

Micro-cycle Teaching Experiments as a Vehicle for Professional Development

Esther M.H. Billings
Grand Valley State University

Lisa Kasmer
Grand Valley State University

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This study used design experiments, specifically micro-cycle teaching experiments (MTE) as a catalyst for practice-based professional development. The MTE incorporated research-based characteristics of effective professional development: it was embedded in the teachers' daily work of planning and enacting lessons, co-constructed with the researcher to build upon students' knowledge, and sustained over time. Pedagogical and mathematical content knowledge were integrated into the planning, implementation, and analysis of these MTEs. In this study, we investigated: To what extent can teachers engage in a MTE as an intentional method for improving teaching? Case studies were used to analyze ways teachers engaged in MTEs and how their teaching was impacted as the result of this experience.

Keywords professional development · reflection · teachers · teaching experiments · mathematical knowledge for teaching

Introduction and Professional Development Perspective

Educators, researchers, and policymakers view professional development (PD) as a vehicle for empowering teachers to develop teacher capacities for improved student learning. Mathematics PD aims to increase knowledge for teaching which involves a solid understanding of both mathematical content and pedagogical methodologies. In this paper, we share the results of a qualitative study using case studies that examines how teachers build these capacities through practice-based PD involving micro-cycle teaching experiments.

Professional Development of Teachers

The primary goal of PD is to improve instructional practice. PD that is carefully designed and implemented can have a high impact on teacher knowledge and skills and student achievement (Darling-Hammond, Wei, Richardson, & Orphanos, 2009). However, in the United States, many teachers report that much of the PD available to them is not useful (Darling-Hammond et al., 2009; DeMonte, 2013; NCTM, 2014); teachers' professional learning is often disconnected from their practice and has limited impact on their students' learning (DeMonte, 2013; Loucks-Horsley, et al., 2010; NCTM, 2014). Research shows that a substantial amount of time (at least 50 hours) of PD is necessary in order for teachers to make changes to their practice, but in the United States, many PD initiatives are shorter in duration (Darling-Hammond et al., 2009;



DeMonte, 2013) resulting in PD comprised of independent, disconnected workshops rather than an ongoing, interconnected sequence of learning opportunities (DeMonte, 2013; Loucks-Horsely et al., 2010). Though PD has at times received a negative reputation in the United States, the educational community still views PD as a crucial vehicle for changing teaching and improving student achievement: “Almost every presentation or speech or conversation about educational reform inevitably includes some reference to the amount of support and training teachers and administrators will need in order to make key reforms real and effective in classrooms” (DeMonte, 2013, p. 1).

Research documents that effective PD for mathematics teachers involves content-focused, job-embedded professional learning that is ongoing and sustained over a period of time, allows for active learning and participation, and involves systemic support (Blank & de las Alas, 2009; DeMonte, 2009; Doerr, Goldsmith & Lewis, 2010; Edmondson, 2009; Kelchtermans, 2004; Loucks-Horsley, 2010; NCTM 2014). Furthermore, PD can have a powerful effect on teachers if it provides opportunities for growth in teachers’: (1) mathematical content and pedagogical knowledge, especially when connecting the study of mathematics to underlying curriculum taught, (2) ability to analyse and respond to student learning, (3) dispositions and beliefs for fostering continued learning including reflection on practice, and (4) opportunities for collaboration (Darling-Hammond et al., 2009; Doerr, Goldsmith, & Lewis, 2010; Edmondson, 2009; Loucks-Horsely et al., 2010; NCTM 2014). Darling-Hammond (1998) also emphasizes professional learning grounded in teachers’ own inquiry.

Practice-based PD is one strategy for professional learning that incorporates all these characteristics. In addition to the characteristics described above, it provides opportunities for teachers to examine authentic questions and teaching dilemmas as well as analyse “artefacts” arising from their own classrooms and educational context (Loucks-Horsely, et al., 2010; Kazemi & Hubbard, 2008). Teaching experiments, specifically micro-cycle teaching experiments, incorporate these research-based characteristics of effective professional development and are one way to support teachers’ growth.

Teaching Experiments

Teaching experiments fall within the broader research paradigm of design experiments (Cobb, Confrey, diSessa, Lehrer & Schauble, 2003). All involve an iterative and cyclical research design (e.g., Cobb, 2000; Gravemeijer, 2004; Lesh & Kelly, 2000; Steffe & Thompson, 2000) in which mathematical learning is examined in relationship to teaching within a broader social context: how does mathematical knowledge develop in the context of the classroom or small-group learning environments? Teaching experiments involve three main components: conjecture (posing questions and making conjectures about teaching or learning), implementation (teaching episodes are planned and taught), and analysis (Gravemeijer, 2004). Analyses of student thinking and the teaching episode itself are used to guide the next teaching episode; through the process of analysis, hypotheses related to teaching and learning are tested and revised. A hallmark of the teaching experiment is the teacher’s active collaboration with the researcher to examine, interpret, and analyze students’ learning (Cobb, 2000; Lesh & Kelley, 2000). Since learning is examined collaboratively and within the context of teaching, many different issues and questions can be explored, and the focus of the study can include the students’, teachers’, or researchers’ thinking (Lesh & Kelley, 2000).

Encouraging teachers to engage in design experiments as part of their everyday work of teaching can be unrealistic because of the time-intensive commitment as well as the research and product expectations. Building on Hiebert, Morris and Glass’s (2003) work of *experiments of teaching* with prospective teachers, we can adapt their analytical design-experiment approach:



lessons are treated as experiments, and this cycle of conjecture, implementation, and analysis/reflection is integrated into the teachers' routine. Artzt, Armour-Thomas and Curcio (2008) also use lessons as a context for prospective teachers to cyclically examine and reflect on their practice. Gravemeijer and Cobb (2006) refer to this iterative cycle as "micro-cycles of design and analysis" where researchers anticipate student learning based upon the designed activities, evaluate student participation and learning, and then revise or develop subsequent activities. Simon (1995) refers to this iterative cycle as a "mathematical teaching cycle," where teachers conjecture, enact, and then revise hypothetical learning trajectories. We call this notion of an iterative cycle of designing, enacting, and analysing lessons as the micro-cycle teaching experiment (MTE), and it frames the foundation for our professional development work with teachers described in the following sections.

Research Question, Design, and Methods

The MTEs used in this study incorporate research-based characteristics of effective professional development. The teachers actively participated in the PD. The use of MTEs stemmed from and was embedded in the teachers' daily work of planning and enacting lessons, co-constructed with the researcher to build upon students' knowledge, and sustained over time. Pedagogical and mathematical content knowledge and attention to student thinking were integrated into the planning, implementation, and analysis of these MTEs. Through this collaborative and iterative process, the teachers had multiple opportunities to reflect on their practice. In this study, we investigated the following research question: To what extent can teachers engage in design experiments – and specifically micro-cycle teaching experiments -- an intentional method for improving teaching?

Lesson Protocols: Prediction Questions and Workshop

This research is based on the premise that using a lesson structure with a specific learning goal in mind that intentionally activates students' schema and prior knowledge about a specific mathematical idea can act as a catalyst for mathematical learning and ultimately enhance teaching. Kazemi and Hubbard (2008) advocate the use of identifying and developing routine instructional activities into PD experiences, activities that regularly occur in the classroom for the purpose of developing disciplinary competence (p. 438). In this study, two different lesson structures or routines were used to elicit students' reasoning: the use of prediction questions and use of a workshop model. Each approach was motivated by a provocation and reflection question that focused on big mathematical ideas.

Prediction, in the context of this study, is defined as reasoning about the mathematical ideas of a lesson using previous knowledge, patterns, or connections *prior* to formal instruction. Students' responses to prediction questions can range from a premature guess, without being based on any plausible reasoning, to a sophisticated reasoning process connecting relevant topics (Kasmer, & Kim, 2011). One lesson routine used by teachers in this study involved posing one or two prediction questions at the onset of the lesson. Students would respond to the question along with supportive reasoning, and the teacher would record student responses, without confirming accuracy or misinformed thinking. These responses provided informal assessment information that the teacher could then use to inform the teaching of the rest of the lesson. At the conclusion of the lesson, the teacher posed the same prediction question as a way for students to reflect upon their learning.



The second lesson routine in this study involved workshop, an instructional model used extensively in language arts (Calkins, 1994; Cambourne, 1988; Keene & Zimmermann, 1997) as well as other disciplines, including mathematics and science (Heuser, 2002; Hoffer, 2012; Wedekind, 2011). The workshop model in this study focused on creating a problem-solving environment and included three key components: connect back, activity, and summarize/reflect. In the *connect back* phase, learners activate schema (Keene & Zimmerman, 1997) by recalling what they currently know, making connections to past experiences or knowledge before they are introduced to the theme or main question of the lesson. During the *activity* phase, learners actively solve problems, explore ideas, and construct new knowledge. In the final *summary/reflection* phase, learners share their thinking, including strategies, conjectures, and challenges and reflect about what they learned. Teachers facilitate this discussion to consolidate the experience, connecting back to the goal of the lesson.

Both lesson structures inspire students to activate their prior knowledge at the onset of a lesson. This sharing of students' knowledge immediately informs teachers of students' readiness, prior knowledge, misconceptions, and ill-formed ideas. Such immediate formative assessment of student knowledge can help teachers to monitor student learning, adjust teaching to meet students' learning needs, resolve discrepancies, and address the lesson's learning goal.

Data Sources and Qualitative Analysis

Case study was chosen as a method to analyse and report the findings of this study in order to provide a holistic description of the teachers' use of MTEs and provide "insight, discovery, and interpretation" (Merriam, 2009, p. 42). Case study is an ideal research methodology for exploring both "how" and "why" questions (Yin, 2008) as well as complex social constructs, in this study the impact of teachers' PD learning in mathematics classroom, consisting of multiple variables (Merriam, 2009). Case study provides the opportunity for researchers to examine contemporary phenomenon within a real-life context, using multiple sources of evidence to build upon previous research (Yin, 2008) and present a "rich picture" of phenomenon (Hamilton, 2011). Merriam (2009) notes that case study is particularly useful for studying educational innovations and evaluating programs.

Ten teachers, all with at least eight years teaching experience, volunteered to participate in this study because they were self-motivated to engage in professional learning and wanted to improve their teaching and either had a professional connection with the researchers or trusted a colleague's recommendation who had previously worked with the researcher. The 10 participants engaged in the PD using two different lesson structures: eight used a lesson routine incorporating prediction questions and two used a workshop model. The eight teachers, three male and five female, who engaged in the PD focused on prediction questions taught in upper primary and lower secondary classrooms (grades 4-8) at three different schools. The two female teachers participating in the workshop routine PD co-taught in a combined, kindergarten and grade 1 classroom, in an urban, racially diverse K-5 school.

At the onset of the Prediction PD, one researcher met with each teacher to discuss the purpose and potential effectiveness of the use of prediction questions, including sharing relevant research. Next, the researcher taught a model lesson incorporating the use of prediction questions in each teacher's classroom followed by a debriefing session that provided an opportunity for the teacher to ask clarifying questions and discuss the researcher's instructional choices based on the students' responses to the prediction questions. For the remainder of the study, each teacher in the prediction MTE taught 8-10 lessons. Teaching the prediction lessons varied by teacher in order to accommodate their scheduling needs. For some teachers, lessons were taught approximately once per month over the course of one school year,



other teachers taught these lessons weekly or bimonthly over one semester. During these lessons the teachers incorporated prediction questions, created by the researcher in conjunction with the teacher and based on the lesson goals, using the following protocol. The teacher posed one or two prediction questions at the beginning of the lesson. Students responded in writing, providing a response and explanation. The teacher previewed students' papers and elicited prediction responses with supportive reasoning. During this response time, the teacher did not confirm the accuracy of prediction responses. Instead, the teacher was to assess students' responses, make instructional decisions pertaining to ways to address misconceptions, ill-formed ideas, and proceed with the lesson. In the summary segment of the lesson, important mathematical ideas were coalesced and misconceptions revealed as the teacher encouraged students to resolve discrepancies between initial prediction responses and the learning that occurred through solving the problem. The researcher observed each lesson and collected field notes, samples of student work, and video recorded the majority of lessons. After each lesson, the teacher would debrief with the researcher about the lesson. During this debriefing session, the researcher and teacher discussed students' thinking and the instructional moves made by the teacher relative to achieving the lesson goals and students' prediction responses. Then alternative instructional moves were considered to better achieve the lesson goals and plan for future lessons. The researcher took field notes after the debriefing sessions to document the interactions. Teachers also kept an electronic reflection journal, summarizing and reflecting about their experiences with prediction questions.

At the onset of the Workshop PD, one researcher met with each teacher to ascertain her personal professional learning goals; what mathematical and pedagogical challenges and questions did each want to investigate through this four month PD? Each teacher's goals and questions were refined and revisited throughout the study. One researcher acted as a participant observer in their classroom an average of three days per week over the time period of four months. Throughout this time, the researcher took observation notes about the children's mathematical thinking and the teachers' instructional decisions; the teacher and researcher would discuss these observations in light of the teacher's professional learning goals during regular debriefing sessions, at least one per week. The researcher also regularly participated in weekly lesson planning and weekly school-mandated reflections meetings with the teachers and each teacher kept a weekly electronic reflection journal to document questions, challenges, or growth in their teaching knowledge. During this time, the workshop routine for structuring a lesson was implemented. After two months, each teacher engaged in a focused MTE for a one-week period. During this MTE, the following cycle was used: (1) Each teacher established a question about her teaching and specific learning goal she wanted to investigate with her students. (2) The teacher and researcher collaborated to plan a lesson by identifying, creating, or modifying mathematical tasks and tools (i.e., representations, a manipulative) to meet these specific learning goals. A workshop structure was used to plan the lesson and included anticipating student responses and formulating an overarching reflection question designed to help students consolidate learning and identify mathematical relationships. (3) The teacher taught the lesson. The researcher documented the thinking and learning processes that emerged through this instructional phase, including student-student, student-teacher, and individual interactions via field notes, video recording, and collecting samples and photos of students' work. (4) After the lesson, the teacher and researcher debriefed for 30-60 minutes; they analysed the lesson and assessed the children's conceptions and misconceptions. The researcher also took field notes during the debriefing. This information was used to notice trends, make conjectures, form theories, reframe and revise their questions, and utilised as the starting point to plan the next lesson. (5) This cycle of asking questions, planning, collecting and analysing data was repeated daily during this week. The teachers also participated in an audio-recorded



interview after the one-week MTE in which they reflected over their experience and learning during the MTE. For the remaining two months of the study, each teacher and researcher continued to engage in this iterative cycle of conjecturing, implementing, and analysing at least once a week. Together the researcher and teacher identified learning goals and used a workshop structure to plan each lesson.

The analysis of the qualitative data for case studies involved identifying recurring patterns and themes (Merriam, 2009). The initial phase of data analysis included watching/listening, coding, and analysing the video data and field notes from teaching episodes including the teachers' journals and data from the debriefing sessions, and identifying categories and themes related to what each teacher paid attention to and how the teacher enacted the planning, teaching, and reflection of the lesson. Our analysis and creation of categories focused on the qualitative characteristics of each teacher's decision making within the Mathematics Teaching Cycle (Simon, 1995), namely: the ability to conjecture about the intended learning goal, plan learning activities, predict students' thinking and understanding, and then consider revisions for future instruction. Components guiding our analysis included: (1) students' thinking is taken seriously and given a central place in both the design and implementation of instruction (2) teacher's knowledge evolves alongside students' growth in knowledge; as students develop in mathematical knowledge, the teacher learns about mathematics, learning, teaching, and the mathematical thinking of students. After this initial analysis, and identification of a large number of categories, the researchers directed their attention to those categories that appeared most often and used them as a springboard for further analysis. As the data were read and reread, the researchers anticipated how new data added to the existing categories or suggestions, and made additions, deletions, or combinations of categories in order to establish themes. Data for the cases were drawn from various sources to triangulate the data (Denzin, 2006) including: classroom observations and corresponding field notes, video-episodes of classroom lessons, student work samples, the teachers' written entries in their reflection journal, and field notes and/or audio recordings of the teacher interviews or debriefing sessions. By examining, categorizing, testing, and recombining evidence to draw empirically based conclusions (Yin 2008), we chose representative cases to characterise the teachers' engagement with MTE and how it affected their teaching.

Case Studies and Findings

In order to provide the reader with a descriptive analysis of the different types of professional learning that occurred through this PD, we chose three prototype cases. Goldsmith, Doerr and Lewis (2014) stress that the same PD opportunity may impact teachers differently; rather than describe whether or not MTE is an effective type of PD, we wanted to instead describe the effects this PD had on individual teacher learning. Since the study incorporated two distinct lesson routines (prediction questions and workshop), we chose representative cases from each type of MTE. Furthermore, we chose cases to illustrate the variety of ways teachers engaged in the MTE and the types of professional learning that emerged through this PD. In this section, we share a classroom vignette from the three teachers typical of their interaction with the MTE. For the Prediction MTE, the teachers' use of prediction questions remained fairly consistent throughout the research study. The episodes described took place approximately halfway through the study.



For Case of Jack (Prediction MTE)

Jack had 12 years teaching experience and taught eighth grade. At the onset of the study, Jack's lessons were teacher-directed. He would present the material in an organised fashion; students listened and took notes -- there was minimal sharing of students' thinking. In this vignette, Jack's students were in the middle of a unit leading to the study of the Pythagorean Theorem; previously they had completed lessons exploring the area and side lengths of the squares drawn from the legs of a right triangle. To begin this lesson that focused on finding the length of a diagonal line segment by using the segment to construct a square, Jack asked this prediction question: "Can you predict the length of this line segment?" He then drew a line segment on dot grid paper using the overhead projector [see Figure 1]. Students spent a few minutes quietly making predictions and writing their responses.

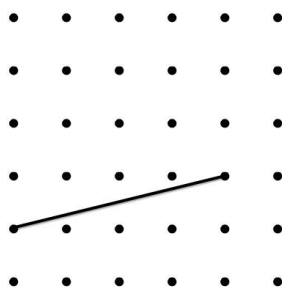


Figure 1. Predict the length of this line segment.

Jack then led a whole class discussion, asking his students "What did you guys have for your predictions? What did you come up with?" A variety of students volunteered to share their predictions: responses included 2, $\sqrt{2}$, 4, 5, $4\frac{1}{3}$ and $4\frac{1}{2}$. One student, James, raised his hand immediately and said, "I counted the number of dots over and up". Jack responded by asking, "So what number did you get, what do you think the length of the line segment is? Give a number." James responded "five". Jack then clarified James's strategy to the rest of the class, "Five, and you did that by counting the number of dots over and up? Four over and 1 up or 1 down and 4 to the left?" James agreed. Other students raised their hands to share. Frank said, "I did $\sqrt{2}$ because I remembered the area of the triangle we found [last time]". Nathan suggested "Four, because 4 times 1 equals 4". Willis, another student jumped in and said "Four, I counted spaces between the dots". Jack asked a clarifying question: "What dots?" Willis stated, "I counted the ones on the page ... up top". Jack pointed to the spaces between the dots above the line on the overhead in response to Willis's clarification. Jack then asked, "Any other guesses besides 4 and 5?" Jason offered his prediction: " $4\frac{1}{3}$. Since it's diagonal, it's longer that way." Jack asked Jason to repeat what he said and Jason re-explained that he knew the length of the straight line segment below was 4, and since it has to be longer than 4 he guessed it was about $\frac{1}{3}$ more. Jack then repeated Jason's idea, pointing to overhead and pantomiming a straight line segment of four (under the drawn diagonal) and saying it looked shorter than the diagonal line and asked Jason why it would be less than 5. Jason explained, "It just didn't look like five; if you spun the line segment around [pull the diagonal line straight down], it just didn't look like it would be 5."

Next, Jack instructed his students to get out their textbook, and explained, they would need to figure out the length of different lines in this lesson. He gave students about five minutes to read the text and then presented a five minute mini-lesson to the class, demonstrating on the overhead projector how to determine the length of a line segment measuring $\sqrt{2}$ by constructing a square, using the given length as one side and making sure to draw the additional side lengths at 90 degree angles. He partitioned the area of the square and asked the students to find the area. Students did this, and he modelled their strategy of partitioning the square into four equivalent triangles, each of area $\frac{1}{2}$ to determine that the total area of the square was two. He then told them to find the square root of two to find the side length, explaining they needed to know what times itself equals two. Students responded by saying "square root of 2" and others, "one point four one four". At this point Jack summarized:

So what you need to do here for all line segments is to find the length of each of those and use this process. If we know the area of a square, then the side length just has to be the square root of the area, and what I want you to do is use the square root and decimal approximation.

Jack then circulated around the room, noticing what students were writing as they calculated lengths and intervening when he saw the students making errors; he asked focused, literal questions to get students to see where they had made calculation errors. For example, Jack noticed Aaron's work and asked, "Is this right? No, I don't think so. You added wrong." Jack observed that Aaron had partitioned a square into triangles, but incorrectly calculated the areas. As Aaron explained his thinking, he revealed he was both incorrectly determining the side lengths of the triangle by counting dots and using an incorrect formula to calculate the area of a triangle. Instead of exploring Aaron's understanding of length or area, Jack responded by telling Aaron the length of the leg of the triangle and then coached Aaron step by step to calculate the area of the triangle using the correct formula. However, as soon as Jack left, Aaron continued to make the same type of errors as he calculated the area and length the rest of the lesson.

Jack continued to use this type of questioning throughout the lesson as he checked and affirmed students' correct answers, reminding some students of formulas or suggesting to students that they recalculate the side length or area. When students counted dots to determine the length of the segment, Jack showed them what the correct measure would be (though many of these students continued to keep counting dots rather than find the distance between dots to determine length). Jack did not conclude the lesson with any summary or consolidation of ideas, nor did he integrate the prediction question back into the discussion.

After the lesson, when debriefing with the researcher, Jack had little to say about the specifics of students' mathematical thinking, even when prompted. He recognized that he had forgotten to re-ask the prediction questions; he had run out of time but did not see this as particularly problematic. He enthusiastically stated that the lesson went really well and that there was nothing he could do to improve it. He was excited to see so many students participating and sharing their ideas with him and each other. Additionally, his electronic journal entries focused primarily on changes in classroom culture – students were participating and sharing ideas on a regular basis. Since his students stayed on-task, completed the lesson, and communicated with him and each other throughout the lesson, he did not see additional ways for improvement, even when prompted with additional questions from the researcher. Throughout the remainder of the study, Jack did not show additional shifts in the way he enacted the prediction questions routine. It is noteworthy to mention that Jack typically reposed the prediction question at the end of the lesson to have students check their work, but he



did not use this time to help student reconcile the discrepancies that emerged with their initial prediction responses and during the lesson.

Jack's engagement in the MTE demonstrated some incremental change in this teaching, related to student participation and communication. However, he did not pay careful attention to the *mathematical* nature of these exchanges; instead he focused on the correctness of the students' responses rather than analyse the particularity of their thinking. Also, he did not directly use his students' mathematical thinking to redirect any portion of the lesson. During the planning of the lesson, Jack identified a learning goal with the researcher focused on mastery of specific mathematical content. He began his lesson with a prediction question and took the time to have students explain and reveal their thinking. As students shared answers to this prediction question, Jack asked questions to get the students to clarify their thinking and also restated their ideas to make sure that both he and the rest of the class accurately understood each student's ideas. In this portion of the lesson, he effectively asked questions to reveal students' pre-conceptions of how to find the length of the segment. Thus, revealing a shift in his teaching disposition about the role of student communication in the mathematics classroom. However, after posing and exploring students' thinking in regard to the prediction question, Jack's lesson became teacher-directed. Instead of asking students to explain their thinking like he did when students responded to the prediction question, Jack reverted to telling them what to do or asked leading questions to get the students to use a proper procedure. He did encourage his students to talk to each other and to work out problems, but he did not specifically use the information gained to help students confront their ill-conceived ideas. For example, James's prediction response indicated that he (and perhaps others) wanted to count dots to calculate length. When Jack circulated among the class, at least three different students, including Aaron, were consistently using this counting strategy to calculate length that, in turn, gave incorrect areas. Instead of asking questions to focus students' attention and generate alternative solution strategies, or interpreting the meaning of "length" or "area" as amount of space taken up by the shape, using squares as a measuring tool, he instead asked questions to get students to plug in the correct numbers to a formula or correctly apply a procedure. Discourse was used to help students in small groups work through the process of how to calculate the length, but not utilised to generate conjectures or propose alternate strategies or grapple with why procedures worked. Jack did not make the connection that he was asking different types of questions of his students during the prediction question and exploration phases of his lesson. He also did not integrate the informal assessment data about students' misconceptions and questions into his instruction and small-group interaction with his students. Furthermore, Jack did not use the prediction question to unify the lesson, nor did he return to this question/learning goal as a means to have his students consolidate their knowledge, reconcile differences of their thinking between the beginning and end of the lesson, or reflect about what they had learned at the end of the lesson; returning to the prediction question at the end of the lesson is a fundamental part of the lesson as it provides an opportunity to consolidate and reflect on what was learned and explicitly confront ill-formed or misconceptions that surfaced (initially and throughout the lesson). Though prompted, Jack did not show demonstrative growth in his ability to notice and analyse students' thinking and use students' thinking as the foundation for exploration during the lesson.

Case of Linda (Prediction MTE)

Linda was considered an ideal participant due to her familiarity with and experience using the curriculum as well as her years of teaching practice. She had taught middle school mathematics for 11 years, had incorporated new approaches to teaching and promoting classroom discourse,



and was interested in new ways to promote student learning. Prior to engaging in the MTE, Linda made attempts to elicit student thinking but often struggled with connecting students' responses to the lesson learning goals.

Linda's seventh grade students were studying exponential relationships. In prior lessons, students explored linear functions and investigated exponential growth with whole number growth factors. In this lesson, students were introduced to compound growth; they explored patterns of change caused by compound growth of the value of a coin collection. Specifically, students considered a coin collection with an initial value of \$5,000.00 increasing in value by 6% each year. Linda asked students to generate a table showing the value of the collection each year for 10 years and determine the growth factor of this situation. The following prediction questions were posed:

Do you predict the increase in coin value would be linear, exponential, or something else? Explain why.

What do you predict the value of this coin will be after 10 years if the initial value is \$2,500.00? How do you know?

Students wrote their predictions responses while Linda circulated around the classroom. During this time, Linda previewed the students' responses in order to plan which responses she would highlight during the whole class discussion. After five minutes of writing, Linda asked the students to share their responses. One student responded:

The increase in coin value would be something else. The reason it's not linear is it doesn't always increase the same amount. The reason it's not exponential is the numbers don't double or triple. In this case every year goes by and the value increases, but 6% comes out of the higher value every year to add on.

This student seemed to regard exponential growth with whole number growth factors and perhaps those that only double and triple, but recognized the addition of 6% value each year from the previous year. Linda stated, "That is an interesting idea", her typical response to students' prediction responses. Linda did not ask clarifying questions, as she only intended for students to share their prediction responses at this point in the lesson. Another student responded:

Exponential because it's not going at a constant rate and its growth factor is 1.06 from 2500 to 2650 and also 2650 to 2809 has a growth factor of 1.06. So if it keeps multiplying by 1.06 then it's exponential growth not something else.

Linda again stated: "That is an interesting idea, and you've made a connection to what you've learned previously in other lessons." Another student suggested, "It will be exponential, I can't explain why – but I multiplied, so that tells me it's exponential but after 10 years it will be about \$275,000.00." Linda asked the student to come to the board and explain his thinking. The student generated this table (see Figure 2):

Year	0	1	2	3	4	5	6	7	8	9	10
Value	2500	4000	6400	10,240	16,384	26,214	41,943	67,108	107,374	171,798	275,000

Figure 2. Student generated table of values.

Linda noticed many of the students nodding in agreement, while others appeared as though they were unsure of this response. Linda then asked the class, "Does this seem like a reasonable amount -- that, if you start with a value of \$2,500.00 in only ten years, you would have almost \$275,000?" Some students agreed that this would be reasonable, and some did not agree. Linda then proceeded to explain the difference between multiplying by 6% and 60%, as she determined this was the cause of their confusion. After this brief instruction, the students proceeded to the exploration portion of the lesson. Linda moved in and out of the student groups, asking questions about the difference between linear and exponential relationships, with specific emphasis on the characteristics of their tables, graphs, and equations. She also asked students to consider what they knew about the percent of a whole, and how that might influence the growth factor in an exponential relationship. Students were struggling with the idea of the percent of increase added to 1 to determine the growth factor. She asked questions such as, "What would happen if I multiply \$2,500.00 by 6%? Do I get a smaller number or a larger number? Why is this?" "What does this amount [\$150.00] represent when I multiply \$2,500.00 by 6%", and "How can this situation be increasing in value if we multiply by 6%? Something doesn't seem right, what do you think?" Linda was never heard giving away answers; rather she continually asked questions to get students thinking about their work. Linda prompted another group to consider previous lessons: "Think about what you've done before, how did you figure out what the growth factor was when it wasn't given?"

At the conclusion of the group work, Linda brought the class together to revisit and change their prediction responses if necessary and summarize the lesson. First, as she had done previously, Linda asked the students whether they thought this situation was linear, exponential, or something else. Most of the students agreed that this was in fact an exponential relationship. She then asked for students to explain their reasoning. One student explained he drew a graph from the table, and the graph "looked like a j, like the other exponential graphs, so I knew it had to be exponential". Another student suggested that this was a situation of repeated multiplication, not repeated addition, so it has to be exponential. Linda continued to call on students to share their revised prediction responses along with their supportive reasoning. Most students accurately determined the value of the collection after 10 years, but some students had difficulty articulating the meaning of 1.06 as a growth factor. At this point, Linda moved to summarize the lesson.

As a result of the students' prediction responses and the subsequent group work, Linda realized some students did not understand what 1 represented in the growth factor (1.06). They could compute the growth factor, but had difficulty articulating why 1 was needed. Consequently, Linda began by asking a student who did understand the role of 1 to come to the board and share what 1 represented in the growth factor 1.06. Linda then asked another group to explain their thinking in a different way. Due to the time she devoted to ensuring her students understood this important concept, Linda did not complete the summary segment of the lesson. However, this summary was completed the following day.

Linda's participation in the MTE enabled her to notice and pay more careful attention to her students' mathematical thinking. Linda's use of the prediction questions elicited student thinking and her students' conceptions and misconceptions informed her instructional decisions throughout the lesson. The prediction questions Linda posed helped her and the students maintain focus on the intended learning goal of the lesson (recognizing that compound growth situations are exponential and understanding the growth factor of compound growth situations). When Linda realized that some students still struggled to understand why the growth factor was 1.06 instead of .06, she adjusted her plan in order to take her students' thinking into account and made the instructional decision to devote time to this concept, knowing that she could not complete the entire summary of the lesson during the class



period. Linda demonstrated the hallmarks of a reflective practitioner. When Linda and the researcher met to debrief the lesson, without any prompting, Linda immediately revealed

I never should have done the mini-lesson on 6% and 60%. I gave away too much. It would have been a much more interesting summary of the lesson with the some students ending with \$275,000 ($2,500 \times 1.60^{10}$) and some students ending with \$4,477 ($2,500 \times 1.06^{10}$). I was worried they would really struggle, but that would have been okay.

Linda shared, "I need to think hard about how to help students make sense of the meaning of the growth factor of compound growth situations." She also wondered about her students' ability to recognize that 6% is 0.06 and not 0.60. Was this a misconception she needed to address earlier in the school year? Did the students have an opportunity to move fluidly between decimals and percentages? Through such questioning, Linda was also considering revisions or subsequent activities to help support her students' learning as a result of witnessing her students' mathematical thinking.

Mary (Workshop MTE)

Mary, a teacher for eight years, taught a combined class of kindergarteners and first graders. At the onset of the study, Mary's instruction was very teacher-directed though her math lessons were interactive and often hands-on. As children engaged in the active part of the lesson, she would use the time to prepare another lesson or catch up on paperwork rather than take the time to observe the children's mathematical thinking. Mary did expect her students to share their ideas, though when she called on students they were expected only to share answers, not explain their mathematical thinking. Mary acknowledged that her decisions to call on students were motivated by classroom-management (to refocus a students' attention) rather than building mathematical ideas. In her teaching interview, Mary described her teaching before starting this PD as focusing on teaching topics and developing skills, rather than focused on learning and overall mathematical goals.

Throughout this MTE experience, Mary identified her ongoing desire for increasing the ways students engaged and communicated in mathematics; she wanted the types of engagement she experienced while teaching reading and writing mirrored in her maths lessons. Furthermore, in the weekly interactions, Mary and the researcher would plan maths lessons, focusing on setting learning goals, anticipating student responses, and identifying specific reflection questions to help the children consolidate their learning. Mary's focus during the one-week intensive MTE occurring about the halfway through the study (week eight) was to have students actively communicate their mathematical thinking. This lesson showcases a shift in Mary's teaching. She continued to use this lesson planning approach and enactment throughout the rest of the study.

Mary began planning by analysing her students' prior mathematical experiences in the past two weeks: they had explored addition through story problems and games using pictures, counters, and ten frames to model addition. Mary established a learning goal: "see" a systematic way to find all the combinations of 10 (using 2 addends). She then created a corresponding reflection question "How do I know I found them all?" and decided the key ideas she would highlight during the class summary discussion. She set the problem in a familiar literary context and hypothesized that using a new model (linking cubes of two different colours) would help the children see ten facts in a new way and find patterns. She posed this question: Would a new model (linking cubes) help the children find all the combinations?



Mary introduced the children to lesson by providing a contextual and mathematical purpose – help Mr. Wonka, a character in a book they had been reading, find all the different ways to make packages of two types of chocolate (where each package has ten chocolates). Since all the children knew $5+5=10$, she demonstrated building a package with five green and five yellow linking cubes and wrote the corresponding number sentence on a recording sheet. The children next engaged in the activity and constructed different 10-packages. Some used guess and check, a few built a package then switched colours to make a corresponding package (but not systematically), and others thought of a ten fact and then built the package. Only one group found all the packages by organizing them into an ascending staircase pattern and constructing missing “stairs.” Based on her observation of students’ challenges to find multiple packages, Mary began the summary time by reviewing ways to make 10. She used the packages Lillie’s group created to frame the discussion: they had found most, but not all 11 combinations. Mary held up a 10-package, and together they counted to establish the total and number of chocolates of each colour. Next, Mary called on a child to say the associated ten fact. Then, she physically placed each package next to the number sentence on their recording sheet. Mary pulled out a 5 yellow/5 green package; there was already a 5/5 package on the floor, and as she went to lay it on top of the package already made, Cal noted, “That’s the equals. It’s the same thing, only backwards. You see, if you put the yellow on the green, and the green on the yellow, that’s backwards; it turns it around.” Mary, responded, “It sure is” and rotated the tower so that opposite colours aligned. At that point, the group’s towers were all displayed and Mary said, “I notice that you have a $2+8$ on your sheet, but you didn’t build that one.” As the class assisted her to build the 2 green/8 yellow package, Cal exclaimed: “Hey! It’s the equals again!” Mary invited Cal to show this relationship; he placed the $8/2$ and $2/8$ packages on top of each other so 8 yellow and 8 green cubes aligned. Mary then asked if any other packages matched up. Abe noticed that the $10+0$ and $0+10$ and $3+7$ and $7+3$ pairs matched up. Since there was only one package with $9+1$ (9 green/1 yellow), the children immediately suggested building the match and together they constructed it. Though they had seen a commutative relationship, Mary also wanted the students to “see” a different systematic way to order the packages. She asked the children to find a pattern after ordering the $0/10$, $1/9$, $2/8$ and $3/7$ packages into a staircase pattern. A number of children exclaimed, “it goes 0, 1, 2, 3”. Abe volunteered to arrange the packages so the ascending colour increased by one each time. Together the class counted the number of packages and seemed convinced they had found all possible combinations of packages because of the increasing staircase pattern.

As Mary debriefed with the researcher after the lesson, she was excited: the summary discussion was mathematically focused and successfully built upon the students’ knowledge evidenced during the lesson. She also reflected that thinking about her mathematical goal and reflection question first was key; it helped focus the lesson, anticipate students’ thinking and prepare questions. Her hypothesis of the role of the model in helping the students see mathematical structure was affirmed. She was also excited to hear and see the different types of reasoning that the students had used. She then acknowledged that this planning structure of choosing a mathematical learning goal and associated reflection question was key for promoting mathematical learning.

Mary began this MTE by analysing student thinking and considering research related to using relational thinking to develop children’s fact knowledge. She chose her learning goal because she recognized the central role of 10 and observed her children did not have automaticity with all their ten facts and conjectured using a different representation would help them “see” mathematical structures. Her ability to notice and build upon students’ mathematical thinking was honed; children’s sharing of mathematical thinking was highlighted throughout the lesson. By observing students’ strategies during the lesson, she led a discussion



that both reviewed ways to make 10 and focused on noticing patterns and mathematical structure (commutative property and compensation – as you increase one addend by 1, decrease the other by 1). She intentionally chose a group that did not have the entire collection of packages so that they could use their pattern analysis to construct all the combinations and notice relationships. Through this MTE, Mary’s pedagogical content knowledge of representing and building conceptual understanding of part-whole relationships, especially 10 grew, as did her focus for planning. In consequent lessons, Mary initiated planning by first asking what her learning goal and reflection question would be, focusing on mathematical structure and representation (a shift from prior practice) and anticipating ways to motivate children to share ideas to forward the mathematical purpose of the lesson.

Discussion and Results

In examining these cases, all the teachers demonstrated evidence of change in their professional learning, though as Doerr et al. (2010) and Goldsmith et al. (2014) explain, often the change is incremental and iterative. Through this ongoing process of examining their own teaching, all of the teachers’ dispositions of what effective mathematical instruction looks like expanded to include more student communication in their classroom. Jack’s shift was a smaller increment than the others as he focused on the need for more communication. He did not grow, like the other two teachers, in his ability to notice and analyse the mathematical nature of the conversation, nor use the students’ knowledge to actively inform instruction. Linda responded to student confusion about the meaning of a non-whole number growth factor by redirecting the summary part of her lesson and Mary used the students’ analysis of patterns to direct the summary discussion. Both Linda and Mary used students’ thinking to inform and direct the lesson as they asked questions, chose examples, and used students’ work and ideas to consolidate understanding, and their ability to do this continued to develop throughout the PD. Furthermore, anticipating student thinking, especially in light of a learning progression, was key in the teacher’s ability to address students’ ill-formed ideas and build on students’ ideas. While teachers in both MTEs successfully anticipated and monitored student thinking throughout all phases of the lesson, one difference between the two MTEs was that in the Prediction MTE, the teacher and researcher did not explicitly discuss anticipated student thinking while it was explicitly discussed in the Workshop MTE which might explain why Jack did not actively use this knowledge to redirect and directly inform his instruction.

As Clarke and Hollingsworth’s (2002) model of teacher growth suggests, growth in one aspect of teachers’ knowledge and practice can promote growth in other areas. While all the teachers demonstrated they could listen closely to a student’s thinking and through this PD experience made this thinking even more visible to the other students, it was crucial that this *attention to student thinking impacted the lesson*, and that *the teacher made instructional decisions in response to student thinking*. Through this ongoing, iterative PD process of planning, enacting, and reflecting about their own lessons with the researcher, both Linda and Mary grew in their ability to learn to notice and analyse students’ mathematical thinking, and through this process gained a more accurate understanding of students’ mathematical understanding and misconceptions, a crucial component of effective PD (i.e., Doerr, et al., 2010).

The teachers’ ability to critically reflect about their practice in light of student learning was a key factor for how the MTE impacted their teaching; through both the individual and collaborative reflective process, the teachers developed and articulated an awareness of how their decisions impacted student learning (either positively or negatively). Even though Jack was provided with similar support as other teachers in the Prediction MTE, the planning,



debriefing and journaling opportunities did not lead him to closely examine his teaching and notice discrepancies. Throughout this PD, Jack did not shift in his awareness and ability to reflect about the students' mathematical thinking – he assumed everything was fine because many of his students could produce correct answers by the end of a lesson. Mary and Linda, on the other hand, were much more aware and willing to discuss and examine how their decisions, both in planning and instruction, impacted student learning.

The ongoing, cyclical routine of setting an intentional mathematical learning goal, and planning ahead of time how to connect this goal with a summary part of the lesson, was also critical, and contributed to the teachers' growth in noticing and building upon students' thinking. The prediction question itself naturally lent itself to this purpose, but *only* if the teacher used it to promote consolidation and reflection and tied it back to the students' thinking, like Linda did. For Mary, using the workshop structure meant she had to generate a separate reflection question, but formulating this question with the researcher focused her planning and provided an opportunity to explore pedagogical and mathematical content. After the intensive one-week MTE, Mary continued planning each lesson by considering what she hoped the reflection discussion would look like. For the remainder of the study, Mary and Linda continued to set intentional learning goals while planning for instruction, listened to students' thinking, attempted to make adjustments to instruction as a result of students' evolving mathematical knowledge, facilitated a discussion summary at the end of the lesson, and reflected over their instruction. However, they did not always possess the necessary knowledge (i.e., of mathematical learning progressions, mathematical content or awareness of student strategies) to make the changes or instructional choices they wanted to make. This awareness and desire for support in developing these aspects of knowledge for teaching suggests the need for readily available, high-quality resources that teachers like Mary and Linda could access that would provide this necessary background knowledge when it is not possible to collaborate with a colleague with mathematical and pedagogical expertise. Closely related to goal setting was the teacher's disposition of what it meant to teach mathematics: did the teacher view mathematics instruction as teaching mathematics topics or skills, as Jack appeared to, or, did the teacher envision the mathematics they were teaching within a larger learning progression of mathematical concepts and skills, like Mary and Linda, whose perspective continued to incrementally shift in awareness through the process of the MTE?

The teachers' varied responses to the MTE confirm other professional learning research supports research documenting that teachers respond differently to PD opportunities (Goldsmith et al., 2014); however MTEs did provide an opportunity for professional learning and growth of *all* the participating teachers, though it must be noted that it required a significant time commitment from the teachers and researchers. As these cases showed, the MTE provided an active, collaborative opportunity for teachers to consider and change their own practice, supporting Kazemi and Hubbard's (2008) recommendation of using job-embedded artefacts and routine instructional activities as a focus for professional learning. All of the teachers deepened their view of the role of student communication in mathematics teaching and persisted in encouraging their students to share their thinking during the lesson and Mary and Linda deepened mathematical and pedagogical content knowledge of the mathematics they teach. The teacher's willingness and ability to reflect critically about his/her practices was a key factor influencing the professional learning and types of growth of the teacher engaging in the MTE.

While the results of this case study research cannot generalise to the entire teaching community given the limitations of this study, these cases do suggest that teachers can engage in design experiments, and describe how MTEs provide an intentional PD opportunity strategy that leads to mathematics teachers' professional learning and growth. More research is needed



to explore the effect of MTEs if teachers' participation in this type of professional learning is compulsory. Further, more research is needed related to how MTEs can be used to deepen teachers' dispositions, knowledge and skills, especially in terms of scalability: given the time commitment as well as individualized and collaborative nature of the MTE, how might this type of PD become integrated more systemically to involve entire schools and districts, rather than individual teachers and classrooms?

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Authors

Esther Billings
Grand Valley State University
Mathematics Department
1 Campus Dr.
Allendale, MI 49401 USA
email: billinge@gvsu.edu

Lisa Kasmer
Grand Valley State University
Mathematics Department
1 Campus Dr.
Allendale, MI 49401 USA
email: kasmerl@gvsu.edu

