Balancing classroom management with mathematical learning: Using practice-based task design in mathematics teacher education

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In this paper we present the results from a study conducted in a UK institution in which 21 mathematics pre-service teachers engage with two practice-based tasks featuring incidents where classroom management interferes with mathematical learning. We investigate their considerations when they make decisions in classroom situations and how these tasks can trigger their reflections on the teaching and learning of mathematics. In our analysis we used the constructs of *social* and *sociomathematical norms* (Cobb & Yackel, 1996) and *Teaching Triad* (Jaworski, 1994). Results indicate commendable norms pre-service teachers aspire to establish in their classroom, such as peer respect, value of discussion and investigative mathematical learning. However, they often miss the opportunity to engage students with metacognitive discussions and mathematical challenge as they focus on behavioural issues or endorse dichotomous and simplistic views of mathematical learning. We credit these tasks with allowing insight into pre-service teachers' considerations and we propose their further implementation in teacher education programs.

Keywords· mathematics teachers • classroom management • mathematical learning • social and sociomathematical norms • teaching triad

Introduction

Classroom management often interferes with working towards commendable learning goals (Levin, Hammer & Coffey, 2009; Kersting, 2008; Mitchell & Marin, 2014). In mathematics this is particularly acute when teachers are striving for balancing what Jaworski (1994) calls the three vertices of a *Teaching Triad*: *mathematical challenge*, *sensitivity to students*, *management of learning*. Our research program (Biza, Nardi & Zachariades, 2007; Nardi, Biza & Zachariades, 2012) examines teachers' priorities when they make decisions in the secondary mathematics classroom with a particular focus on this balance. We use practice-based and research-informed tasks in which we invite teachers to consider a mathematical problem and typical student responses to the problem. Tasks are in the format of fictional, yet data-grounded, classroom scenarios. We collect written responses to the tasks and conduct follow-up interviews. Recently we extended this research program, initially conducted in Greece, to using these tasks in the mathematics teacher education program based in our institution (Post-Graduate Certificate in Education in Mathematics, thereafter PGCE) in the UK (Biza, Nardi & Joel, 2014; Biza, Joel & Nardi, 2015).

The tasks that we focus on in this paper engage pre-service mathematics teachers who attended the PGCE program of our institution with realistic classroom scenarios. The scenarios combine seminal mathematics learning and teaching issues with classroom behaviour issues (e.g. classroom management, conflicts between students or between students and teacher). In this paper we present outcomes of this endeavour, especially in relation to the following research questions:

(I) What are pre-service teachers' considerations when they make decisions in situations where classroom management interferes with mathematical learning?

(II) How can this type of practice-based tasks that combine focus on classroom management and mathematical learning issues trigger pre-service teachers' reflections regarding the teaching and learning of mathematics?

In what follows, we present the theoretical underpinnings of our study, its methodology and two tasks. Then, we introduce data and results obtained through the use of these tasks with a cohort of pre-service mathematics teachers. Finally, we discuss the above research questions in the light of these results and conclude with implications of our study for research and teacher education.

Stimulating and probing teacher reflection through engagement with situation-specific tasks

The overall aim of the study we draw on here is to refine typologies that describe teachers' knowledge and beliefs — such as Shulman's (1987) constructs of *pedagogical content knowledge* and Hill and Ball's (2004) *mathematical knowledge for teaching* — and explore how such knowledge and beliefs transform into pedagogical practice. Our aims resonate with the consideration of beliefs as important factors influencing practices (McLeod & McLeod, 2002) and we are aware that explorations of teachers' beliefs and their relation to practice acknowledge the overt discrepancy between theoretically and out-of context expressed teacher beliefs about mathematics and pedagogy and actual practice (e.g. Speer, 2005; Thompson, 1992). Our research sets out from the assumption that teacher knowledge is better explored and developed in situation-specific contexts. There is a growing number of studies in mathematics teacher education research underpinned by this assumption. For example, Speer (2005) claims that methods used in collecting and analysing data can affect beliefs attributed to teachers and suggests that, instead of discussing beliefs and teaching practices in the abstract, a discussion on concrete context — in this study: classroom video data — can provide a shared understanding between researchers and participating teachers on those beliefs that are attributed by researchers to teachers. Zazkis, Sinclair and Liljedahl (2013) propose lesson plays — short stories written in the form of a dialogue between teacher and students — for research and teacher development purposes. These plays draw on specific perceptions of a particular student (or group of students) regarding a mathematical topic and describe a fictional classroom situation in which this topic is discussed. Lesson plays have been used with prospective teachers to support reflection on their future actions and, although the involvement in this activity cannot replace real teaching experiences, it can help teachers develop a larger teaching repertoire.

In our research, we invite teachers' comments on tasks based on classroom scenarios (Biza *et al.*, 2007; Nardi *et al.*, 2012) that they are likely to experience in their lessons. Our tasks start from a mathematical problem that students are likely to encounter in typical secondary mathematics lessons, followed by fictional student responses to this problem. Participants (teachers, pre-service or in-service) are invited to: solve the mathematical problem; consider the purposes of its use in the lesson; reflect on the fictional student responses; and, describe the feedback they would provide to the students. Through these tasks we have been exploring pre- and in-service teachers' knowledge of mathematics and mathematical teaching, especially in terms of its gravitation towards: certain types of mathematical thinking; certain types of pedagogy; and, certain types of didactical practices as evident in the feedback they state they would offer to students. Teacher responses to these tasks as well as the discussion in follow-up interviews have elicited not only insight into the teachers' mathematical knowledge but also a spectrum of considerations that feature when teachers make their decisions on how to react in a classroom situation. We have presented analyses of these considerations in the terms of Toulmin's (1958) and Freeman's (2005) model for analysing *warrants* for informal arguments. Our classification of these warrants (Nardi *et al.*, 2012) distinguishes between: epistemological and pedagogical *a priori* warrants; professional and personal *empirical* warrants; epistemological and curricular *institutional* warrants; and, *evaluative* warrants. Our 7-tier classification aims to encompass the complex set of personal, professional, pedagogical and epistemological nexus of considerations that underpin teachers' decisions in the classroom.

Until recently our scenarios had focused on teaching situations that capture key mathematical issues (such as formation of mathematical concepts, use of definitions, visualisation, mathematical argumentation, etc.) — and less on other key issues such as classroom management. The strand of our study that we draw on in this paper aims to refine our study of aforementioned teacher considerations in a way that addresses the complexity of classroom situations within which the teacher needs to deal concurrently with issues that pertain to mathematical learning as well as to classroom management.

In this paper, we propose a new version of the scenarios that expand the aims of the previous one in the following two ways. Firstly, we aim to explore teachers' pedagogical knowledge regarding classroom behaviour management, especially in relation to the teaching of mathematics — such as dealing with a mathematical learning issue while a misbehaviour incident occurs. Secondly, we aim to trigger teachers' reflection on their own considerations on the teaching of mathematics and their role as a teacher. We envisage that prospective teachers' engagement with this type of task, through their written responses first and the discussion afterwards, can support meeting the above aims. There are two strong influences on our analyses: Jaworski's (1994) *Teaching Triad* and Cobb and Yackel's (1996) *social and sociomathematical norms*. We outline these influences briefly — as well as some recent works that explore the complex nature of dealing with mathematical learning and classroom management issues concurrently — in what follows.

Teaching as a balancing act

Jaworski's (1994) Teaching Triad consists of three "domains" of activity in which teachers engage with: *management of learning* (ML), *sensitivity to students* (SS) and *mathematical challenge* (MC). ML describes how the teacher organises the classroom learning environment (e.g., groupings, planning of tasks, setting ways of working). SS describes teacher knowledge of students and attention to their needs and in particular the ways that he/she interacts with individual students and guides group interactions. Sensitivity to students has been shown to relate to both the affective — e.g., offering praise, encouraging students to participate (SSA) — and the cognitive — e.g., judging appropriate questions, inviting explanation (SSC) — domain. MC describes the challenges offered to students to engender mathematical thinking and activity. This includes tasks set, questions posed, and emphasis on metacognitive processing. Studies that followed Jaworski's (e.g., Potari & Jaworski, 2002) showed that the above elements are closely interrelated and suggest that a balance between sensitivity to students (in both the cognitive and the affective domains) and mathematical challenge is an indicator of effective mathematics teaching. Our study builds on research (e.g., Zaslavsky & Leikin, 1999) that has deployed the Teaching Triad to characterise teaching in the context of secondary mathematics teacher education.

Of importance to us is to address the need — highlighted by the third author of this paper, leader of the PGCE program in our institution at the time the study was being conducted — for providing pre-service teachers with opportunities to reflect upon the challenges of balancing SS, MC and ML in their teaching; and, to do so during the program's sessions and "away from the busy-ness of classrooms" (Biza *et al.,* 2015, p. 36). We see the task-based approach of our study as an apt way to address this need.

There is a growing body of work in mathematics teacher education research that addresses the complexities of this balancing act. For example, Levin *et al.* (2009) observe that new teachers struggle to attend to their students' ideas because teacher education typically focuses on classroom management and curriculum coverage. When commenting on their own teaching, prospective teachers stress that they often prioritise concern about their instructional and management moves and, thus, often fail to notice the substance of pupil reasoning (Kersting, 2008; Mitchell & Marin, 2014). Mitchell and Marin (2014) report that teachers find discussing their concerns with peers useful and describe a video club in which four student teachers utilised the Mathematical Quality of Instruction (MQI) analysis framework to code each other's lessons and to discuss their coding in facilitated group sessions. The authors found that participants became better at noticing important aspects of mathematics more generally and MQI components more specifically. They also adopted a less judgmental stance toward what they noticed. Finally, their self-reported beliefs and practices altered; they credited their participation in the video club for incorporating more opportunities for students to engage with mathematical content and ideas.

Another example of research that addresses the complexities of balancing classroom management with management of mathematics learning is Goodell's (2006). She reports analyses of sessions in which groups of pre-service teachers reported critical incidents from their teaching placements to each other, and then chose one incident to report to the whole class. Each pre-service teacher then submitted a written report of ten critical incidents. Analysis of the incident reports found that the issues raised focused on four main areas: teaching and classroom management; student factors such as pre-requisite knowledge, understanding, resistance and motivation; issues concerning relationships with colleagues, students and parents; and, school organisational issues such as policies and access to resources.

A key influence on our study has been the work of Leatham and Peterson (2010a, 2010b). Leatham and Peterson (2010a) is a monograph that describes a teacher education program designed as a collaboration of pre-service teachers and cooperating teachers (mentors) that addresses five common problems with traditional teacher education programs: lacklustre outcomes, focus on survival over technique, focus on self, isolation, and lack of direction. Traditional teacher education often revolves "around classroom management and not how to craft and carry out a lesson in a way that would engage students in meaningful mathematical activity" (p. 225). The same researchers in another study (2010b) asked 45 cooperating teachers about the main purpose of teacher education, particularly the school placement part.

Participants indicated: classroom management, experience with real teachers, classrooms and behaviour problems. Only one cooperating teacher mentioned the value of experience with real students' thinking. This outcome indicates that cooperating teachers did not perceive the aims of the teaching program they offer in the same way as the program's creators, described as "to craft and carry out mathematics lessons that effectively anticipated, elicited, and used mathematical thinking" (p. 226). The authors note that a "common survival tactic is to take minimal ownership of lesson materials and to present lessons that rely on a transmission model of teaching, requiring only that students be quiet and pay attention" (p. 228) and note that "one problem with the traditional structure of student teaching is that it encourages a focus on student teachers' own actions as teachers and thus makes it difficult for them to decenter in order to focus on their students" (p. 229).

We note that Leatham and Peterson (2010a, 2010b) seem to perceive mathematical thinking as a priority over classroom management. Our experience and anecdotal evidence from mentors and experienced teachers indicates that classroom management is a priority for many teachers. Our study is underpinned by the belief that these two need to be addressed concurrently, one (classroom management) out of sheer necessity and the other (students' mathematical thinking) as an ultimate pedagogical imperative.

Our analyses deploy constructs that have the capability to probe into the classroom environment. Most pronouncedly we draw on the constructs of social and sociomathematical norms (Cobb & Yackel, 1996). Social norms govern the overall interaction in the classroom, and they include rules regarding students' participation in discussion, group work, and critique of other students or teacher. Sociomathematical norms govern the classroom interactions that are specific to mathematics, and they include rules such as "what counts as a different mathematical solution, a sophisticated mathematical solution, an efficient mathematical solution and an acceptable mathematical solution" (*ibid.*, p. 178). Since their introduction, both constructs have been used extensively in the analysis of classroom practices and teacher education/professional development. For example, Cobb, Stephen, McClain and Gravemeijer (2001) use them as "an interpretive framework for analysing communal and individual mathematical activity and learning" (p. 119) in a classroom-based design research. Similarly, in a teaching experiment, McClain (2002) discusses the evolution of sociomathematical norms in relation to the acceptance and the justification of mathematical arguments and highlights the importance of "the relation between the negotiation of classroom social and sociomathematical norms and the students' mathematical development" (*ibid.*, p. 226). Clark, Moore, and Carlson (2008) introduce the sociomathematical norm of *speaking with meaning* in a professional development program for secondary mathematics and science teachers. Further, van Zoest, Stockero and Taylor (2012) see sociomathematical norms together with *professional norms* (which are analogous to social norms in the context of learning mathematics) to address the needs of the teaching profession.

We now present: the two tasks; how we used them to generate pre-service teacher reflections in the context of the PGCE sessions in our institution; and, our rationale for their design.

The Tasks

The tasks are based on realistic classroom scenarios and have three parts: a brief description of the classroom context (including Year and attainment level in accordance with the national curriculum specifications in place at the time of the study) and the mathematical problem; a dialogue that occurs in a fictional classroom; and, a series of questions in which participants are invited to reflect and respond as the teacher of this class. In this paper we discuss two of these scenarios. Both combine classroom management with mathematical learning issues that can occur in a mathematics classroom and aim to explore whether participants would identify – and how they would address – these issues.

In the first scenario (*Polygon task*, Figure 1) the teacher is entering a class (Year 10, high attaining) that is used to an instrumental and competitive working style. The teacher aims to introduce a more investigative and relational approach that "includes justifications for the used rules and the relations amongst them". The class is invited to investigate the sum of the angles of a polygon in a Dynamic Geometry environment and conclude with the formula. After students derive the formula through investigation, the teacher asks the class to justify the correctness of this formula. In the dialogue that follows, students are satisfied with a formula that works for "all the polygons" they tried and they cannot see the necessity of justifying the correctness of this formula ("It isn't necessary"). They consider the teacher's request as a waste of time taken off from their formula practising time and react rather rudely to the teacher.

With the Polygon task, we aimed to generate reflection on the following learning and teaching issues: investigative approach to mathematics; relations between mathematical investigation and mathematical justification; the formula and the validation of its correctness; and, teaching for *relational* and *instrumental understanding* (Skemp, 1976). Classroom management issues concern the already established culture in the mathematics classroom and teachers' flexibility to change this culture. Also, with this task we wanted to raise the issue of student misalignment with teacher lesson plans and see how the pre-service teachers would address these misalignments.

Class X is a high attaining group which you have taken over at the start of Year 10. So far Class X has been taught mathematics as a list of rules and they have been practising the application of these rules in a range of examples. These students have learnt to perform well in a competitive classroom environment in which they work on tasks and they are rewarded for the correctness and rapidness of their work. In your teaching you aim to instigate a different approach that includes justifications for the used rules and the relations amongst them.

In a session on the sum of the angles of a polygon, you have asked the students to

- work with a Dynamic Geometry software in order to sketch polygons with 3, 4, 5, 6, 7, … sides and
- report the number of sides and the sum of the angles in a table, in order to conclude with a general rule about the sum of the angles of a polygon.

After a couple of trials the students conclude that the sum equals 180^{o} multiplied by the number of sides minus two and verify this rule with trials of polygons with several numbers of sides.

At that point you ask the students to explain why this rule is correct and the dialogue below follows:

YOU:	Why is this formula correct? Can you give any explanation?
STUDENT A:	It works for all the polygons we tried.
YOU:	How do you know that this will work for all polygons?
STUDENT B:	It isn't necessary. What we need is a formula that works.
STUDENT C:	Yes, we spent so much time playing with the software. If you had given us the formula and a list of problems to work on, by now we would have got more done.
STUDENT A:	Practice makes perfect.

Questions:

a. What do you think are the issues in this situation?
b. What are you going to say to each one of these students?
c. Are you going to change your approach? Justify your response.

Figure 1. Polygon Task

In the second scenario (*Simplification task*, Figure 2) the class (Year 10, middle attaining) has to calculate an algebraic expression for specific values of *p* and *c*. Two students, student A and student B, approach the problem differently: student A, substitutes the values from the beginning, whereas student B simplifies the expression first and then substitutes the values. When student A acknowledges her difficulty in simplifying expressions, student B judges student A in an offensive ("you are thick") and dismissive ("what can I expect from you anyway?") way. Both solutions are correct. However student B's approach demonstrates proficiency in important algebraic skills which student A does not seem confident with applying. Student A instead puts herself in the slightly tedious and potentially risky position of working on extensive arithmetic operations.

With the Simplification task, we aimed to generate reflection on the following learning and teaching issues: existence and acceptance of different types of solutions; appreciation of the simplification-first solution vs the straight-to-substitution solution; and, dealing with difficulties with algebraic expressions. Classroom management issues concern: students' mutual respect; sharing and critiquing ideas in a classroom; and, dealing with situations of misbehaviour.

In a Year 10 middle attaining class you have invited the students to solve the following problem:
When p=2.8 and c=1.2, calculate the expression: $3c^2+5p-3c(c-2)-4p$.
After working on the problem for some time you invite the students to share their solution with the class. The dialogue below follows:

YOU:	Ok, let's see what we can do with this question. Who wants to share their answer with me?
[Student A and Student B raise their hands at the same time.]	
YOU:	Student A?
STUDENT A:	I found 10.
YOU:	How did you find 10?
STUDENT A:	I substituted the values 2.8 and 1.2 in the expression. It took me ages.
YOU:	Thank you Student A! [To the class] Does everyone agree?
STUDENT B:	I have the same answer but I did it so much quicker.
YOU:	Go on…
STUDENT B:	I worked out the expression before substituting the numbers and I ended up with a much simpler expression: $p+6c$. Then I substituted the values 2.8 and 1.2 and I found 10, easy!
STUDENT A:	I like the way I did it; I don't like simplifying.
STUDENT B:	My solution is brilliant, yours takes ages. You cannot work with letters because you are thick [Some students are giggling] … what can I expect from you anyway? [Some students are laughing].

You heard what Student B said …

Questions:
a. How are you going to respond to Student A, to Student B and to the whole class?
b. What do you think are the issues in this situation?
c. How are you going to deal with these issues in the future?

Figure 2. Simplification Task

In both tasks we wanted to see how pre-service teachers see themselves in similar situations and how they perceive their agency in the classroom in relation to dealing with classroom management and mathematical learning. For this reason the phrasing of the tasks is personalised (e.g. "*you* have taken over").

We now present the educational context in which this study was conducted as well the methodological approach in data collection and analysis.

Context and Methodology

In the UK, where this study was conducted, there are numerous distinct training routes in achieving Qualified Teacher Status (QTS), with School Centred Initial Teacher Training (SCITT) and university based training the most dominant. SCITTs need to be accredited by Higher Education Institutions (HEIs), although the training year is nearly all school based. The university based training route leads to a secondary PGCE that confers QTS and is often associated with Masters level credits. This route has a compliance expectation of a minimum 24 weeks in two schools with a further, but not statutory, 12 weeks of study at university. Participants in this study were 21 pre-service teachers following a mathematics PGCE in a UK university.

Data collection took place in a series of half-day teaching sessions that the first author contributed to the program. In these sessions pre-service teachers were invited to respond to a series of tasks, including the tasks in Figures 1 and 2. We collected the participants' written responses to each task. Whole-class discussion of the tasks ensued. This study has been approved by the Research Ethics Committee of our institution and agreed upon with the participants. The data we present here originate in their written responses to the Polygon and the Simplification tasks (Figures 1 and 2).

In the analysis we present in the next two sections we are particularly interested in the norms (social and sociomathematical) pre-service teachers want to establish in their classroom and how they would establish these norms. We note that it was not always possible to distinguish social from sociomathematical norms with watertight precision and that their simultaneous consideration has more potential to describe the complexity of the teaching practice we want to address with this study. We conclude our analysis with summarising participants' responses in terms on the three "domains" of the Teaching Triad (ML, SS, MC).

Data and Analysis

The Polygon Task

In the polygon task the class is asked to identify a formula that works for all polygons and to establish why the formula works (Figure 1). In their responses to this task, all participants highlighted that students are not used to or expected to — and therefore reluctant to — ask *why* in mathematics. Instead, they are conditioned to an instrumental approach and unwilling to engage with mathematics in an exploratory manner, especially with the use of ICT. Most of the responses offered a juxtaposition between the instrumental understanding in which students are used to, and trained in, and the relational understanding that will give them more insight into how things work. Also, a reflection is offered on respondents' intentions to introduce a teaching approach towards relational understanding and to launch a shared understanding (between the students and themselves) of what mathematics is and how mathematical truth is secured.

We consider these views as mostly related to the socio-mathematical norms participants aspire to establish in their classroom and we discuss these further below. However, before doing so, we would like to embed these views in how participants see the overall classroom social environment as this has been established throughout students' school experience and the role of the teacher in this environment. What, for example, do students expect from the teacher? Are students ready to engage with an investigative activity?

Participant [14], for example, mentions that the "class isn't used to [the] new style of teaching as they prefer to be given a formula and practise it". In a similar spirit, participant [20] writes: "They [students] are expecting being spoon-fed a rule/procedure and to then master its application through practice". It seems that students' learning habits in mathematics are attributed to the overall studying habits and the expectation that the school will prepare them for exams. Participant [12], for example, writes that

> […] there isn't any autonomous learning embedded within the students' practice. As a result of this, they are not enthusiastic about maths and only want the information that will help them pass the exam. The lack of autonomous learning leads the students to make assumptions based on small amount of data.

Participant [21] concurs that students are more interested in "getting work done quickly than understanding how things work" and participant [20] says: "The students are also 'programmed' to do work quicker for rewards-sudden change from this". Additionally, some participants commented on the use of ICT in the classroom and how this can affect students' behaviour. Participant [09], for example, mentions that the "element of using ICT could also create distractions and enable off topic discussions".

These statements indicate participants' views on the established social norms in the classroom, not necessarily for mathematics, that are characterised by: practice on tasks; instrumental approaches with fewer investigations; competition; reward for the quickest response; preparation for the exams; and, potentially limited use of technology.

We now focus on the learning and teaching issues that pertain to mathematics. These pre-service teachers intend to establish a shared understanding of what mathematics is (sociomathematical norm) in their classroom and acknowledge their responsibility in doing so. Participant [20], for example, emphasises the different perspectives between teacher (understanding) and students (application of rules) and notes the abrupt shift from one to the other:

> [I would] perhaps have a clear discussion with the students of what maths is (for me) and what it is for them and why. Also, discuss then what the expectations are and that I'm the teacher now and it can't be all as before.

Let us see also how pre-service teachers would establish a sociomathematical norm of an investigative classroom which aspires at identifying mathematical justification and relational understanding. Participant [02], for example, diagnoses students' unwillingness to investigate mathematical ideas and their perception of mathematics as a repetitive use of ready-made rules:

> They don't see the point and value in investigating mathematical concepts when someone has already done this for you. They see maths as a repetition of using ready-made rules to calculate given questions.

She suggests "testing with a shape of 100 sides" with the help of a computer software. Generally, she praises students for self-discovery and states her resilience that the beauty of mathematical discovery will prevail over instrumentalism.

Participant [16] mentions that students are unsettled by the "change of tack" in the lesson. The "teacher should have explained the investigative nature of the task BEFORE the event" [her capitals]. She would give "TINY" or "HUGE" polygons to trial. She would encourage deriving, not just using, the formula. In her response to student B she writes:

> Ah, but what you need is to understand WHY it works. Maths is about patterns — you could discover the make up of the world if you knew how. Lets get you working on the WHY and the HOW. That will make you into a mathematician, not just a technician. [her capitals]

She insists that a pre-introduced "framework of discovery" is necessary. This script stands out for its insistence on the need for an explicit articulation of the new norm in advance: "I would've explained all the nuances beforehand. Given examples of patterns. Discussed algebraic expressions. Then set the task a framework of discovery."

Similarly, participant [01] notes that students are used to the pursuit of quick answers and they "are not interested in why and thus won't develop a deeper understanding". This will "hinder them further on in their relationship with mathematics". She will not relent under the pressure from the students and will strive for a smoother move away from instrumental to "deeper" understanding. A similar consideration on the gradual transition towards such understanding comes also from participant [21]: "If they are used to one way of thinking/one type of maths class maybe changes should be introduced slowly". However neither participant [01] nor [21] offer more insight on how this gradual transition can be achieved.

Participant [18] also observes that students' understanding is instrumental and that they have no interest in conceptual or relational understanding. He would respond to them with stressing the importance of knowing why, of having a formula that works for all (including large) polygons: "[in his response to student C] You may have had more <u>practice</u>, but I think you will <u>learn</u> more this way if you understand the theory behind the rule" [his emphasis]. It seems that he appreciates the contribution to students' learning that comes from understanding the theory behind the rules. However, later in question (c), he doubts this contribution in students' progress when he claims that, although he would "persist with the relational teaching for a while", he "would definitely be prepared to return to a more instrumental style of teaching if [he] felt the progress of the students was being negatively affected in any way". In this script, regressing to an instrumental approach is an option, if this affects "the progress of the students". This is also one of the many scripts where the two approaches are perceived in a rather simplistically dichotomous manner.

Furthermore, participant [15] is uncertain about the prospect of changing an approach that is efficient for high exam results, especially if these results — even at the expense of understanding and enjoyment — constitute the aim of schooling:

> Previous teaching was prescriptive: learn the rule, apply it. While this may be appropriate if the sole goal is to get students through their exams, it means their understanding (and probably enjoyment) of mathematics is very limited. Is it possible to change the way that the students learn? Will they accept learning by investigation?

Although most of the responses raised the importance of investigating the *why* behind the formula, and some mentioned that trial of few cases is not enough to justify the correctness of this formula, none offered a clear explanation on how they would establish this correctness. Many scripts mentioned that mathematics is about identification of "patterns" and suggest trial of more cases, sometimes extreme such as "tiny" or "huge" (participant [16]). Only few responses mentioned some proving activity. One of these was from participant [20] who responded to student B as follows:

> Yes but how do you know this formula works for all polygons. Perhaps you haven't thought of one where the formula doesn't work. You are right that it is not necessary to try all polygons but how can you prove this formula works for all of them?

This is one of the few cases in which there is an effort towards mathematical generalisation of the observed pattern but without any further details on how this generalisation would have been pursued.

Regarding the rather disrespectful tone of the students' response to the teacher, surprisingly, we found only two responses raising this issue. Participant [21] mentions that

"Student C's response may come across as rude" without addressing this issue in her response to student C later in question b. Participant [11], furthermore, raised the issue of teacher undermining by the students:

> The students are too used to being in a very instrumental learning environment. So, when asked to investigate and think more in depth about their explanations they struggle. I think that because they're struggling, and they're not necessarily used to not being able to answer questions, they start to undermine the teacher with their comments.

We see this as a subtle comment on a situation in which successful students are facing a novel for them experience of not being able to respond to a mathematical problem and deal with this uncomfortable position by undermining the teacher. This undermining might be distracting for a newly qualified teacher who has not developed robust confidence yet and is not trained to anticipate similar situations.

In the language of the Teaching Triad, the majority of the participants discuss the established classroom practice that prioritises instrumental over relational approaches. They suggest a management of learning (ML) that considers this practice and introduces students gradually to a new, more investigative approach (SSA, SSC). They also suggest the mathematical challenge (MC) of investigating the relations and the reasons behind the mathematical formula but they do not offer insight into how they would do this. Also, they do not grasp the opportunity to discuss proof and proving and they restrict the discussion to pattern identification (MC). Additionally, through endorsing a dichotomous approach to instrumental and relational understanding, they miss the opportunity for a metacognitive discussion of benefits and drawbacks of both approaches (SSC). We stress that only a few responses discussed students' disrespectful reaction to the teacher and that the pre-service teachers who participated in this study seem to aspire at establishing sociomathematical norms of investigative and relational approaches to mathematics. Although they are aware of the difficulties of this endeavour in the current school culture, they intend to establish a social norm of gradual change towards less learning through spoon-feeding by transferring the agency of this learning to the students. However it is unclear how they would establish these norms.

The Simplification task

All respondents addressed student B's ill-behaved reaction to student A in the simplification task (Figure 2). Almost all respondents reprimand student B for disrespecting student A, some in public and some in private. Some participants reflected on the situation as an indication of individual student misbehaviour and suggested disciplinary measures for student B's punishment, such as: "consequences", "sanctions", "warnings", "sent out", "exclude from class", "peer courts" or "detention". Participant [9], for example, considers this as a "fairly common misbehaviour in a lesson, due to the level of disruption and attention that pupils can steal away from a member of staff" and she feels the need to protect the class from this disruption through penalising student B. Other pre-service teachers reflect on the situation not only as an indication of individual student misbehaviour but also as a sign of a concerning classroom culture and address this issue as such. In general, across the scripts, there is a clear priority to establish certain social norms in the classroom. The norm that these teachers aspire to establish is of a respectful classroom, a "no put down zone" (participant [18]) with a teacher who "will not tolerate classroom bullying" (participant [1]).

The responses varied regarding the emphasis put on the mathematical aspects of the incident, especially in relation to the value of the two approaches to the problem and the potential difficulties of student A with simplification. We considered these aspects as a good opportunity for participants to reflect on the sociomathematical norms they would establish in their classroom. In their responses, all participants spotted that both solutions are correct. However, not all participants addressed student A's difficulties with simplification. Also, not all participants discussed the differences between the two solutions. Eleven out of the 21 responses include evidence of at least one of the following: the two solutions are not of equal value; student A has difficulties with algebra; the response addresses student A's difficulties in question (a). The remaining 10 responses consider the two solutions of equal value and, although three mention that student B's solution can be seen as quicker, they do not address student A's unease with simplification. They focus mainly on behavioural aspects of the incident.

We scrutinised responses further to identify how they would react in a similar situation and, especially, how they would establish particular social and sociomathematical norms in their classroom. Characteristically, participant [3] in her response to question (a), she would say to student A:

> I like that you have acknowledged that your method takes a long while, whilst it gets you the correct answer, which is great; can you see that simplifying may make it easier for you and save you time especially if I gave you a much much longer complicated expression.

Then, she would praise student B for the efficiency of their approach but she would reprimand them for their treatment of student A:

> Both solutions to the problem are good solutions they both gave correct answers, student A's solution took a lot longer as they were working with really complicated arithmetic rather than simplifying this doesn't make student A thick so I don't want to hear you use that again.

Also, she would enrol student B towards helping student A:

> I like how you have simplified the expression to get a quicker easier method so maybe you could try and help student A with simplifying as it's something student A doesn't like and it will help with your understanding too.

In her response to question (b), she identifies issues related to the class environment with references to lack of "respect" and "self-esteem". Later, in question (c), her tackling of these issues is similarly concretely targeted. She would "constantly instil a positive respectful classroom environment encouraging all students to offer answers" and she would "try to encourage students to help another and discuss methods". We see this response as characterised by pedagogical specificity and consistency, especially regarding the establishment of respect through collaborative work. To her, this collaboration appears as panacea to the disrespect mind-set.

Participant [14], in question (b), mentions that the issues of this situation are: "Student A prefers numbers to algebra. Student B is quite rude" and "[b]oth of these need dealing with". It is evident that participants [3] and [14] spotted both the mathematical and behavioural aspect of the scenario. Both appreciate the algebraic approach as more efficient and, especially for participant [14], this approach is necessary for the exams: "[f]rom a teacher's point of view student A needs to be taught how to simplify for the purpose of non-calculator papers". Pedagogically, participant [3] aims to "encourage students to help each other and discuss methods" whereas participant [14] wants to "ensure that the rude speaking out is unacceptable and not welcome in the class and therefore doesn't happen again" (question (c)).

The balance in the consideration of both mathematical and behavioural issues that we saw above was not in much evidence in other scripts. For example, participant [6] writes in question (a):

> I would want to highlight that calling someone else stupid is unacceptable. As this was mentioned in front of the whole class and some people laughed I would make the point of addressing the whole class with this. I would also speak to student B after the lesson regarding their behaviour & lack of respect for their peers.
>
> I would then go on to say that both these methods can be used and both get to the same outcome. However it is up to each individual as to which they use, depending on their preferred method.

In question (b) she mentions: that the "[i]ssues are that students believe there is only one way to answer a question"; and, "[i]ssue of the lack of respect from Student B and other members of the class giggling towards Student A. Need to ensure rules are obeyed". And, in question (c): "Make it clear that respect is extremely important and that there are many ways to solve mathematical problems it is about finding ways you are comfortable with you don't all have to work the same way"[her emphasis]. For this participant the two solutions are equivalent and it is up to the student to decide which one to use. In this sense she does not address student A's difficulty with algebra and her response leans more towards addressing behavioural issues. Although, she seems to be aware of students' belief in the existence of "only one way to answer a question" is problematic, she does not prioritise in her response the establishment of a sociomathematical norm that can accept, deal with and juxtapose different solutions.

Generally, pre-service teachers indicate the norms they wish to establish in their classrooms: a social norm of a respectful classroom and a sociomathematical norm of the acceptance of different solutions for the same problem. Participant [4], for example, wants to "create a culture of discussing, sharing and involving each other and make sure no student goes against this culture". Similarly, participant [12] "would always re-iterate that students should be supportive of each other's views and give constructive criticism". Neither [4] nor [12] suggest how they would establish this culture in their class though. Participant [11], on the other hand, is more specific when she suggests "prompting cards/discussion templates so that the students are aware of how to argue their point without being disrespectful".

Overall, the pedagogical goal of establishing a collaborative, participatory and engaging culture in the classroom is transparent in most of the responses we received. What is not always transparent, however, is how this culture would be established (see for example [4] and [12]). As our experience of teacher education suggests, implementation of pedagogical goals in the class is always a challenge that pre-service teachers need to be prepared to face, react to, and reflect on.

In the language of the Teaching Triad, all the pre-service teachers raised the classroom management issues and the management of learning (ML). Also, with sensitivity to students' affect (SSA) they spotted the effect these issues may have on students' motivation, participation, and self-confidence. With these considerations in mind, they aspire to the establishment of social norms in the classroom that will address these issues. However, in many of the responses, at least three opportunities for mathematical learning seem to have been missed: to discuss the existence of more than one solution (MC); to raise students' metacognitive awareness of the benefits and drawbacks that alternative solutions may have (SSC); and, to address student A's difficulties with simplification (SSC). As a result, the sociomathematical norms these pre-service teachers aspire to establish in their classroom are not always clear. Instead, they are often blurred by their disproportionate attentiveness towards classroom management issues.

Conclusions

In this paper we present the results from a study in which 21 pre-service teachers engaged with two tasks based on classroom scenarios in which classroom management interferes with mathematical learning. Our analysis of the responses to the two tasks suggests the following insight into our two research questions.

With research question I, we aim to identify pre-service teachers' considerations when they make decisions in situations where classroom management interferes with mathematical learning. The analysis highlights that most pre-service teachers discuss social norms they aspire to establish and maintain in their classroom, such as peer respect, value of discussion, investigative learning and less spoon-feeding. In parallel, they discuss the sociomathematical norms they envisage for their classroom, such as acceptance of different solutions and the investigation of the *why* and *how* in mathematical procedures. Also, they mention the degree to which they feel responsible for doing so. Our analysis also highlights that many participants prioritise classroom management issues and either ignore entirely — or make limited reference to — mathematical learning issues of the incident (Simplification task). Further, very often participants address learning with simplistic dichotomous approaches (instrumental vs relational understanding in the Polygon task, or simplification or not in the Simplification task) by missing the opportunity to address these issues with their students and thus develop metacognitive awareness. We note however that the pre-service teachers respond to these tasks quite early on in their training year and we expect that their understanding (for example, of some theoretical constructs such as instrumental and relational understanding) would be much more polished by the end of the year. In any case the pre-service teachers' engagement with these tasks alerts us to where such polishing is necessary — and where more subtle deployments of such theoretical constructs can be celebrated. We see some of these subtle deployments in the few but remarkable cases where the pre-service teachers distinguish between types of mathematical understanding that are likely to yield good results in exams or foster an appreciation of mathematics as a discipline etc., and acknowledge merits in both.

With research question II, we aim to investigate how this type of practice-based tasks, that combine focus on classroom management and mathematical learning issues, can trigger pre-service teachers' reflections regarding the teaching and learning of mathematics. Overall, we credit this type of task with allowing insight into the pre-service teachers' considerations regarding their intended practice. However, while in their written responses the pre-service teachers outline their overall aspirations in broad and generally clear brushes, the majority do not elaborate how they would materialise these aspirations. Using additional tools, such as group discussions or focused group interviews — a phase of our project not reported in this paper — is a valuable complement to the written responses. For example, in the context of the tasks discussed in this paper, our analysis highlights that the pre-service teachers' overall commendable intended pedagogical approaches are somewhat marred by clichéd tendencies in some responses (such as a simplistically dichotomous perception of relational and instrumental understanding). Our analysis also highlights that few responses hinted at exploiting the opportunities in the scenarios for mathematical challenge. Engaging the pre-service teachers with post-written-response discussions — and analysing their shifting perspectives in the light of such discussions — is one direction that our study is now taking.

Furthermore, we note that different tasks elicit different responses from the pre-service teachers, even with regard to issues that may at first appear similar. For example, our analysis highlights the overwhelming attention to classroom behaviour issues in the Simplification task in the pre-service teachers' responses. Yet, classroom behaviour issues were barely mentioned in the pre-service teachers' responses to the Polygon task. Therefore general inferences about tendencies in the priorities of individual pre-service teachers must be drawn with caution — if drawn at all. Also, as we often repeat (Biza *et al.*, 2007; Nardi *et al.*, 2012; Biza *et al.*, 2015), these analyses are of pre-service teachers' *intended* practice only.

At present, our project is also entering a further phase in which pre-service teachers, after several occasions of engagement with tasks such as the ones presented in this paper, are invited to compose their own scenarios. In doing so, we intend to explore, in a more open manner, the vista of pre-service teachers' concerns as they experience teaching in their first school placements and as they prepare for their first teaching job. So far, we have steered their attention towards the issues highlighted by our own scenarios. We suspect that there are pre-service teacher anxieties and conundrums that our tasks are yet to tap into.

In conclusion, with the study presented in this paper we shift the attention of our research program to what we see as an innovative direction that combines, and concurrently considers, classroom management and mathematical learning issues in teacher education and research. With this study we aim to raise awareness of practices that tend to dominate teacher education programs such as a compartmentalised structure and assessment of teaching. In the UK, for example, the expectations of teachers as they enter the profession (HEA, 2012, pp. 6-13, standards 1-8) suggest that classroom management is assessed separately from quality of learning — and there is no explicit reference to how to establish and maintain the balance between the two. We stress that this is not an issue that relates to UK teacher education programs only — see, for example, analogous observations made by Leatham and Peterson (2010a) in the US context. We envisage that this strand of our research program will give further insight into the complexity of balancing effective classroom management with high quality mathematical learning, and into how this balance can be achieved in the preparation of new mathematics teachers.

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