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Schema Construction among Pre-service Teachers and the Use of IT in Mathematics Teaching: A Case Study

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Recent developments about cognitions underlying mathematical learning are beginning to suggest that the activation and appropriate use of prior knowledge by students is, to a large measure, controlled by the quality of organisation of that knowledge. Thus, teaching needs to support the construction of well-connected mathematical knowledge. An important assumption here is that teachers need to construct a repertoire of subject-matter knowledge that is rich and well connected before they can help their students build similar mathematical knowledge. Thus, mathematics knowledge building is an important issue in teacher preparation programs. This paper reports on a study about the knowledge state of a pre-service teacher who planned to use computers in the teaching of linear functions. The results of the study indicate the existence of gaps in the student teacher's subject-matter knowledge. Significantly, there was also a lack of important connections between his understanding of linear functions and the instructional use of a computer software. Knowledge gaps and implications for classroom students' acquisition of mathematical schemas and mathematics teacher education programs are examined and discussed.

There is an emerging consensus that students should be given sufficient space in acquiring new knowledge and exploiting this knowledge in performing various mathematical tasks both in and outside the classroom. This view about the role of students in the learning of the contents and processes of mathematics has been well articulated in major reform documents which have developed standards for mathematical understanding and the effective use of technology in fostering that understanding (National Council of Teachers of Mathematics, 1989, 2000).

The renewed interest in knowledge-related aspects of mathematical performance has prompted mathematics educators and teachers to invest considerable effort in helping students develop a better grasp of the subject-matter and, thereby, promote deeper levels of conceptual understanding and an appreciation of the power of mathematics. Concurrent developments in the area of cognitive psychology and domain expertise have had significant effects on our understanding of why and how deeper levels of processing of mathematical information by the teacher and the student are necessary for optimal levels of mathematical performance. However, little effort has gone into utilising this knowledge about teachers' conceptual understanding in our examination of how teachers' plans and actions impact upon student learning. Specifically, there is little data on the question of the relationship between pre-service teachers' understanding of mathematical concepts and how this would affect their instructional use of computers in the classroom.

Schemas and Mathematics Activity

Cognitive analysis of mathematical performance involves the description of the quality of the knowledge that helps students make progress in problem-solving and other mathematical tasks. Research evidence shows that students who have developed a well organised mathematical knowledge base not only show deeper levels of understanding of mathematics concepts, but also are able to retain this knowledge for a longer period, and access it when it is required (Cooper & Sweller, 1987).

A central concept to emerge from this stream of research is the notion of schema. A schema refers to the network of mathematical and related knowledge that is built around a core concept (Mayer, 1992). Defined in this manner, a schema may consists of other concepts that are linked to the core concept as well as information about procedures that are appropriate when students are required to work with these concepts. For example, students could build a schema around the concept of ratios. Such a schema could include concepts that are linked to ratios such as numbers, fractions and percentages. Further, when a student has to solve problems involving ratios and related concepts he or she will need to use procedures for the transformation of equations, simplification of fractions, algebraic manipulation or drawing of a diagram to visualise some aspect of the ratio problem. This latter set of information that is not directly related to ratios but nevertheless is required when solving ratio problems is also built into a ratio schema. As students' expertise increases, the quality of the ratio schema can be expected to become more complex and powerful. These types of schemas have been argued to play a key role in helping students categorise and solve problems (Owen & Sweller, 1989).

Mathematical schemas also play a crucial role in making meaning with incoming information. Currently, there is considerable emphasis on teaching for meaningful learning (Clarke, 1997; Steffe, Cobb & Richards, 1988; Lowrie, 2002), the assumption being that when students make sense of the mathematics they are exposed to they will enjoy mathematics and appreciate its relevance. Despite this growing importance of and agreement with meaningful learning, we are not well informed about what happens when students strive to construct meaning and what role, if any, previously learnt mathematics plays in the process. Information about the nature of schemas that students activate could provide considerable insight into mechanisms underlying meaningful learning of mathematics concepts and rules.

More recent investigations about mathematical thinking and problem solving have focused on the role of schemas in problem modelling and representation. A number of studies of school mathematics and science have highlighted the critical role played by schemas in assisting students analyse and investigate problems. Chinnappan (1998), in his investigation of geometry showed that schemas that contain information about trigonometric ratios and strategies for algebraic manipulation play an important role in assisting students construct advanced models of plane geometry problems. Likewise, Schoenfeld and Hermann (1982) found that successful students used elaborate schemas in the categorisation of problems involving polynomials. Working within the domain of kinematics, Chi, Feltovich and Glaser (1981) demonstrated that expert problem solvers invoked schemas that were built around important physics principles when they attempted to solve problems. In a related study on the solution of two-step problems, Nesher and Hershkovitz (1994) reported the reliance on schemas by the successful students. Taken together, the results of the above studies provide considerable support for the view that mathematical schemas constitute important knowledge structures that we as educators of future mathematics teachers need to consider in our planning, teaching and assessment activities.

Teacher Knowledge and Schemas

The foregoing discussion about the role of schemas in mathematics learning clearly demonstrates that teacher actions in the classroom need to involve students in the performance of tasks that have the likelihood of enhancing the development of schemas. The extent of knowledge and skills that teachers bring to the mathematics classroom not only influence their plan for teaching segments of the curriculum but also the delivery of the subject matter both of which have direct implications for encouraging students to construct schemas. What follows is a brief examination of some work on teacher knowledge and its potential impact on the development of students' mathematical schemas.

Developments in the area of mathematical teacher expertise (Chinnappan, 1994; Clark & Peterson, 1986; Schoenfeld, 1992) indicate that there are three major components which could be related to the knowledge base of teachers: mathematical content knowledge, the organisation of this knowledge and the blend of knowledge of content and pedagogy. *Mathematical content knowledge* includes information such as mathematical concepts, rules and associated procedures for problem solving, that is, the subject-matter knowledge. *The organisation of the content knowledge* refers to the links that teachers construct between the various components of the content knowledge. The *blend of content and pedagogical knowledge* includes understandings about why some students experience difficulties when learning a particular concept while others find it easy to assimilate, knowledge about useful ways to conceptualise and represent concepts (Feiman-Nemser, 1990) and the quality of explanations that teachers generate prior to and during instruction (Leinhardt, 1987). This latter knowledge has also been labelled as *pedagogical content knowledge* (Shulman, 1986).

The interaction between teacher knowledge and student schema building has been studied in the context of numerous mathematics topics including function (Norman, 1993; Wilson, 1994). In the high school curriculum, the understanding of a function, the various forms of functions, and their applications are essential for satisfactory progress in other areas such as calculus and analytical geometry, and higher mathematics that students could encounter in their tertiary studies. Curriculum Standards (National Council of Teachers of Mathematics, 1989) have identified features of functions that are indicative of depth in students' understanding of functions, namely, modelling real-world problems using functions, classifying and describing functions. Included in these understandings are representation of functions, translations among multiple representations of functions and the application of technology in the investigations of functions (Wilson, 1994).

While some progress has been made in our understanding of graphical representation of functions (Even, 1993; Leinhardt, Zaslavsky & Stein, 1990) there is a dearth of information about how teachers could exploit this form of representation in bringing about deeper levels of understanding of functions and their attributes among the students. Investigations of teachers' knowledge of functions and the teaching of functions have provided less information on the nature of knowledge that teachers access and use when computer aids are used in the teaching/learning process, and the possible effect that this could have on students' ability to construct function schemas.

Information Technology and Schema Building

The instructional use of Information Technology (IT) is an emerging area in mathematics teachers' professional development programs. With the increasing support for using technology during instruction, there is a need to articulate advantages conferred by technological tools in the learning process. Kaput (1986) argued that computer-supported learning is pedagogically more powerful because students experiment with mathematics concepts and procedures in a dynamic environment resulting in a high level of engagement with prior knowledge. Such engagement could also involve students creating and modifying computer-generated objects such as graphs, sketches and manipulatives such as Dienes Base Ten Blocks. The increase in knowledge activation and use of prior knowledge could be expected to have a profound effect on schemas that students build about a concept. This line of reasoning suggests that a mathematics teacher who aims to utilise computers during teaching will have to draw on a more complex pedagogical content knowledge schema.

Research Question

Within the context of teacher knowledge, the above analysis raises the question, "Do present mathematics pre-service curriculum programs support knowledge construction in pre-service teachers of the type that has been shown to be instrumental in fostering classroom students' ability at building and using highly organised schemas?" More specifically, "What is the nature of the professional knowledge base of our student teachers who aim to use IT in the near future?"

The principal aim of the investigation reported in this paper was to generate data that would throw light on the nature of knowledge that a student teacher had developed during the course of his training, and examine possible consequences of the quality of that knowledge for the development of schematic knowledge about functions. In this case study, The knowledge states of a pre-service teacher who was asked to teach the topic of linear functions with the aid of a particular piece of computer software were explored. In attempting to describe the knowledge base, the study focused on (a) his understandings about the concept of linear functions,

(b) the relationships between knowledge of linear function and other areas of mathematics, (c) knowledge about the teaching and learning of linear functions and (d) understandings about the use of a computer-based graphing tool to foster the development of schemas related to functions.

Method

The methodology used was a descriptive case study (Yin, 1998). As it involved a single-case design, the participant was the primary unit of analysis. According to Yin (1998, p. 236), while this approach suffers from the 'issue of selectivity', the direct focus on a particular case can be used to generate 'insightful' data.

Participant

The participant in the present study was a 25-year-old male who was completing the third year of his BEd(Secondary) program. He is referred to as Michael in this report. Michael volunteered to participate in this study. Prior to the study, Michael had completed four secondary mathematics methods subjects all of which emphasised constructivist principles in mathematics teaching and learning. Before this study, he had no formal teaching experience. Michael spent two weeks observing a Year 10 advanced mathematics class in his second year of the course as part of his practicum requirement. This observation included two lessons on linear functions. Discussions with Michael and his practicum supervising teacher indicated that he had developed an awareness not only of students' difficulties with functions but also their beliefs about mathematics in general. During the two years prior to the study, he had also completed mathematics discipline requirements for the BEd(Secondary) which included calculus, analytic geometry and statistics. Michael had no prior experience in the use of computer software in learning or teaching mathematics. He had, however, used software called *Derive* to solve problems in his first year calculus tutorials.

Material and Procedure

The investigator met Michael on two occasions. During the first meeting which lasted about sixty minutes, he was trained in the use of a graphing software, *ANUGraph* (Smythe & Ward, 1987) for Macintosh. The investigator introduced the software and showed the various parts of the menu. Michael was given ample time to experiment with this tool and raise questions about its capability and limitations.

Towards the end of the first meeting, Michael was given three focus questions to think about for the next session. The first two questions were related to characteristics of linear functions and the relationships between linear functions and other concepts in mathematics. The third question asked him to think about ways in which he would use *ANUGraph* to teach linear functions to the group of students that he observed during his second year of practicum. As part of this question, he was also asked to anticipate the type of difficulties these students would encounter in learning about linear functions via the software, and how he would help them. It was expected that Michael's understanding of the students, albeit limited, would provide insight into that part of his knowledge about students' difficulties not only with the concept of linear functions but also making sense of the concept within the *ANUGraph* environment. This hypothetical teaching situation was expected to provide an important context for activating this pre-service teacher's pedagogical content knowledge schema.

During the second session, Michael was given 20 minutes to work with *ANUGraph*, and invited to raise any questions. Following this activity, he was asked to respond to the above-mentioned three questions. The investigator probed responses that were not clear. In relation to question 3, Michael was encouraged to explain and justify the strategies that he might adopt if he were to use *ANUGraph* in his teaching of linear functions.

The interview session was audio taped and transcribed. The transcripts were then analysed for evidence of three groups of knowledge: content knowledge about linear functions, organisation of this content knowledge, pedagogical content knowledge and links between these knowledge components and the use of *ANUGraph* for teaching purposes. Quantitative analysis of Michael's responses examined instances of knowledge activation.

Results

Table 1 shows some examples of the three knowledge components that were considered to be important for the construction of schemas of linear functions. All three examples in the *Content knowledge* category were relevant to visualising and sketching linear functions. In the category of *Organised content knowledge*, the examples presented show that Michael had a good command of concepts and the relationships between these concepts. He was able to demonstrate how scales used in the coordinate system could influence the positioning of the x- and y-coordinates of a point, and reflected upon the link between the equation of a line ($y=2x$) and the slope of that line.

Table 1
Selected examples of Michael's knowledge base

Knowledge Component	Selected Examples
Content knowledge	Equivalent equations
	Substitution
	Steepness of a slope
Organised content knowledge	Linear functions can be graphed
	Scaling is shown on x and y axis
	y = 2x means the ratio of y:x is 2:1
	For every point on the line you go one unit across and two units up
Pedagogical content knowledge	If the students are allowed to play with the software they will pick up things by discovery
	Students learn by themselves

The example presented here involving the category of *Pedagogical content knowledge* suggests that Michael expected the particular group of students to be motivated by the use of *ANUGraph* and that they would explore other aspects of linear functions without much intervention from him. However, he did not identify or conjecture about the type of mathematical concepts that the students would discover by themselves. This suggests that he did not direct his student activities towards the building of specific schemas.

Table 2 shows the results of analysis of instances of knowledge activation. Michael had built up a reasonable amount of knowledge about linear functions in all the three areas that were hypothesised at the beginning of this study. He was able to access 36 items of content knowledge, 12 items of organised units of content knowledge and 12 items of knowledge that showed he was aware of the learning and teaching of the concepts mentioned during the interview.

The above frequencies were subjected to a second analysis in order to examine how well Michael was able to deploy his subject matter knowledge and pedagogical knowledge in a learning environment that was supported by *ANUGraph*. For example, Michael talked about the algebraic representation of a straight line, that is, $y = mx + c$ which was recorded as part of his Content knowledge (Column 2, Table 2). However, the question remained about how he would explore and expand students' schemas about straight lines with the aid of *ANUGraph*. Data relevant to this issue were generated by determining instances of items of knowledge activated (KCF) that were revisited in his discussions involving use of *ANUGraph*. These instances appear in Column 4 of Table 2 (frequency of KCSF). The proportions of KCSFs in relation to KCFs are expressed as percentages in Column 5 of Table 2.

Table 2
Frequency of activation of knowledge components

Knowledge Component (KCF)	Frequency of KCF	Knowledge Components Related to Software (KCSF)	Frequenc y of KCSF	KCSF as a Percentage of KCF
Content Knowledge	36	Content Knowledge	13	36[a]
Organised Content knowledge	12	Organised Content knowledge	2	17
Pedagogical Content knowledge	12	Pedagogical Content knowledge	2	17

[a] percentages have been rounded.

Table 2 shows that despite the training given in the use of the software, Michael did not make sufficient use of many of the options available within *ANUGraph*. This is evidenced by the fact that only 36% of the content knowledge was related to use of the software. This situation, for example, was illustrated more clearly in his explanation about plotting a linear function. Michael discussed two ways of plotting a linear function. Firstly, he outlined a method of determining two points on the graph when the y-intercept and gradient are given, and then joining these two points. In the second method, Michael suggested that students could plot a linear function by using a set of values for x and y coordinates. From a teaching viewpoint, both these approaches are sound and they do provide an insight into connections between geometric and algebraic representation of linear functions. However, the construction of these forms of the linear function could be significantly enhanced by the *ANUdata* option that is available in the software menu. This feature not only provides an efficient method to plot a linear function from a table of x and y values, it also encourages students to determine the equation of the straight line. It appears that Michael's limited use of the software would likely lead to reduced opportunities for students to investigate this procedure more fully and to develop a schema that shows links among a set of ordered pairs of x and y values and their symbolic relation.

The data presented in column 4 of Table 2 suggest that there are gaps in Michael's knowledge about ways in which the software could have been used to build on or highlight the links among components of knowledge associated with linear functions. One such relationship involves the solution of two linear equations that could be achieved with dramatic effect by plotting the two equations and determining the coordinates of the point of intersection. The coordinates of the point of intersection could be read easily by using the *show-coordinates* and *zoom* options available in the software menu.

The investigator expected Michael to make a few comments about the teaching and learning aspects of linear functions, and how the implementation of his chosen approaches would enhance or hinder student participation and learning outcomes. As shown in Table 2, Michael made 12 remarks that were related to the pedagogical content area. In almost all of his explanations, Michael seemed to be preoccupied with how *he* would learn linear functions with little consideration to the expectations, abilities, beliefs and attitudes of the students. What is equally interesting is the minimal connections that were made between the pedagogical content knowledge and the use of the software itself.

Figure 1 shows two linear functions that were drawn by Michael in his attempt to compare steepness of lines. While this effort clearly constitutes an important strategy in the use of the software, Michael did not exploit this situation to show important relations. One such relation could be that coordinates of any point on the lines should satisfy the algebraic relationships which are represented by the respective equations. This relationship could be readily illustrated by using the *show-coordinates* option which allows one to move points P and Q along the respective straight line graphs, and by investigating how this transformation affects the relationships between the coordinates.

In reference to the function $y=x$, Michael made the following observation:

> Graph of y=x, line going straight through and we would expect that (the angle) it will be equal to 45 degrees.

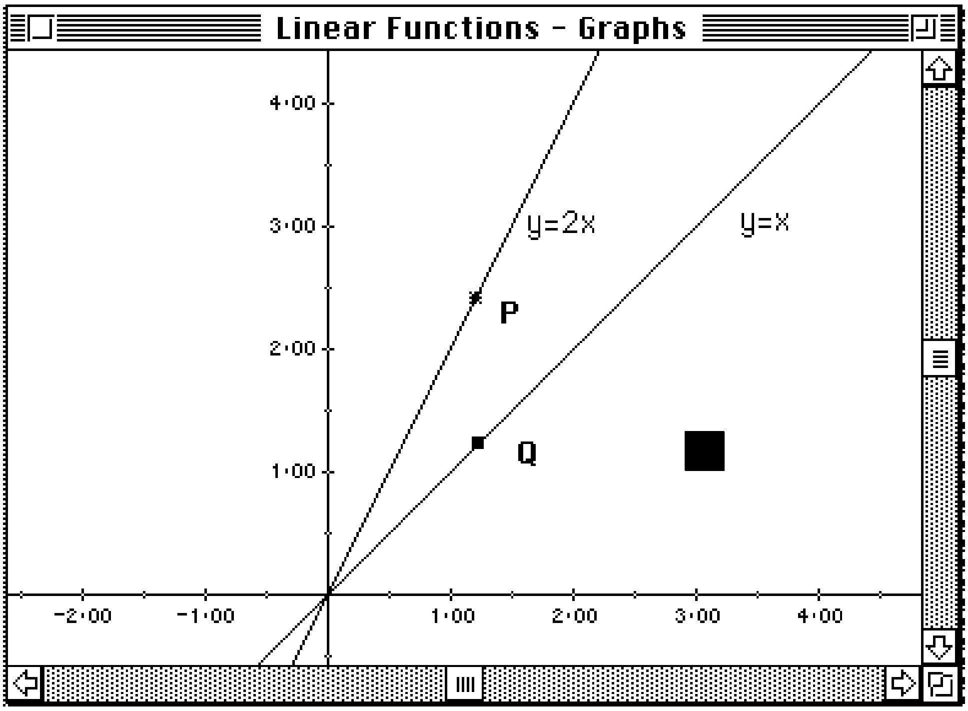

Figure 1. Comparison of steepness

The above statement again shows that Michael expected students to work out that the angle between the graph and the x-axis has a magnitude of 45 degrees. This point could have been made more dynamically by using the cursor to find out the coordinates of, say point Q, and asking students to use the right-angled triangle created by dropping the vertical and horizontal segments. This activity could then be followed up by applying the tangent ratio to the angle in question. Such an approach has the potential to encourage students to appreciate not only the power of the software better, but more importantly, to assist students to build a schema that would help them relate knowledge about right-angled triangles, trigonometry, gradient and linear function that was acquired in a non-Cartesian system, and to knowledge that that was embedded in the Cartesian system.

Discussion

The purpose of this study was (a) to generate data about the nature of a student teacher's knowledge about functions and the teaching of functions in a computer-supported learning environment and (b) to explore possible implications of that knowledge for the construction of mathematical schemas among students. The quality of the participant's knowledge was assessed with reference to a conceptual framework for teacher knowledge that included three major components: *mathematical content knowledge*, *organisation of mathematical content knowledge* and *pedagogical content knowledge*. Analysis of the student teacher's knowledge base suggests that he has a reasonably well-developed knowledge in the content area of linear functions. This was evidenced by his ability to provide at least three ways to represent linear functions and several examples to illustrate this point.

In the second area of interest concerning organisation of knowledge of functions, one could detect a number of gaps in the knowledge base of the participant. For instance, Michael did not make any connection between linear functions and solution of linear functions via graphical or any other means. This gap in the knowledge is particularly significant given that it could form an important node in the schema about linear functions which students are expected to learn. In addition, Michael failed to tap into the software options that could have made the above relationship clearer to the students in the classroom. Prawat (1989) argued that a well organised knowledge structure aids in the accessing and use of that knowledge flexibly. It seems that Michael's knowledge about linear functions, their solutions and facilities in the software was not organised in ways that would help him access it easily before and during instruction. He might therefore experience problems in constructing alternative representations of the concepts, and in activating available content knowledge during the teaching process.

Interview data generated in the study suggested that Michael's teaching plan was less concerned with potential difficulties that could be experienced by his students while they attempted to assimilate knowledge about linear functions with prior knowledge about algebra and geometry. Additionally, there were few instances during which he took into consideration students' attitudes to and beliefs about the topic, and how these factors could impact on their use of *ANUGraph* for independent investigation of linear functions. Knowledge about the learner and how the learner would process content knowledge (Peterson, 1988) constitutes a critical factor in the acquisition and further development of schemas. Thus, on the basis of what Michael said during the interview, it would seem that that he was not aware of the importance of understanding the learner in the learning/teaching situation.

Data analysis relevant to the issue of the relationship between this student teacher's subject-matter knowledge about linear functions and the instructional use of the computer software showed that Michael was competent in performing routine operations such as constructing an equation for a function and graphing it with the aid of *ANUGraph*. He showed an understanding of how the visual features that were built into the software could be utilised for the purposes of illustrating the gradient of not only a particular function but also of a family of linear functions. However, he did not extend this important feature of the software in order to solve problems or extend students to pose novel problems involving the construction of linear functions. These activities have significant pedagogical value for schema development (Clements & Battista, 1994; Kaput, 1986) and they could be facilitated by the appropriate use of the software. For example, as mentioned earlier, the software has the facility to generate a linear equation for a given set of ordered pairs by using the *ANUdata* option. Despite being alerted to the availability of this option, Michael did not make use of this information. This option would challenge students to make new connections between two representations of linear functions (tabular and algebraic). Failure to draw on this facility could deprive students of an excellent opportunity to build schemas that capture important mathematical links about linear functions in a theoretical and a practical context. The ability to move flexibly between tabular and algebraic representations is considered to be indicative of deeper understandings of functions and their use in solving problems (Chinnappan, 2001; Moschkovich, Schoenfeld & Arcvi, 1993). Furthermore, *ANUGraph* has the potential to be used as a tool for testing conjectures about linear functions. It appears that the limited and, in a sense, superficial use of the computer software by the student teacher would not generate learning outcomes that were conducive to the enlargement and enrichment of knowledge networks that one could associate with a sophisticated linear functions schema.

The limited number of connections that Michael established between his content and pedagogical knowledge could also have a significant effect on the quality of instructional explanations that he is able to provide during actual teaching. According to Leinhardt (1987), a sound knowledge of the subject matter or parts of the subject matter, an understanding of students' prior knowledge and their attitudes toward the topic of instruction constitute the building blocks of superior instructional explanations. It, therefore, appears that Michael's explanations about linear functions could suffer from (a) his disjointed and limited knowledge of this area, (b) insufficient exploitation of the software and (c) his lack of concern about students' weaknesses, strengths and prior knowledge that they might bring to the learning situation.

While one has to be careful in drawing general conclusions on the basis of this single-subject design case study, the results here allow one to form a tentative picture about the relationship between one pre-service teacher's knowledge base and its effect on schema building among students. At the beginning of this report it was argued that schema building constitutes an important aim of classroom mathematics instruction, and that teacher actions need to be directed towards this learning activity. The results of this study seem to suggest that in order for pre-service teachers to play an active part in schema generation among classroom students they need to draw on well-organised and automated sets of schemas from their own store of knowledge as there is a link between teachers' knowledge and students' learning outcomes.

The subject of this study did not appear to have developed a well-integrated body of knowledge about the mathematical content, technology and potential learning difficulties of his students. If this indeed is the case, one may expect his teaching actions would not promote the construction of the type of mathematical schemas that Sweller (1989) deemed necessary for problem-solving success. That said, one has to acknowledge that data generated in the present study were based on a hypothetical teaching situation. It is possible that a real teaching experience with *ANUGraph* could reveal a more complete picture about Michael's knowledge schemas.

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