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Measuring Pre-service Primary Teachers' Knowledge for Teaching Mathematics

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This paper reports on the knowledge for teaching mathematics of 294 pre-service primary teachers from seven Australian universities participating in a project aimed at establishing a culture of evidence-based improvement of teacher education. The project was funded by the Australian Learning and Teaching Council. Rasch measurement techniques were used to validate and obtain performance measures on an overall Teacher Knowledge scale and three subscales (beliefs, content knowledge, and pedagogical content knowledge). The relative difficulties of items on each of the three subscales are discussed and differences between the participants' performances on each subscale and the overall scale according to level of education (prior to their pre-service teacher education course), previous mathematics study, course type, mode of study, and confidence to teach mathematics at the grade levels for which they were being prepared, are examined. The findings contribute to the establishment of an evidence-base for pre-service teacher education, and they also raise questions about the knowledge with which pre-service teachers leave teacher education, and current understandings of how important aspects of the knowledge teachers need can be measured.

Teacher education in Australia has come under increasing scrutiny due to the emergence of a national, rather than State-based, education policy environment. This new environment features a national assessment program in literacy and numeracy (NAPLAN) for school students in Years 3, 5, 7, and 9 and a national curriculum in mathematics, English, science, and history, from Foundation to Year 10. In addition, national professional standards for teachers and a national system for accreditation of pre-service teacher education programs (Australian Institute for Teaching and School Leadership [AITSL], 2011a; 2011b) have replaced State-based frameworks. The AITSL Program Standards stipulate that teacher education providers must be able to demonstrate, at the time of initial program accreditation, that graduates will meet the Graduate career stage of the national professional standards. These latter developments appear to be underpinned by assumptions about the importance of quality in pre-service teacher education programs and the contribution of such programs to teacher quality. However, until recently there existed little research on the opportunities to learn provided by teacher education programs and their relative effectiveness (Tatto, Lerman, & Novotna, 2010). This paper arises from a study that aimed to provide an evidence-base for effective models of pre-service mathematics teacher education. It reports the outcomes of pre-service teacher education programs in terms of the knowledge for teaching mathematics demonstrated by a sample of prospective primary school teachers in seven Australian universities.

Factors Affecting the Outcomes of Teacher Education

Internationally, there is increasing interest in the effectiveness of mathematics teacher preparation programs and the influence of program structure on prospective mathematics teachers' knowledge. Only a few years ago, Blömeke, Felbrich, Müller, Kaiser, and Lehmann (2008) noted that the few empirical studies that had been reported in this field tended to be small-scale, short-term, and conducted within the researchers' own institutions and with their own students. According to Blömeke et al., the main problem was that teacher education research "lacks a common theoretical basis, which prevents a convincing development of instruments and makes it difficult to connect the studies to each other" (p. 719). These authors tackled the theoretical problem by proposing a model for measuring effective teacher education in terms of the professional competence of future teachers. Professional competence was conceptualised as comprising professional knowledge, professional beliefs, and personal characteristics, with each of these components of competence acknowledged to have several inter-related dimensions. Blömeke et al. went on to propose factors that influence the development of professional competence, including individual characteristics of future teachers, institutional characteristics of teacher education, and the systemic features of teacher education within specific countries. The resulting model of professional competence and factors that influence its development provided the theoretical framework for an international comparative study of mathematics teacher education in six countries, "Mathematics Teaching for the Twenty-First Century (MT21)" (see Schmidt et al., 2008, for a report on findings).

Developments since Blömeke et al.'s (2008) review include probably the best known study of prospective teachers: the Teacher Education and Development Study in Mathematics (TEDS-M). TEDS-M was an international quantitative comparative study of the preparation and competencies of primary and secondary mathematics teachers carried out in 15 countries from 2006-2009, aimed at understanding how national policies and institutional practices affect the outcomes of mathematics teacher education. Data were collected about the characteristics of prospective teachers as well as outcome measures of their professional competencies. The conceptual model of teacher professional competencies was similar to that proposed by Blömeke et al. (2008) in that it recognised both cognitive abilities (professional knowledge) and affective-motivational characteristics (beliefs, professional motivation, self-regulation) as key criteria for measuring effective teacher education (see Döhrmann, Kaiser, & Blömeke, 2012, for a discussion of the conceptualisation of mathematics competencies in the TEDS-M study). Information was also gathered about future teachers' opportunities to learn, defined in terms of the content and teaching methods experienced during teacher education, and relationships between opportunities to learn and knowledge outcomes were examined. The significant role of teacher knowledge within conceptual models of teacher professional competencies makes it important to develop clear definitions and well-justified measures of the knowledge required for mathematics teaching.

Professional Knowledge Required for Teaching

It has long been recognised that teachers require more than knowledge of the subject matter for effective teaching. Shulman (1987) suggested seven categories of knowledge that might make up the knowledge base for the teaching profession: content knowledge, pedagogical knowledge, pedagogical content knowledge (PCK), curriculum knowledge, and knowledge of students, educational contexts, and the purposes of education. In mathematics education there has been much interest in the nature of PCK, which is concerned with the most useful ways of representing and formulating mathematics that make it comprehensible to others. Chick, Pham, and Baker (2006) used a framework developed from classroom observations to investigate the PCK of individual teachers. The framework describes aspects of PCK that are "clearly PCK" (e.g., knowledge of student ways of thinking about a mathematical concept), "content knowledge in a pedagogical context" (e.g., knowledge of the connections between mathematical topics), and "pedagogical knowledge in a content context" (e.g., knowledge of strategies for engaging students). The description of these categories highlights the interconnectedness of mathematics content knowledge (MCK) and PCK and the difficulty of distinguishing between them.

In an effort to tease out the relationship between PCK and content knowledge, Deborah Ball and her colleagues have introduced the notion of Mathematical Knowledge for Teaching (MKT) (Ball & Bass, 2000; Ball, Lubienski, & Mewborn, 2001; Hill, Ball, & Schilling, 2008). They argue that MKT has two strands, comprising subject matter knowledge and PCK. These researchers conceptualise PCK as made up of knowledge of content and students, knowledge of content and teaching, and knowledge of curriculum.

The TEDS-M study distinguished among MCK, mathematics pedagogical content knowledge (MPCK), and general pedagogical knowledge (Döhrmann et al., 2012), with only MCK and MPCK assessed in all countries that participated in TEDS-M. The MCK test categorised items according to the level of difficulty, ranging from novice (content typically taught at the grade level to be taught by the future teacher), to intermediate (one or two grades in advance of the grade level the future teacher will teach) and advanced (three or more years beyond the grade level the future teacher will teach). The MCK test was designed to align with the TIMSS assessment framework, and so it addressed the cognitive domains of knowing, applying, and reasoning together with (for future primary school teachers) the content domains of number, geometry, algebra, and data. Even so, it was a challenging task for the TEDS-M researchers to reach agreement about what constituted MCK for future teachers, since different mathematical content domains receive different emphases in countries around the world.

Conceptualising MPCK in the TEDS-M study proved to be still more difficult because of the different theoretical, educational, and cultural traditions of the participating countries. Nevertheless, two sub-domains of MPCK were agreed upon: curricular knowledge and knowledge of planning for mathematics teaching and learning; and knowledge of enacting mathematics for teaching and learning (Döhrmann et al., 2012). The first sub-domain refers to more than knowledge of the mathematics curriculum, and includes "the ability to identify the key ideas in learning programmes, seeing connections within the curriculum, establishing appropriate learning goals and knowing different assessment formats" (p. 329), as well as selecting teaching approaches and anticipating student responses. The second sub-domain refers to the ability to analyse and diagnose student thinking and to interpret and evaluate their mathematical solutions and arguments, as well as to explain concepts and guide classroom discourse.

Although much of the research on the knowledge required for effective teaching has concentrated on describing and categorising that knowledge, there is evidence that MCK and PCK work together in developing professional competence. For example, in a study conducted in Germany with a representative sample of Grade 10 classes and their mathematics teachers, Baumert et al. (2010) found that the level of teachers' mathematics PCK was a significant predictor of students' mathematical achievement whereas teachers' levels of MCK were not. However, these authors argued that both forms of knowledge are essential, since content knowledge forms the basis for development of pedagogically-oriented knowledge for teaching mathematics.

Professional Beliefs

The TEDS-M study framed teacher professional competencies in terms of both cognitive and affective characteristics, the latter including beliefs about mathematics and beliefs about the teaching and learning of mathematics (Döhrmann et al., 2012). In this study, beliefs about mathematics were held to be important for guiding the application of professional knowledge in practice. Beliefs of future primary teachers were investigated using an instrument comprising 12 Likert-style items that distinguished between a dynamic perspective (Mathematics as a process of enquiry) and a static perspective (Mathematics as a set of rules and procedures) (see Felbrich, Kaiser, & Schmotz, 2012). The cultural orientation of the participating countries—either individualistic or collectivistic—was also measured using a separately validated instrument. Although there were strong variations in belief-perspectives within countries, between-country differences were also apparent and these seemed to be related to the individualistic-collectivistic orientation of a country.

Rather than treating beliefs as a separate component of teachers' professional competencies, Beswick, Callingham, and Watson (2012) argued that beliefs should be included in any conception of teacher knowledge because these constructs are so closely intertwined in the context of practice. They developed a written profile to measure the beliefs, content knowledge, and PCK in mathematics of a sample of middle school teachers. A Rasch analysis demonstrated that this profile measured a single underlying construct, thus validating their holistic conception of knowledge for mathematics teaching.

How do Opportunities to Learn Influence Knowledge Outcomes in Teacher Education?

The TEDS-M study revealed great variation across countries and programs in opportunities to learn tertiary-level mathematics (Tatto & Senk, 2011). Knowledge outcomes also differed significantly between participating countries and between teacher education programs within countries (Blömeke, Suhl, Kaiser, & Döhrmann, 2012). For example, Singapore primary teachers participating in the TEDS-M study ranked first or second in MCK and MPCK, depending on pre-service program structure. However, it was not possible to establish connections between these strong knowledge outcomes and measures of opportunities to learn used in the TEDS-M study (self-reported opportunities to learn tertiary level mathematics, school level mathematics, and mathematics pedagogy within NIE's teacher education program) (Wong, Boey, Lim-Teo, & Dindyal, 2012). In contrast to the Singapore findings, the United States TEDS-M researchers found clear evidence of a relationship between opportunities to learn both MCK and MPCK (Schmidt, Houang, & Cogan, 2012). It may be that this relationship is more important in countries like the US that have highly decentralised education systems and where there is substantial within-country variation in the regulation of teacher preparation.

The study reported here took into account research summarised above in developing a survey for measuring pre-service primary school teachers' knowledge for mathematics teaching. The survey included items concerned with MCK, PCK, and mathematical beliefs as outcome measures. Previous research confirms that teacher knowledge, and especially PCK, is difficult to measure. Multiple choice items are effective for use with large samples, but, as Chick (2011) noted, responses are elicited out of context and they do not give detailed information about the reasons for the choices made. Lesson observations and interviews are better suited to a deeper examination of the different aspects of PCK as they are manifested in practice, but these methods are time consuming and less effective for larger samples of teachers. While acknowledging the limitations of surveys for investigating PCK, the present study makes a contribution to the evidence-base informing pre-service teacher education and suggests further questions concerning the operationalisation and measurement of knowledge for teaching.

The Study

The 2-year Australian Learning and Teaching Council (ALTC) funded project, Building the Culture of Evidence-based Practice in Teacher Preparation for Mathematics Teaching (CEMENT), involved seven universities representing all states and the Northern Territory and providing a diverse range of teacher education programs. The project's aims, as reported by Callingham et al. (2011, pp. 901-902), were to provide:

1. Evidence-based changes to mathematics education within the participating universities;

2. Recommendations about effective models of teacher education for teaching mathematics;
3. Processes for bringing about change at unit and course level; and
4. Progress towards a national culture of evidence-based practice in relation to mathematics teacher education.

The study adapted its conceptual framework from that used in the TEDS-M study (Tatto et al., 2008). The framework identifies three domains assumed to comprise factors affecting outcomes of teacher education programs. The first domain is concerned with the *characteristics of future teachers* and includes background information such as age, gender, location, qualifications, study experiences, attitudes to and beliefs about mathematics and mathematics learning. The second domain considers the characteristics of teacher educators, including classroom teaching experience, qualifications, and beliefs about mathematics, mathematics teaching and learning, and the needs and capacities of the pre-service teachers with whom they work. The third domain encompasses *characteristics of teacher education programs*—mode and level of delivery (e.g., internal, external, or mixed mode; undergraduate vs. postgraduate); the balance between subjects teaching mathematics content, mathematics pedagogy, and general pedagogy; and the duration and organisation of practicum sessions. The outcomes of teacher education programs are conceptualised as beginning teachers' beliefs and knowledge relevant to teaching mathematics.

The research reported here investigated different aspects of prospective primary school teachers' knowledge for teaching mathematics, and compared these outcome measures for groups of participants based on a range of future teacher characteristics and teacher education program characteristics. It provides starting points for the participating universities to examine their mathematics education practices and directions for further research into the relative effectiveness of a range of teacher education models and emphases in mathematics education. It thus contributes to aims 1, 2, and 4 of the project.

Instrument

An online multiple-choice survey was used to obtain data in a cost-effective, readily scored way. The 84-item instrument comprised 9 beliefs statements and 1 confidence item with which respondents indicated the extent of their agreement on 5-point Likert-type scales; 45 items concerned with MCK; and 29 designed to address PCK. The MCK and PCK item numbers include the multiple parts of several questions. Each pre-service teacher who undertook the survey was offered all of the beliefs and confidence items along with a randomly generated set of 10 MCK and 11 PCK questions, some of which included multiple parts making the numbers of items responded to greater than 10 and 11 respectively. This design enabled the approximate time to complete the survey to be kept to 45 minutes while allowing a greater number of items to be used and evaluated. The mean numbers of MCK and PCK items responded to by each participant were 22 and 19 respectively, with each MCK item answered by an average of 97 participants, and each PCK item answered, on average, by 119 participants. Qualtrics survey software (www.qualtrics.com) was used to deliver the survey to participants using a unique web-link.

The process of survey development, described in detail by Beswick and Callingham (2011a), was collaborative. It drew upon the extensive combined expertise of the nine mathematics educators involved in the project, several of whom had published work related to the nature and development of beliefs and knowledge for teaching mathematics (e.g., Beswick et al., 2012; Chick et al., 2006). The nine beliefs items comprised three each related to the nature of mathematics, mathematics teaching, and mathematics learning and were modified from existing sources (e.g., Thompson, 1984; Van Zoest, Jones, & Thornton, 1994).

The MCK items reflected content in each of the three content strands of the Australian Curriculum: Mathematics (Australian Curriculum Assessment and Reporting Authority [ACARA], 2012), namely Number and Algebra, Statistics and Probability, and Geometry and Measurement. The numbers of items designed to address each strand broadly reflected the amount of content in each strand in the curriculum and hence approximately half of the MCK items addressed Number and Algebra and one quarter covered each of the other two strands. Care was taken to include approximately even coverage of curriculum content from Foundation (the first year of schooling) to Year 7 (the first year of secondary school in most Australian states and territories).

The PCK items were intended to capture the following aspects of the construct: (1) analysing/anticipating/diagnosing student thinking, (2) constructing/choosing tasks/tools for teaching, (3) knowledge of representations, and (4) explaining mathematical concepts. These aspects of PCK are consistent with and further explained by the sub-categories of Chick et al.'s (2006) framework concerned with; student thinking and misconceptions, understanding the cognitive demands of tasks and the use of resources to support teaching, knowledge of using representations to model or illustrate a concept, and knowledge of strategies for teaching a concept. They also align with the TEDS-M sub-domains of curricular knowledge and knowledge of planning (aspects 2 and 3) and knowledge of enacting mathematics for teaching and learning (aspects 1 and 4) (Döhrmann et al., 2012). Typical items were presented in the context of a briefly described scenario with possible teacher responses from which participants could choose.

Examples of items are shown in Table 1. The MCK items and many of the PCK items were scored right or wrong. Some PCK items, however, did not lend themselves to dichotomous scoring. The PCK item shown in Table 1, for example, was scored A = 0, B = 2, C = 2, D = 1 because B and C were both seen as helpful responses in terms of developing understanding. D represented a sophisticated approach that could be meaningful for students with appropriate prior knowledge but judged less likely to be helpful than B or C.

Participants

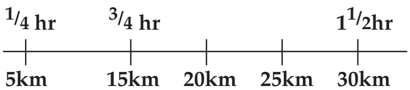
Cohorts of primary pre-service teachers in each of the seven participating universities were invited to complete the survey. Data were received from 294 pre-service teachers. Because the participants were volunteers and data about the relevant complete populations is not available, it is impossible to establish the extent to which the sample is representative of Australian pre-service teachers. The following details, however, provide a picture of the characteristics of the participants. More than three-quarters (n = 228, 77.6%) were full-time students and just under one-quarter (n = 66, 22.4%) were part-time. Approximately equal numbers were studying on-campus (n = 129, 43.9%) and off-campus (n = 131, 44.6%), with the remainder (n = 33, 11.2%) undertaking a mix of on- and off-campus study. Most were planning to graduate in 2011 (n = 80, 27.2%) or 2012 (n = 79, 26.9%). A further 37% were aiming to graduate in 2013 (n = 58, 19.7%) or 2014 (n = 51, 17.3%). Smaller numbers were aiming for graduation in 2015 (n = 13, 4.4%) or later (n = 12, 4.1%). Most (n = 132, 44.9%) were enrolled in a Bachelor of Education (BEd) (a 4-year course), one third (n = 98, 33.3%) in a combined degree program (at least 4 years), 46 (15.6%) were undertaking a Master of Teaching (MTeach) degree (2 years), and 17 (5.8%) were engaged in a Diploma of Education (DipEd) (1 year).

The highest educational level prior to enrolment in their current course was secondary for 133 (45.2%) respondents. Sixty-five respondents (22.1%) held a bachelors degree and 39 (13.3%) and 33 (11.2%) respectively held advanced diplomas or certificates. Twelve (4.1%) had a post-graduate degree and a further 10 (3.4%) a graduate diploma. One reported primary education as his/her highest prior educational level.

Because of the range of educational jurisdictions in which the universities were located and the consequent variety of subject names, Year 11 and 12 mathematics study options were worded as follows: Year 11/12 Mathematics or statistics subject that didn't count towards university entrance; Year 11/12 general mathematics or statistics course that counted for university entrance; and Year 11/12 specialist mathematics or statistics intended for mathematics study at university. For brevity these options are referred to in this paper as non-pre-tertiary, pre-tertiary, and specialist. Two thirds (n = 196, 66.6%) of the participants described the highest level of mathematics or statistics studied prior to their current course as Year 12. Of these the majority (n = 140, 47.6% of the total number of participants) had studied pre-tertiary mathematics. Forty-three (14.6%) had studied non-pre-tertiary mathematics and 13 (4.4%) had studied specialist mathematics. Forty-four (15.0%) reported their highest level of mathematics as Year 10 and a further 40 (13.6%) had studied some mathematics or statistics as part of a bachelors degree. Two (0.7%) reported studying mathematics at a Technical and Further Education (TAFE) college and 12 (4.1%) indicated that they could not remember the highest level of mathematics or statistics studied. None had studied mathematics at post-graduate level.

The responses to the confidence item (shown in Table 1) were used as an additional background variable. Most participants indicated that they were fairly confident (n = 159, 56.2%) to teach at the grade levels that they would be qualified to teach, with a further 38 (13.4%) describing themselves as completely confident. Most of the remainder (n = 58, 20.5%) indicated they were "A little confident" and 14 (4.9%) chose each of "Don't know", and "Not at all confident".

Table 1
Examples of items

<table>
<tr><td>Beliefs</td><td>Periods of uncertainty and confusion are important for mathematics learning.
Strongly Disagree Disagree Neither Agree nor Disagree Agree Strongly Agree</td></tr>
<tr><td>Confidence</td><td>Please rate your confidence to teach mathematics at the grade levels that you will be qualified to teach on the following scale.
Not at all confident A little confident Don't know Fairly confident Completely confident</td></tr>
<tr><td>Mathematics Content Knowledge (MCK)</td><td>Steve buys a shirt that is discounted by 10% on the ticket. A sign on the rack stated, 'Discounted by a further 15%'. This is the same as a discount of what percentage off the original price of the shirt?
A) 12.5% B)15% C) 17.5%
D) 23.5% E) 25%</td></tr>
<tr><td>Pedagogical Content Knowledge (PCK)</td><td>A teacher sets the following proportional reasoning task for an upper primary class:
Bill and Ben were out on a Sunday morning bike ride. After three quarters of an hour they passed a sign that showed they had ridden 15 kilometres since they left home and that they still had 25 kilometres to reach their destination. How long will it take them to get there?
Which of the following representations is most helpful for the teacher to develop the students' understanding of proportional reasoning in solving this problem?
A) Cross multiplying
<table>
<tr><th>Time (hr)</th><th>Distance (km)</th></tr>
<tr><td>$^3/_4$</td><td>15</td></tr>
<tr><td>χ</td><td>25</td></tr>
</table>
C) Ratio table
<table>
<tr><td>Time</td><td>$^3/_4$ hr</td><td>$^1/_4$ hr</td><td>1hr</td><td></td></tr>
<tr><td>Distance</td><td>15km</td><td>5km</td><td>1km</td><td>**25m**</td></tr>
</table>
B) Double line number


D) Find the unit rate:
Riding 15 km in $^3/_4$ hr is equivalent to riding 1 km in $^3/_4 \div 15$ hr.</td></tr>
</table>

Data Analysis

Rasch measurement models use the interaction between persons and items to position each item and person against an underlying construct on the same genuine interval scale. The scale units are logits, the logarithm of the odds of success (Bond & Fox, 2007). Because the items that comprised the instrument used in this study had differing structures the specific model used was Masters' (1982) Partial Credit Model. Rasch models are based on three assumptions. These are, (1) that the items work together to measure an underlying construct, (2) the underlying construct is measureable such that higher item scores are indicative of more of that construct, and (3) that items contributing to the construct are independent of one another (Bond & Fox, 2007). The initial step in using Rasch measurement is, therefore, to establish the extent to which these assumptions hold.

To this end, Rasch analyses of all 84 items and then each of the three sets of items (Beliefs, MCK, and PCK) were conducted using Winsteps (Linacre, 2011) with a view to establishing scales in relation to which relative item difficulties and person performance measures could be calculated. In each case infit mean square values for both items (INMSQI) and persons (INMSQP), available from the Winsteps output, were used to evaluate the fit to the Rasch model (Bond & Fox, 2007). Fit values between 0.77 and 1.3 logits are generally acceptable with 1.0 representing the ideal value (Keeves & Alagumalai, 1999). Winsteps also provides person and item reliability indices and separation statistics. Low person separation (<2) and person reliability less than 0.8 indicate uncertainty about the ordering of the ability measures of the respondents (Linacre, 2011). Similarly, low item separation (<3) and item reliability less than 0.9 indicate uncertainty about the ordering of the items in relation to their difficulty (Bond & Fox, 2007). Person reliability is enhanced by lengthening the test (i.e. having participants respond to more items) and increasing the range of abilities represented by the respondents. Item reliability can be improved by increasing the number of respondents (Linacre, 2011). Measures of the performance of each pre-service teacher were obtained for each of the overall survey and the Beliefs, MCK, and PCK subscales. These measures were used to compare groups based on responses to the confidence item, course type (e.g., BEd, MTeach), university, full or part time enrolment, mode of study, level of mathematics studied, prior educational level, and anticipated graduation year, using t-tests and analyses of variance.

Results and Discussion

Eighty of the 84 items formed a scale with satisfactory fit values for both items and persons ($INMSQ_I$ = 0.93, $INMSQ_P$ = 0.97), suggesting that these items provided a measure of an underlying construct that we called Teacher Knowledge. The Cronbach α reliability statistic was 0.89. Person separation (0.75) and person reliability (0.36) were low, whereas item separation (12.57) and item reliability were high (0.99). This means that the precise person ability ordering is uncertain. Nevertheless, the acceptable Cronbach α suggests that the survey data provide a sound basis for looking for between group differences in Teacher Knowledge. Because the focus of the study was on providing evidence about the effectiveness of pre-service teacher education, the item reliability and the consequent confidence in the order of item difficulty, is of greatest importance. The nine beliefs items showed excellent fit to the Rasch model ($INMSQ_I = 0.99$, $INMSQ_P = 1.01$) and acceptable reliability ($\alpha = 0.79$). Again, the person separation (0.60) and person reliability (0.27) were low but the item separation (13.47) and reliability (0.99) were high. All but one of the 45 MCK items comprised a scale that fitted to the Rasch model ($INMSQ_I = 0.99$, $INMSQ_P = 0.97$). Cronbach α was 0.65. As for the overall Teacher Knowledge scale and Beliefs scale, the person separation (1.06) and person reliability (0.58) were low, whereas the item separation (3.54) and item reliability (0.93) were high. Finally, 28 of the 29 PCK items also comprised a scale with excellent fit to the model ($INMSQ_I = 0.99$, $INMSQ_P = 1.00$) and Cronbach α of 0.58. The person separation (0.43) and person reliability (0.15) were low although the item separation (9.06) and item reliability (0.99) were high. The lower Cronbach α and person reliability measures for MCK and PCK scales reflect the high apparent levels of missing data resulting from the random assignment of subsets of each of these item types to survey respondents and the effective shortening of these parts of the survey that this entailed. In this context, the reliability statistics are acceptable.

Relative Difficulty of the Different Scales

Figure 1 shows box plots of the distribution of pre-service teachers' performance on each of the three scales, Beliefs, MCK and PCK. As was the case for pilot data reported by Callingham et al. (2011), the median performance declined across the scales. However, in contrast to the pilot data, performance on the Beliefs scale was lower and there is greater overlap between the performance of the pre-service teachers in relation to MCK and Beliefs, and between MCK and PCK. Nevertheless, pre-service teachers found endorsing belief statements and, to a lesser extent, correctly answering mathematical content questions less difficult than responding appropriately to the PCK items. Items from each scale that proved most and least difficult are discussed in the following sections.

Beliefs. The easiest beliefs statement to endorse (that is, the statement with which participants were most likely to agree) both related to mathematics teaching. They were; "Teachers must be able to represent mathematical ideas in a variety of ways" and "The teacher must be receptive to the students' suggestions and ideas". The most difficult statements to endorse were; "Acknowledging multiple ways of mathematical thinking may confuse children" and "The procedures and methods used in mathematics guarantee right answers". Apparently the pre-service teachers' openness to multiple representations and student contributions was accompanied by resistance to the notion that there are "right" answers in mathematics. Beswick and Callingham (2011b) reported a similar result and referred to Brown, McNamara, Hanley, and Jones' (1999) finding that pre-service teachers are influenced by their own experiences of learning mathematics that commonly include fear of getting incorrect answers, to explain a similar finding. They speculated that mathematics teacher education helped reduce pre-service teachers' fear of failure, and that the associated emotional relief contributed to the attractiveness of statements that suggest multiple acceptable possibilities even if the implications had not been thought through.

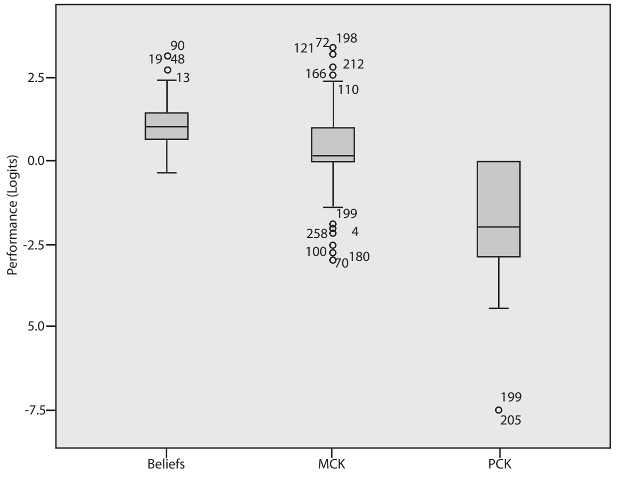


Figure 1. Distribution of pre-service teachers' performance measures on three subscales with outliers shown

Two items relating to the nature of mathematics, "Mathematics is a beautiful and creative human endeavour" and "Mathematical ideas exist independently of human ability to discover them", were the next most difficult to endorse, and equally so. Again this mirrors the finding of Beswick and Callingham (2011b) and suggests that the nature of mathematics may receive little attention in teacher education courses.

Mathematical content knowledge (MCK). The easiest items involved identifying pictures of a square, rectangle, non-rectangular parallelogram, and a non-square rhombus as parallelograms. In each case more than 96% of respondents answered correctly. The most difficult items included one requiring respondents to decide whether a parallelogram "has diagonals that cross at right angles when adjacent angles are different" is always, sometimes or never true (25.0% correct). The easy items involved shape recognition without reference to properties and thus required geometric thinking at Van Hiele's Visual level at which the appearance, rather than properties, of shapes are the basis on which they are recognised (Siemon et al., 2011). The most difficult geometry item, however, required abstract/relational geometric thinking (Siemon et al., 2011) in order to analyse the relationship between properties of a parallelogram.

Two items related to generalisations expressed algebraically were also among the five most difficult. These involved deciding whether a - b = b - a always, sometimes, or never (25.3% answered correctly); and whether $a^2 < a$, is always, sometimes, or never true (28.9% correct). Algebra requires a greater level of abstract thinking than does arithmetic (Malisani & Spagnolo, 2009) and hence the difficulty of these items is not surprising. Another item among the five most difficult required respondents to decide whether the product of an odd and an even number is sometimes, always, or never odd (30.9% correct). The need to know the meaning of "product" may account for many of the incorrect responses. The inclusion in this group of an item that asked pre-service teachers to determine the length of a line marked above a broken ruler with zero not aligned with the end of the line (25.5% correct) is more surprising.

Overall the results for MCK suggest that although teacher education needs to attend to sophisticated concepts involving abstract thinking, the weak mathematics knowledge of entering pre-service primary teachers (Mays, 2005) means that simple knowledge and skills, such as the correct use of a ruler, cannot be taken for granted. We are also aware that subtle variations in wording can influence the interpretation of items. For example, in the MCK item shown in Table 1 the percentage discount asked for could be interpreted as just the 15% of the 90% of the original price (and hence 13.5% of the original price) rather than the total discount off the original price (and hence 23.5%). Since 13.5% was not an option respondents who arrived at that conclusion may have been prompted to realise that 23.5% was the desired response but it is possible that some of these respondents were disadvantaged by the omission of 13.5% as an option or wording that clearly indicated that it was the total discount that was meant.

Pedagogical content knowledge (PCK). The easiest PCK item was shown in Table 1 and involved proportional reasoning. Most (84.9%) pre-service teachers responded with one or other of the responses that were scored 2. Only 4.4% chose the response scored 0. The transition from purely additive thinking to being able to reason proportionally when appropriate, is recognised as a key milestone in students' mathematical development (Sowder et al., 1998). This recognition may have led to the teaching of proportional reasoning receiving relatively more attention than other topics in teacher education programs. Alternatively, the result could be an artifact of the scoring of the options provided: only one was scored '0' and this may have been readily recognised as a meaningless procedure based on pre-service teachers' recollections of learning mathematics. The next easiest item involved a teacher's response to a scenario in which student calculators yielded differing answers to a calculation involving addition and multiplication. In this case 58.5% of responses scored 2 and a further 39% were scored 1.

In contrast, just 12.2% of the pre-service teachers provided an appropriate response to a student's representation of the sum of $^1/_4$ and $^1/_4$ that suggested a total of $^2/_8$. This was the most difficult item. Chick (2011) described the controversy this item generated among the researchers to illustrate the inherent subjective judgements involved in devising PCK items: certain approaches are privileged above others when, in fact, more than one teaching response could be seen as optimal depending upon the precise circumstances. The difficulty of efficiently measuring PCK, particularly aspects of it that depend upon teaching responses to the minute-by-minute unfolding of classroom events, seems unlikely to be easily overcome.

Between Groups Analyses

Comparisons of performance on the Teacher Knowledge scale and each of Beliefs, MCK and PCK were made between groups identified by responses to the confidence item, course type, university, full or part- time enrolment, mode of study, level of mathematics studied, prior educational level, and anticipated graduation year. Analyses of variance and t-tests were used to identify significant differences. No differences for any of the scales were found in relation to university, full or part-time enrolment, or anticipated graduation year. In addition, none of the groups considered differed in relation to either Beliefs or PCK. The significant differences that were found are described in relation to each background variable in the following sections.

Differences according to confidence level. For overall Teacher Knowledge there were differences in performance between pre-service teachers who were "Not at all confident" (M = -6.65) or "A little confident" (M = -6.48), and those who were "Fairly confident" (M = -6.12), or "Completely confident", (M = -5.93), $F(4, 278) = 10.84$, $p < .01$. Those who chose "Don't know" (M = -6.40) to describe their level of confidence performed less well than those who were "Completely confident" ($p < .05$). The associations between confidence and Teacher Knowledge may reflect the pre-service teachers' awareness of either the extent of their mathematical or other knowledge or at least of their capacity to learn about sophisticated ideas.

Pre-service teachers who were "Not at all confident" (M = -0.15) performed less well in terms of MCK than those who were "Fairly confident" (M = 0.60), or "Completely confident", (M = 0.64), $F(4, 278) = 4.54$, $p < .05$. Those who were "A little confident" (M = 0.14) demonstrated less MCK than those who were "Fairly confident", $p < .05$. As for Teacher Knowledge, awareness of one's level of MCK is likely to affect one's confidence to teach mathematics and so the association between MCK and confidence is unsurprising.

Differences according to course type. On Teacher Knowledge those enrolled in a Diploma of Education (M = -5.78) performed better than those enrolled in either a combined degree (M = -6.25) or BEd (M = -6.24), $F(3, 289) = 4.09$, $p < .01$. Differences were found between the same groups for MCK, with DipEd pre-service teachers (M = 1.06) demonstrating greater MCK than those undertaking a combined degree (M = 0.30), $F(3, 289) = 3.99$, p < 0.01, or a BEd (M = 0.42), $p < .05$. These differences, and particularly those for in MCK, are not easy to explain because the sample comprised primary pre-service teachers: those studying for a DipEd would not necessarily have studied mathematics or statistics as part of a prior degree.

Differences according to mode of study. Differences related only to MCK with pre-service teachers studying in mixed mode (M = 0.04) performing less well than those studying off-campus (M = 0.55), $F(2, 290) = 4.13$, $p < .05$. There is no apparent explanation for this association, however, because relatively few of the participants were studying in mixed mode (n = 33, 11.2%) and so the result warrants caution. Possibly of greater significance is the lack of difference between on- and off-campus study modes.

Differences according to highest level of mathematics or statistics completed. Pre-service teachers who had studied mathematics or statistics as part of a bachelor degree (M = -5.97) had greater Teacher Knowledge than those whose highest mathematics level was Year 12 non-pre-tertiary (M = -6.38), $F(6, 287) = 3.05$, $p < .05$.

In terms of MCK, pre-service teachers who reported their highest mathematics level as Year 10 (M = 0.14) did less well than those who had completed Year 12 specialist mathematics (M = 1.06), mathematics at TAFE (M = 2.16), or bachelor degree level mathematics (M = 0.79), $F(6, 287) = 5.74$, $p < .05$. Completion of non-pre-tertiary mathematics at Year 12 (M = 0.10) was associated with lower MCK performance than Year 12 specialist mathematics (M = 1.06), mathematics at TAFE (M = 2.16), $F(6, 287) = 5.74$, $p < .05$, or bachelor degree level mathematics (M = 0.79), $F(6, 287) = 5.74$, $p < .01$. Unsurprisingly, greater MCK was associated with having studied more and higher level mathematics.

Differences according to highest prior education level. Prior educational level impacted on Teacher Knowledge performance, with those who had completed a bachelor degree (M = -6.03) performing better than those whose highest educational level was secondary school (M = -6.28), $F(5, 286) = 2.25$, $p < .01$.

Differences were also found for MCK, with pre-service teachers with bachelor (M = 0.69) or post-graduate degrees (M = 1.01) demonstrating greater MCK than those whose highest educational level was either certificate (M = 0.26) or secondary level (M = 0.35), $F(5, 286) = 2.49$, $p < .05$. Again, because the participants were primary pre-service teachers it cannot be assumed that the additional study included mathematics.

Factors Not Associated with Knowledge Differences

None of the factors of university, full-or part-time study, anticipated graduation year, or on- and off-campus study modes was associated with differences in pre-service teacher performance on any of the scales. The only one of these that might be expected to impact teachers' knowledge would be anticipated graduation year, with those nearer to graduation having more knowledge than those further from the end of their course. This would amount to greater opportunity to learn which was associated with greater MCK and PCK for US, but not Singaporean, pre-service teachers in the TEDS-M study (Schmidt et al., 2012; Wong et al., 2012). The courses studied by participants in this study varied in length from 1 to at least 4 years. A pre-service teacher planning to graduate in the next year, although near the end of his/her course, could be in the first, second, or fourth year of the program, with consequent implications for the opportunities to learn that they had experienced. The inclusion of part-time students in the sample further complicates the interpretation of anticipated graduation year. To the extent that anticipated graduation year is a valid measure of the amount of mathematics education received, the apparent lack of impact on knowledge is of concern. More detailed information than was provided by the survey used in this study—about the nature, and placement within courses of mathematics content and mathematics curriculum units, along with details of by whom and how they are delivered—is needed to unpack whether, and to what extent, pre-service teacher mathematics education influences teacher knowledge.

Scales for which There Were No Between-Group Differences

There were no between-group differences for either Beliefs or PCK in spite of the diversity of the participants' backgrounds and of the courses and institutions in which they were enrolled. As discussed in the previous section, more detailed information about teacher education programs that provides a more nuanced view of pre-service teachers' experiences than anticipated graduation year are needed to better evaluate the impact of teacher education on these aspects.

The difficulty of changing beliefs has been well established (e.g., Ambrose, 2004; Lerman, 1997). Nevertheless, an aim of at least some teacher education programs is to influence the beliefs of pre-service teachers towards those considered helpful in terms of underpinning student-centred teaching practice. Several studies have reported success (e.g., Ambrose, 2004; Beswick, 2006) but most, including Ambrose (2004) and Beswick (2006), have been accompanied by scepticism about the longevity of apparent change. The results of this study show no impact of time in teacher education courses, as measured (albeit imperfectly) by anticipated graduation year, on pre-service teachers' beliefs. One possible explanation of this result is that the pre-service teachers had not reflected on the nature of mathematics in order to construct beliefs about it that could provide a coherent basis for their views about teaching and learning the subject. Although happy to adopt the rhetoric of student-centred teaching, they may not, as suggested by Beswick and Callingham (2011b), have gone beyond the emotional attractiveness of such statements to consider their implications.

The absence of an association between performance on PCK items and confidence, even though there was an association between confidence and MCK, suggests that pre-service teachers may be aware of, and concerned about, the need to know mathematics content but less aware of the extent of their PCK or of its importance. It could also reflect greater familiarity with mathematics content tests compared to tests of their PCK.

PCK remains poorly understood and difficult to operationalise and measure. These facts underpinned the difficulty of devising items to measure it that were of an appropriate level of difficulty. In contrast to MCK, there is little agreement or confidence about measures of PCK (Chick, 2011). These difficulties, along with the facts that pre-service teachers are unlikely to have considered the concept of PCK prior to their teacher education course, and that it may or may not be explicitly discussed even in that context, explain why pre-service teachers could be expected to be less aware of their ability in terms of PCK than in relation to MCK. Although PCK has been a topic of interest to mathematics educators since Shulman (1987) introduced it, much of the work in the area to date has focussed on conceptualising the construct (e.g., Chick et al., 2006; Hill et al., 2008) as a necessary pre-cursor to measuring it, and this work is far from complete. The items developed and used in this study contribute to this work and the findings emphasise the importance of continued research on PCK—its meaning, measurement, and development.

The lack of any relation to PCK according to either prior education level or highest mathematics or statistics studied is consistent with PCK being uniquely addressed by education courses such that differences in entering knowledge and experience are immaterial. Nevertheless, Baumert et al.'s (2010) reminder of the importance of MCK as the basis of PCK suggests that MCK is also an important concern of teacher education. In addition, the data suggest that neither how nor where (mode, full or part time, course type, university) education study is undertaken make a difference to the level of PCK acquired. The findings suggest that efforts to improve the quality of graduating teachers in relation to the kind of knowledge that is central and unique to the act of teaching (as well as most connected with student outcomes as asserted by Baumert et al.) should not be overly-focussed on input variables or on gross structural aspects of courses and study modes. Rather, research addressing, in detail, the ways in which PCK manifests in teaching; the thinking, skills, and mathematics knowledge that underpin it; and how its development among pre-service teachers can be enhanced is very much needed.

Conclusion

This study is timely in relation to national agendas around school curricula (ACARA, 2012), and national teacher and teacher education program standards (AITSL, 2011a; 2011b). The survey results reported provide the beginnings of an evidence base in Australia for the ongoing improvement of the quality of mathematics teacher education and hence of graduates.

Of particular relevance are the relatively few associations between the background variables and aspects of teacher knowledge that were measured, with only MCK and the overall measure, Teacher Knowledge, differing according to any. Unsurprisingly, the more mathematical or general education that the pre-service teachers had experienced—essentially opportunities to learn (Tatto & Senk, 2011)—the better their performance on both measures. Equally unsurprisingly, greater confidence to teach mathematics at the level they would be qualified to teach it was also associated with more MCK and Teacher Knowledge. PCK performance, which, in contrast to MCK, has been shown to predict student outcomes, showed no difference in relation to any of the grouping variables. There were, similarly, no differences in relation to beliefs. Whereas influencing beliefs is recognised as difficult (Ambrose, 2004) there is an expectation that PCK can be learned. Nevertheless, both constructs may not be well operationalised by multiple choice survey items such as used in this study

(Chick, 2011). The need to test theories using large-scale data collection, however, means that the effort to develop and validate readily scored survey instruments cannot be abandoned. The relative difficulty of the PCK items and ease of the beliefs items for the participants in this study suggest a need to devise PCK and beliefs items that better match the ability of primary pre-service teachers; that is, easier PCK items and more difficult beliefs items. The difficulty of agreeing on the scoring of PCK items (Chick, 2011) is consistent with Beswick et al.'s (2012) conception of teacher knowledge as including beliefs in that it highlights the role that the researchers' beliefs played in judging the relative merits of teaching responses in terms of the PCK that they evidenced. Items that present scenarios that include more of the detail and complexity of the contexts in which PCK is enacted and which offer options that also imply particular pedagogic stances, or beliefs about mathematics teaching and learning, may be a way forward.

The absence of associations of any of the knowledge measures with the structural aspects of teacher education programs or even with anticipated graduation year could be an artefact of the measures or of the lack of precision of anticipated graduation year as a proxy for the opportunity to learn that respondents had experienced. Interview data from the larger study may provide important insights into validity of the measures and the subtleties of the impacts of teacher education programs. Whether or not this proves to be the case, the current policy environment makes evidence-based teacher education a priority.

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