A case study of teachers' development of well-structured mathematical modelling activities

Micah Stohlmann
University of Nevada, Las Vegas

Cathrine Maiorca
California State University, Long Beach

Charlie Allen
University of Nevada, Las Vegas

Received: 23 November 2015 Accepted: 22 March 2016

This case study investigated how three teachers developed mathematical modelling activities integrated with content standards through participation in a course on mathematical modelling. The class activities involved experiencing a mathematical modelling activity, reading and rating example mathematical modelling activities, reading articles about mathematical modelling activities, and in-class discussion and feedback. We describe the teachers' development process and how well structured the activities were based on six principles of mathematical modelling activities. We also describe the teachers' interpretations of the six principles connected to their modelling activity. Two of the teachers were able to develop modelling activities that met the six principles while one teacher did not meet the generalizability and self-assessment principles. Developing mathematical modelling activities is a difficult task and the class activities allowed the teachers to go through revisions with their mathematical modelling activities to allow the opportunity for the activities to be properly structured.

Keywords · in-service elementary teachers · mathematical modelling · model-eliciting activities

Mathematical modelling is increasingly becoming an essential integrated part of mathematics education. In Sweden, mathematical modelling is one of seven mathematical abilities to develop in students. Germany includes mathematical modelling as one of six compulsory competencies (Blum & Borromeo Ferri, 2009). In the United States, mathematical modelling is one of eight Standards for Mathematical Practice. Australia has mathematical modelling as part of the concepts and techniques that students should know in the National Mathematics Curriculum (Australian Curriculum, Assessment and Reporting Authority, 2015).

Modelling links classroom mathematics to the mathematics that is used in everyday life. While elementary teachers value routine real world problems, they are less likely to value more "messy" real world mathematical modelling (Vale, 2002). This is compounded by the fact that mathematical modelling has been implemented sparingly (Ng, 2013; Winter & Venkat, 2013) and has not been used much in teacher education (Doerr, 2007). One of the main barriers to more large-scale implementation of mathematical modelling is teacher training (Burkhardt, 2006). When teachers have been exposed to more challenging tasks they come to value them and place more emphasis on the teacher as a facilitator and students sharing their ideas (Clarke, Roche, Cheeseman, & van der Schans, 2014).

In order for modelling to be successfully implemented in K-12 classrooms, teachers need to understand what the essential features of mathematical modelling are and be able to use modelling activities integrated with content standards. The purpose of this study is to describe teacher developed modelling activities that were integrated with the U.S. Common Core State Mathematics Standards (Common Core State Standards Initiative [CCSSM], 2010) and evaluate how well-structured they were. Model-Eliciting Activities (MEAs), a specific type of mathematical modelling activity, are often developed by researchers (e.g. Big Foot MEA, Lesh & Doerr, 2003; Historic Hotels MEA, Aliprantis and Carmona, 2003). It has been found that these types of activities can be difficult to develop because all six principles of MEA development (Table 1) need to be met (Lesh & Doerr, 2003). There is one research question that guided this study: To what extent did the teachers understand how to develop a MEA based on the six principles of MEA development?

Table 1

Principles for Guiding MEA Development

Principle	**Description**
Model Construction	Ensures the activity requires the construction of an explicit description, explanation, or procedure for a mathematically significant situation
Generalizability	Also known as the Model Share-Ability and Re-Useability Principle. Requires students to produce solutions that are shareable with others and modifiable for other closely related situations
Model Documentation	Ensures that the students are required to create some form of documentation that will reveal explicitly how they are thinking about the problem situation
Reality	Requires the activity to be posed in a realistic context and to be designed so that the students can interpret the activity meaningfully from their different levels of mathematical ability and general knowledge
Self-Assessment	Ensures that the activity contains criteria the students can identify and use to test and revise their current ways of thinking
Effective Prototype	Ensures that the model produced will be as simple as possible, yet still mathematically significant for learning purposes (i.e., a learning prototype, or a "big idea" in mathematics)

(Lesh, Hoover, Hole, Kelly, & Post, 2000)

There is little research on how teachers can be supported to develop mathematical modelling activities. At the elementary level, teachers have developed tools for use with Model-Eliciting Activities (MEAs). These tools include an observation tool for observing students while working on MEAs and a metacognitive teacher self-coaching tool for implementing MEAs (Berry, 2010). A class of secondary teachers that designed MEAs found them to be quality supplementary materials for a mathematics class, close to real-life problem solving, and reported that they helped to develop communication skills and mathematical discourse (Yu & Change, 2011). At the college level Moore & Diefes-Dux (2004) found that in development of mathematical modelling activities faculty go through cycles of expressing, testing, and revising their ideas that are similar to what students go through when they work on mathematical modelling activities. This iterative process, an important part of the development process, can ensure that mathematical modelling activities are properly structured.

In this study, with a small group of three teachers in a class, we hypothesized that with support the teachers would be able to write mathematical modelling activities in one-week's time. In a week-long class focused on mathematical modelling the teachers could go through iterations to develop their activities. The structure of the class activities was designed, through different experiences and feedback, to help teachers understand how to develop a specific type of mathematical modelling activity, Model-Eliciting Activities (MEAs). MEAs are well-structured activities that support quality pedagogy with best practices in mathematics teaching and learning. With the growing emphasis of mathematical modelling in mathematics education there is a great need for research on how to help teachers understand mathematical modelling and to have well-structured activities that can be used to give students quality mathematical modelling experiences.

There are several different interpretations of mathematical modelling, and in the next section we describe the theoretical perspective used for this study. The **Realistic** perspective has the goals of solving real world problems, understanding the real world, and promotion of modelling competencies. The **Contextual** modelling perspective has subject-related and psychological goals. **Educational** modelling can be differentiated into **Didactical** modelling, structuring of learning processes and its promotion, and **Conceptual** modelling, concept introduction and development. The **Socio-critical modelling** perspective is driven for critical understanding of the surrounding world. Next, the **Epistemological** perspective looks for theory development. Finally, a meta-perspective is the **Cognitive** modelling perspective, which aims to analyse cognitive processes and emphasizes modelling as a mental process such as abstraction or generalisation (Kaiser & Sriraman, 2006). The Models and Modelling Perspective is aligned with the **Contextual** perspective.

In particular, "Our definition of mathematical modelling is an iterative process that involves open-ended, real world, practical problems that students make sense of with mathematics using assumptions, approximations, and multiple representations. Other knowledge besides mathematics can be used as well. Mathematical modelling curricula should have multiple acceptable models that can be developed" (Stohlmann & Albarracin, 2016, p.1).

Theoretical Perspective: Models and Modelling

One of the most important characteristics of the Models and Modelling Perspective (MMP) is that those who are experts in their field tend not only to do things differently, but see or interpret things differently. The development of these interpretation systems or models is an invaluable part of what it means to have expertise in a field (Lesh, Carmona, & Moore, 2009). Similarly, having teachers understand what makes mathematical modelling activities well-structured enables them to ensure students have quality experiences with mathematical modelling.

MMP is based on the idea that learners do not engage only their mathematical understandings in solving problems. When learners interpret situations mathematically, they do not just engage their logical systems, but also their beliefs, values, and feelings, depending on how the situation is interpreted. In the MMP learners also integrate knowledge from more than one subject. This knowledge can be drawn from context specific experiences (Lesh et al., 2009). Similarly, when developing mathematical modelling activities teachers can engage their prior experiences and affective dispositions in developing their modelling context. If students do not have experience with a realistic context, background knowledge is important to orient them to the problem.

In the MMP mathematical ideas and modelling competencies are developed simultaneously. The students' abilities to solve "real life" problems are drawn on throughout the learning process and not only after the content is learned (Lesh & Zawojewski, 2007). In this perspective "the kind of mathematical understandings and abilities that are needed involve dynamic, iterative and graphic ways of thinking that are quite different than those that have been emphasized in traditional schooling" (Lesh, & Doerr, 2003, p. 12). As students iteratively develop their solutions during modelling activities, so teachers can go through an iterative process in their development of these activities. Through this process teachers can become more aware of the situations that can enable them to "see" the thinking of their students and build on their students' demonstrated knowledge (Lesh & Zawojeski, 2007).

Model-Eliciting Activities

Model-Eliciting Activities (MEAs) are activities that are embedded in the Models and Modelling Perspective (Lesh & Zawojeski, 2007). These modelling activities are client driven, open ended, realistic problems that are developed based on six principles (Table 1). The way that students are required to use mathematics in an MEA is similar to the way students are required to use mathematics in real life situations (Lesh, & Doerr, 2003). Often while completing an MEA students will refine, revise, and extend powerful math constructs (Lesh, & Doerr, 2003). When developing MEAs teachers can test and revise their MEAs based on the six principles to ensure proper development.

MEAs are implemented starting with an opening article or video, followed by readiness questions to help students become familiar with the real-world context and the problem statement. Next students work in groups to solve the problem. They then present their ideas to the whole class. Finally, in their small groups they are given time for revision of their models and for reflection.

Follow-up activities to MEAs can be used to help students formalize the models that they developed. Lesh et al. (2009) have called these activities Model-Exploration Activities (MXAs). MXAs are teacher-led guided activities that help students to focus on the "big" mathematical ideas they worked with during an MEA (e.g. proportionality or measures of centre). These activities help to develop the shareability and reusability of the conceptual tools that students have begun to develop.

When MEAs are done at the beginning of a unit, they can be used to assess students' prior knowledge and as a way for students to see the need for understanding the "big" idea the MEA was developed around. Follow-up activities to MEAs can be done in two main ways to make the mathematics the students used in their models explicit. A follow-up activity can focus on students using the model they developed in a similar realistic context. A follow-up activity could also be more solely focused on the mathematical content. Either way the students would further develop, test, and revise their ideas around a significant mathematical idea.

It is worth noting that Julie & Mudaly (2007) have described two approaches to modelling: modelling as vehicle and modelling as content. The MEA approach used is more in line with the modelling as a vehicle approach, in which the particular curricular content controls the choice and pursuit of the activity though students are free to develop their solutions using any mathematics they see fit that meets the needs of the problem statement. Modelling as content entails the construction of mathematical models of natural and social phenomena without the prescription that certain mathematical concepts, procedures or the like should be the outcome of the model-building process. Also, in this approach the reality situation is the starting point and the mathematical problem has to be constructed. Most modelling has been done with the modelling as a vehicle approach and we recognize that both approaches have benefits.

Methods

Study design

This research was a multi-case study (Yin, 2003) of three elementary teachers enrolled in a week-long master's level class on mathematical modelling. They were the only ones enrolled in this class. All three teachers were Caucasian females. One of the teachers, April, had half a year experience substitute teaching in elementary and middle school classes and was looking to enroll in a doctoral program. Another teacher, Melia, had taught Grade 5 for five years and was the grade level leader at her school. The third teacher, Maddie, had taught Grade 2 for seven years.

The mathematical modelling master's level class was developed by the lead author with the main goal of facilitating teachers' understanding of Model-Eliciting Activities and how to develop these activities (See Table 2). On day one the teachers experienced an MEA and follow-up activity that focused on characteristics of proportional situations. In the broad sense, not particular to MEAs, mathematical modelling was also discussed. On the second day teachers learned about the 6 principles of MEA development and then evaluated MEAs based on these principles. On day 3, since all of the teachers had an elementary focus, they each read and summarized an article by Lyn English in which various MEAs were described. On day 2 and day 3 the instructor discussed with the teachers their ideas for their MEAs and on day 4 the teachers received feedback on a draft MEA. Finally, on day 5 the teachers shared their final MEAs and follow-up activities.

Table 2

Mathematical Modelling Summer Class Description

Day	Main Topics
1	• Teachers participated in the Bigfoot Model-Eliciting Activity (MEA) (Stohlmann, 2012) and a follow-up activity focused on the "big" idea of proportionality. The Bigfoot MEA is a modification of the Big foot MEA (Lesh & Doerr, 2003). • The essential features of mathematical modelling were discussed (Stohlmann & Albarracin, 2016).
2	• Teachers shared their MEA ideas. • The 6 principles for MEA development and the evaluation of example MEAs based on these principles was done.
3	• Teachers shared their MEA ideas. • Teachers discussed summaries of and reactions to three articles (English, 2003; English & Watters, 2004; English, 2008). • Teachers discussed how the Standards for Mathematical Practice are integrated with MEAs (Stohlmann, Maiorca, & Olson, 2015).
4	• Feedback time for the teachers' MEA drafts. Also, teachers discussed integrated Science, Technology, Engineering, and Mathematics (STEM) education and how this can be integrated with MEAs (Stohlmann, Moore,

		& Cramer, 2013).
5	•	Final MEA and follow-up activity sharing.

On day 1, the essential features of mathematical modelling that were discussed were, (a) start with a real world problem, (b) work from key questions, (c) make sense of the problem with mathematics often involving assumptions and approximations, (d) ensure the mathematics is accurate and makes sense in the realistic situation, (e) goal of clear verbal and written communication throughout often including multiple representations, (f) modelling is an iterative process that involves open-ended problems, and (g) reflection on mathematics used or the modelling process (Stohlmann & Albarracin, 2016).

Data collection and analysis

Data for this study was collected over a five-day period and included researcher field notes, teacher reflections on the MEA development process, and drafts of the teachers' MEAs. The participants were required to develop a Model-Eliciting Activity and follow-up activity that focused on a "big" idea in the U.S. Common Core State Mathematical Standards. One researcher took field notes during the class sessions and the teachers wrote two reflections on their MEA ideas before creating their first draft of their MEA. Participants also completed a final reflection form that described the individual experiences they had while creating the modelling activity and how they thought their MEAs addressed the six principles of development (Table 1). For the cross-case analysis the MEAs that were developed by the teachers were evaluated using the six principles of MEA development. All three researchers coded the final MEAs to determine if the six principles were met and provided an explanation for their coding. The researchers used the description of the six principles from the literature as the basis for their coding (e.g. Lesh et al., 2000). The Krippendorff's alpha coefficient of inter-rater agreement was .954, where 1 indicates perfect reliability and 0 the absence of reliability. Once coding differences were calculated, the raters came to agreement on the discrepancies so that 100% agreement was reached.

Results

Cases

Maddie

Maddie is a Caucasian female who had 7 years experience teaching second grade. Maddie's MEA idea from the beginning was based on a second grade U.S. Common Core State Mathematics standard, *2.G.2 Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.* While the context of the MEA changed, the standard or content focus stayed the same throughout. Initially, Maddie focused on how many people could fit into a room. She wanted to use squares to represent people on a larger rectangular piece of paper. On the second day she was still a little unsure of what would qualify as a MEA as she wrote, "If I chose a number of squares that could be put into a various number of arrays, different groups could come up with different ways to organize them, therefore creating different number sentences. This could lead into a discussion of efficient ways to count them???" For this problem there would be multiple ways of counting the squares, however, there would be the same correct answer. Also, she was unsure if the context would be the right realistic context to engage students. Melia mentioned she could use the context of organizing desks in a room. Based on this idea the instructor also gave Maddie a copy of the book, *Spaghetti* *and Meatballs for all* (Burns, 2008), to see if that could help with the context. In this book a host of a party makes different table arrangements based on the number of guests that arrive.

At the beginning of the third class, Maddie was focused on using the best possible room arrangement as the context for her MEA. She wanted to know if there might be a video about how rooms are set-up in the best possible arrangement to hold a specific number of occupants. She was not able to find such a video. She also wrote about students having to organize desks in a classroom and use a number sentence to describe the number of desks. She still wanted to use "a number of squares that could be put into a various number of arrays; (and) different groups could come up with different ways to organize them, therefore creating different number sentences." However, this time she added that groups would also have to write a letter to explain why their room arrangement was the best.

The differences in Maddie's final MEA draft (See Table 3) and her rough draft were the addition of one additional detail and ensuring she met the generalizability principle for MEA development. In Maddie's rough draft she further refined her MEA ideas focused on arranging a classroom. She decided to use the book *Spaghetti and Meatballs for All* as the opening reading in order to have students start to think about different arrangements and what makes an arrangement efficient. In the problem statement she added a more realistic room arrangement by including a teacher's desk, five desks for computers and walking space. On reviewing the draft in class Melia commented that Maddie's MEA did not meet the generalizability principle so Maddie included a section about guidelines that could be used in any room arrangement. In the follow-up activity students would work further with the ideas related to arrays and efficient ways to count them. As an extension, students would be tasked with creating different arrays given a set number of connecting cubes. Maddie found creating this follow-up activity to be the easiest part of the assignment.

Table 3

Maddie's Final MEA

Room Arrangement MEA

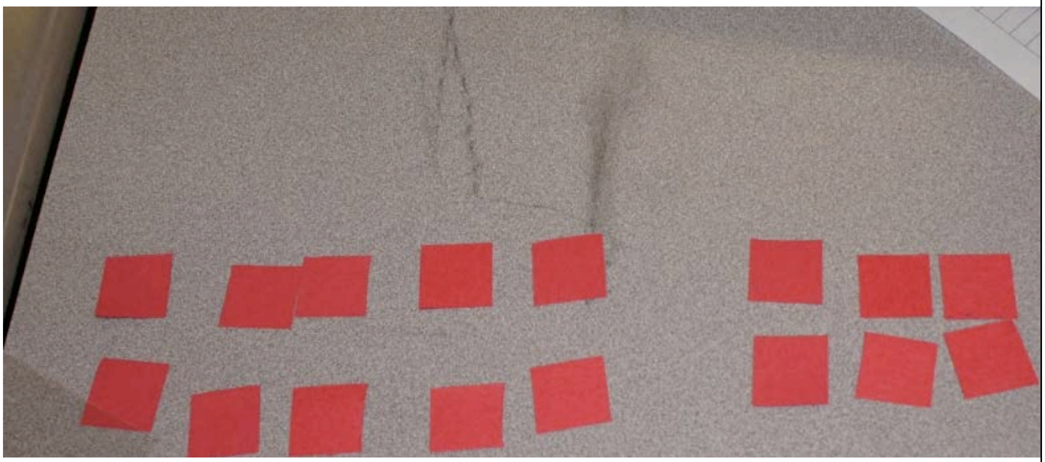
Opening Reading: The teacher will read aloud Spaghetti and Meatballs for All!

Readiness Questions

- Why was Mrs. Comfort worried about the table arrangements? Should she have worried about it?
- Was the original table arrangement more efficient? Explain your answer.
- What other ways could Mrs. Comfort have arranged the tables and chairs to seat all of the guests and herself? (32 total)? Draw or write your answer.

Task (introduction)- You will be given a piece of paper with a large rectangle drawn on it (Teacher note: large enough to fit exactly 40 squares). You will also be given several small squares to use for filling in the rectangle. Once you have filled in the rectangle, explain (in words) how you filled in the rectangle and how many total squares you used. Once most or all groups have completed this task, groups will be asked to share their explanations and solutions. Groups who wish to revise their explanation or final solution should do this now.

Task/Problem Statement- Pretend the rectangle on the paper is your classroom. How could you arrange the room so that there are enough desks for everyone (20 students), a desk for the teacher, and five desks for computers. *Remember*- there needs to be walking space in the room!! In writing, explain how you arranged the room and why it is most efficient. Include details/guidelines that could be incorporated into any room arrangement (i.e. space for movement, optimal space for activities etc.) Afterwards, write a letter to me (the teacher) persuading me to pick your way of arranging the classroom. Be sure to include why your way is



most efficient.

Follow-up Activity

I would introduce and explain that objects arranged into equal groups are called ***arrays***. I would then give students a paper that has four examples of arrays and ask students how they could count the various arrays to find the total number of objects in the first two examples. I would assume that some students would count one-by-one, some would count columns, some would count rows, and some might find some other way of grouping the objects to count the total. I would write the various strategies used by students on the board so they could refer to them. Looking at the arrays on the paper, I would ask students to identify the most efficient way to count them. Grouping objects by row or column should stand out as an efficient way to count the total. Students would then use this strategy to count the total for the next two examples.

Extension: Students will be given connecting cubes and will be asked to use only a specified number of them to make an array (i.e. Take 6 cubes and make an array). Students will be asked to explain how they created their array, noticing that there are different ways to create an array with the same number of cubes.

Melia

Melia was in her fifth year teaching the fifth grade. She chose to build her Model-Eliciting Activity around a few of the U.S. Common Core standards for statistics. Even though the standards are sixth grade standards, Melia noted that in the past these concepts were covered in her fifth grade classes. Her initial thoughts were to create an activity that would focus on the ability to calculate central tendencies for a given set of data. Melia expressed a desire to present her MEA to her fifth-grade class using a video clip and a short article that would include the presentation of a problem to be solved. For the type of data to be analysed by her class, she chose temperature. Consequently, her desire was to use a video clip pertaining to weather forecasting. She further expressed a preference for a video about a 7-day forecast so her students could observe a collection of changing data. Finally, Melia wanted to include a follow-up activity that involved the average local temperature for a 7-day period.

In the second reflection on her MEA project, Melia still wished to use temperature and weather forecasting to create a statistics activity focused on the mean of a set of data. She confirmed her decision to use a video clip involving a 7-day weather forecast. Instead of an article, as originally intentioned, she decided to use a brief section from a fifth-grade science textbook that discussed weather forecasting. After the reading, her plan was to task students with assisting a local meteorologist by scripting a written 7-day forecast, and let one student assume the role of the meteorologist presenting the forecast to the class. A follow-up activity was planned that involved using central tendencies to solve a problem.

Melia's final draft of her MEA (Table 4) was the same as her first draft and listed three Common Core standards that could be addressed.

6.SP.A.1- Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers.

6.SP.A.2- Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.

6.SP.A.3- Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

The opening science article focused on how data is collected to assist in weather forecasts. Once students would answer the readiness questions, they would then be shown an actual 7-day Las Vegas Forecast video. The class would then discuss *Predicting the Forecast Task*, which explicates the scenario of assisting the meteorologist. Each group would be given two sheets. The first, a *7-Day Forecast*, consists of a top half with sections for Sunday through Saturday, and the bottom half with lines for writing and the subheading "Description of Process." The second sheet, *7-Day Forecast Graph*, is basically graph paper. Upon completion of these two sheets, each group is to choose a "Lead Meteorologist" to present that group's findings.

The follow-up activity uses another worksheet titled *Exploring More with Central Tendencies*, which requires filling in definitions for mean, median, mode, and range, and solving two word problems concerning temperatures and determining these statistical measures. Melia's follow-up serves well as an assessment of students' knowledge of how to determine mean, median, mode, and range, as well as reinforcing the definitions of these terms. Additionally, the second word problem requires the same process as the MEA, but for five days as opposed to seven and in a different location than Las Vegas. However, keeping the follow-up focus on temperature does not transfer any skills learned, statistics or model-building, to a different type of scenario.

Melia enjoyed creating an MEA and felt once her topic was selected the entire process came naturally to her. She did find two things that were difficult parts of writing the MEA: "figuring out how to involve a client" and structuring in the generalizability principle. Overall she felt that "MEAs seem like very efficient ways to get students to think critically."

Table 4

Melia's Final MEA

Weather Forecasting MEA

Opening Reading: Article on how weather forecasts are made

Readiness Questions

Answer the following questions based on the information you just read about how forecasts are made.

1. What are the tools that are used to gather information about weather?
2. What is the range between the highest and lowest temperatures in the USA? The world?
3. Why do you think it is important to track storms with the use of radar?

Predicting the Forecast Task

Weather Channel Meteorologist Mike Seidel needs you and your classmates help in predicting next week's 7-Day forecast. Mr. Seidel has been analyzing temperature data from last year to help him predict the weather for next week. He noticed that the *average temperature from last year was 95°.*

Based on the average temperature from last year Mr. Seidel needs you to find out a way to predict what the temperature of each of the next 7 days will be. Keep in mind that the temperatures you come up with need to be reasonable and realistic with the weather and temperature that is typical for Las Vegas.

Once you have identified your 7-Day temperatures, you will need to create a weather forecast in which you will display your solution. You will also need to explain your thinking so that Mr. Seidel will be able to apply the same process again with future temperature predictions.

Mr. Seidel would also like a graphical representation of your data collected.

Follow-up Activity

Exploring More with Central Tendencies

Provide definitions in your own words for the following terms of central tendencies.

- Mean ____
- Median ____
- Mode ____
- Range ____

Using the knowledge you have about Central Tendencies, solve the following problems on your own. When finished, share and compare your answers with your group members.

1. When Sarah was looking at the weather forecast for her trip to Hawaii, she noticed the predicted high temperatures for the days she would be there were 89°, 81°, 89°, 84°, and

92°. What can Sarah expect the average temperature to be during her trip? Looking at this set of data, identify the median, mode and range.

2. Suppose the average temperature for a 5-day period in the Rocky Mountains was 43°, what could be an appropriate set of data to represent the temperature for each of the 5 days? Using the temperature data you created, identify the median, mode and range.

April

April is Caucasian female who had experience teaching as a long term substitute in the elementary grades and middle school. Initially she felt that MEAs would be a little confusing to create. She wanted clarification of how MEAs could be used at the end of a unit to connect the knowledge that a student might use to develop their MEA solution. For her MEA development, she wanted to start with a realistic context and then see what mathematics content could be connected. She tried to think of other legends similar to the Bigfoot MEA that could be used. She also thought about using a conspiracy theory. Most importantly, she felt that her MEA needed to be written about a mathematical concept that could be approached in numerous ways and encourage the use of several problem solving skills.

After the second class April felt a lot more comfortable with writing her MEA. She found it beneficial to look at example MEAs and discuss how they met the 6 principles of development. She also felt that reading English & Watters' (2004) article on mathematical modelling with young learners had a role in her feeling more comfortable because it gave her a format that she could follow. April decided that the topic of her MEA should be running errands. She wanted to provide the students with a map and GPS that would help them determine their route. She thought that the students could organize their MEA around real life constraints such as time, proximity, or order that errands should be completed. She felt that this MEA was realistic and would connect a real life scenario to computing time and distance, as well as provide students with examples of how mathematics is used outside of the classroom.

Several suggestions were given to April after the class reviewed her first MEA draft. There were no readiness questions for students to answer while viewing the opening video on how to be "green" while driving. Melia noted that the generalizability principle was not met. The instructor noted that without a map included the self-assessment principle would not be met, because there was no way for students to complete the MEA or to compare solution strategies. No follow-up activity was included in the draft MEA either. In the final, MEA readiness questions were added as was a follow-up activity, but the other suggestions were not used (Table 5).

April felt that the hardest part of developing an MEA was differentiating it from a traditional classroom problem. She stated that "an MEA is more of a project where students are able to use any reasonable means necessary to solve the problem, as opposed to class examples where the students are told what to do." She also found it difficult to create a situation that was realistic and would address specific mathematical concepts. April came to feel that, "being able to show students how a topic is applicable in real life" is a key characteristic of an effective math teacher.

The follow-up activity for the Errand MEA consists of repeating the same task for another dimension: gas used, miles travelled, or time spent. There is no transfer of the concepts learned creating this model to another domain. Doing so would certainly challenge the students' creativity by requiring them to build their own optimization model.

Table 5

April's Final MEA

Errand MEA **Opening Video:** Video on how to be "green" when driving. Video: http://www.youtube.com/watch?v=27m5hgmnRYU 1. What two things does consolidating trips help save? 2. How often should you check tire pressure? 3. Should you travel at the speed of traffic or at the speed limit? Why? After the video, the students will be given a list of errands needed to be run, a map of the city that has all of the below noted locations marked, scratch paper and a pencil, current gas prices, and access to a computer for any other information that they might want or need. They need to list the errands in any order that they choose to complete them in, that makes logical sense. The most important part of the list is that it is the most efficient in either usage of fuel, time, or miles traveled, whichever is assigned at that time. Errands need to be run: - Drop off dry cleaning, takes one hour - Go to the ATM - Get gasoline - Pick up dry cleaning - Get take out for dinner - Get hair cut - Drop off mail After the errands have been listed, the students need to list about how much gas they think they would use, the time that they would spend, and how many miles they would use. An explanation is also needed for why the errands were listed in the assigned order. It is important that patterns are noted and an explanation is provided for why they think the patterns occur because they will need to use that information for the follow up activity. **Follow-Up Activity:** Students will need to reorder their list according to another dimension on the list. Explanations and evidence are needed to prove the list order.

Integration of MEAs with 6 Principles

We coded the teachers' final MEAs to determine if they met the 6 principles. The teachers also wrote explanations for how they believed their MEAs met the 6 principles. Maddie and Melia's MEAs met all 6 principles while April's MEA did not met the generalizability and self-assessment principles. Maddie's explanations were in line with our explanations for how her MEA met the 6 principles (Table 6). For the reality principle the explanations had a different focus in that Maddie took a broader view that her MEA would help students understand how to plan for something and problem solve which could fit many realistic contexts. Our interpretation focused on the immediate context of arranging tables or a classroom. Both explanations tie into the reality principle.

Table 6

Maddie's MEA Connections to the 6 Principles

Principle	Participant's explanation	Researcher's explanation
Model Construction	"The questions that follow the read aloud require students to explain why they think one way of arranging the tables was more efficient than the other. In addition, students were required to show and explain how their arrangement of the classroom would be beneficial to all."	**Principle was met:** This was met in the problem statement. "How could you arrange the room so that there are enough desks for everyone (20 students), a desk for the teacher, and five desks for computers."
Generalizability	"Creating guidelines that could be followed for the set-up of any room allows students to make the connection between this activity and its real context outside of the classroom."	**Principle was met:** This was met through the problem statement. "Include details/guidelines that could be incorporated into any room arrangement (i.e. space for movement, optimal space for activities…etc.)"
Model Documentation	"When explaining how to set up the room, students are modelling with squares and explaining in writing how their arrangement is efficient."	**Principle was met:** This was met in the problem statement. "In writing, explain how you arranged the room and why it is most efficient." "Afterwards, write a letter to me (the teacher) persuading me to pick your way of arranging the classroom. Be sure to include why your way is most efficient."
Reality	"The activity demonstrates the complexity of planning for something. Students should begin to understand that problem solving and creating efficient ways to manage them (problems) is important in many contexts."	**Principle was met:** This principle is met through the book, *Spaghetti and Meatballs for all*, the opening questions and in the problem statement. "How could you arrange the room so that there are enough desks for everyone (20 students), a desk for the teacher, and five desks for computers. **Remember**-there needs to be walking space in the room!!" "Afterwards, write a letter to me (the teacher) persuading me to pick your way of arranging the classroom. Be sure to include why your way is most efficient."

Self-Assessment	"Giving guidelines for walking space and keeping general classroom activities in mind (movement around the room), students can look at their arrangement of the arrangement of others to determine if it is an efficient use of space."	**Principle was met:** This principle was meet through the materials the students had to determine if their room arrangement had walking space and was efficiently arranged. Also, "as groups present, other groups may decide to make some changes to their arrangement-as long as they can explain the necessity of the revisions."
Effective Prototype	"The arrangement students come up with can be used as a way to introduce arrays."	**Principle was met:** In the follow-up activity the mathematical big idea is clearly stated: "I would introduce and explain that objects arranged into equal groups are called ***arrays***. I would then give students a paper that has four examples of arrays and ask students how they could count the various arrays to find the total number of objects in the first two examples." In the introduction students create an array and count the 40 squares. Also, in arranging their desks for the main MEA students could make various arrays and count them.

Melia's MEA met all six principles and her explanations were in line with what we described as well (Table 7). In addition to the 7-day forecast Melia decided to have students construct a graphical representation to further meet the model construction and model documentation principles. As long as students understood the concept of average temperature they could self-assess the mathematics of their 7-day forecast. However, students would have to also use information they could collect, the knowledge of other group members, other groups, and the instructor to determine if their 7-day forecast was reasonable based on Las Vegas weather.

Table 7

Melia's MEA Connections to the 6 Principles

Principle	Participant's explanation	Researcher's coding and explanation
Model Construction	"Even though students are doing math the entire time with this activity, I decided that the modelling of mathematics was not displayed too much. Due to this, I decided to have the	**Principle was met:** This was met through the problem statement: "*Based on the average temperature from last year* Mr. Seidel needs you to find out a way to predict what the temperature of each of the 7 days will be. " "Once you have

	groups of students create some sort of graphical representation of the data set they came up with.”	identified your 7-day temperatures, you will need to create a weather forecast in which you display your solution.”
Generalizability	“To ensure generalizability, the students are required to explain their process in detail so that the meteorologist will be able to use that process in the future to predict other weekly forecasts.”	**Principle was met:** This was met through the problem statement: “You will also need to explain your thinking so that Mr. Seidel will be able to apply the same process again with future temperature predictions.”
Model Documentation	“This goes hand-in-hand with generalizability because the students are being asked to provide a written explanation of their thought process. Also, students are creating a graphical representation of the data.”	**Principle was met:** This was met in the problem statement: “you will create a weather forecast in which you display your solution. You will also need to explain your thinking…Mr. Seidel would also like a graphical representation of your data collected.”
Reality	“Reality is shown through the entire activity process. Students are constantly aware of the weather and temperature throughout the day. Especially in Las Vegas, people focus on the weather in order to decide their daily activities.’	**Principle was met:** The opening article about weather forecasting and readiness questions meet the reality principle. Also, the context of a real weather forecast and a real meteorologist contribute to the reality principle.
Self-Assessment	“As students are working through this task, they are always encouraged to test and challenge their thinking. When presenting the forecasts at the end, students are also asked to question each others work and to revise their own processes if needed.”	**Principle was met:** Groups will be given the opportunity to share their solutions, question each other, and revise their work.
Effective Prototype	“As a Common Core State Standard, 6th grade students are asked to manipulate data sets and find their central tendencies. This is what my MEA and follow-up activities focus on.”	**Principle was met:** The problem statement has students use average temperature and the follow-up activity explicitly focuses on measures of central tendency.

April’s MEA did not meet two principles, generalizability and self-assessment, and an explanation for a principle that was met was not in line with the principle. April’s explanation for the generalizability principle was articulate. However, this was not evident in her MEA.

There was no evidence that students would have to describe their model in-depth so that it could be used in similar situations. Her explanation for the self-assessment principle was not accurate and this principle was not evident in her MEA. Her explanation focused on students using their model in the follow-up activity instead of on whether students could determine if their model met the needs of the client in the MEA. Based on the information provided in the MEA there was no way for students to create their model or self-assess if it met the needs of the client because there was no map included. While April met the effective prototype principle, her explanation does not focus on any mathematical "big" ideas that the MEA could be used to develop. There are mathematical ideas in the MEA such as ratios, but these were not mentioned.

Table 8

April's MEA Connections to the 6 Principles

Principle	Participant's explanation	Researcher's explanation
Model Construction	"The students need to decide what factor is the most important to them when running errands, either time, gas, or mileage, then find a way to order the errands that reflects the decided factor. It is pivotal that they are able to explain their ordering when listing them off."	**Principle was met:** This was met in the problem statement: "They need to list the errands in any order that they choose to complete them in, that makes logical sense. The most important part of the list is that it is the most efficient in either usage of fuel, time, or miles traveled, whichever is assigned at the time."
Generalizability	"Students had to develop a route of their own that they would take when trying to get all of the errands completed. Their route had to have a theory behind it, so randomly choosing the order would not work, and it had to be the most efficient in either the mileage, time, or gas. Once a method was found to complete all of the tasks, the students would be able to use it in other examples that were similar."	**Principle was not met:** Based on the written MEA and the follow-up activity there was no evidence for using a model developed in a similar situation.
Model Documentation	"The students need to have a visual to demonstrate the order in which they would complete the tasks. If students chose to make their route based on the least amount of miles driven, a map would be a good way to show that. The method that they use to demonstrate their thinking is	**Principle was met:** This was met in the problem statement: "After the errands have been listed, the students need to list about how much gas they think they would use, the time that they would spend, and how many miles they would use. An explanation is also needed for why the errands were listed in the assigned order."

	up to them, but it needs to be explicit when observed."	
Reality	"It is very common to have a list of tasks that need to be accomplished and as students get older, they will see how this type of thinking will be helpful in all kinds of scenarios. Unfortunately, nothing is too easy, so the students were given the complications of needing to keep the order in mind when forming their errand list."	**Principle was met:** This was met through the opening video, video questions, and the context of running errands.
Self-Assessment	"The self-assessment portion becomes a lot more evident in the follow up activity where students have to use their methods in repeated reasoning activities. This will really demonstrate if their problem solving abilities were of the highest caliber and if they could be used in other scenarios."	**Principle was not met:** There was no evidence that students could self-assess their models in order to revise them if needed. Using a different variable fuel usage, time, or miles traveled could lead to a different type of model.
Effective Prototype	"The students are just required to list their errands and the reasoning for why they are in the listed order. The reasoning for this activity is not going to require too much knowledge that is very in-depth, but other factors could be included that would make it more challenging."	**Principle was met:** While it was not stated explicitly we could see several big mathematical ideas that could be made explicit after this MEA including ratios and proportions.

Discussion

This case study investigated the extent to which teachers could develop mathematical modelling activities based on six principles of development (Lesh et al., 2000). In developing their MEAs the teachers went through various levels of an iterative process that was aided by prior knowledge, experiencing an MEA, discussing example MEAs, reading articles about MEAs, and in class discussion and feedback on their MEA ideas. These are important aspects to support teachers in their MEA development. Moore & Diefes-Dux (2004) also found that the developers of MEAs go through revisions of ideas similar to what students go through when they work on mathematical modelling activities.

The teachers' development process varied as the ideas for an MEA came easiest to Melia. Melia had no revisions between the first and final draft of her MEA and offered more in-class feedback to the other two teachers on how they could meet the six principles. Maddie took some time to get the right context to fit with her MEA, but was able to create a draft MEA that only needed to be modified for the generalizability principle. April initially stated she was confused about how to develop an MEA, had several ideas for a context, and then needed the most revisions from her rough draft. April's MEA was also written more as a guide for teachers rather than being addressed to the students who would complete the MEA.

Both Maddie and Melia developed an MEA that met the six principles of development, while April did not meet the self-assessment and generalizability principle. April received feedback in class for modifications she could make to her MEA to meet these two principles but did not make these changes. While self-assessment is supported through the knowledge of other group members and other groups, an MEA must contain enough information for participants to determine when their solution meets the needs of their client (Lesh et al., 2000). Generalizability is often incorporated by including information in the problem statement that the solution should be explained in detail so that it could be used for similar situations. In Melia's follow-up activity students could use their model developed to create a 5-day forecast for the Rocky Mountains. It is interesting to note that April explained the generalizability principle well in her reflection but did not include this in her MEA. Both Maddie and Melia were able to explain how their MEAs met the six principles of development while April's explanation of the self-assessment principle and effective prototype principle were not accurate.

The self-assessment principle can be a difficult principle to meet. April did not incorporate the self-assessment principle and did not explain it well. Even an expert elementary grades MEA researcher found in one of her MEAs that the self-assessment principle was probably not fully integrated because there were insufficient criteria for students to assess their progress and determine whether their final model met the client's needs. Children also might have lacked the needed scientific background knowledge to have access to the MEA as this MEA was interdisciplinary (English, 2009). Future research can focus on ways of supporting this principle in MEA development. One strategy is for MEAs to contain a second set of data by which students test their models.

April might have been able to develop a general mathematical modelling problem but struggled with aligning her activity with the six principles of MEA development. Future research can focus on a modelling class that exposes teachers to the different perspectives of modelling (Kaiser and Sriraman, 2006) and lets them be open with the modelling activity that is developed.

Developing MEAs with the six principles is still important because the six principles provide structure for teachers in their development process. Martin-Kniep & Uhrmacher (1992) found that one of the obstacles teachers face in curriculum development can be a lack of a framework to guide the process. The six principles for MEA development have a strong research base and provide a useful structure.

While this study focused on the U.S. Common Core State Standards that the teachers had to align a modelling activity with, other countries have standards that need to be met as well. If teachers are involved in curriculum development, then there is a greater likelihood other teachers will use the curricula (Bidwell, 1985). There is a greater chance that teachers will implement mathematical modelling if they can meet content standards as well.

Limitations and Future Research

The one-week summer class helped the teachers understand MEAs and how to develop MEAs, but it remains to be seen which of the aspects of the class were the most beneficial: experiencing an MEA, reading and rating example MEAs, reading articles about MEAs, reflection, or the in-class discussion and feedback. We maintain that all are important aspects for teachers to robustly understand how to develop mathematical modelling activities, but future research can see which aspects are the most beneficial.

Providing support for the structure of the one-week class in this study, the six country collaborative Learning and Education in and through Modelling and Applications (LEMA) project had a focus on helping teachers understand what modelling is, experiencing modelling activities, creating modelling activities, classifying modelling activities, and reflection (Garcia & Ruiz-Higueras, 2011). Similarly, Borromeo Ferri & Blum (2010) suggest that teachers should have experience in learning about mathematical modelling, solving and creating modelling problems, and reflection on modelling.

Research involving teachers' development of MEAs is valuable and should continue. When teachers implement MEAs they see how powerful this method of modelling is because all of their students are able to demonstrate knowledge, including the low-achieving students (Lesh and Doerr, 2003). It has also been found that through MEAs, elementary students are able to develop concepts far more advanced than would have been taught in the traditional classroom (English, 2006).

A review of research on elementary grades mathematical modelling was conducted and provides insight into how mathematical modelling curricula still needs to be developed to include other content. Future research can focus on the development of mathematical modelling curricula with the content of geometry, fractions, place value, decimals, and equations and expressions (Stohlmann & Albarracin, 2016).

Future research can focus on larger groups of teachers as they go through the development process of MEAs and subsequently evaluate how well-structured they are based on 6 principles. Also, future research could focus on teachers' subsequent implementation of their developed MEAs and the understandings demonstrated by students. Teacher beliefs through this process, and any change in teaching practices in the teachers' non-MEA lessons, could also be investigated.

References

Aliprantis, C., & Carmona, G. (2003). Introduction to an economic problem: A models and modelling perspective. In R. Lesh and H. M. Doerr (Eds.), *Beyond constructivism* (pp.225-264). Mahwah, NJ: Lawrence Erlbaum Associates.

Australian Curriculum, Assessment and Reporting Authority (2015). Australian curriculum: Mathematics. Retrieved from http://www.australiancurriculum.edu.au/mathematics/curriculum/f-10?layout=1

Berry, B. (2010). An investigation of teachers' shared interpretations of their roles in supporting and enhancing group functioning. In R. Lesh, P. Galbraith, C. Haines, & A. Hurford (Eds.), *Modelling students' mathematical modelling competencies* (pp.471-480). New York: Springer.

Bidwell, C. (1985). The school as a formal organization. In J. March (Ed.), *Handbook of Organizations* (pp.972-1022). Chicago, IL: Rand McNally.

Blum, W., & Borromeo Ferri, R. (2009). Mathematical modelling: Can it be taught and learnt? *Journal of Mathematical Modelling and Application, 1*(1), 45-58.

Borromeo Ferri, R., & Blum, W. (2010). Mathematical modelling in teacher education—Experiences from a modelling seminar. *Proceedings of the sixth Congress of the European Society for Research in Mathematics Education* (pp. 2046-2055). Lyon, France: INRP. www.inrp.fr/editions/cerme6

Burkhardt, H. (2006). Modelling in mathematics classrooms: Reflections on past developments and the future. *ZDM, 38*(2), 178-195.

Burns, M. (2008). *Spaghetti and meatballs for all!* New York: Scholastic.

Clarke, D., Roche, A., Cheeseman, J., & van der Schans, S. (2014). Teaching strategies for building student persistence on challenging tasks: Insights emerging from two approaches to teacher professional learning. *Mathematics Teacher Education and Development, 16*(2), 46-70.

Common Core State Standards Initiative (2010). *Common Core Standards for Mathematics*. Retrieved from http://www.corestandards.org/assets/CCSI_Math%20Standards.pdf.

Doerr, H. (2007). What knowledge do teachers need for teaching mathematics through applications and modelling? In W. Blum, P. Galbraith, H. Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education* (pp.357-364). New York: Springer.

English, L. D. (2003). Mathematical modelling with young learners. In S. J. Lamon, W.A. Parker, & S. K. Houston (Eds.), *Mathematical modelling: A way of life* (pp. 3-18). Chichester, England: Horwood Publishing.

English, L. D., & Watters, J. J. (2004). Mathematical modelling in the early school years. *Mathematics Education Research Journal*, *16*(3), 59-80.

English, L. D. (2006). Mathematical modelling in the primary school: Children's construction of a consumer guide. *Educational Studies in Mathematics*, *63*(3), 303-323.

English, L. D. (2008). Introducing complex systems into the mathematics curriculum. *Teaching Children Mathematics*, *15*(1), 38-47.

English, L. D. (2009). Promoting interdisciplinarity through mathematical modelling. *ZDM*, *41*, 161-181.

Garcia, F., & Ruiz-Higueras, L. (2011). Modifying teachers' practices: The case of a European Training course on modelling and applications. In G. Kaiser, W. Blum, R. Ferri, & G. Stillman (Eds.), *Trends in teaching and learning of mathematical modelling* (pp.569-578). New York: Springer.

Julie, C., & Mudaly, V. (2007). Mathematical Modelling of social issues in school mathematics in South Africa. In W. Blum, P. Galbraith, H-W. Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education: The 14th ICMI study* (pp.503-510). New York: Springer.

Kaiser, G., & Sriraman, B. (2006). A global survey of international perspectives on modelling in mathematics education. *ZDM*, *38*(3), 302-310.

Lesh, R. (2010). Tools, researchable issues & conjectures for investigating what it means to understand statistics (or other topics) meaningfully. *Journal of Mathematical Modelling and Application, 1*(2), 16-48.

Lesh, R., Carmona, G., & Moore, T. (2009). Six sigma learning gains and long term retention of understandings and attitudes related to models & modelling. *Mediterranean Journal for Research in Mathematics Education, 9*(1), 19-54.

Lesh, R., & Doerr, H. (2003). Foundations of models and modelling perspective on mathematics teaching, learning and problems solving. In R. Lesh & H. Doerr (Eds.), *Beyond constructivism: Models and modelling perspectives on mathematics teaching, learning, and problem solving.* Lawrence Erlbaum Associates, Inc.: Mahwah, NJ.

Lesh, R., & Zawojewski, J. (2007). Problem solving and modelling. In F. K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 763-804). Reston, VA: National Council of Teachers of Mathematics.

Martin-Kniep, G., & Uhrmacher, P. (1992). Teachers as curriculum developers. *Journal of Curriculum Studies, 24*(3), 261-271.

Moore, T. J., & Diefes-Dux, H. A. (2004). *Developing model-eliciting activities for undergraduate students based on advanced engineering content.* Paper presented at the Frontiers in Education Conference, Savannah, GA.

Ng, K. (2013). Initial perspectives of teacher professional development on mathematical modelling in Singapore: A Framework for Facilitation. In G. Stillman, G. Kaiser, W. Blum, & J. Brown (Eds.), *Teaching mathematical modelling: Connecting to research and practice* (p.427-436). New York: Springer.

Stohlmann, M., & Albarracin, L. (2016). What is known about elementary grades mathematical modelling. *Education Research International, 2016,* 1-9.

Stohlmann, M., Maiorca, C., & Olson, T. (2015). Preservice secondary teachers' conceptions from a mathematical modeling activity and connections to the Common Core State Standards. *The Mathematics Educator Journal, 24*(1), 21-43.

Stohlmann, M., Moore, T., & Cramer, K. (2013). Preservice elementary teachers' mathematical content knowledge from an integrated STEM modeling activity. *Journal of Mathematical Modelling and Application, 1*(8), 18-31.

Stohlmann, M. (2012). YouTube incorporated with mathematical modeling activities: Benefits, concerns, and future research opportunities. *International Journal of Technology in Mathematics Education, 19*(3), 117-124.

Vale, C. (2002). What do education students value in primary mathematics curriculum? *Mathematics Teacher Education and Development, 4*, 28-41.

Winter, M., & Venkat, H. (2013). Pre-service teacher learning for mathematical modelling. In G. Stillman, G. Kaiser, W. Blum, & J. Brown (Eds.), *Teaching mathematical modelling: Connecting to research and practice* (pp.491-500). New York: Springer.

Yin, R. K. (2003). *Case study research: Design and methods* (3rd ed.). Thousand Oaks, CA: Sage Publications.

Yu, S., & Change, C. (2011). What did Taiwan mathematics teachers think of model-eliciting activities and modelling teaching? In G. Kaiser, W. Blum, R. Ferri, & G. Stillman (Eds.), *Trends in teaching and learning of mathematical modelling* (pp.147-156). New York: Springer.

Authors

Micah Stohlmann
University of Nevada, Las Vegas, Department of Teaching and Learning, 4505 S. Maryland Parkway,
Box #453005, Las Vegas, NV, 89154, USA
Email: micah.stohlmann@unlv.edu

Cathrine Maiorca
California State University, Long Beach, College of Education, 1250 Bellflower Boulevard, Long Beach, CA, 90840, USA
Email: Cathrine.Maiorca@csulb.edu

Charlie Allen
University of Nevada, Las Vegas, Department of Teaching and Learning, 4505 S. Maryland Parkway,
Box #453005, Las Vegas, NV, 89154, USA
Email: cmallen@unlv.nevada.edu