL rather than course taking patterns. However, the TEDS-M does have the potential to provide more information on the relationship between teacher knowledge and OTL through additional secondary analysis of the data set.

In summary, we know that content and pedagogical content knowledge are important for teacher preparation. While the opportunities to learn mathematics may be one way to develop teacher knowledge, the relationship between types of knowledge, and how different mathematics courses influence the development of mathematics teachers' knowledge, remains unclear. Our work continues the tradition of investigating the relationships between and development of CK and PCK internationally by using the TEDS-M data to examine the relationship between teachers' knowledge with the opportunity to learn mathematics. We add to the field by showing how distal measures of teacher knowledge, such as opportunities to learn mathematics, might inform our understanding of the relationship between CK and PCK. Once again, our analysis is framed around the following research questions:

1. For future primary and secondary mathematics teachers, what is the relationship between content knowledge and pedagogical content knowledge?
2. For future primary and secondary mathematics teachers, what is the relationship between these knowledge constructs and opportunities to learn mathematics?

Method

The TEDS-M[4] research study was conducted by the International Association for the Evaluation of Educational Achievement, the International Study Center at Michigan State University, and the Australian Council for Educational Research. The TEDS-M study investigated how teachers were prepared to teach mathematics, the nature of teacher education programs, and differences in teachers' content and pedagogical content knowledge within and across 17 countries. Data was collected from representative samples from teacher education programs in each country: future primary teachers, lower secondary teachers, and teacher educators.

Our analysis of this data focuses on data from future primary (N = 15,163) and secondary (N = 9,389) teachers in their last year of teacher preparation. While there was variation between countries about how each teacher education program was structured, all future teachers were enrolled in an institution that offered formal opportunities in learning to teach mathematics. These teachers came from 17 countries: Botswana, Canada, Chile, Chinese Taipei, Georgia, Germany, Malaysia, Oman, the Philippines, Poland, the Russian Federation, Singapore, Spain, Switzerland, Thailand, the U.S., and Norway. Although we examined data from all of these countries, we report extensively on fourteen of the participating countries. We excluded the three remaining countries (Botswana, Canada, and Norway) from our analyses because of small

[4] For additional details about the study, see Tatto and Senk (2011).

sample sizes and/or problems with data collection (e.g., difficultly aggregating different populations in the country; see Tatto and Senk (2011) for details).

Instruments

TEDS-M collected a variety of information on future teachers' mathematics CK, PCK, and mathematics coursework taken. We focus on two of the TEDS-M instruments most relevant to our research questions: (1) a mathematical knowledge for teaching assessment and (2) an opportunities to learn survey.

Mathematical knowledge for teaching assessment. To measure future teachers' content and pedagogical content knowledge, the TEDS-M research team developed a mathematical knowledge for teaching assessment by drawing from work by several international scholars on the constructs (e.g., An, Kulm, & Wu, 2004; Conference Board of the Mathematical Sciences, 2001; Even & Ball, 2009; Hill, Rowan, & Ball, 2005; Pepin, 1999; Schmidt et al., 2007). Using this research, the team was able to design items and instruments to measure the CK and PCK of preservice teachers who intended to teach primary or lower-secondary school.

The CK items measured fundamental mathematics assumptions, definitions, concepts, and procedures, and included future teachers' knowledge of number (e.g., whole numbers, fractions, decimals, patterns, ratios), geometry (e.g., geometric shapes, geometric measurement), algebra (e.g., algebraic expressions, equations, inequalities, formulas), and data (e.g., data organisation and representation, data reading and interpretation). For example, future teachers were asked to determine the area of the walkway around a rectangular swimming pool based off of a provided diagram. The CK items were a mixture of multiple choice and constructed response, providing a measure of teachers' knowledge of mathematics at and above the content level they were intending to teach. Future primary teachers answered 74 CK items and future secondary teachers answered 76 CK items.

The PCK items measured future teachers' ability to identify and explain students' errors and thinking on example problems. For example, future teachers analysed the validity of students' proofs justifying why multiplying three consecutive natural numbers results in a multiple of 6. The PCK items were a mixture of multiple choice and constructed response with varying degrees of difficulty. There were 32 PCK items administered to future primary teachers and 27 items administered to future secondary teachers. It is worth noting that PCK can only be measured in a limited way without observing future teachers in a classroom, and this paper-and-pencil assessment likely only captures certain aspects of teachers' PCK. However, this method of assessment was the only feasible way to measure PCK with such a large sample in a large number of countries.

Future teachers' knowledge scale scores were estimated using Item Response Theory. The scale scores were calculated on a logit scale and standardised to have a mean of 500 and standard deviation of 100 for equally weighted countries (see Appendix B of Tatto (2012) for details). Since each group answered items appropriate for their grade levels, scaling was done separately on the scores for each. This means that, one limitation of this data is that CK and PCK scores *cannot* be compared between future primary and secondary teachers (TEDS-M User's Guide). Each teacher had a scale score for CK and a scale score for PCK. The mean CK score and mean PCK score for each country was calculated by finding the mean scale scores across all the future teachers in that country at that level (primary or secondary).

Opportunities to learn survey. As part of the Opportunities to Learn (OTL) survey, future teachers were asked whether or not they had studied particular topics at the university (or tertiary) level within their teacher preparation program, as well as topics typically taught at the primary or secondary school level. If a future teacher had indicated that he or she had studied a topic, the score given was a 1, and 0 otherwise. Then the total number of topics each teacher studied was calculated by summing these values. An average number of topics studied for each country was calculated by finding the mean number of topics studied across all future teachers in that country. Providing an OTL survey allowed researchers to determine variations in curricula in teacher preparation programs across countries. In the current study, we focus on teachers' opportunities to study specific mathematics topics to see if such opportunities relate to teachers' CK and PCK. One limitation of this measure is that it is based on future teachers' self-report, which may not always accurately recall what was studied. Moreover, the measure does not indicate the depth with which each topic was covered (e.g., over an entire course or as part of a broad mathematics course). However, this OTL survey was developed from extensive previous research that found these scales to have strong validity and reliability using techniques such as exploratory and confirmatory factor analyses, scale reliability analyses, and experts' judgments (Schmidt et al., 2007; Tatto, 1996; Tatto et al., 1993). This past research has found OTL to be an appropriate measure of future teachers' opportunities, and was the more feasible way to obtain this information with such a large international sample.

Data Analysis

The IEA International Database Analyser software (Tatto, 2012) allowed us to calculate means and correlations using the appropriate estimation weights. Estimation weights were needed to obtain design-unbiased estimates of population features (Tatto, 2012). Calculating correlations allowed us to capture the strength of the relationships between constructs and compare how these relationships varied. Because of the very large sample size, all correlations we report are statistically significant ($p < .01$). As a result, instead of relying upon statistical significance as an indicator, we mostly highlight relationships at or above the international average.

Results

We begin by reporting the means of and correlations between future teachers' CK and PCK scores for each participating country. We then investigate differences in correlations between countries to further understand these relationships. Finally, we discuss correlations between the number of topics future teachers had the opportunity to learn and future teachers' CK and PCK.

Relationship between content and pedagogical content knowledge

The mean differences between content and pedagogical content knowledge assessment scores at the country level for future primary and secondary teachers were comparable, within a one-half standard deviation (50 points) of each other. The largest differences in favour of CK were for Chinese Taipei future primary teachers with a 30.97-point difference and the Russian Federation future secondary teachers with a 27.53-point difference. The largest differences in favour of PCK were for the U.S. future primary teachers with a -25.85-point difference and Chilean future secondary teachers with a -39.33-point difference. Yet some differences were quite small, and the overall international average showed a difference between CK and PCK of -1.91 for future primary teachers and -.90 for future secondary teachers. Recall that all scores were standardised to have a mean of 500 and standard deviation of 100.

As discussed above, previous analysis of this same data found that focusing only on mean scores obscured the relationship between CK and PCK (Blömeke, Suhl, & Kaiser, 2011). We also observed that the means at the country level obscured the very large variation in the sample, which led us to investigate the relationship at the teacher level. We noticed that on the teacher level, the discrepancies (the difference between CK and PCK for each teacher) were much greater, with some as large as 200 points – such as when an individual teacher had a very high CK score with a low PCK score. Thus, although the overall country-level mean assessment scores seemed to be relatively close, individual teachers' knowledge scores were often very different. This, in turn, led us to investigate the correlations between these constructs.

We calculated the overall teacher-level correlations between CK and PCK (Table 1). Overall, the international averages were moderate for future primary and secondary teachers. At the primary level, Poland, Germany, the Russian Federation, Thailand, and the U.S. had correlations above the international average. At the secondary level, Germany, the Russian Federation, Poland, the U.S., and Singapore had correlations above the international average. As we looked at these correlations, we noted that in several cases, the countries did not necessarily have small differences between overall country-level knowledge assessment scores, as stated above, so the fact that their correlations between CK and PCK were higher illustrates that there was a smaller range of values at the individual teacher-level.

Table 1
CK and PCK Correlations by Country

	Primary	S.E.	Secondary	S.E.
Chile	.46	.03	.51	.03
Chinese Taipei	.43	.04	.45	.04
Georgia	.38	.03	n/a	n/a
Germany	.62	.03	.70	.03
Malaysia	.44	.05	.52	.04
Oman	n/a	n/a	.44	.04
Philippines	.34	.04	.37	.10
Poland	.68	.01	.67	.05
Russian Federation	.58	.05	.68	.04
Singapore	.34	.04	.55	.04
Spain	.41	.03	n/a	n/a
Switzerland	.38	.03	.40	.08
Thailand	.50	.03	.50	.03
United States	.48	.03	.64	.03
International Average	**.46**		**.54**	

Note. Due to the very large sample size, all correlations were statistically significant ($p < .01$). Oman did not have data on future primary teachers, Georgia had a very low sample size of future secondary teachers, and Spain did not have data on future secondary teachers. Consequently, these correlations are marked with "n/a."

Relationship between Opportunities to Learn Mathematics and Teacher Knowledge

Overall, correlations between OTL and teacher knowledge were low, with some moderate correlations in specific countries.

Future primary teachers reported having the opportunity to learn 13.29 total mathematics topics on average (SD = 5.08, range 0 to 24), and future secondary teachers reported having the opportunity to learn 18.42 total mathematics topics on average (*SD* = 4.91, range 0 to 24). The total number of topics participants reported having the opportunity to learn was not highly correlated with teacher knowledge (CK or PCK) for future primary or secondary teachers (Table 2). For future primary teachers, the average correlations between OTL and CK and PCK were low (international averages $r = 0.11$ and $r = 0.08$, respectively). Countries with correlations above the international average for OTL and CK included Chinese Taipei, Germany, Poland, and Singapore. Those countries with correlations above the international average for OTL and PCK were Chinese Taipei, Georgia, Germany, Poland, and Singapore. However, most countries had correlations between OTL and PCK that were close to zero.

Looking at future secondary teachers, the average correlations between OTL and CK and PCK were also low (international averages $r = 0.15$ and $r = 0.08$, respectively). Those countries with correlations above the international average for OTL and CK were Chinese Taipei, Germany, the Russian Federation, and the U.S. Those countries with correlations above the international average for OTL and PCK were Chile, Chinese Taipei, Germany, the Russian Federation, and the U.S. These results suggest that OTL overall was not generally associated with higher levels of teacher knowledge, except in a small subset of countries.

Table 2
Correlations of Total Math Topics Studied and Teacher Knowledge

	Primary Total Topics		Secondary Total Topics	
Country	CK	PCK	CK	PCK
Chile	.07	.05	.15	.13
Chinese Taipei	.25	.19	.19	.19
Georgia	.08	.09	n/a	n/a
Germany	.45	.36	.44	.24
Malaysia	.06	.04	.05	-.09
Oman	n/a	n/a	-.03	-.08
Philippines	-.09	-.10	.03	.05
Poland	.57	.42	.00	-.03
Russian Federation	.02	.03	.21	.17
Singapore	.14	.15	.10	.03
Spain	.07	.00	n/a	n/a
Switzerland	.08	.01	.02	.08
Thailand	.04	.04	.14	.10
United States	-.02	-.05	.54	.39
International Average	**.11**	**.08**	**.15**	**.08**

Note. Due to the very large sample size, all correlations were statistically significant ($p < .01$). Oman did not have data on future primary teachers, Georgia had a very low sample size of future secondary teachers, and Spain did not have data on future secondary teachers. Consequently, these correlations are marked with "n/a."

Summary

Content and pedagogical content knowledge scores had moderate correlation with each other. Additionally, both measures of teacher knowledge had low correlations with OTL, but the correlations were higher for CK than PCK. A subset of countries often had correlations above the international average: Germany, Poland, the Russian Federation, and the U.S. had the highest correlations between CK and PCK for both future teacher populations. Within this group, only Germany attained correlations above the international average for all categories of teacher knowledge and OTL. Poland had correlations above the international average for only future primary teachers, while the Russian Federation and the U.S. had correlations above the international average for only future secondary teachers. Chile, Chinese Taipei, Singapore, and Thailand also had some correlations close to or above the international average for CK and PCK, but these were inconsistent. These findings, possible causes, and implications for teacher preparation programs are explored next.

Discussion

Our analysis of the TEDS-M data sought to understand the relationship between content knowledge, pedagogical content knowledge, and opportunities to learn mathematics. We were interested in these relationships and differences that might occur between participating countries. Our analysis yielded three main findings: 1) there is large individual variation in teachers with how closely related their CK and PCK are, suggesting CK might not have to develop before PCK; 2) the relationships between CK and PCK are only moderate and may not be as strongly linked as previously thought; and 3) the OTL mathematics has a weak relationship with CK and an even weaker relationship with PCK. While OTL mathematics is not equivalent to the number of mathematics courses taken and does not include an evaluation of the quality of those courses, this measure does give some indication that, within these teacher preparation programs, exposure to more mathematical topics may not increase one's CK and PCK. We now discuss these findings.

Relationship between Content and Pedagogical Content Knowledge

The correlation between content and pedagogical content knowledge in most of the TEDS-M countries displayed was moderate. In particular, out of the 14 countries, only four had correlations above .60, with only two obtaining these correlations for *both* future primary and secondary teachers. As discussed above, past research has shown that CK and PCK are highly correlated for particular populations of teachers, and develops either concurrently (Krauss et al., 2008), or with CK acting as a prerequisite for PCK (Baumert et al., 2010; Even, 1993). Therefore, our findings of moderate correlations between CK and PCK was not anticipated.

One potential explanation for these findings could be methodological or instrumentation problems with the TEDS-M instruments. Although TEDS-M was a very large and complex study, published technical reports of TEDS-M (e.g., Tatto, 2012) indicate that the study had national and regional partners in each country to help alleviate potential difficulties by testing and piloting all their measures extensively. TEDS-M also used well-established analysis techniques to adequately deal with the complexities inherent in this kind of study. Therefore, we do not think methodological issues are the main cause for the current findings.

Another explanation is that the correlations between content and pedagogical content knowledge were low to moderate due to challenges inherent in the assessment of these constructs. As discussed above, inconsistent evidence supports the relationship between CK and PCK. The results from this large, diverse sample of future teachers suggest that prior findings showing a high positive correlation between content and pedagogical content knowledge may need further investigation, particularly in how CK and PCK were defined and measured in those studies and what sample of teachers was used.

Beyond these potential measurement issues, other compelling implications from our current findings support and challenge prior studies analysing content knowledge, pedagogical content knowledge, and their relationship. For example, our results contradict Buschang et al.'s (2012) findings that CK and PCK are tightly linked and develop together. Our findings support the idea that teachers with high CK may not always have high PCK and vice versa (Blömeke et al., 2011) because the correlations overall were low to moderate rather than strongly negative or strongly positive. Hence, our findings challenge research that suggests that CK is necessary, but not sufficient, for PCK (e.g., Depaepe et al., 2013). Our results indicate that teachers with low CK can still have high PCK. Perhaps sufficient experience in other areas that influence PCK, such as teaching experience and extensive reflection on instructional practice (Hauk et al., 2010; Kleickmann et al., 2013), can compensate for lower CK when developing PCK. In particular, the low to moderate correlations in our analysis indicate that CK may not necessarily need to develop before PCK (e.g., Baumert et al., 2010). Hence, future reforms should consider the best ways to foster content and pedagogical content knowledge as distinct constructs rather than working under the assumption that developing CK is a necessary prerequisite for developing PCK. In other words, just knowing mathematics does not necessarily help one become an effective mathematics teacher.

Correlations between content and pedagogical content knowledge. To further explore possible reasons for the low to moderate correlations between content and pedagogical content knowledge, we examine how these constructs develop differently between countries. We would like to note that we are unable to speak to differences within countries because the TEDS-M dataset does not allow for analyses of these within country explanations. As a result, we would be left speculating about possible reasons for the correlation results, and rather than do this, hope that scholars within each country will further explore these possibilities in more depth.

In considering between country differences, we focus on seven countries with correlations between CK and PCK that are at or above the international averages: Chile, Germany, Poland, the Russian Federation, Thailand, Singapore, and the U.S. Generally, for these countries, the relationship between teacher knowledge for future secondary teachers was higher than for future primary teachers, with the exception of Poland and Thailand, who had equal correlation. We now examine correlations between countries to explain our findings about the relationship between CK and PCK.

Between country differences. When considering the differences between countries, we noticed consistently higher correlations for future teachers in Germany, Poland, and the Russian Federation. This could be due to the nature of educational reforms and the purposeful attention paid to the sequence of content and pedagogy in these countries. As discussed previously, the Bologna Declaration and the subsequent reforms created specific policies related to teacher knowledge. Russia embraced teacher education focusing on principles of knowing and being able to teach mathematics well. Germany developed a core curriculum for teachers to organise and connect the subject-centred and education-centred elements of the knowledge of new teachers. However, why did no other countries involved in the Bologna Process show similar correlations?

Beyond Germany, Poland, and the Russian Federation, we found a group of countries (the U.S., Chile, Singapore, and Thailand) with correlations above the international average for only future primary or only future secondary teachers. How do these countries approach primary and secondary teacher preparation differently? The U.S. had correlations close to the international average for future primary teachers, but attained the fourth highest correlation for future secondary teachers. One reason for this could be the requirements currently in place for primary versus secondary teacher training. In particular, most future primary teachers in the U.S. need only take a select few content courses along with multi-disciplinary methods courses, while future secondary teachers typically major in mathematics. Therefore, in considering the patterns in the correlation between CK and PCK, future research might investigate how connectedness between content and pedagogical content knowledge is addressed in both primary and secondary teacher preparation programs. We now look at the relationship between opportunities to learn mathematics and teacher knowledge.

Relationship between Teacher Knowledge and Opportunities to Learn Mathematics

Contrary to expectations from the extant literature, we found that neither content nor pedagogical content knowledge was strongly correlated to the opportunities to learn mathematics in teacher preparation programs. In fact, most countries had correlations at or below .25 for OTL and teacher knowledge.

However, our results did show modest differences in the correlation between CK and OTL compared with the correlation between PCK and OTL. For the future primary teachers, the international average correlation for OTL and CK was .11, and for PCK was .08. For the future secondary teachers, the international average correlation for OTL and CK was .15, and for PCK was .08. Such differences might be a result of the lack of an explicit focus on PCK in traditional mathematics courses. As noted in the opening sections, the literature suggests that developing PCK requires specially designed tasks and courses (e.g., Lim-Teo, Chua, Cheang, and Yeo, 2007; Youngs & Qian, 2013), leading to expected low impact on PCK from typical mathematics content courses.

Therefore, from our findings and reading of the literature, while it is surprising that CK and OTL had overall low correlations, it is perhaps less surprising that PCK and OTL also had low correlations because there are many more factors thought to influence PCK. In fact, our findings support Wilson et al.'s (2001) and Young & Qian's (2013) findings that PCK and mathematics course taking are not strongly related. To unpack the reasons for these findings, we again considered both measurement issues as well as differences in teacher education programs between participating countries.

Differences in correlations between teacher knowledge and opportunity to learn mathematics. One might interpret the unexpected findings of low to modest correlations between knowledge and OTL by considering measurement issues, particularly the ways that TEDS-M assessed OTL. Recall that future teachers were asked to indicate whether or not they had studied particular mathematics topics within their teacher preparation programs. This type of measure has been used in prior work (e.g., Schmidt, Cogan, and Houang, 2011; Tatto and Rodriguez, 2012), and OTL is thought to be a reasonable proxy for the number of mathematics courses taken (e.g., with the assumption that those who have studied more topics have taken more mathematics content courses). However, self-reports of OTL may not be an adequate measure for content coverage, in that the survey provided no guidance on what it meant to "study" a topic in a math course. For example, two respondents from the same country who both indicated that they had studied geometry would be considered equivalent, even if one respondent took a full-year geometry course while the other only had a single week of coursework on geometry. Furthermore, it is not clear whether students' self-reported recall of which topics they studied is a reliable indicator of which topics they had actually studied - asking future teachers to remember topics they may have covered in mathematics courses three years ago may yield questionable and inconsistent answers. However, beyond these potential measurement issues, our results still have important implications for how the OTL may relate to future teachers' knowledge.

The overall low correlation between CK and OTL may have occurred not only because of the complex relationship between the two constructs (Tatto & Senk, 2011), but also because of the nature of the mathematics taken within teacher preparation programs. In fact, the benefits of certain mathematics courses for teacher knowledge may depend on the type of course and the type of student teachers may encounter in the future (Ingersoll, 2007; Monk, 1994).

To unpack differences between countries, we report on eight countries whose correlations were at or above the international average for more than one of the six calculated correlations for knowledge and OTL. Chinese Taipei and Germany had correlations above the international average for *both* future primary and secondary teachers, but Germany's correlations were notably higher than the international average. Chile, the Russian Federation, Thailand, and the U.S. had correlations above the international average for *only* future secondary teachers, with the U.S. having notably higher correlations. Finally, Poland and Singapore had correlations above the international average for *only* future primary teachers, but Poland's correlations were notably higher than the international average.

Between country differences. To understand the notable correlations between teacher knowledge and OTL between countries, we consider the influence of policy changes in Europe to teacher education reforms and the nature of their teacher preparation programs. As discussed previously, the Bologna Process prompted changes in teacher preparation programs throughout Europe through a stronger academic orientation, among other changes (Bauer & Prenzel, 2012). For example, in Germany, future teachers spend three to four years at university studying mathematics while simultaneously participating in a practicum experience. Following these studies, future teachers spend two years student teaching. In short, the first phase of training is theory/knowledge oriented connected to university and the second phase is practice/skill oriented connected to a school. By working to better coordinate the two phases, it is possible that the opportunities to learn mathematics within these programs provides observable benefits to teacher knowledge in the form of consistent and notable correlations between teacher knowledge and opportunities to learn mathematics.

However, it is worth noting that other countries involved in the Bologna Process did not have notable correlations, and only Poland and the Russian Federation had any correlations above the international average. This begs the question as to how the Bologna Process and subsequent reforms were instituted differently across the region and how these differences have impacted future teachers' knowledge as well as their mathematical studies. To further understand these differences, one might consider how the Russian Federation's requirements for teachers to know and teach mathematics well influence the relationship between CK, PCK, and OTL; why this relationship is higher more often for their future secondary teachers; and why this relationship is different than in neighbouring Germany.

On a related note, the U.S. attained notable correlations only for future secondary teachers. One explanation of the notable correlations between teacher knowledge and OTL in the U.S. may be the emergence of a new type of content course, which we have referred to as a *connections course* (Murray & Star, 2013). Connections courses have the explicit goal of helping future teachers make connections between the mathematics they will teach and the mathematics they learn in college. In surveying the landscape of connections courses offered by mathematics departments in the U.S., [Authors] note that there appear to be at least two types of such connections courses – *secondary mathematics from an advanced standpoint* and *tertiary mathematics with connections*. *Secondary mathematics from an advanced standpoint* courses cover school mathematics at a level of depth, detail, and rigor suitable for undergraduate mathematics students, but explicitly highlight and investigate the connections to K-12 mathematics. Such courses may help future teachers see the importance of the mathematics they are required to take as undergraduates while enabling them to contemplate pedagogical strategies that might support students' thinking in ways that can make advanced mathematics more accessible. *Tertiary mathematics with connections* courses begin with college-level content and make explicit connections to secondary mathematics. Such courses cover advanced mathematics (such as Abstract Algebra), but do so by focusing on the importance of these topics within school mathematics. Both types of courses work to make the connections between the advanced and secondary mathematics, but from different vantage points. It is possible that the emergence of connections courses in the U.S. may account for stronger relationships found between CK, PCK, and OTL because such courses appear to be explicitly designed to foster these relationships.

In summary, our findings contradict past research studies that suggest content and pedagogical content knowledge can necessarily be developed through mathematics courses (Baumert et al., 2010; Bolyard & Moyer-Packenham, 2008; Kleickmann et al., 2013; Schmidt et al., 2007; U.S. Department of Education, 2010). Nevertheless, when considering how to influence knowledge development in different countries, perhaps more specific types of courses, such as connection courses, could be implemented rather than the blanket recommendation given by NCTM, the MET II report, and the Bologna Process for more mathematics content courses. Even so, while it is important to discuss the similarities and differences between teacher preparation programs in different countries, we must point to the fact that context matters. Specifically, we must recognise that various educational contexts lead to different course structures, program requirements, and instructional approaches being more effective in some countries over others.

Conclusion

The main finding from our analysis was that the correlations between content knowledge, pedagogical content knowledge, and the opportunity to learn mathematics were modest and often low globally. The TEDS-M data suggest that our understanding of how teachers develop the knowledge critical for effective mathematics teaching is complex and worthy of future study. However, we acknowledge limitations with the data that limits our ability to make additional claims. For example, the TEDS-M data set is not intended to examine variation within a country beyond examining correlations between constructs. That is, the data is not aggregated by institution, thus we cannot make claims about within country differences between teachers from various programs or regions. Nevertheless, we note promising research addressing within country variations, extending TEDS-M data to further explore teacher knowledge, including studies in Germany (e.g., the Teacher Education and Development Study Follow Up [TEDS-FU]), the U.S. (e.g., Schmidt, Houang, & Cogan, 2011), and Chinese Taipei (e.g., Hsieh, Lin, & Wand, 2012).

In terms of global implications for teacher education, our results ask teacher education programs to reconsider the common assumption that CK will lead to PCK. While further causal research is necessary to understand the complexity of exactly why CK and PCK were correlated only in a few countries and situations, the teacher education community should note that the relationship between CK and PCK is messy. As content and pedagogical content knowledge continue to be recognised as important components of mathematics teacher development, we must further investigate the relationship between these constructs and how to best foster them in international mathematics teacher education.

References

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, *59*(5), 389–407. http://doi.org/10.1177/0022487108324554

Baumert, J., Kunter, M., Blum, W., Brunner, M., Voss, T., Jordan, A., … Tsai, Y.-M. (2010a). Teachers' mathematical knowledge, cognitive activation in the classroom, and student progress. *American Educational Research Journal*, *47*(1), 133–180. http://doi.org/10.3102/0002831209345157

Baumert, J., Kunter, M., Blum, W., Brunner, M., Voss, T., Jordan, A., … Tsai, Y.-M. (2010b). Teachers' mathematical knowledge, cognitive activation in the classroom, and student progress. *American Educational Research Journal*, *47*, 133–180. http://doi.org/10.3102/0002831209345157

Blömeke, S., Suhl, U., & Kaiser, G. (2011). Teacher education effectiveness: Quality and equity of future primary teachers' mathematics and mathematics pedagogical content knowledge. *Journal of Teacher Education*, *62*(2), 154–171. http://doi.org/10.1177/0022487110386798

Bolyard, J. J., & Moyer-Packenham, P. S. (2008). A review of the literature on mathematics and science teacher quality. *Peabody Journal of Education*, *83*(4), 509–535. http://doi.org/10.1080/01619560802414890

Bukova-Güzel, E., Kula, S., Uğurel, I., & Özgür, Z. (2010). Sufficiency of undergraduate education in developing mathematical pedagogical content knowledge: Student teachers' views. *Procedia-Social and Behavioural Sciences*, *2*(2), 2222–2226.

Conference Board of the Mathematical Sciences. (2012). *The Mathematical Education of Teachers II* (Vol. 17). Providence, RI: American Mathematical Society.

Dalgarno, N., & Colgan, L. (2007). Supporting novice elementary mathematics teachers' induction in professional communities and providing innovative forms of pedagogical content knowledge development through information and communication technology. *Teaching and Teacher Education*, *23*(7), 1051–1065. http://doi.org/10.1016/j.tate.2006.04.037

Depaepe, F., Verschaffel, L., & Kelchtermans, G. (2013). Pedagogical content knowledge: A systematic review of the way in which the concept has pervaded mathematics educational research. *Teaching and Teacher Education*, *34*, 12–25. http://doi.org/10.1016/j.tate.2013.03.001

Ferrini-Mundy, J., Floden, R., McCrory, R., Burrill, G., & Sandow, D. (2005). Knowledge for teaching school algebra: Challenges in developing an analytic framework. Presented at the American Education Research Association, Montreal, Quebec, Canada.

Hauk, S., Jackson, B., & Noblet, K. (2010). No teacher left behind: Assessment of secondary mathematics teachers' pedagogical content knowledge. In *Proceedings of the 13th Conference on Research in Undergraduate Mathematics Education*. Retrieved from http://sigmaa.maa.org/rume/crume2010/Archive/HaukNTLB2010_LONG.pdf

Hill, H. C., Rowan, B., & Ball, D. L. (2005a). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal*, *42*(2), 371–406.

Hill, H. C., Rowan, B., & Ball, D. L. (2005b). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal*, *42*(2), 371–406. http://doi.org/10.3102/00028312042002371

Ingersoll, R. (2007). A comparative study of teacher preparation and qualifications in six nations. *University of Pennsylvania GSE Publications*. Retrieved from http://repository.upenn.edu/gse_pubs/145

Karp, A., & Vogeli, B. R. (Eds.). (2010). *Russian mathematics education: History and world significance*. World Scientific.

Kleickmann, T., Richter, D., Kunter, M., Elsner, J., Besser, M., Krauss, S., & Baumert, J. (2013). Teachers' content knowledge and pedagogical content knowledge: The role of structural differences in teacher education. *Journal of Teacher Education*, *64*(1), 90–106. http://doi.org/10.1177/0022487112460398

Krauss, S., Brunner, M., Kunter, M., Baumert, J., Blum, W., Neubrand, M., & Jordan, A. (2008). Pedagogical content knowledge and content knowledge of secondary mathematics teachers. *Journal of Educational Psychology*, *100*(3), 716–725. http://doi.org/10.1037/0022-0663.100.3.716

Leikin, R. (2004). The wholes that are greater than the sum of their parts: Employing cooperative learning in mathematics teachers' education. *The Journal of Mathematical Behaviour*, *23*(2), 223–256. http://doi.org/10.1016/j.jmathb.2004.03.006

Lim-Teo, S. K., Chua, K. G., Cheang, W. K., & Yeo, J. K. (2007). The development of diploma in education student teachers' mathematics pedagogical content knowledge. *International Journal of Science and Mathematics Education*, *5*(2), 237–261. http://doi.org/10.1007/s10763-006-9056-5

Lo, J.-J., Hung, C., & Liu, S. (2002). An analysis of teacher education reform in Taiwan since 1994 and its potential impact on the preparation of mathematics teachers at the elementary school level. *International Journal of Educational Research*, *37*(2), 145–159. https://doi.org/10.1016/S0883-0355(02)00057-5

Monk, D. H. (1994). Subject area preparation of secondary mathematics and science teachers and student achievement. *Economics of Education Review*, *13*(2), 125–145. http://doi.org/10.1016/0272-7757(94)90003-5

Moon, B., Vlăsceanu, L., & Barrows, L. C. (2003). *Institutional approaches to teacher education within higher education in Europe: current models and new developments*. European Centre for Higher Education.

Murray, E., & Star, J. R. (2013). What do secondary preservice mathematics teachers need to know? Content courses connecting secondary and tertiary mathematics. *Notices of the American Mathematical Society*, *60*(10), 1297–1300.

National Council of Teachers of Mathematics. (2005, July). Highly qualified teachers: A position of the National Council of Teachers of Mathematics. Retrieved from http://www.nctm.org/uploadedFiles/About_NCTM/Position_Statements/qualified.pdf

National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel*. Washington, D.C.: U.S. Department of Education. Retrieved from http://www2.ed.gov/about/bdscomm/list/mathpanel/index.html

Neumann, K., Fischer, H. E., & Kauertz, A. (2010). From PISA to educational standards: The impact of large-scale assessments on science education in Germany. *International Journal of Science and Mathematics Education*, *8*(3), 545–563. http://doi.org/10.1007/s10763-010-9206-7

Nilssen, V. L. (2010). Guided planning in first-year student teachers' teaching. *Scandinavian Journal of Educational Research*, *54*(5), 431–449. http://doi.org/10.1080/00313831.2010.508909

Piccolo, D. (2008). Views of content and pedagogical knowledges for teaching mathematics. *School Science and Mathematics*, *108*(2), 46–48.

Schmidt, W. H., Cogan, L., & Houang, R. (2011). The role of opportunity to learn in teacher preparation: An international context. *Journal of Teacher Education*, *62*(2), 138–153. http://doi.org/10.1177/0022487110391987

Schmidt, W. H., Tatto, M. T., Bankov, K., Blömeke, S., Cedillo, T., Cogan, L., … others. (2007). *The preparation gap: Teacher education for middle school mathematics in six countries*. Mathematics Teaching in the 21st Century, Center for Research in Mathematics and Science Education, Michigan State University. Retrieved from http://www.educ.msu.edu/content/sites/usteds/documents/MT21Report.pdf

Seymour, J. R., & Lehrer, R. (2006). Tracing the evolution of pedagogical content knowledge as the development of interanimated discourses. *The Journal of the Learning Sciences*, *15*(4), 549–582.

Schmidt, W. H., Cogan, L. S., Houang, R. T., & McKnight, C. (2009). Equality of Educational Opportunity: A Myth or Reality in US Schooling. *Education Policy Center, Michigan State University*.

Shi, X., & Englert, P. A. (2008). Reform of teacher education in China. *Journal of Education for Teaching*, *34*(4), 347–359.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, *15*(2), 4–14. http://doi.org/10.3102/0013189X015002004

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review*, *57*(1), 1–23.

Stefimova, N. (2010). The preparation of mathematics teachers in Russia: Past and present. *Russian Mathematics Education: History and World Significance*, *4*, 279.

Tatto, M. T. (2012). *Teacher Education Study in Mathematics (TEDS-M): Technical report*. Amsterdam, NL: International Association for the Evaluation of Educational Achievement.

Tatto, M. T., Peck, R., Schwille, J., Bankov, K., Senk, S. L., Rodriguez, M., ... & Rowley, G. (2012). *Policy, Practice, and Readiness to Teach Primary and Secondary Mathematics in 17 Countries: Findings from the IEA Teacher Education and Development Study in Mathematics (TEDS-MM)*. International Association for the Evaluation of Educational Achievement. Herengracht 487, Amsterdam, 1017 BT, The Netherlands.

Tatto, M. T., & Rodriguez, M. (2012). *Evaluating mathematics knowledge for teaching as an outcome of pre-service teacher education: Lessons from the high performing countries in the TEDS-M study*. Michigan State University, USA: TEDS-M ISC.

Tatto, M. T., Schwille, J. S., Senk, S., Ingvarson, L. C., Peck, R., & Rowley, G. L. (2009). Teacher Education and Development Study in Mathematics (TEDS-M): conceptual framework: policy, practice, and readiness to teach primary and secondary mathematics.

Tatto, M. T., & Senk, S. (2011). The mathematics education of future primary and secondary teachers: Methods and findings from the teacher education and development study in mathematics. *Journal of Teacher Education*, *62*(2), 121–137. http://doi.org/10.1177/0022487110391807

Weng, F. Y. (2004). Centralization and Decentralization in Educational Governance in Taiwan. In M. Ka-Ho (Ed.), *Centralization and Decentralization* (pp. 39–58). Springer Netherlands. https://doi.org/10.1007/978-94-007-0956-0_3

Wilson, S. M., Floden, R., & Ferrini-Mundy, J. (2001). *Teacher preparation research: Current knowledge, gaps, and recommendations*. East Lansing, MI: Center for the Study of Teaching and Policy, University of Washington.

Youngs, P., & Qian, H. (2013). The influence of university courses and field experiences on Chinese elementary candidates' mathematical knowledge for teaching. *Journal of Teacher Education*, *64*(3), 244–261. http://doi.org/10.1177/0022487112473836

Authors

Eileen Murray
Montclair State University
1 Normal Avenue, Montclair, NJ 07043, USA
email: murrayei@mail.montclair.edu

Kelley Durkin
Vanderbilt University
Peabody Research Institute
230 Appleton Place, PMB 181, Nashville, TN 37203, USA
email: kelley.durkin@vanderbilt.edu

Theodore Chao
The Ohio State University
1945 North High Street, Columbus, OH 43210, USA
email: chao.160@osu.edu

Jon R. Star
Harvard Graduate School of Education
6 Appian Way, Cambridge, MA 02138, USA
email: jon_star@gse.harvard.edu

Rozy Vig
email: rozy314@gmail.com