The Role of ICT in Supporting the Development of Professional Knowledge during Teaching Practice

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Prospective mathematics teachers often start their practice with images of learning situations quite different from current curriculum orientations. Such is the case of Fabio, who, at the beginning of his practicum, proposed mostly exercises and promoted unidirectional communication. We analyse the contribution of a teacher education setting based on ICT (email and forum) in developing his professional knowledge, looking especially at tasks and forms of communication. The qualitative and interpretative case study of this prospective teacher is based on interviews and on the analysis of the documentation generated in the teacher education setting. The results show that, in addition to exercises, the prospective teacher began to suggest more open and challenging tasks, and to promote contributive communication in his classroom. In planning his lessons, Fábio's professional knowledge showed improvement in the diversity of tasks that he adopted, in the attention he gave to student activity, in the adequacy of instructional materials that he used, and in the forms of communication that he promoted, which involved more students in the classroom activities. By reflecting on his actions, he developed his ability to gather information about his practice, how to interpret it, question it, and reshape it. The reconstruction of particular parts of the practice helped Fabio to expand his professional knowledge and to see practice as a source of learning to teach.

Introduction

This study looks at integrating information and communication technology (ICT) in a tool for supervising prospective mathematics teachers in their teaching practice. Teaching practice is one way of acquiring practical knowledge about ways of teaching mathematics (Viseu & Ponte, 2012). Developing practical knowledge is action-based, learning about practical questions linked to contact with students and the acquisition of contextual knowledge with respect to planning the content of the school curriculum.

Teaching practice is the culmination of the teacher education process, whose structure and content can be based on an analysis of the teaching activity. Jaworski and Gellert (2003) and McDuffie (2004) believe that it is as or more important to know how prospective teachers should acquire the competencies they should have than to know what such competencies are. As teacher educators for higher education institutions in Portugal we want to look at the situations that emerge from each prospective teacher's practice, relative to the construction of this knowledge. Recommendations have emerged from research for supervisors to support prospective teachers to prepare lessons, solve problematic situations, and undertake more monitoring and trips to schools (Caires, 2001). Various obstacles that can hinder supervisors' efforts to help prospective teachers, such as the distance from the schools to the university and timetable incompatibility can be overcome through the use of ICT. These technological resources are powerful tools for distance communication and can help with mutual aid (Ponte, 2002). This potential of ICT sparked our interest in finding out what contribution electronic means of communication could make to developing elements of pedagogical content knowledge of prospective teachers, if used in the supervisory process.

Pedagogical Content Knowledge

The research emphasised the knowledge teachers need in order to teach, centring on what they need to know and how and how this knowledge is acquired via university courses and classroom experiences (Carter, 1990). This question came into the spotlight after the critique by Shulman (1986) in light of the research trend to be more concerned with understanding pedagogical aspects than with teachers' knowledge of the content, and, in particular, ways of making it easier for students to understand. On recognising that there is a specific knowledge to teach, Shulman organises it into content knowledge, general pedagogical knowledge, pedagogical content knowledge, and curriculum knowledge. Of these categories Shulman singles out pedagogical content knowledge and curriculum knowledge, which he sees as essential to teaching by mixing knowledge of the content and pedagogy required to teach it. The interest in this type of knowledge lies in the connection established between the knowledge of the content and the practice of teaching (Ball, Thames, & Phelps, 2008).

Some authors criticise the notion that Shulman gives to pedagogical content knowledge. They say that Shulman's notion of pedagogical content knowledge relates more to a declarative conception of the teacher's knowledge than to a conception of knowledge guided towards action or included in practice. Santos and Ponte (2002) suggest that the knowledge teachers need to be able to teach contains elements other than content knowledge and pedagogical knowledge, which are related to curriculum knowledge and its management: they need knowledge of the students and their learning process, of the main ends and objectives of the curriculum, of content organisation, of materials, and of assessment methods.

Deciding on how and when to address a particular mathematics topic, guiding students as to what they have to do, listening to and commenting on their ideas, determining the validity of a mathematical argument or the adequacy of mathematical representations, and establishing links between the topics tackled in other subjects or in mathematics itself are examples of tasks for which content knowledge is crucial (Osana, Lacroix, Tucker, & Desrosiers, 2006). It is important for teachers to properly understand both the mathematics curriculum and their students if these activities are to be developed efficiently. Knowledge of the curriculum involves knowing the school programmes, the range of materials to be used in teaching, and the pros and cons of using such programmes and materials in the classroom (Shulman, 1986). It also requires knowing how mathematics contents interact, what approaches are recommended, the ends and objectives, and the indications relevant to assessing students' knowledge (Canavarro, 2003). Knowledge of the students involves knowing their interests and problems (Santos & Ponte, 2002) and how they learn and develop their ideas in mathematics (Kilpatrick, Swafford, & Findell, 2001). When planning and in class, teachers need to pay heed to what students know, understand how to answer their questions and statements and how to decide what to do with the various ideas that students come up with (Kilpatrick et al., 2001).

Of these professional elements, the nature of the tasks and forms of communication in class are crucial to making mathematics content understandable to students. Considering the degree of challenge offered by different tasks, Ponte (2005) classifies them as exercises, problems, and research and exploration. Exercises are generally tasks that are solved mechanically and repetitively by applying a formula or algorithm that leads directly to the answer. They enable students to practise and consolidate the knowledge they are acquiring. Problems involve non-routine situations for which students do not have an instant solution process. Research and exploration tasks, according to Ponte (2005), are more open and require students to participate in the "specific formulation of the questions to be solved" (p. 15). In these tasks students look for regularities, establish and test conjectures, discuss and communicate their conclusions orally or in writing.

In these activities, the way teachers lead the communication—verbal and written—may or may not encourage students to voice their doubts and justify their ideas. According to Brendefur and Frykholm (2000) communication in the classroom can take different forms: uni-directional, contributive, reflective and instructive. In uni-directional communication the teacher dominates discussions, basically using closed questions and creating few opportunities for students to give their strategies, ideas, and opinions. Contributive communication involves informal interaction among the students and between teacher and students, when they contribute with elements to solve a task. In reflective communication the teacher and students use their mathematics conversations to go more deeply into the discussion topic. Instructive communication tries to go further to help students to construct and modify their mathematical knowledge. Reflective and instructive communication are conceptually different from uni-directional and contributive communication in that the focus of the discourse shifts from the conveying of information to the production of meanings.

Reflection

The connection between reflection and practice led Korthagen (2001) to value practical problem situations as an element of professional development. Mewborn (1999) sees these situations as favouring the reconstruction of teachers' practical knowledge, enriching and changing their thinking patterns and performance standards in light of the references that give it meaning.

Aspects that tend to cause difficulties for the development of reflection in prospective teachers on their pedagogic practice include: (i) difficulty in gaining the ability to look back and learn from classroom experience, since it is complex and often jeopardised by the lack of time and aloofness of certain supervisors (Pultorak, 1993); (ii) the notion that reflection is not part of teachers' work (Hatton

& Smith, 1995); (iii) the non-existence of reflexive practices during university studies (Ponte, 2002); and (iv) beliefs acquired as students (Korthagen, 2001).

Smyth (1989) suggests that these problems can be overcome by means of *questions of a pedagogical nature*: (i) *Description*: What do I do? What do I think? What is my teaching like? (ii) *Interpretation*: What does it mean? What theories and conceptions are expressed in my teaching? (iii) *Confrontation*: Why do I think/act as I do? What are the causes and reasons? (iv) *Reconstruction*: How might I modify my thinking and practice? Description is the first step for prospective teachers to develop reflection on their practice, revealing what they do or feel. This description is only meaningful if it is guided by an interpretation of their performance, so as to make them aware of their system of appreciation, essential to confrontation with alternative discourses.

Hatton and Smith (1995) note the role of written communication in the development of the ability to reflect, by giving a "voice" to prospective teachers to make their own thoughts and actions heard. One way of enhancing written communication stems from discursive interactions mediated by the computer (Wu & Lee, 2004).

ICT in the Supervision of Teaching Practice

The importance of preparing prospective teachers in accordance with current recommendations for mathematics education has prompted the researchers' attention. One way that research has found useful is to integrate ICT in supervision for pedagogical practice. A study by Souviney and Saferstein (1997) on the possible use of electronic communication in the supervision of prospective teachers found that the use of such technology by the participants had grown considerably over time. The authors noted that the potential of email and the internet was particularly important in facilitating the revision of lesson plans and units, favouring collaborative work between the prospective teachers when drafting tasks for the classroom, helping to sustain ongoing dialogue about what happens during teaching practice, and as a means to think about problems arising during professional practice.

The potential of ICT led Cornu and White (2000) to argue for a change from traditional supervision—in which university supervisors observe classes and make comments—to more facilitatory models in which supervisors join prospective teachers in their lesson preparation. This model involves the prospective teachers contacting their supervisor by email and, once a week, contacting a colleague. Most prospective teachers thought that supervision via email reduced the stress felt when supervisors come to the school, increased their responsibility and autonomy, and facilitated contact with the university supervisors when preparing lesson plans, sharing experiences, and getting more systematic feedback.

Bodzin (2000) says that when prospective teachers share their ideas and experiences through ICT it helps eliminate the isolation that they often feel during teaching practice. With respect to the role of an open forum, Bodzin finds that the prospective teachers think that this helps them to feel less inhibited about asking questions and offering comments than in a face-to-face situation.

Because the forum is open to the public, however, it limits the participation of the prospective teachers when messages relate to topics not covered in the classroom, and there is not always time to use the feedback they receive.

The new means and opportunities to learn from practical teaching offered by communication via computer prompted Wu and Lee (2004) to consider how it could be used in the pedagogical practice of prospective teachers. They developed a teacher education setting comprising: a resource section, where prospective teachers put teaching materials; a discussion forum, as an incentive for them to talk about issues that arise in their teaching practice; and a personal folder where they could put their work and which contained parts of videos of the teaching practice that the authors recorded. The prospective teachers felt that they learned from the comments they received about their teaching, and from the more and less positive aspects of the different teaching styles they saw. Because they could watch their teaching videos over again their thoughts were not based solely on recalling what had happened, but were also related to what they saw, and their comments were contextualised with parts of the video.

Another study on the use of virtual supervision through a discussion forum and email communication was undertaken by Ponte et al. (2005). The prospective teachers found the forum the better of the two resources. The authors believe that the difference lay in the mandatory nature of participation in the forum (two messages every two weeks), whereas email communication relied more on individual initiative. Some participants only used this means when strictly necessary, preferring face-to-face conversation with their supervisor.

ICT helps supervisors to work with their prospective teachers in planning lessons and discussing practical issues (Putnam & Borko, 2000), which could be reflected in a change of conceptions on the teaching of mathematics that tells students how they ought to think and act for teaching that involves them in the development of their mathematical knowledge.

Research Methodology

We have prepared a tool to help the training of future teachers by face-to-face meetings and through the use of ICT (Figure 1).

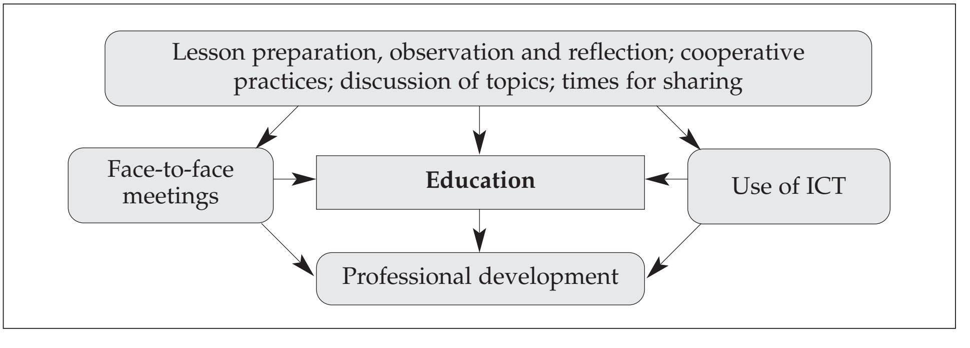


Figure 1. Components of teacher education setting for prospective teachers

Face-to-face meetings were weekly, some only with prospective teachers from each school[1] (for the preparation of lessons to be observed and discussing them) and others with all the prospective teachers supervised by the first author of this paper (for discussing texts and planning activities).

Among the range of resources offered by ICT, email and discussion forums were permanent means of communication that supplemented face-to-face meetings. Email was used for the preparation of the prospective teachers' lessons and development of their thoughts on observed lessons. At the pre-observation stage participants sent a lesson proposal and a discussion on their ideas and dilemmas to their supervisor. At the post-observation stage, the prospective teachers sent their thoughts on their action, after the face-to-face discussion.

In the forum, the prospective teachers shared and discussed situations relating to their practice. The forum had two areas: (i) *Norms* area where some of the *Norms for the school mathematics curriculum and assessment* (National Council of Teachers of Mathematics (NCTM), 1991) and *Professional norms* (NCTM, 1994) were discussed[2]; and (ii) Free area where issues related to the prospective teachers' practice were discussed.

One prospective teacher, Fábio, was chosen from the nine involved in the placements. The choice was made based on the relevance of the information obtained about his pedagogic practice, and a case study was prepared using Fábio. The study followed an interpretive qualitative approach, through the importance ascribed to human activity in the experiences in which each individual develops their meanings (Erickson, 1986). The register of the case study was narrative to provide knowledge of his ideas and experiences during his teaching practice placement, based on the perceptions of the prospective teacher.

Data were collected in two audiotaped interviews, one at the start of teaching practice (E1) and the other at the end (E2), from email messages sent in the preparatory and reflection stages of the lessons in which Fábio was observed, from the observation of those classes, and from messages posted in the forum. The email messages sent by the prospective teacher are given in order of receipt (e_n), followed by the initial of the prospective teacher and direction of communication from the prospective teacher (F) to the supervisor (S). Analysis of the data suggested several structuring topics of Fábio's discourse. New interpretations of the empirical material meant that the data were placed in the following basic categories that order, structure, and systematise the information (Miles & Huberman, 1994): (i) aspects of the teaching-learning of mathematics; (ii) aspects of the use of ICT; (iii) the preparation, practice, and reflection stages of the lessons observed; and (iv) reflection on ICT input from the teacher education set-

1 In Portugal, at this time, after four years at university taking mathematics and education courses, prospective teachers ended their education with a placement in a local school. Prospective teachers were sent in groups to different schools, some of them quite a way from their university. Each prospective teacher worked with two classes and taught some lessons in a class for which one of the school's cooperating teachers is responsible. This model has recently been changed under the Bologna process and the placement is now in classes of a cooperating teacher.

2 The NCTM Norms mentioned in this study are the translation of the original versions done by the Association of Portuguese mathematics teachers.

ting. For this report, the information was taken from each category that revealed the nature and structure of the tasks undertaken by Fábio and the forms of communication he used in the classroom. The case is presented as: (1) Fábio's teaching practice; (2) Tasks carried out by Fábio in the classroom; and (3) Mathematics communication in Fábio's lessons. Regarding his teaching practice, of the six lessons that were observed the most significant information from three lessons that he taught in his cooperating teacher class is presented. This information covers the preparation, observation and reflection stages of the observed lessons. The information on the preparation and reflection comes from the email interaction between the prospective teacher and his supervisor. Regarding the lesson preparation stage, the information is presented in terms of: "Concerns" and "Strategy" expressed by the prospective teacher, and "Suggestions" from the supervisor.

As the supervisor is one of the authors of this paper, the credibility of the interpretations is backed by various elements, such as: (1) comparing information from a variety of sources; (2) not adding to or altering the sense and meaning of the participants' statements; and (3) involving the prospective teacher in the data analysis, after assessment of the teaching practice activities. Once it was written, the prospective teacher had the chance to read the case study, which enabled him to comment on the degree to which he identified with the person portrayed and the appropriateness of the supervisor's interpretations of the meanings ascribed by the latter.

Fábio's Teaching Practice

Fábio is a prospective teacher without any prior experience of teaching, a rather shy personality with few words, but very open to discussing classroom situations. He did his teaching practice with year 7 and year 8 students, and with a year 9 class of which his cooperating teacher was in charge, where he taught one lesson in the first term, two lessons in the second term and three in the third term.

Solving Problems with Systems of Equations

Preparation of the lesson. Fábio planned his first year 9 observation lesson by identifying the content to be tackled, the students' prior knowledge and the programme objectives ($e_6F\rightarrow S$) (see Table 1).

Table 1
Fábio's planning for a year 9 lesson

Concerns	Strategy
• Knowing the content to be taught and the prior knowledge students should have. • I have examined manuals to "study" the material. • I chose some tasks that I found suited to the goals.	• Teaching format "explanatory and discovery". • Presentation of the transparency and solving problems with the intermediate steps.

In the roles he assigned to the teacher and student he tended to express a conception of teaching that valued it as meaning that students keep abreast of and supplements its processes: "telling the students the steps to follow to solve problems" ($e_6F \rightarrow S$).

Supervisory interaction. As the solution of the systems of equations from the problems adapted by Fábio was also the solution of those problems, in my (Viseu) email interaction with him ($e_6S \rightarrow F$) while preparing this class I warned him that in that case he would not be able to trigger discussion about whether or not that solution matched the problem's context (see Table 2). Engaging students in solving problems involves listening to their strategies and the different responses.

Table 2
Suggestions sent to Fábio from his supervisor

Suggestions ($e_6S \rightarrow F$)
• Stress that the solution of a system which expresses a given problem is not always the solution to that problem.
• Listen to the different solution strategies, ask if anyone can suggest a different solution.
• The students can make a note in their exercise books of the most important aspect covered in the lesson in the Final Synthesis/Discussion.

Practice in the classroom. Fábio started the lesson by projecting a transparency showing the first problem:

> In a class of 30 students the difference between the number of boys and twice the number of girls is 6. How many girls and boys are there in the class?

The transparency showed the steps to follow to solve it: "1. Read and understand the problem; 2. Identify the unknowns and give them letters; 3. Express the unknowns in the problem by equations; 4. Solve the resulting system; 5. Check and criticise the solution; 6. Draft an answer".

Fábio: What does the problem tell us? What should we do first?

(Fábio removes the transparency showing the first step)
We first have to understand the problem.

Student: There are 30 students.

Student: There are twice as many boys as girls.

Student: The number of boys less twice the number of girls is 6.

After a few students had had their say, Fábio showed the second step in solving the problem. He suggested using *x* for the number of boys and y for the number of girls. In the third step, he enters the two equations that express the problem on the board, to give the relevant system. The other steps, shown on the slide, were carried out by a student on the board. After recording the answer to the problem, Fábio presented another problem:

> Ana and Pedro are brother and sister who collect stamps. Between them they have 72 stamps. If Pedro gives 5 of his stamps to his friend Eduardo, the brother and sister will each have the same number of stamps. Comment on your solution.

A student went to the board to label the variables and another expressed the problem in a system of equations. Several hands were raised when asked to solve the system, and one student solved it without telling the class what she did, and without answering the problem. The solution of the system is a pair of decimal values that does not answer the problem context. Fábio concluded that the problem has no solution, and proposed another problem:

> A garage accommodates cars and motorbikes. If there is a total of 23 vehicles and a total of 78 wheels, calculate how many motorbikes there are in the garage.

After reading it Fábio asked another student, who had his help, to go to the board and write the relevant system of equations on it with the letters *w* and *k*. To try and get other students to join in Fábio asks a student to go to the board and solve the system. After a couple of minutes hesitation she asked the teacher if "the letters could be *x* and *y*?" The common labelling of variables with these letters meant that the change to w and k made it harder for her.

Reflection. When he analysed his actions, Fábio showed his dissatisfaction when he realised that "I should have allowed time for the students to participate" ($e_8F \rightarrow S$). He queried his attitude at certain points in the class where he could have taken more advantage of the students' intervention: "I gave the first step instead of getting a solution from the students" ($e_8F \rightarrow S$). The prospective teacher sees that, although he let the students have some time to answer the tasks, he did not always take advantage of their activity. When the students were not keeping up it appears that instead of trying to see why they were having difficulties, he was trying to give them the answer.

In the last problem of the class, in his interaction with some students in their places, he suggested that they assign the letter *w* to the number of cars and the letter *k* to the number of motorbikes. His concern with changing the letters of the unknowns was deliberater, so that they would not always be *x* and *y*. The inconsistency of teaching language, however, seems to have led to conflicts in the students' processes: "the girl who solved the system had difficulties because the letters were different, I saw that I ought to have looked at it with the students" ($e_8F \rightarrow S$). So Fábio sees the importance of questioning certain procedures in his own action that can affect the students' activity.

Inscribed Angle of a Circle

Preparation of the lesson. When studying the angles of a circle the exploratory nature of the topic meant that Fábio wanted to use dynamic software to involve the students in finding the relative relations of the angles at the centre and the inscribed angles (e30F→S) (see Table 3).

Table 3
Fábio's concerns about the lesson and teaching strategies

Concerns	Strategy
• I give ready-constructed figures, because I'm afraid that the students will go astray in the constructions. • Getting the students' attention since they are involved in the computer and don't notice me.	• Teaching "in pairs". • Setting the students to measure it so that they can familiarise themselves with the concepts. • The questions will be answered by students on the board.

Fábio indicates that he is afraid the students will not manage to construct the figures to be explored with Geometer's Sketchpad (GSP). When organising his lesson plan he tries to follow some of the methods in the Geometry Norm (NCTM, 1991), which shows that he values, in his teaching practice, the mathematics education literature.

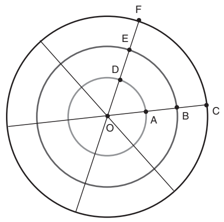
Supervisory interaction. In the proposed plan he sent, Fábio reveals that he is more concerned about informing the students about which GSP commands to use than to present tasks of an exploratory nature that would enable them to take advantage of this resource. In the course of our exchange of emails I mentioned some aspects that I thought were relevant to a lesson held in a computer room (e23S→F) (see Table 4).

Table 4
Suggestions sent to Fábio from his supervisor for the inscribed angle of a circle lesson

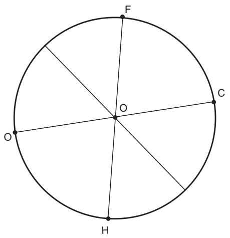
Suggestions
• Working with GSP the emphasis is not on the computer but on the work on the tasks. Get the students to justify the observations they note.
• In the Final Synthesis/Discussion devise a task to check if the students have understood the topic.
• When preparing the lesson, question your steps: what am I aiming at with each task? How much do the students already know? What questions should I ask? Do the questions challenge the students? What answers should they give? And if someone give such-and-such an answer what should I do? How do I guide the students' work? And what about discussion time?

By bringing ready-constructed figures he removed the conceptual value from what is inherent to the constructions sought. It only makes sense to use computers if the strategy followed is focused on student activity.

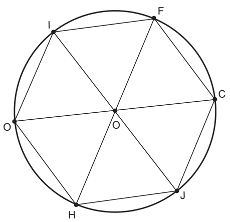
Based on my suggestions Fábio rethought his plan and sent a new one that stressed the task that he adapted for the Synthesis/Discussion time (see Figure 2).



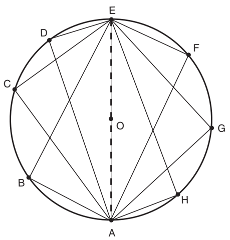
1. What's the relation between $A\hat{O}D$ and $C\hat{O}F$?
2. What's the relation between the arcs AD, BE and CF?
3. What is the relation between $A\hat{O}D$ and the arc CF?



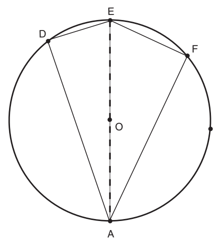
4. Suppose that $F\hat{G}C = 30^o$. Calculate $F\hat{O}C$ and the length of arc CF.



5. What can you say about chords FC and GI? And about arcs FC and GI?
6. Compare triangles FOC and GOI.



7. What is $E\hat{G}A$? Why?



8. Suppose that $D\hat{E}F = 115^o$. What can you say in relation to $D\hat{A}F$?

Figure 2. Presentation of Sketches and questions for the students to answer

Practice in the classroom. Fábio started the lesson by handing out a sheet with the tasks. He wanted the students to explore the first two tasks on the computer, working in pairs, and to establish the relations between the inscribed angles of a circle and the length of the corresponding arc (instructions to students are shown in Figure 3).

1. Show, in a circle, two inscribed angles with the same corresponding arc.
   1.1 Calculate these angles and the length of the corresponding arc.
   1.2 Drag the angles drawn. What do you see?

2 Show three angles inscribed in a semi-circle. Calculate each angle and the length of the corresponding arc. What do you see?

3 Draw in a circle two angles at the centre, of the same size.
   3.1 Draw the chords that connect the ends of the corresponding arcs of each of the angles at the centre. What do you see?
   3.2 What can you say about the triangles that you have formed?

Figure 3. Instructions to students for the inscribed angle in a circle task

After this exploration period Fábio gave the students the task he had devised for the Final Synthesis/Discussion, shown above:

Students: Angles and are angles at the centre. They're equal.

Fábio: What's the relation between AD, BE and CF?

(No-one answers) Luís said that they are the same size.
What about the arcs?

Student: They're equal.

Fábio: Why?

Student: Because the angles are equal and the lengths of the corresponding arcs are equal.

Students: AOD and arc CF have the same measure.

Student: Because arc CF is the arc corresponding to angle AOD and so it has the same measure.

Fábio: What is the measure of angle FOC, knowing that?

Student: It's .

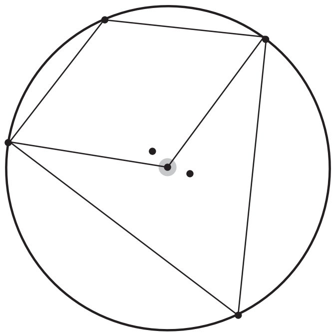
Fábio: Why?

Student: 30° x 2 = 60°..

Fábio: Why did you multiply by 2?

Student: Because the angle is at the centre and the other is inscribed and has the same corresponding arc.

As the students answered the questions without explaining their answers Fábio tried to ask them about their statements. Once the task was completed, he challenged the students to establish the relation between the opposite angles of a quadrilateral inscribed in a circle (see Figure 4).



Look at this figure:

What are the relations between angle ABC and angle ADC?

Figure 4. Fábio's challenge presented to students

Student: The inscribed angle is half the measure of the angle at the centre.

Fábio: Let's relate angles ADC and β. What has happened?
(Nobody answers) What angle is ADC?

Students: Inscribed.

Fábio: And what about angle β ? What is the relation between their arcs?

Students: They're equal.

Fábio: $\beta = 2A\hat{D}C$. How much is $\alpha + \beta$?

Students: 360º.

Fábio: Why?

Student: It's four right angles.

Fábio: But you know that $\alpha = 2A\hat{B}C$, and so…

Student: $2A\hat{B}C + 2A\hat{D}C = 360^o$.

Fábio: Do you see it?

Student: No.

Fábio: (Repeated his explanation). How can we simplify this expression?

Student: $2A\hat{B}C + 2A\hat{D}C = 360^o$ «» $2(A\hat{B}C + 2A\hat{D}C) = 360^o$
«» $A\hat{B}C + A\hat{D}C = 180^o$

Fábio: What do we call angles ABC and ADC?

Students: Supplementary. The sum of two angles inscribed in a circle is 180º.

Student: If we just moved one of the chords, would the angles still be opposed?

Fábio: Let's see. (Manipulates the GSP figure) What figure do you have?

Students: Ah, it only occurs with quadrilaterals.

Fábio guided the students' answers with questions, trying not to induce them to give the right answer and stimulating them to discuss their answers.

Reflection. In the interaction with the students. Fábio realised that "most of the students only did the constructions and did not answer the questions" ($e_{31}F \rightarrow S$), which did not give them the information about what to do to carry out the next task. When he saw that the best students took part more, he questioned his performance:

> Student participation fell short of my expectations, they hardly ever asked me anything. This makes me wonder whether they understood the content or not. ... I should get round this situation by asking students questions directly so that the more participatory ones don't answer. ($e_{31}F \rightarrow S$)

Comparing what he wanted to do with what actually happened, Fábio found alternatives that would broaden student participation. So it seems that stimulating the learners' participation does not come from one-off changes to the classroom environment but from systematic work throughout all the lessons.

Relations between the trigonometric ratios of an angle

Preparation of the lesson. Fábio thinks of using a problem that involves the leaning Tower of Pisa to teach the trigonometric ratios of an acute angle of a right-angled triangle ($e_{46}F \rightarrow S$) (see Table 5).

Table 5
Fábio's concerns about the lesson and possible teaching strategies

Concerns	Strategy
• Students having problems with the practical activities and the synthesis. • Preventing the better students from stopping the weaker ones from participating, I'll try to direct questions to individuals.	• To calculate the tilt of the Tower of Pisa. The answer will be verified in GSP. • To establish the basic trigonometric formula. • To present a Sketch to see if the formula works for other values.

The prospective teacher wanted the students to infer the trigonometric relations of an acute angle by completing a table with the values of the measure of 10º, 20º and 30º.

Supervisory interaction. To organise the information that enables the students to infer the trigonometric formulae I suggest to Fábio that he should insert the columns for the relations between the trigonometric ratios of the angles considered in a table: "To connect the parts of the lesson, put the inclination of the Tower of Pisa (5.2)º in a table. Create columns for the students to relate the trigonometric ratios of the angle they are considering" (e_{39}S→F).

Fábio emailed back (e_{47}F→S) to say that he was working on a task that would enable the students to apply the trigonometric formulae to solve problems (see Figure 5).

I'm sending a proposal that I'm still going to arrange a problem situation that will fit the diagram.

Find the measure of length AE, without calculating angle α

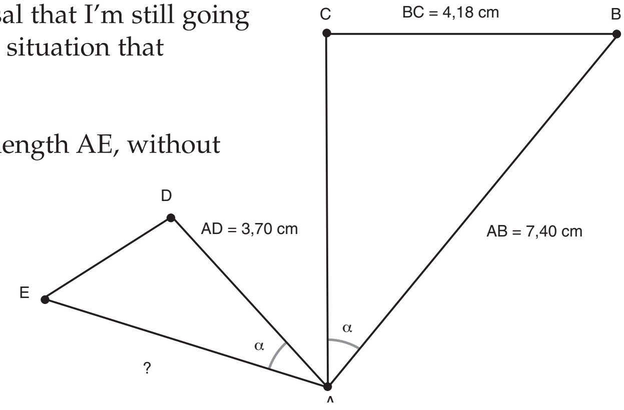


Figure 5. Fábio's task requiring the application of trigonometric formulae

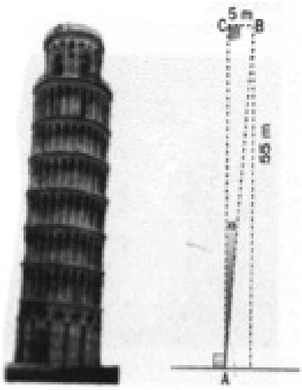
When I realised that he intended to adapt a figure to a real given context I told him that he could work in GSP with digital images, which helped him to include the following proposal in the final plan (see Figure 6).

Imagine you're in Santarém. You know the distance from Leiria to Castelo Branco (112 km), from Santarém to Castelo Branco (124 km) and from Santarém to Lisbon (63 km). Find the distance from Santarém to Trindade, without calculating angle α.

Figure 6. Suggested proposal from supervisor

Practice in the classroom. Fábio began the class by asking the students to work out the tilt of the Tower of Pisa, projecting Figure 7 in GSP.

A construção da famosa Torre de Pisa concluiu-se em 1284. Ao terminar verificou-se que a parte mais alta da torre se separava da vertical cerca de 90 cm. Actualmente, esta separação é de 5 m e a altura da torre é de 55 m. Qual o ângulo formado pela torre com a vertical?



[Translation: The famous Tower of Pisa was finished in 1284. It was then noticed that the top of the tower deviated from the vertical by around 90 cm. The deviation is currently 5 m and the tower is 55 m tall. What angle is formed by the tower and the vertical?]

Figure 7. Projected image of the Leaning Tower of Pisa

Fábio: Can you see how to solve the problem?

Student: We know that that side is 55m that one is 5m.

Fábio: What do we want to find?

Students: It's the tangent. The tangent of α is equal to the length of the opposite side over the length of the adjacent side.

Fábio: $\tan\alpha = {}^{5}/_{55}$ «» $\tan\alpha = 0.09$.

What we want to know is the angle whose tangent is 0.09. At the end of the book we have a table that helps us calculate the value of this angle.

Student: It gives 5.

Fábio: What do we have to do?

$\tan\alpha = 5_{55}$ «» $\tan\alpha = 0.09$ «» $\alpha = \tan^{-1}0.09$ «» $\alpha = 5.2°$

The students completed a table with the values for the sine, cosine, tangent, ratio of the sine to the cosine of the same angle and the sum of the squares of the sine and cosine of the same angle, for values of 5.2º, 10º, 20º and 30º.

Fábio: Find regularities.

Students: Sine divided by the cosine equals the tangent.

Fábio: What you're saying is tanα = . Do you all agree?

Students: Yes.

Fábio: Are there any more regularities?

Student: $\sin^2\alpha + \cos^2\alpha$ is always 1.

Fábio: What can we generalise from these results?

After some students derived the trigonometric formulae Fábio changed the measurements of the triangle and the size of the acute angle under consideration, so that they could find out if those formulae were valid for any acute angles. After working out two practical situations the class was ready to finish. Nevertheless Fábio suggested that the students examine the statement: "twice the sine of an angle is equal to the sine of twice the same angle". When examining the expression obtained for an angle of 10º, it reduced—contrary to what was expected—fromto . This equation was achieved by rounding off and the process of comparing numbers with decimal places deserved to be discussed with the students. Taking three decimal places, the thousandth place says that the equation derived is not valid since 2sin(10°) = 0.347 and sin(2x10°) = 0.342.

Reflection. Fábio questions how he examined the results obtained: "I used GSP to check the results to show the values one line at a time. I should have asked the students to say what values they had got and compared them with the right ones" ($e_{51}F\rightarrow S$).

When trying to use the practical activities to help the students to see the usefulness of the formulae derived, Fábio realised that this did not happen: "I thought at first that the applicability and usefulness of the formulae would be underscored by solving the practical activities. But some students asked me: What's the point of this?" ($e_{51}F\rightarrow S$)

In the discussion on the statement "Twice the sine of an angle is equal to the sine of twice the same angle", Fábio saw that the question did not prompt discussion among the students. When he analysed the guidance he gave for solving this task he saw the effect of ill-considered rounding-off, when transforming "2sin(10°) = sin(2x10°)" into the equation "2 x 0.17 = 0.34": "When analysing the situation I realised that this answer had a mistake" ($e_{51}F\rightarrow S$).

Tasks Proposed by Fábio in the Classroom

Fábio started his teaching practice with "exercises" as the basis (Interview E1). In the first period "exercises predominated because of that idea that the more that students solve things the more they learn" (Interview E2). Problems sometimes recurred in some lessons, such as the lesson on solving systems of two equations with two unknowns. By solving exercises and problems he showed, at the start of his practicum, a concept of teaching that emphasised the activity of the teacher.

During his teaching practice he used exploratory tasks for the students to "establish relations" ($e_{21}F\rightarrow S$) and "discover properties" ($e_{30}F\rightarrow S$). Examples are having the students study the angles of a circle, and exploring the relations between trigonometric ratios of the same acute angle. The exploratory nature of the tasks enabled the students to use dynamic geometric software, gather data, and establish relations. Fábio gradually changed the tasks he chose; instead of only working from the textbook he tried to involve the experiences and interests of the students, challenging them to think and encouraging mathematical communication: "For students to develop discursive skills we must first give them tasks that get them to think and infer. So the tasks should relate to the interests and experience of students and challenge them to think" (NA (*Norms* area of the forum), 17.1.04).

From the second period, Fábio explained the assumptions of some of the tasks that he took from the mathematics education literature when he said, for instance, "I'm complying with the geometry Norm (NCTM, 1991) as the students are finding relations ... constructing, measuring, comparing and transforming geometric figures" ($e_{30}F \rightarrow S$).

For Fábio, the nature of some tasks he brought to the classroom helped to promote interaction among the students. He specifically mentioned working with colleagues from other schools and with his supervisors, when choosing these tasks: "I began with more routine exercises, but during the year, through emails and the forum, I got suggestions from supervisors and colleagues that helped me to implement tasks of different kinds" (E2). He also mentioned the discussion forum about Norms, on topics that helped prepare tasks to encourage "student participation" (E2). The use of routine exercises gave way to more challenging tasks that were likely to get the students to discuss and explain their results.

Mathematics Communication in Fábio's Lessons

Fábio started his practicum by embarking on uni-directional communication in the belief that students played a "rather passive role in the teaching-learning process" (E1). He thought that students mostly gathered the information processed by the teacher. He realised that he did not always pay heed to the students' processes and that he took the initiative when solving tasks. In the lesson on "Solving systems of equations", for example, rather than exploring the solving strategies with the students, Fábio guided the solution using the steps he had prepared beforehand.

The experience gained from his teaching practice led him, after the first period, to favour more intervention by the students in the classroom activities: "The students should have the chance to explore and discuss their ideas" (NA, 22.2.04). He began to value student interaction as a way of learning with and from others and as a way of promoting mathematical communication: "There are sometimes discussions that I play no part in, when the students explain things to one another, clarifying their doubts" (NA, 7.3.04). He tried to foster contributive communication, motivating the students to discuss their results and answers, to explain their reasoning, and only intervened when necessary. But he saw that he did not always manage to control the students' discussion of ideas properly. He thought that this improved during the course of his teaching practice. For example, in the lesson on "Inscribed angle of a circle", when explaining some of the options he chose to handle the activities, Fábio saw that he should pay more heed to what students do and say.

Over the year, he stressed the role of the teacher through the questions he asked when promoting mathematical communication: "What questions, how they are asked, the right time for each one, the departure point to prompt the reasoning from the students and to organise their ideas" (NA, 3.1.04). Such questions demonstrate the teacher's ability in the way and when they ask them, either to mathematically challenge the students or to guide them. In this guidance he considered that the teacher should "organise the students' discourses so that they listen to one another's ideas so that they can refute them" (NA, 17.1.04). When involving the students in constructing the collective thinking of the class about what was being done and said, at certain points in the lesson he wanted to promote reflective communication.

To enhance the students' activity, Fábio mentioned the reading and discussion of the *Norms*, which helped him to clarify his view of the roles of the teacher and the students in the teaching and learning of mathematics: "At the start of the placement I focused mostly on the role of the teacher, and this changed as I talked to the supervisors and read the *Norms*" (E2).

Conclusion

The complexity of a teacher's professional knowledge means that certain elements of it can only be developed and become meaningful with experience. This knowledge includes the nature of the tasks and the forms of communication promoted in the classroom. Regarding the tasks, Fábio started out by mostly choosing exercises so that the students could apply their knowledge. Brown and Borko (1992) and Osana et al. (2006) consider that the unfamiliarity with open-ended tasks means that prospective teachers tend to use the same tasks that they worked on as students. As the teaching practice progressed, he used other types of tasks, such as exploratory ones. He aimed to use the problems as Ponte (2005) reports, to develop the students' reasoning ability. Fábio stressed the inductive aspect of the exploratory tasks by giving the students the chance to establish relations, properties, and definitions of the concepts addressed. As Ponte (2005) notes, this helped to develop his self-confidence.

The use of open-ended tasks was an important element for prospective teachers, both in teaching practice and in university classes (Brown & Borko, 1992). This support could lead to tools like those from this study, perhaps via Moodle, which might stimulate a culture of cooperation between prospective teacher in preparing their tasks, in sharing classroom experiences, and in discussing mathematics education texts on the role of tasks in student learning. The elements that helped Fábio's education to some extent correspond to what Boero, Dapueto, and Parenti (1996) call "collective learning", based on practice. The face-to-face and virtual interaction developed over the teaching practice motivated Fábio to use different kinds of task in the classroom. Tasks that are more closed and focused on the teacher's activity began to give way to more exploratory and open–ended tasks that encourage student participation (NCTM, 2007). After reflecting on his practice with respect to using these tasks, the prospective teacher developed his knowledge of the teaching process and broadened his perception of the effect of each kind of task on the students' activity and how he organised the lesson activities.

Regarding the forms of communication he used in the classroom, Fábio first employed uni-directional communication (Brendefur & Frykholm, 2000), and was more concerned with informing and explaining than giving the students time to solve the tasks set and paying attention to their activity. As the teaching practice progressed he implemented an increasingly contributive type of communication (Brendefur & Frykholm, 2000) by engaging the students in the lesson activities, and in the presentation and justification of their conclusions and processes. He listened more carefully to the students' answers and posed questions that would help them to define the concept in question (Nicol, 1999). But he did not present the answers and did not always utilize the ones given by the students, probably because he did not understand some of them.

Sometimes, when he incorporated the outcome of a discussion of the students' results and processes in his teaching, Fábio promoted reflective communication (Brendefur & Frykholm, 2000). He prompted the students to explain their answers, often asking "why"? "what do you have to do?" or "does everyone agree?". His lack of teaching experience and the value he placed on the teacher's authority in constructing mathematical knowledge were obstacles to involving the students in building the significances of the concepts addressed. The transformation of the students' various forms of thinking into a whole class thinking requires deep professional knowledge, which only comes with practice. Despite his inexperience, the prospective teacher was not afraid to take risks by involving the students in the classroom, trying to answer the questions they asked with other questions, or putting them to the class. Engaging the participation of students in the communication that is developed in the classroom is a didactic aspect that has attracted special attention from the mathematics education community. The reading and discussion of articles on discourse in the classroom and the observation of lessons by experienced teachers are strategies that prospective teachers can use to develop their ability to encourage different forms of classroom communication (Viseu & Ponte, 2012).

The changes Fábio made as he proceeded were based on the knowledge he acquired from systematising the approaches recommended in the school programme and the *Norms* (NCTM, 1991, 1994). They were also due to his reflections on his performance and the interactions with the participants in the teacher education setting, especially during lesson preparation. In these interactions, email and the forum helped with task preparation and discussion of the pros and cons of the structure, and level of challenge, of tasks undertaken in the classroom. Reading and discussion of the *Norms* in the forum, the supervisory interaction, and the probing nature of the forum and email messages helped to change how Fábio communicated in the classroom; he gave less emphasis to his interventions and started to pay more heed to those of the students.

The way Fábio followed his supervisors' suggestions to incorporate different types of task into his strategies and foster different forms of classroom communication show that those responsible for preservice teacher education must take care to monitor prospective teachers in their teaching practice year. This study suggests that the face-to-face occasions supplement electronic communication and enhance such monitoring. ICT offers prospective teachers systematic support and helps to banish the fear felt by many at the start of their career and to develop a teaching practice in keeping with current curricular guidance. Reflection and discussion about what he wanted to happen or about what happened in the classroom were, for Fábio, a "kaleidoscope" of experiences that generated the development of his pedagogical content knowledge.

References

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*, 389-407.

Bodzin, A. (2000). Preservice science teachers and internet telecommunications tools: Issues to consider. *Proceedings of the Annual Meeting of the Association for the Education of Teachers in Science*, Akron, OH, 610-634.

Boero, P., Dapueto, C., & Parenti, L. (1996). Didactics of mathematics and the professional knowledge of teachers. In A. J. Bishop, K. Clements, C. Keitel, J. Kilparick, & C. Laborde (Eds.), *International handbook of mathematics education* (pp. 1097-1122). Dordrecht: Kluwer.

Brown, C., & Borko, H. (1992). Becoming a mathematics teacher. In D. A. Grouws (Ed.), *Handbook of research in mathematics teaching and learning* (pp. 209-239). New York: Macmillan.

Brendefur, J., & Frykholm, J. (2000). Promoting mathematical communication in the classroom: two preservice teachers' conceptions and practices. *Journal of Mathematics Teacher Education, 3*, 125-153.

Caires, S. (2001). *Vivências e percepções do estágio no ensino superior*. Braga: Grupo de Missão para a Qualidade do Ensino Aprendizagem, Universidade do Minho.

Canavarro, A. P. (2003). *Práticas de ensino da Matemática: Duas professoras, dois currículos*. Tese de Doutoramento em Educação, Faculdade de Ciências da Universidade de Lisboa.

Carter, K. (1990). Teachers' knowledge and learning to teach. In W. Houston (Ed.), *Handbook of research on teacher education* (pp. 291-310). New York: MacMillan.

Cornu, R. L., & White, B. (2000). *Email supervision in the practicum: What do student teachers think?* Retrieved 12 July 2002 from http://www.atea.schools.net.au/ATEA/papers/lecornuetal.pdf.

Erickson, F. (1986). Qualitative methods in research on teaching. In M. Wittrock (Ed.), *Handbook of research on teaching* (pp. 119-161). New York, NY: Macmillan.

Hatton, N., & Smith, D. (1995). Reflection in teacher education: towards definition and implementation. *Teaching and Teacher Education, 11*(1), 33-49.

Jaworski, B., & Gellert, U. (2003). Educating new mathematics teachers: Integrating theory and practice, and the roles of practicing teachers. In A. J. Bishop, M. A. Clements, C. Keitel, J. Kilpatrick, & F. K. S. Leung (Eds.), *Second international handbook of mathematics education* (pp. 829-875). Dordrecht: Kluwer.

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.) (2001). *Adding it up: Helping children learn mathematics*. Washington, D. C.: National Academy Press.

Korthagen, F. (2001). A reflection on reflection. In F. Korthagen, J. Kessels, B. Koster, B. Lagerwerf, & T. Wubbels (Eds.), *Linking practice and theory: The pedagogy of realistic teacher education* (pp. 51-68). Mahwah, NJ: Lawrence Erlbaum.

McDuffie, A. R. (2004). Mathematics teaching as a deliberate practice: An investigation of elementary pre-service teachers' reflective thinking during student teaching. *Journal of Mathematics Teacher Education, 7*(1), 33-61.

Mewborn, D. S. (1999). Reflective thinking among preservice elementary mathematics teachers. *Journal for Research in Mathematics Education, 30*(3), 316-341.

Miles, M. B., & Huberman, A. M. (1994). *Qualitative data analysis: An expanded sourcebook*. Thousand Oaks: SAGE.

National Council of Teachers of Mathematics (NCTM) (1991). *Normas para o currículo e a avaliação da Matemática escolar*. Lisboa: APM e IIE.

National Council of Teachers of Mathematics (NCTM) (1994). *Normas profissionais para o ensino da Matemática*. Lisboa: APM e IIE.

National Council of Teachers of Mathematics (NCTM) (2007). *Princípios e normas para a matemática escolar*. Lisboa: APM.

Nicol, C. (1999). Learning to teach mathematics: Questioning, listening, and responding. *Educational Studies in Mathematics, 37*, 45-66.

Osana, H., Lacroix, G., Tucker, B. J., & Desrosiers, C. (2006). The role of content knowledge and problem features on preservice teachers' appraisal of elementary tasks. *Journal of Mathematics Teacher Education, 9*(4), 347-380.

Ponte, J. P. (2002). As TIC no início da escolaridade: Perspectivas para a formação inicial de professores. *Revista Portuguesa de Formação de Professores, 2*, 9-16.

Ponte, J. P. (2005). Gestão curricular em Matemática. In GTI (Ed.), *O professor e o desenvolvimento curricular* (pp. 11-34). Lisboa: APM.

Ponte, J. P., Oliveira, P., Varandas, J. M., Oliveira, H., & Fonseca, H. (2005). *Exploring the role of virtual interactions in pre-service mathematics teacher education*. Paper presented at the Congress of European Research in Mathematics Education (CERME) 4, Sant Feliu de Guíxols, Spain.

Pultorak, E. G. (1993). Facilitating reflective thought in novice teachers. *Journal of Teacher Education, 44*(4), 288-295.

Putman, R. T. & Borko, H. (2000). What do new views of knowledge and thinking have to say about research on teacher education? *Educational Researcher, 21*(1): 4-15.

Santos, L., & Ponte, J. P. (2002). A prática lectiva como actividade de resolução de problemas: Um estudo com três professoras do ensino secundário. *Quadrante, 11*(2), 29-54.

Shulman, L. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 3-14.

Smyth, J. (1989). Developing and sustaining critical reflection in teacher education. *Journal of Teacher Education, 30*(2), 2-9.

Souviney, R., & Saferstein, B. (1997). Email communication and clinical supervision: The Internet Project. *Journal of Computing in Teacher Education, 14*(1), 21-27.

Viseu, F., & Ponte, J. P. (2012). A formação do professor de Matemática, apoiada pelas TIC, no seu estágio pedagógico. *Bolema, 26*(42A), 329-357.

Wu, C.-C., & Lee, G. C. (2004). Use of computer-mediated communication in a teaching practicum course. *International Journal of Science and Mathematics Education, 2*(4), 511-528.

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