Mathematics Teacher Educators' Orientations Towards Academic and School Mathematics

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The study reported in this article examines how mathematics teacher educators (MTEs) mobilise and articulate reference practices in their pedagogical communication with teachers. Reference practices designate the constellations of norms, epistemic values, artefacts, and validation criteria that MTEs invoke in their professional work. Pedagogical communication refers to the processes by which MTEs anticipate, select, sequence, and legitimate educational messages. Using a qualitative approach, data were generated through classroom observations (recorded through field notes and video recordings), and course documents in two Brazilian settings: a Professional Master's programme for in-service teachers and a prospective mathematics teacher education programme. Selected video-recorded segments were transcribed, and the field notes and transcripts were coded line by line through thematic analysis procedures. Two contrasting orientations were identified. In the first, academic mathematics functioned as the reference practice: objects, sequences, and validity criteria followed formal definitions and proofs. In the second, school mathematics served as the reference: curriculum prescriptions, textbooks, and evidence of student reasoning organised the formative experience. Three analytical dimensions characterise these orientations: the source of objects and tasks, the prevailing epistemic values, and the validation criteria invoked. Drawing on these findings and prior literature, a slider representation that maps the range of positions MTEs may assume along a continuum between academic and school mathematics is proposed. The slider offers a tool for the analysis of professional practice and promotion of teacher reflection.

Keywords • mathematics teacher education research • reference practices • pedagogical communication • academic mathematics • school mathematics

Introduction

Recent research has increasingly focused on mathematics teacher educators (MTEs), broadening the field beyond schoolteachers to include the professionals who prepare them. This shift raises key questions regarding the specific nature of MTEs' work (Beswick & Goos, 2018; Chapman, 2021; Goos, 2025). In this context, a question arises: what distinguishes MTEs' work from that of mathematics schoolteachers, and how is this distinction enacted in communication across prospective and in-service teacher education contexts? Although models of professional knowledge have offered productive descriptions of MTEs' work, cataloguing knowledge domains falls short of accounting for the situated, relational nature of MTEs' work in practice (Chapman, 2021; Goos, 2025). The concept of expertise may address this limitation by considering knowing and doing, foregrounding how professional competence is developed and enacted within social and institutional arrangements (Chapman, 2021; Goos, 2025).

Pedagogical communication occupies a central place within this conception of MTE expertise. It is through oral and written interaction with teachers that MTE expertise is enacted and made tangible (Barbosa & Chapman, 2024). The term, as adopted here, designates the processes by which MTEs anticipate, select, sequence, and legitimate educational messages, deciding which content to address, in what order, and by the standards something counts as valid within a formative setting (Barbosa & Chapman, 2024). Pedagogical communication focuses on the work of constructing formative experiences for professionals who are likely to become teachers. Articulating these communicative dimensions is consequential because doing so exposes the practices, norms, and artefacts that MTEs rely on when educating mathematics teachers.

In carrying out this communicative work, MTEs mobilise "reference practices"—understood as constellations of socially situated norms, epistemic values, modes of reasoning, artefacts, and validation criteria that MTEs invoke, explicitly or tacitly, to make arguments, select examples, design tasks, and provide guidance. The concept draws on Wenger's (1998) account of shared repertoires and boundary crossing, thereby illuminating contexts in which MTEs and teachers negotiate meanings across teacher education programmes and school classroom settings (Bakogianni et al., 2021; Wenger, 1998). Reference practices, in this sense, are the bodies of socially organised activity from which MTEs draw when determining what is worth addressing, how to sequence content, and what qualifies as a legitimate contribution in a teacher education setting.

Three scopes of pedagogical communication can be distinguished: (a) mathematics; (b) the teaching of mathematics; and (c) academic research (Barbosa & Chapman, 2024). Each scope implies a different constellation of reference practices. This study draws on the first two scopes. The third scope, academic research, was not examined because the observed sessions did not focus on research production or on the communication of research practices. Academic mathematics, organised around formal definitions, proofs, and university-level artefacts, serves as a reference practice within the scope of mathematics. School mathematics, anchored in curricular prescriptions, classroom routines, textbooks, and evidence of student reasoning, represents mathematical knowledge as it is organised and enacted in classroom instruction and serves as a reference practice within the scope of teaching mathematics. By examining these two reference practices, the study traces how each of the two scopes is realised in the communicative work of MTEs. These scopes may be connected through what Barbosa and Chapman (2024) and Barbosa (2025) called "bridge-building": arrangements in which MTEs deliberately integrate or sequence elements from different scopes of pedagogical communication.

Despite growing interest in MTE expertise, relatively little is known about how specific reference practices are mobilised and articulated when MTEs communicate with teachers. Some studies described dimensions of expertise and discussion routines in methods courses (Barbosa & Santos, 2026; Kastberg et al., 2024), but they rarely traced how a given reference practice, such as academic or school mathematics, shapes the selection of content, the sequencing of tasks, and the criteria by which contributions are judged valid. This gap constrains understanding of the sources that inform MTEs' communicative decisions and, by extension, the formative experiences constructed. It also leaves unexamined a question with direct practical implications: whether and how an MTE's orientation towards one reference practice rather than another shapes what teachers encounter during their preparation.

The objective of the study conducted was to examine how MTEs mobilise and articulate academic mathematics and school mathematics as reference practices in their pedagogical communication with teachers. Fieldwork research was conducted in two institutional settings in Brazil: a Professional Master's programme offering content-oriented courses for in-service mathematics teachers, and a prospective mathematics teacher education programme. The research question is:

> *How do mathematics teacher educators mobilise and articulate different reference practices in their communication with teachers?*

By attending to the objects MTEs select, the learning sequences they construct, and the validity criteria they invoke, the study offers a more detailed account of the sources that inform MTE expertise. The study also advances a conceptual representation for elaborating the orientations MTEs may assume between these two reference practices, offering a tool for both analysis of professional practice and promotion of teacher reflection.

Literature Review

Although research related to MTEs has grown recently, important areas remain under-explored (Chapman, 2021; Goos, 2025; Superfine et al., 2024). Recent syntheses map the research that has made considerable progress in describing what MTEs know and do, while acknowledging that central questions about the nature of this work remain open (Chapman, 2021; Goos, 2025; Superfine et al., 2024). Much of the progress has come through characterising MTE knowledge as distinct from teacher knowledge: it carries its own dimensions, such as meta-knowledge about teachers' knowledge, and takes shape under specific institutional and material conditions (Beswick & Goos, 2018). Models such as Mathematical Knowledge for Teaching Teachers (MKTT) and Mathematics Teacher Educators' Specialised Knowledge (MTESK), derived from frameworks initially designed for teacher knowledge, have provided structured accounts of what MTEs know or need to know (Martignone et al., 2022; Masingila et al., 2018; Superfine et al., 2020; Superfine et al., 2024). These models are productive, but Chapman (2021) argued that the field needs to move beyond strictly cognitive categories towards perspectives that can account for aspects of MTEs' professional work that may fall outside the scope of such models.

The concept of expertise is one way to move beyond this limitation (Goos, 2025; Helliwell et al., 2026). Rather than isolating knowledge domains, it foregrounds a system of knowledge, dispositions, and practices enacted within specific social contexts (Helliwell & Chorney, 2022; Kastberg et al., 2024). Barbosa and Chapman (2024) defined MTE expertise as "an amalgam of knowledge, social participation, and communication that expresses the educator's specific know-how in their role as a teacher educator" (p. 41). The emphasis is placed on what the MTE does and why, given particular social and material conditions. Expertise, under this view, is not restricted to what MTEs know. It is constituted in and revealed by the ways MTEs mobilise repertoires of practices and meanings when communicating with teachers.

Following Bernstein (2000), MTEs and teachers operate in a pedagogical relationship regardless of the instructional format—whether lectures, collaborative tasks, or field-based activities. This relationship persists even when an MTE does not explicitly address teachers' work, because the organisation of the formative experience itself conveys assumptions about teaching. The pedagogical message is sometimes explicit: MTEs address the professional work of teaching directly, propose tasks linked to classroom practice, or articulate criteria for what counts as valid in a given schooling context. Other times, the message is conveyed implicitly. The way an MTE conducts a class or a session, sequences content, handles a mathematical topic, or responds to a teacher's contribution conveys assumptions about what matters and how professional work should proceed.

This view of pedagogical communication has an important consequence. Someone teaching an advanced mathematics course to teachers, whether or not they identify themselves as mathematics teacher educators, is engaged in pedagogical communication. The way an MTE organises proofs, selects examples, handles errors, and structures the progression of ideas constitutes a lived model of how mathematics can be taught. Teachers do not simply acquire content in such courses, they also observe and likely internalise modes of teaching mathematics that may shape their own teaching (Zazkis & Leikin, 2010). The pedagogical message is carried by the pedagogical practice, not solely by what is said about teaching.

Situating pedagogical communication within related frameworks helps sharpen what the concept captures. Smith and Stein (2018) described a cycle of anticipating, monitoring, selecting, sequencing, and connecting student responses to make classroom mathematical discussions productive. Their framework, developed for school instruction, resonates with the communicative work of MTEs in so far as both involve purposeful decision-making about what mathematical content to foreground and in what order. The difference lies at the institutional level and the breadth of reference. MTEs are not orchestrating student mathematical talk; they are constructing formative experiences for professionals who are likely to become teachers. It is this specificity that the notion of pedagogical communication, when used for MTEs, is designed to capture.

The three scopes of pedagogical communication: Mathematics, Teaching Mathematics, and Academic Research (Barbosa, 2025; Barbosa & Chapman, 2024) orient the MTE's communication towards various sets of concerns and, consequently, towards various reference practices. The bridge-building analogy describes instances in which MTEs intentionally connect two or more scopes, making visible that communication with teachers might not be self-contained. Teacher education, in this framing, draws on and points to other contexts of practice.

Drawing on Wenger's (1998) account of shared repertoires and negotiated meaning, reference practices designate the norms, epistemic values, modes of reasoning, artefacts, and validation criteria that MTEs invoke when communicating with teachers. These practices shape MTEs' selection of objects, sequencing of tasks, and legitimation of what "counts" as valid within a given teacher education setting. Two reference practices are especially important here: academic mathematics and school mathematics.

Academic mathematics refers to the body of knowledge produced and validated within university mathematics, characterised by its styles of proof, modes of generalisation, formal language, and meta-mathematical considerations. It is typically enacted in advanced courses such as Mathematical Analysis, Topology, and Abstract Algebra. The literature on the relationship between university mathematics and teacher education documents both the formative potential of this reference—in broadening conceptual horizons and revealing structural connections across mathematical domains—and the tensions that arise when its legitimacy standards are transposed to teacher education without deliberate mediation (Moreira & David, 2008; Scheiner & Bosch, 2023). Moreira and David (2008) went further reporting that the conflict between academic mathematics and school-teaching knowledge is not a matter of levels but of distinct epistemic commitments. Scheiner and Bosch (2023) compared three theoretical approaches to this relationship, each offering a different account of how, and whether, academic mathematics should connect with teacher preparation. Other studies documented concrete mediation strategies, such as spotlighting central lines of reasoning from advanced subjects in ways that illuminate school-level demands (Wasserman et al., 2019; Wasserman et al., 2023).

School mathematics designates the mathematical practice organised and enacted for classroom instruction, carrying its own didactical artefacts, learning expectations, classroom routines, and validity criteria, among them the use of representations, pragmatic and empirical arguments, and diverse forms of justification. Where academic mathematics settles validity through formal proof and pursues generality as a value, school mathematics is shaped by curricular timelines, students' developmental trajectories, and the practical demands of teaching. A theorem that a university course treats as a logical exercise becomes, in the school context, something to be illustrated and tied to what students already know. This reference practice surfaces when MTEs guide discussions about curricula, textbooks, tasks, and evidence of student learning, engaging with these not as diluted versions of academic content but as artefacts governed by their inherent logic and pedagogical rationale (Moreira & David, 2008). Mobilising school mathematics as a reference tends to reorganise both content sequences and modes of legitimation in teacher education (Moreira & David, 2008). What counts as a good explanation, a well-chosen example, or a sufficient justification are assessed not only by deductive rigour but also by the capacity to support student understanding within the constraints of classroom practice. In schools, content sequencing follows curricular structure rather than disciplinary hierarchy, and the validation of contributions shifts towards criteria such as accessibility, representational clarity, and responsiveness to how students reason.

Within a mathematics teacher education programme, MTEs may occupy a mediating position between academic mathematics, school mathematics, and the teacher education context. Marshman and Goos (2025) described MTEs in these circumstances as "hybrid" MTEs who move between mathematics and mathematics education communities, acting as brokers of meanings, artefacts, and criteria, often belonging partly to both. This movement requires resources to shift ideas, refine language, and reposition problems as the frame of reference changes (Marshman & Goos, 2025). Leikin (2020) added a further dimension to this picture by arguing that the breadth of an MTE's disciplinary horizon, shaped by the depth of their mathematical and psychological knowledge, influences how they frame tasks, assess contributions, and decide what to pursue in a session. MTEs from different professional communities, such as research mathematicians, mathematics education researchers, and expert schoolteachers, operate with different horizons, and these differences translate into distinct perspectives of what teacher education should accomplish and which reference practices take priority. MTEs do not enter settings empty-handed; they bring repertoires developed across practices, most saliently academic and school mathematics, whose norms, artefacts, and validation criteria might be invoked to select content, sequence experiences, and legitimate what counts during teacher education. Positioned at the boundaries between academic mathematics, school mathematics, and teacher education, MTEs function as brokers, translating meanings across these practices and contexts (Marshman & Goos, 2025; Ozmantar & Ağaç, 2023). The ways in which brokering work unfolds in practice, specifically how reference practices are mobilised and articulated in MTEs' communication with teachers, remain insufficiently documented.

Methodological Approach

The present study examines one aspect of a broader research project on the expertise of mathematics teacher educators (MTEs). A qualitative approach (Creswell, 2012) was adopted to trace how MTEs mobilise reference practices in their pedagogical communication as it unfolds in natural settings. The study is not a self-study. The researcher (article author) did not take on the role of a teacher educator in any of the settings observed.

Research Context and Participants

Fieldwork was conducted in two institutional contexts, treated as contrasting cases and referred to here as Context 1 (C1) and Context 2 (C2). C1 comprised Geometry and Arithmetic, whereas C2 comprised Teaching Practice and a supervised practicum; both contexts were located within mathematics teacher education programmes at the same Brazilian university where the researcher was based. The programmes differed in institutional structure, teacher profile, and curricular logic, making C1 and C2 productive contrasting cases for examining how reference practices might operate. In C1, a research assistant observed sessions and produced detailed field notes. In C2, the researcher conducted observations and made video recordings. The Human Ethics Research Committee of the author’s institution granted ethical approval, and all participants provided informed consent.

Context 1

C1 comprised two courses within the Professional Master's Programme in Mathematics (the acronym in Portuguese is PROFMAT), a nationwide postgraduate programme coordinated by the Brazilian Mathematical Society (SBM). PROFMAT serves in-service public school mathematics teachers seeking to deepen their mathematical knowledge. Its curriculum is set nationally and built around advanced mathematics: core courses include Numbers and Real Functions, Discrete Mathematics, Geometry, and Arithmetic, among others. The two courses observed were Geometry, covering the Euclidean plane and spatial geometry from an axiomatic standpoint (triangle congruence, similarity, areas, trigonometry, convex polyhedral volumes); and Arithmetic, which addresses divisibility, the Euclidean algorithm, prime numbers, and congruences through a formal deductive treatment. Both courses aimed to consolidate logical-deductive reasoning at a level of rigour that exceeds what school teaching typically demands. Seventeen in-service mathematics teachers participated across the two C1 courses.

Two MTEs taught the two courses. The geometry MTE held a Doctorate in pure mathematics, had 17 years of experience teaching advanced mathematics in in-service and prospective mathematics teacher education programmes, and was affiliated with the Department of Mathematics, where he also conducted mathematical research and supervised Master's students. The arithmetic MTE held a Master's degree in mathematics and was completing his Doctorate at the time of data collection; he had 15 years of experience in teacher education programmes, particularly in advanced mathematics courses. The researcher had prior professional contact with both MTEs, which facilitated access. Both MTEs regarded their work as crucial to teacher education. In their view, teaching advanced mathematics to teachers is a formative dimension, grounded in the conviction that knowing advanced mathematics shapes how teachers understand and handle the mathematics they teach in schools. This viewpoint was shared by the two C1 MTEs.

It is the case that when a course is organised entirely without any connection to school mathematics, the way the MTE handles ideas, structures proofs, selects examples, and responds to questions conveys modes of doing and teaching mathematics that teachers may carry into their classrooms (Zazkis & Leikin, 2010). The two C1 MTEs, both trained in pure mathematics, operated with a horizon firmly anchored in formal mathematics, which is consistent with the reference practice their communication aimed to mobilise.

Context 2

C2 comprised two components of the prospective mathematics teacher education programme (Licenciatura em Matemática): Teaching Practice and a supervised practicum. In Brazil, Licenciatura programmes typically run four years, combining advanced mathematics, pedagogical studies, and a school-based practicum. Teaching Practice (Prática de Ensino) covered mathematical topics from secondary school and examined both their conceptual structure and pedagogical dimensions. Fourteen prospective mathematics teachers were enrolled. In the supervised practicum, prospective teachers conducted fieldwork in school mathematics classrooms under an MTE's guidance. However, the MTE conducted preparation sessions before sending them to schools. Eleven prospective teachers from the cohort were enrolled in the supervised practicum component.

Two MTEs were responsible for C2. The Teaching Practice MTE had completed a Master's in Pure Mathematics in the 1990s and moved into teacher education shortly after. Years of working closely with prospective and in-service mathematics teachers led her to pursue a PhD in Mathematics Education, which she finished in 2017. Since then, she has been involved in courses with an explicit focus on mathematics education. She also supervises Master's students. The researcher had collaborated with her on research projects previously, which made field access straightforward. The supervised practicum MTE held both a Master's and a Doctorate in Mathematics Education, the latter completed in 2018. She came to the position with experience in in-service teacher education. She had joined the institution two years before data collection began, working within the prospective mathematics teacher education programme.

Data Collection and Analysis

Data collection varied across the two contexts. In C1, a research assistant observed sessions and produced detailed field notes. In C2, the researcher conducted observations and made video recordings. Ethical approval was granted by the Human Ethics Research Committee of the author's institution, and all participants provided informed consent. Observation documented interactions and events as they occurred; document collection provided contextual material for interpretation (Creswell, 2012). A protocol derived from the conceptual framework guided the focus of observations during sessions: the sources from which objects, tasks, and examples were drawn (university textbooks, school curricula, student work); sequencing decisions (textbook order, curricular progression, responses to teachers' questions); the validation criteria invoked or implied when contributions were assessed (formal proof, pedagogical use, curriculum alignment); instances of explicit or implicit reference to school or academic mathematics; and moments where the MTE connected or kept separate different reference practices. Video recording was not permitted in C1, so detailed field notes were written during the sessions. In C2, sessions were video-recorded, and selected segments were transcribed verbatim. The corpus comprised 12 sessions in C1 and 14 in C2, totalling approximately 50 hours of fieldwork.

Analysis followed the procedures of thematic analysis (Terry et al., 2017), combining deductive and inductive strategies. The field notes and selective verbatim transcripts were coded line by line. Selection, sequencing, and legitimation, the communicative dimensions from the literature review, served as sensitising concepts for initial coding. In addition to these deductively informed codes, other codes were generated inductively from the data. For example, "Textbook as sole authority" emerged from observing the MTE in the Arithmetic course, in which teachers read directly from the textbook, setting aside teachers' reports of how they teach the topic. Similarly, "School student error as entry point" captured how the MTE in Teaching Practice used evidence of students' difficulties to organise the discussion. An example of coding illustrates how excerpts were coded and how codes were subsequently grouped into themes. In the Arithmetic course, the field notes record:

> A schoolteacher stated that he teaches this content, but in a different way. [...] The MTE insisted that it is essential to "know more than what we teach"; therefore, "we need to be familiar with the proofs, theorems, and properties that justify the concepts." The other ways of finding out the Greatest Common Divisor (GCD) presented by the teachers were ignored. [Field notes, C1]

This excerpt was coded as "dismissal of school methods" and "formal knowledge as legitimation." Both codes were then grouped under the theme "academic mathematics as reference practice," a recurring pattern in which communication privileged formal mathematical knowledge and pushed the teachers' school-based approaches to the margins. The analytic sequence moved from familiarisation with the full corpus through initial coding, grouping into provisional themes, reviewing the themes against the data, and arriving at the two analytical categories presented in the next section. Episodes were chosen for presentation because they illustrated a theme with particular force: each illustrates instances of a reference practice and enough interactional texture to make the communicative moves visible.

Three measures strengthened trustworthiness. A colleague with expertise in mathematics teacher education research coded independently five episodes from both C1 and C2 using the same analytical framework. We compared results, discussed disagreements, and reached consensus on all contested cases. Portions of the coding were also discussed within the author's research group, where members examined selected episodes and the proposed codes, offering alternative readings that helped refine the analytical categories. Field notes and recordings were triangulated with course documents: syllabi, textbook excerpts, and task sheets.

Results

The thematic analysis produced two analytical categories, academic mathematics as reference practice and school mathematics as reference practice, each capturing a distinct orientation in how MTEs organise their communication with teachers. In what follows, each category is illustrated with episodes from C1 and C2, drawing on field excerpts to make visible how the reference practice operated in each setting.

Academic Mathematics as Reference Practice for Mathematics Teacher Educators

In this category, there were two situations in which the MTEs' communication with teachers was anchored primarily in the practice of academic mathematics, defining what "counts" as valid within the discipline in terms of definitions, propositions, proofs, and classic problems. In both situations analysed, the authority of the discourse flows from the formal deductive register and from the university textbook, which serves as the source for task selection and sequencing. Connections with other reference practices, especially school mathematics, appear only marginally. When school mathematics is mentioned or implied, it is not taken up as an organising reference for the selection, sequencing, or legitimation of the mathematical work.

Both situations refer to two courses within the in-service mathematics teacher education programme referred to in the previous section as C1. The first took place during the Geometry course. It was organised as an initial block of lecture-based classes, followed by time for exercise solving. The field diary summarises this arrangement:

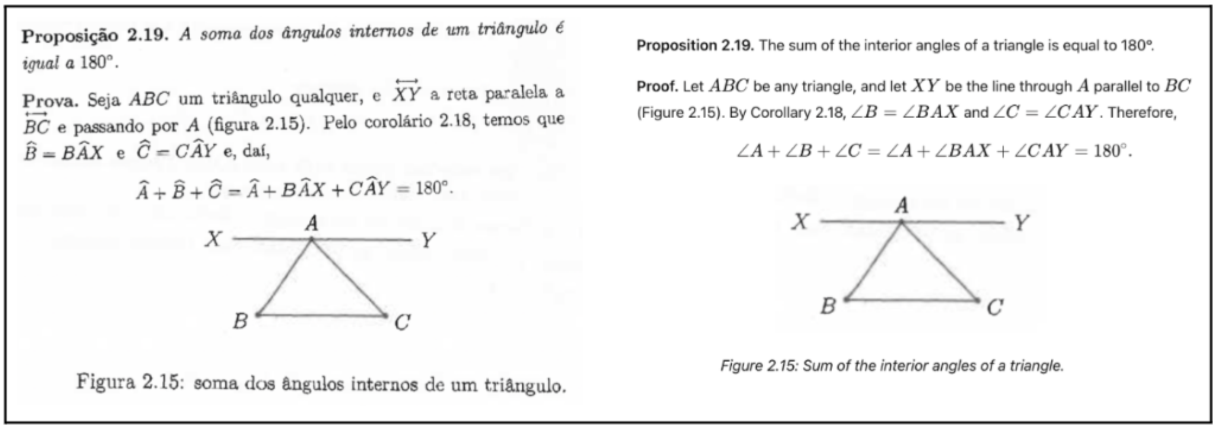
> In the very first meeting, the mathematics teacher educator introduced the topic: Axioms. The lesson unfolded as follows: the MTE wrote definitions/proofs on the board, using examples to illustrate what

was being explained. During this period, the MTE handed out exercises for teachers to work on at home. [Field notes, C1]

After the initial sessions, time use was renegotiated. The field diary summarised this shift as follows:

> This way of organising the class only changed in the session this week. During the session, the MTE proposed two options for how the lesson could proceed. First, they would continue with the content. The second option was to begin solving exercises on the board. […] It was decided that the continuation of Axioms would be handled in extra classes and that this class would be reserved for solving exercises on the board […]. An in-service teacher who volunteered to go to the board wrote a solution, followed by the MTE's correction. [Field notes, C1]

The MTE adopted the textbook on Geometry by Muniz Neto (2013), a source commonly used in mathematics content courses within prospective mathematics teacher education programmes in Brazil. It addresses plane Euclidean geometry with emphasis on triangle congruence, combines straightedge-and-compass constructions with demonstrations of classical results, and provides exercise lists to consolidate techniques and propositions. The language is precise and formal, systematically using symbols, axioms, and logical connectives. Two excerpts illustrate the tone of this textbook. First, the proposition on the sum of the measures of the interior angles of any triangle (see Figure 1).



Proposição 2.19. *A soma dos ângulos internos de um triângulo é igual a* 180°.

Prova. Seja ABC um triângulo qualquer, e $\overleftrightarrow{XY}$ a reta paralela a $\overleftrightarrow{BC}$ e passando por A (figura 2.15). Pelo corolário 2.18, temos que $\widehat{B} = B\widehat{A}X$ e $\widehat{C} = C\widehat{A}Y$ e, daí,

$$\widehat{A} + \widehat{B} + \widehat{C} = \widehat{A} + B\widehat{A}X + C\widehat{A}Y = 180^\circ.$$

Figura 2.15: soma dos ângulos internos de um triângulo.

Proposition 2.19. The sum of the interior angles of a triangle is equal to 180°.

Proof. Let ABC be any triangle, and let XY be the line through A parallel to BC (Figure 2.15). By Corollary 2.18, $\angle B = \angle BAX$ and $\angle C = \angle CAY$. Therefore,

$$\angle A + \angle B + \angle C = \angle A + \angle BAX + \angle CAY = 180^\circ.$$

Figure 2.15: Sum of the interior angles of a triangle.

Figure 1. Excerpt taken from Muniz Neto's (2013, p. 53) with English translation.

The proof, accompanied by a diagram, draws a parallel line and uses angle relations to reach a formal conclusion. The text does not pursue other school-oriented justifications (e.g., paper folding or manipulatives). A second excerpt shows the kind of task valued (see Figure 2). The valuation of the task was inferred from its frequency of use, the MTE's comments, the time devoted to it, the mode of correction, and its centrality in the lesson. For example, the continuation of Axioms was moved to extra classes to prioritise board exercises and the MTE's correction. [Field notes, C1]

1. * Se dois segmentos são iguais e paralelos, prove que suas extremidades são os vértices de um paralelogramo.
2. Seja $ABCD$ um quadrilátero convexo qualquer. Mostre que os pontos médios de seus lados são os vértices de um paralelogramo (sugestão: use o teorema da base média quatro vezes, para concluir que o quadrilátero que tem por vértices os pontos médios dos lados tem pares de lados opostos iguais).
3. Uma reta r passa pelo baricentro G de um triângulo ABC e deixa o vértice A de um lado e os vértices B e C do outro. Prove que a soma das distâncias de B e C à reta r é igual à distância de A a r.

1. *If two segments are equal and parallel, prove that their endpoints are the vertices of a parallelogram.*
2. Let $ABCD$ be any convex quadrilateral. Show that the midpoints of its sides are the vertices of a parallelogram (hint: use the Triangle Midsegment/ Midpoint Theorem four times to conclude that the quadrilateral whose vertices are the side midpoints has pairs of equal opposite sides).
3. A line r passes through the centroid G of a triangle ABC and leaves vertex A on one side and vertices B and C on the other. Prove that the sum of the distances from B and C to the line r equals the distance from A to r.

Figure 2. Excerpt taken from Muniz Neto (2013, p. 89): Portuguese original (top) and English translation (bottom).

The exercises required teachers to perform proofs and apply theorems, in line with the course's orientation. From the standpoint of pedagogical communication, the MTE focused exclusively on the practice of academic mathematics: the validity of what is learned is guaranteed by deductive arguments; examples illustrated techniques; and exercise lists served as practice and a means of checking teachers' mastery of proof techniques. On occasion, the activity approximated school practice when an in-service teacher volunteered to solve an exercise at the board, followed by the MTE's correction, but it remained within an academic framework.

The second situation comes from the Arithmetic course, specifically the class focusing on the Greatest Common Divisor (GCD). The book adopted was "Aritmética" (Arithmetic, in English) by Hefez (2013). It covers topics in elementary number theory. Based on a rigorous formal approach, the book originated from Hefez's lecture notes and is also widely used in Brazil for courses on mathematical content. On the day when the Greatest Common Divisor (GCD) was addressed, the field diary records the following note:

> Before the lesson began, the MTE informed the teachers that they would follow the programme's textbook. Shortly after that, he began the lesson by reading from the Arithmetic book. He drew the teachers' attention to the definition and properties of the Greatest Common Divisor (GCD). After the reading, the MTE made a record on the board. [Field notes, C1]

From there, the MTE presented the uniqueness theorem for the GCD of nonzero integers and introduced Euclid's Algorithm, with an example. According to the field notebook, the format was expository, with no planned interventions implemented. At one point, a contrast emerged between the academic approach and teachers' school practice:

> A schoolteacher stated that he teaches this content, but in a different way. […] Another teacher shared that listing divisors is the most used method […]. […] Besides this method, they say another common approach is prime factorisation. However, the MTE insisted that it is essential to 'know more than what we teach'; therefore, 'we need to be familiar with the proofs, theorems, and properties that justify the concepts', said the MTE. The other ways the teachers presented to find the GCD were ignored. [Field notes, C1]

Although the MTE listened to the teachers' reports, he emphasised the need to master formal propositions and the algorithm. Even when teachers interrupted the flow of the lesson, which contrasted with their approach to teaching the Greatest Common Divisor (GCD) in the school context, the MTE ignored these practices, instead privileging the formal mathematical approach.

This episode presents a selective perspective: formal mathematical knowledge is validated as "what needs to be known," even when the content is at the school level. It also suggests a limit in articulating the teaching practice, even when mathematics teachers interrupt the lesson flow and report how they teach GCD.

The two situations provide insight into how MTEs mobilise and articulate academic mathematics as their reference practice. Regarding the selection, "what" to address with the teachers reflects the formal grammar of mathematics. Sequencing follows university textbook order, and legitimation is predominantly intradisciplinary (proofs, properties, algorithms). When school references arise, they function as footnotes rather than opportunities to explore connections. In these situations, the MTEs primarily drew on academic mathematics as their reference practice, even though the courses were part of an in-service mathematics teacher education programme.

School Mathematics as Reference Practice for Mathematics Teacher Educators

In this analytical category, two situations in which the MTEs' communication with teachers is oriented primarily by the practice of school mathematics were examined. They focus on the curriculum document, textbooks, classroom routines, expected student ways of reasoning, and the difficulties that may be encountered. Unlike an orientation centred on academic mathematics, the reference here is to mathematics teachers' work in schools.

Both situations refer to two settings in the prospective mathematics teacher education programme referred to in the previous section as C2. Teaching Practice focused on the mathematical topics typically covered in high school, discussing both their conceptual and pedagogical aspects. In one class, the MTE presented two skills from the Brazilian Common Core that link arithmetic and geometric progressions to functions (Brasil, 2018). This move shifted the focus from the "content" to a curricular organisation that calls for mathematical connections. The following excerpt captures the reactions of three prospective teachers, identified as Teachers A, B, and C.

MTE:	Especially regarding the skills, is there anything new you don't remember, haven't seen, or haven't considered?
Teacher A:	I hadn't seen this association with functions […]. The textbooks aimed to be as concise as possible, particularly about algorithms.
Teacher B:	In a textbook, there's a section where they treat geometric progression in connection with the exponential function […] and arithmetic progression with the linear function.
Teacher C:	It's mostly what Teacher A said. These topics are presented in isolation, as if they had no relationship, as independent content.
Teacher A:	Maybe I already knew about it […] I don't remember where, but I don't see these topics in the textbooks.

[Video recording, C2]

Given that the prospective teachers were unfamiliar with the relationships between arithmetic and geometric progressions and their connections to linear and exponential functions, the MTE asked the teachers, working in groups, to identify these associations in textbooks. In the following session, the teachers presented their mappings of how such associations appeared in textbooks. Their review suggested that these links appeared in only a handful of textbooks and, when they did, the treatment was limited. The MTE addressed the group, asking whether a deeper treatment would support school teaching, and stressed the importance of reading textbooks with a critical eye.

Across these two sessions between the MTE and the prospective teachers, at least two dimensions of school mathematics were brought into play: the normative curriculum and the textbook. Both were articulated by discussing arithmetic and geometric progressions and their connections to functions, topics prescribed for the high school curriculum. Thus, in the two sessions reported above, the MTE's communication and the tasks assigned to teachers were explicitly related to what occurs or might occur in school mathematics, taking curricular prescriptions and textbook approaches as primary points of reference.

The second situation occurred in a preparatory session for prospective teachers before their supervised practicum. In this context, the MTE organised activities to bring them closer to the reality of school mathematics classrooms. The MTE proposed a fieldwork task to investigate the difficulties school students encounter when solving first-degree equations. The task was as follows:

> Visit seventh- and 8th-grade classes […] and gather written records of how they solve first-degree equations […] collect notebooks, assessments, or administer a short test. Identify the difficulties and consider ways to address them.

[Field notes, C2]

Days later, the prospective teachers engaged in a discussion moderated by the MTE. The excerpt below, which focuses on classroom observations, exemplifies pedagogical communication when school practice served as the reference, identifying the prospective teachers as PT1 and PT2.

MTE:	PT1, what showed up in the records?
PT1:	Many solved $6(x+3) = x + 12$ as if it were $6x + 3 = x + 12$... They seem to ignore multiplying 6 by both x and 3.
MTE:	Hmmm, the distributive property.
PT2:	This also occurred with the responses of the students who applied the test. They forget multiplying.
MTE:	And how do you think we could draw students' attention to the distributive property without being too directive?
PT1:	Remembering the distributive property for them?
MTE:	It might sound straightforward. Maybe you should start with numbers first ... that way, they can see that if the distributive property is not applied, the equation becomes false.

[Video recording, C2]

Here, the MTE treats students' written work as a legitimate source for mathematical discussion with prospective teachers. The task explicitly asked prospective teachers to collect and analyse student responses, bringing their inquiry closer to the realities of school mathematics classrooms. In this situation, the reference practice of school mathematics was unambiguous.

In summary, in both situations, the reference practice was school mathematics, as evidenced by tasks involving the normative curriculum, textbooks, and an analysis of students' written responses. In both cases, the MTEs' communication aligned closely with what happens or could happen in the mathematics classroom. Mathematics did appear, such as the links between arithmetic/geometric progressions and functions or the distributive property, embedded in how the subject was approached within the school context.

Discussion and Final Remarks

The research question is: How do mathematics teacher educators mobilise and articulate different reference practices in their communication with teachers? The two analytical categories (Academic mathematics and School Mathematics) presented above point to contrasting orientations. In one, the MTEs' communication was organised around academic mathematics. Objects, sequences, and validity criteria followed the logic of formal mathematics; school mathematics, when it surfaced, was set aside rather than taken up. In the other, school mathematics provided the organising frame. Curriculum documents, textbooks, and evidence of student reasoning supplied the tasks, determined the sequence, and oriented what counts as a productive contribution.

Three analytical dimensions help characterise these orientations, each emerging from the interplay between the conceptual framework and the data. The first concerns the sources of objects and tasks: whether the MTE draws on university textbooks and formal problems, on school curricula and student work, or on a deliberate combination of the two. The second concerns epistemic values: whether understanding is suggested by formal proof, pragmatic justification tied to classroom viability, or forms that hold both registers in view. The third concerns validation criteria: deductive correctness, pedagogical utility, or their integration.

In C1, all three dimensions converged towards academic mathematics. The MTEs worked exclusively from the university textbook, required formal proofs, and dismissed teachers' school methods as insufficient. In C2, the convergence ran in the opposite direction. The MTEs worked from curriculum documents and student data, valued pedagogical reasoning, and judged contributions by their relevance to classroom practice.

These contrasting configurations were not accidental. They reflect the different disciplinary horizons that each MTE brings to teacher education (Leikin, 2020). The C1 MTEs, trained in pure mathematics and affiliated with a Mathematics Department, operated with a mathematical horizon anchored in formal rigour. In contrast, the C2 MTEs, with doctorates in Mathematics Education and sustained engagement with school practice, operated with a horizon that foregrounded mathematics teachers' daily work. Leikin (2020) argued that an MTE's horizon, shaped by their professional community and training, determines what they see as worth pursuing and what remains outside their field of vision. Hence, it is possible to assume that the reference practice an MTE mobilises is partly a function of the horizon from which they operate.

The findings sit well within the existing literature. The primacy of formal mathematics-oriented pedagogical practice and the marginal role of school mathematics in C1 echo what Moreira and David (2008), Scheiner and Bosch (2023), Wasserman et al. (2019), and Wasserman et al. (2023) described as the distance between university mathematics and the demands of school teaching. The orientation in C2 aligns with work that takes curriculum and classroom evidence as the basis for designing formative tasks (Appova & Taylor, 2019; Ozmantar & Ağaç, 2023). In both cases, the data confirmed that MTEs' expertise, though rooted in teacher education itself, draws on and is shaped by other practices (Beswick & Goos, 2018; Chapman, 2021; Leikin, 2020; Ozmantar & Ağaç, 2023).

Figure 3 presents a generic slider representing a continuum between academic and school mathematics. The MTE marker is movable and may occupy different positions along the continuum, depending on the reference practice mobilised in a particular setting.

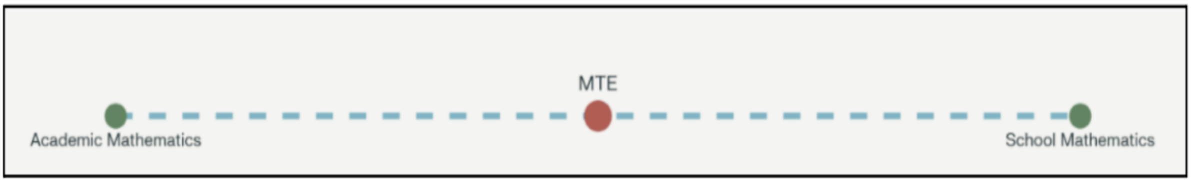


Figure 3. Generic slider representing possible MTE positions along the continuum between academic and school mathematics.

The data anchored the two poles as contrasting orientations; however, they do not exhaust the potential positions MTEs might assume. Theoretically, the three dimensions—data sources, epistemic values, validation criteria—are not binary. Each admits degrees and combinations. For instance, an MTE might draw on an advanced mathematics textbook but choose problems because they illuminate a school-level difficulty. Another might require formal proof during one part of a session and shift to a school-oriented justification when the discussion turns to how a mathematical concept could be addressed to students. Although this type of combination was not observed in the data collected in contexts C1 and C2, there is reason to think that intermediate positions exist. Wasserman et al. (2019) and Wasserman et al. (2023) exemplified how selecting advanced mathematics topics with high potential to illuminate school-level ideas can provide the MTE with the recontextualisation needed to support teachers' professional decisions through representations and problems. It does not abandon academic mathematics. It repositions academic mathematics within a pedagogical rationale oriented towards teaching. The concept of bridge-building (Barbosa, 2025; Barbosa & Chapman, 2024) captures this kind of move: MTEs deliberately connect different scopes of pedagogical communication, for instance, integrating or sequencing elements from academic and school mathematics.

On this basis, I propose replacing the dashed line in Figure 3 with a continuous line (Figure 4). The representation is not a claim that all positions along the continuum are derived from the data collection. It is a theoretical proposition, grounded in the two empirically identified poles and supported by the literature on bridge-building (Barbosa, 2025; Barbosa & Chapman, 2024) and on making university mathematics matter for mathematics teacher education. The slider signals that there are not two fixed alternatives but a range of placements, each defined by a particular configuration of source, epistemic values, and validation criteria.

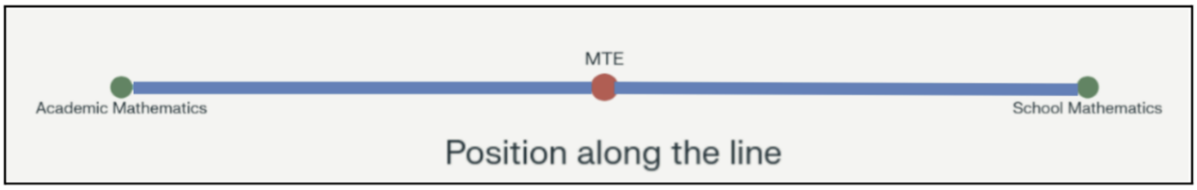


Figure 4. The different positions of the MTE in relation to academic and school mathematics.

What would different positions look like in practice? An MTE near the academic mathematics pole draws consistently on advanced mathematics textbooks, requires formal proof, and validates contributions by deductive standards, as in C1. Near the school mathematics pole, the MTE works on curriculum documents and student data, values pedagogical reasoning, and validates by classroom relevance, as in C2. A potential intermediate position might involve selecting a topic from Abstract Algebra because of its potential to clarify a school-level concept. The educator could present it with formal rigour and then ask teachers to consider how the underlying idea could be made accessible to students, thereby combining academic sources with school-oriented validation. As previously mentioned, this configuration is not documented in the present data collection; it is proposed through a broader understanding by combining data analysis with literature from the field. Mapping such positions empirically is a task for future research.

Figure 4 should be understood as a descriptive tool. There is value in bridge-building as a professional resource for MTEs (Barbosa, 2025; Barbosa & Chapman, 2024), but the purpose here is not to advocate for a particular position. The contribution lies in making visible the range of potential orientations and in providing an analytical language for identifying where a particular MTE's professional practice falls and understanding why it falls there.

The study advances the field in two aspects. First, by foregrounding reference practices as an analytical lens, it moves beyond descriptions of what MTEs know and examines how their communication is organised and legitimated. Second, the slider representation offers a tool for mapping variation in MTE practice. Selection, sequencing, and legitimation shift with the operative reference practice, and the three dimensions proposed here supply concrete criteria for locating positions and identifying what movement along the continuum would require.

Future research could examine how MTEs enact their expertise at intermediate positions, where academic and school mathematics are deliberately combined. The schema could also be extended to include reference practices not examined here, such as mathematics education research. From a professional standpoint, Figure 4 may serve as a reflective instrument. There is the potential for MTEs to use it to locate their practice, examine the coherence between their stated purposes and their communicative moves, and plan bridge-building arrangements consistent with the goals of mathematics teacher education.

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Ethical approval

This study draws on data collected as part of a broader research project for which ethical approval was granted by the Federal University of Bahia, Brazil. Participation was voluntary, and all participants were informed about the aims of the research and the use of the data for scholarly purposes. Informed consent was obtained from all participants. Participants' confidentiality and anonymity were protected throughout the research process, and the data were used solely for academic purposes.

Competing interests

The author declares there are no competing interests.

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