

Tools for Mathematics Teacher Educators' Professional Growth: Conceptualising Inquiry Opportunities for Representations of Teacher Education Practice

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Ongoing professional growth is central to Mathematics Teacher Educators' (MTEs) work, yet they often lack tools to systematically inquire into and learn from their own practice. This multiple case study investigates how two distinct representations of practice — a video-based *Video Stimulated Recall Conversation* (VSRC) and a survey-based *Collaborative Professional Development Survey* (CPDS)—support MTEs in inquiring into their own professional practice. Drawing on the semiotic infrastructure for practice framework, we examined how MTEs reflected on practice across two contexts involving prospective teachers: a video-club in Israel, using the VSRC, and an academic course in the United States (U.S.), using the CPDS. Findings show that both tools surfaced common problems of practice, such as time management and balancing facilitator input, while highlighting different dimensions. The VSRC, by foregrounding temporality and the visibility of specific interactions, illuminated real-time decision-making, specific linguistic choices, and gestures. The CPDS, in contrast, revealed teachers' perspectives and patterns of change over time, offering candid feedback through anonymity. Notably, both tools did not provide artefacts of teacher learning. The study contributes considerations for selecting and designing representations for MTEs' professional growth, underscoring the value of combining multiple, complementary tools that highlight different facets of practice.

Keywords • mathematics teacher education research • representations of practice • mathematics teacher educators • practical measures • video-stimulated recall conversation

Introduction

Mathematics Teacher Educators (MTEs) play a critical role in improving mathematics teaching and learning. Few studies, however, have focused on their lifelong learning, particularly the tools that support MTEs in ongoing inquiry into their practice (Goos & Beswick, 2021). We examine and compare two tools designed to foster MTEs' learning from evidence of their work, drawing on the concept of representations of practice (RoPs). Through a multiple case study, we theorise the interplay between how MTEs' practice is represented through different tools, and the implications for MTEs' professional learning. In doing so, we contribute to the field's understanding of MTEs' professional growth, including a set of key considerations to inform MTEs' selection of tools to collect evidence of their practice.

The first tool is a video-stimulated recall conversation (VSRC, see Appendix 1), in which MTEs who facilitate a video club around mathematics lessons review video-recordings of their own work while using specific observation lenses. The video-club aims at enhancing secondary mathematics teachers' reflective skills and mathematical content knowledge. In the program's workshops, teachers watch and discuss videotaped mathematics lessons of unfamiliar teachers, using a six-lens-framework that focuses discussions on the lesson's mathematical ideas and tasks, the filmed teacher's goals, dilemmas, and beliefs, and the classroom's interactions (see Karsenty & Arcavi, 2017). Drawing on the video-club's design, the video-stimulated recall conversation tool was developed with video as the central representation of practice and incorporates guiding questions inspired by the six lenses.



The second is a tool eliciting teachers' perception of the teacher education learning environment (see Appendix 2), coupled with a protocol for analysing visualisations of the resulting data (see Appendix 3). Designed as a *practical measure* (Takahashi & Norman, 2025), it provides timely, minimally burdensome evidence of features of the professional learning environment that shape teacher learning, including teachers' opportunities to de-privatise their practice, the group's discussion practices, teachers' sense of membership in the community, and relevance to teachers' own contexts, while supporting MTEs' inquiry into their practice.

Crucially, each tool captures distinct dimensions of MTE practice through fundamentally different representational means. Both have the potential to support MTEs' inquiry and professional growth, as demonstrated in our previous studies (Nieman et al., 2023, 2026; Schwartz et al., 2021), albeit in different ways. In this article, we examine the features of each tool that afforded MTEs' professional learning. Building on the idea that comparison is a powerful mechanism for identifying patterns, trends, and themes that may not be apparent when examining a single tool and context, we ask:

- Which problems of teacher education practice does each tool foreground, and which are backgrounded?
- How do the tools afford and limit MTEs' opportunities for sensemaking about problems of practice?

Answering these questions is key to better specify how MTEs learn with representations of practice. To address these questions, we first provide an overview of representations of practice and how they are currently used for MTEs' growth and discuss what we mean by problems of practice. We then present the theoretical framework for analysing how different tools and contexts foreground and background MTEs' problems of practice. Building on this foundation, we compare cases that illustrate the use of distinct representations of practice, highlighting the opportunities and challenges each affords. Finally, we discuss implications for MTEs' professional learning.

Background: Representations of Practice and Mathematics Teacher Educators' Growth

The knowledge base about MTEs has expanded significantly in recent years, showing that their work requires forms of expertise beyond what classroom mathematics teachers need (Chorney et al., 2025; Goos & Beswick, 2021), such as meta pedagogical content knowledge (meta-PCK) (Chick & Beswick, 2018), knowledge about teachers' professional identity (Escudero-Ávila et al., 2021), knowledge of adult learning (Murray & Male, 2005), and specific discussion practices (Kastberg et al., 2025). We are interested in how such expertise can be developed, particularly through the use of representations of one's practice.

Representations of practice (RoPs)—accounts of practice such as video recordings or vignettes—are central to professional education (Grossman et al., 2009). They make aspects of practice visible and thus available for practitioners' inquiry, especially when paired with structured conversational routines. To date, the field has primarily attended to representations of teaching practice that support teachers' inquiry into their work or that of their colleagues'. In this article, we focus on representations of MTEs' practice, designed to support MTEs' inquiry into their own facilitation in initial teacher education settings.

Classic examples of RoPs—such as video recordings (Hollingsworth & Clarke, 2017), written cases (Dreher et al., 2021), storyboarded lessons (Herbst & Chazan, 2011), and student work (Kazemi & Franke, 2004)—have long helped render aspects of practice visible for professional reflection and analysis. More recently, the field has broadened to include immersive formats such as extended-reality environments and 360° video (e.g., Kosko et al., 2021). A key point when examining representations is that the potential of any medium lies not only in what it represents, but also in how it is taken up—how potential meanings are co-constructed in interactions around the medium. Thus, we conceptualise RoPs as comprising both the medium of representation (e.g., video) and the protocols through which the medium is mediated (see also the layers of RoPs suggested by Weingarden & Ehrenfeld, 2025).



Although research on teacher learning with RoPs is abundant, far less is known about how engaging with RoPs shapes MTEs' and facilitators' professional growth. Some studies describe activities and tools designed to support MTEs' growth, such as the use of video (Lesseig et al., 2017) or scripted professional learning scenes (Shure et al., 2022), yet research on these tools and activities is limited. Moreover, existing reports on RoP use rarely examine critical questions about the nature of the RoPs themselves: Which aspects of practice are represented, and how? What choices shape their preparation (Hall, 2000)? Attending to these questions is essential for understanding the kinds of learning opportunities RoPs can create for MTEs.

Addressing such questions requires closer attention to the representational forms themselves. In this regard, Herbst et al. (2016) distinguish the media used in RoPs: found (e.g., classroom student work), transformed (e.g., edited video), and designed (e.g., storyboarded frames). Each type affords different balances of authenticity and control: a found RoP provides a contingent, context-specific portrayal, whereas a designed RoP allows creators to select what and how to represent. Building on this distinction, we propose an additional category: summarising RoPs, namely, representations that synthesise and visualise data about practice. For example, the EQUIP tool can generate reports about participation patterns by race or gender (Reinholz & Shah, 2018). Similarly, surveys can elicit learners' perceptions of a lesson or teacher education session, and their results can be represented in ways that visualise aggregated data about participants' experiences. Practical measures of students' experiences in mathematics discussions, for instance, give teachers aggregated insights into students' views on instruction (Jackson et al., 2025).

In this article, we analyse two tools that function as RoPs for MTEs' inquiry: a survey-based summarising RoP and a video-based transformed RoP, each with distinct representational capacities.

Theoretical Framework

Building on this overview of existing RoPs and the gaps in understanding their use for MTEs' professional inquiry, we turn to a theoretical framework that conceptualises how representations both foreground and background certain problems of practice. The underlying assumption is that representing practice is never neutral: it is a deliberate act of design, which involves semiotic choices. From a semiotic perspective (Halliday & Matthiessen, 2004; Kress & Van Leeuwen, 1996), RoPs are multimodal texts that combine linguistic, visual, and other semiotic resources to construct meanings about practice (Herbst et al., 2024). Choices about what to include, omit, highlight, or background shape how practice is interpreted and what inquiries are likely to occur. Representing, thus, positions viewers in particular ways, affording certain forms of inquiry while constraining others. In this sense, RoPs serve as both windows into- and constructions of- practice, carrying the imprint of the representer's goals and tacit expertise (Herbst et al., 2024). This is done, for example, by deciding on "what is made visible" (Little, 2002)—the aspects of practice represented and thus available for inquiry—and the "degree of completeness and specificity" with which they are portrayed (p. 934).

To discuss these issues analytically we draw on the semiotic infrastructure for practice (SIP) framework (Herbst et al., 2024), which builds on tools from multimodal Systemic Functional Linguistics (SFL; Halliday & Matthiessen, 2004). The SIP framework outlines several multimodal systems of choices involved in representing practice. While originally developed and applied to storyboarded lessons—for example lessons designed by MTEs (Schwartz et al., 2025)—we use it here to analyse RoPs based on different media. Although the framework articulates a broad set of semiotic systems (Herbst et al., 2024), we focus on those most relevant to our analytic goal: examining how RoPs support MTEs' professional inquiry. Because the goal was not to analyse the representations themselves, we examine how particular representational features shape MTEs' inquiry into their practice. We therefore foreground the three systems that most directly influence what aspects of practice become available for sensemaking when engaging with the RoPs—*visibility*, *temporality*, and *individuality* (Herbst et al., 2024).

Visibility concerns the level of detail and the specific features of practice that are foregrounded or backgrounded (as is also described in Little, 2002), and the ways these features are rendered perceptible



(e.g., through transcription, depiction of inscriptions, selective omission). Temporality involves how the pacing and sequencing of events in a lesson or a school year are represented, shaping whether moments are condensed, elongated, or reordered. Individuality concerns how characters, settings, or objects are differentiated in order to convey distinct perspectives or mathematical approaches. For example, whether the students represented in a lesson are individualised by their names, gender, mathematical ideas, and so on. Examining RoPs through these lenses enables a more fine-grained analysis of how different representations support inquiry into MTEs' problems of practice (Schwartz et al., 2025).

Problems of practice are persistent, recurrent, and complex challenges that resist straightforward solutions (Lampert, 2001), such as *balancing participation* and *mathematical goals*. For MTEs, such problems arise in the context of teacher education—for example, choosing a task to use with teachers and supporting them in attending to both mathematical and pedagogical dimensions (Zazkis & Marmur, 2021). Additional problems include balancing responsiveness to teachers' needs with adherence to program requirements or making tacit aspects of teaching visible without oversimplifying them. Like their classroom counterparts, these are not technical problems with definitive solutions but enduring dilemmas that require ongoing professional judgment.

By combining the lenses of visibility, temporality, and individuality with the notion of problems of practice, we examine the interplay between how RoPs are framed and how they are interpreted. This approach enables us to examine how various representational means shape what becomes visible as a problem of practice how it can be addressed.

Before returning to this analytic framework in the Methods section, we first describe the focal tools, how they were selected for analysis, and the contexts in which they were developed and used.

The Focal Tools

This paper focuses on tools that are deliberately placed at the centre of attention for analysis, reflection, or inquiry for MTEs. They are selected as the objects through which practitioners examine and make sense of their practice. In this section we examine two focal tools designed to foster MTEs' learning from evidence of their work: Video Stimulated Recall Conversation (VSRC) and Collaborative Professional Development Survey (CPDS). First, both tools were intentionally designed to surface persistent, high-leverage dimensions of MTE practice that are otherwise difficult to access or discuss, making them productive for examining how representations support professional inquiry. Second, the tools differ in form and in theoretically meaningful ways related to the representational dimensions in the framework—visibility, temporality, and individuality. These differences make the tools useful for examining how different aspects of representation afford or constrain inquiry into MTE practice. Finally, although other tools could support MTE learning, we focus on these two because our involvement in their design and implementation provides analytic access to the representational decisions embedded in the tools and how they function as representations of practice.

The Video Stimulated Recall Conversation

The first author, Schwartz, participated in a research and professional development project, VIDEOM-LM (Karsenty & Arcavi, 2017; Schwartz & Karsenty, 2020), which developed a video club aimed at enhancing secondary mathematics teachers' reflective skills and mathematical knowledge for teaching. In the video-club sessions, teachers watch and discuss videotaped mathematics lessons of unfamiliar teachers, using a six-lens-framework that focuses discussions on the lesson's *mathematical ideas* and *tasks*, the filmed teacher's *goals*, *dilemmas*, and beliefs, and classroom *interactions*.

Building on this design and the centrality of video in the program, the program developed a video-stimulated recall interview (VSRI) protocol in which MTEs reviewed video recordings of their own facilitation with a researcher, pausing to reflect on key decisions, roles, and goals. This protocol has proven useful not only as a research tool (e.g., Schwartz et al., 2023) but also for supporting MTEs'



professional growth (Schwartz et al., 2021). Building on this potential, it was later adapted into a video-stimulated recall conversation (VSRC) between an MTE and a mentor—often an expert MTE. The distinction here is important: while the interview functions primarily as a data collection method, the conversation is designed to serve as a collaborative learning opportunity. Indeed, thinking about VSRC as a professional growth tool is in line with the growing use of video in one-on-one coaching and in coaches' preparation (e.g., Amador et al., 2025; van der Linden et al., 2022).

We consider the VSRC a transformed tool, rather than a found one, as it uses edited video clips to highlight key decision points of MTEs during a video club session. The choices involved in filming and editing—such as focusing the camera on the MTE, or selecting excerpts of the discussion (rather than, for example, teachers' mathematical problem solving)—shape the particular emphases of the tool.

To illustrate how these design choices play out in practice, we describe the original VSRI protocol (which later became VSRC, see Appendix 1). Video-club sessions were recorded, and a few days later the MTE and a researcher watched selected episodes of whole-group discussions together. Before watching, they reviewed a one-page schematic overview of the session activities and their timestamps in the video. The MTE was then asked to pause the video whenever they noticed an important decision, a facilitation role being enacted, or another moment worth reflecting on. These sessions typically lasted 90–120 minutes and focused the MTE's reconstruction of their in-the-moment decision making, with the goal of eliciting underlying resources, orientations, and goals (ROG; Schoenfeld, 2010) that shaped their decisions. Importantly, unlike in the video-club sessions, where the videos feature lessons taught by unfamiliar teachers, the VSRC requires MTEs to examine their own practice and reconstruct the rationales behind their choices.

The Collaborative Professional Development Survey

The second author, Nieman, collaborated on a cross-institutional project aimed at supporting and investigating the implementation of instructional improvement strategies in middle-grades mathematics across university-district partnerships (Jackson et al., 2025). The project designed the CPDS as a practical measure to support MTEs in inquiring into their practice. Unlike research or accountability measures, practical measures are intended to enable practitioners to engage in ongoing inquiry, in which they generate and analyse data to further specify a problem of practice, and test and revise local theories regarding how to address the problem. Administering practical measures and analysing the resulting data is intended to be timely, minimally burdensome, and integrated in educators' ongoing practice (Takahashi & Norman, 2025).

The CPDS is a short teacher-facing survey (3–4 minutes) that elicits teachers' perspectives on features of the professional learning environment known to shape teachers' learning opportunities (Appendix 2). Teachers' responses are aggregated and visualised as a representation of MTEs' practice, providing information about aspects of professional learning within their purview to influence. The CPDS was designed and refined through 18 cycles of piloting, interviews, and revisions to ensure the items were interpretable, meaningful, and actionable (Nieman et al., 2023, 2026).

In use, MTEs administered the CPDS after teacher education sessions and then engaged in conversations with colleagues to analyse PTs' responses. These conversations supported them to interpret teachers' experiences, connect survey data to facilitation decisions, and set goals for future practice. Analyses showed that the value of the CPDS lay not only in the resulting data but also in the nature of MTEs' conversations. Through these conversations, MTEs supported and challenged one another to consider how planning and facilitation decisions shaped teachers' opportunities to learn, while adopting a learning stance towards teaching and their facilitation (Nieman et al., 2023). To support this, the project developed a protocol (Appendix 3; <https://pmr2.org/resources>) that orients MTEs to connect teachers' survey responses with what occurred in the teacher education session and analyse facilitation. For example, the protocol includes questions such as: "What happened in the session that might explain teachers' learning and survey responses? What else happened in the session



that might explain teachers' learning and survey responses?" Based on their analysis, MTEs identified facilitation strengths, goals, and next steps.

Methods

This study employs a qualitative, multiple case study design (Stake, 2013), focusing on two cases that highlight different aspects of MTEs' use of RoPs drawn from their own work. Comparing these cases enables a deeper understanding of the affordances and shortcomings of each tool.

Contexts and Participants

We analyse two cases: one in which an MTE facilitating a teacher video-club used a VSRC, and another in which three MTEs co-facilitating a teacher education course used the CPDS. In both cases, MTEs engaged with evidence from practice in conversation with another MTE. Next, we provide more details on the two contexts. Pseudonyms are used to identify participants to maintain anonymity and confidentiality.

Video stimulated recall conversation context

This case took place in a teacher education college program that used a video-club to highlight mathematics teachers' problems of practice and introduce key content topics. The seven-session, in-person video-club included 17 elementary and secondary prospective teachers (PTs). The sessions were facilitated by one MTE, Naomi, an experienced mathematics teacher who also served as the mentor teacher for the PTs during their field placement at her school. Naomi had experience in school-based leadership, teacher mentoring, and supporting teachers' learning in practice-based settings, but this was her first time facilitating the video-club program.

Naomi was selected from the broader research study because she implemented a professional learning structure adapted for PTs from practicing teacher settings. Her dual role as mentor teacher and facilitator, together with her reflective engagement with the video-club discussions, made this case particularly relevant for examining how representations of practice supported inquiry into teacher education practice.

The research group, including the first author, were not involved in facilitating the sessions. Their role focused on documenting the sessions and conducting the follow-up conversations with Naomi.

Collaborative professional development survey context

The second case took place in a university-based program designed to support mathematics prospective teachers (PTs) in developing equitable teaching practices. The five-session virtual program included six secondary PTs and two mentor teachers. It was co-facilitated by three MTEs (Brita, Neil, and Adam), all faculty in mathematics education with prior experience facilitating teachers' professional learning. While each had experience working with PTs, this was their first year collaboratively facilitating the PT-focused program.

Brita, Neil, and Adam were selected from the broader research project because they were among the few participants in the larger project working specifically with PTs rather than practicing teachers, making their use of the CPDS particularly relevant to the aims of the present analysis.

The researchers (including the second author) were members of the broader project team but were not involved in facilitating the sessions. Their role focused on supporting administration of the CPDS and documenting facilitators' reflective conversations.

Data Sources

The primary data sources included records of MTEs' use of the CPDS and the video-stimulated recall conversation.

- *VSRC context*: Data include two 90-minute conversations, held five months apart (Sessions 2 and 6 out of seven sessions), in which the MTE analysed a video recording of her session in relation to her facilitation. These conversations were video recorded, transcribed, and accompanied by researchers' memos.
- *CPDS context*: Data include two 60-minute conversations, held six months apart (Sessions 1 and 5 out of five sessions) in which MTEs analysed PTs' responses to the CPDS, focusing on their facilitation practice. Each conversation was video recorded, transcribed, and accompanied by researchers' field notes.

Data Analysis

We analysed the video-recorded data in relation to the specific artefacts involved (video recordings of the video-club, and PTs' responses to the CPDS protocol). The analysis examined how MTEs engaged with these artefacts as representations of practice, and how these engagements afforded or constrained their opportunities for inquiry.

First, we transcribed the conversations and annotated the recordings to capture non-verbal cues, such as pointing. We then identified episodes defined by shifts in focus. For each episode, we posed the following guiding questions:

1. Which aspects of MTEs' practice are discussed, and what problems of teacher education practice emerge?
2. How are these topics discussed? Specifically, what evidence do MTEs draw on, and how does this evidence relate to each tool's representational features?

We attended to issues of researcher positionality throughout the analysis. Each author initially analysed the case with which they were most familiar, drawing on contextual knowledge of the setting and participants. The other researcher then probed these interpretations to ensure claims were grounded in the data rather than prior assumptions about the MTEs or tools.

Following the guiding questions, we carried out three layers of coding:

- *First layer*: Identification of aspects of practice discussed and the problems of practice that emerged. This included recurring concerns and dilemmas, with attention to what seemed troubling or unresolved. We also noted features beyond content (pauses, tone, moments of surprise) that revealed how MTEs engaged with the issues. After coding separately, we met to compare and refine categories.
- *Second layer*: Identification of the evidence MTEs drew upon. We examined multimodal data from the conversations, including gestures, references to artefacts, and instances when the MTEs paused the video in the VSRC or shifted focus to a different item in the CPDS. These actions highlighted what counted as significant evidence and how tools mediated reflection.
- *Third layer*: Integration across contexts and coding layers: We compared how problems of practice emerged and discussed when represented through different tools. For example, we examined how issues such as time management were framed differently when MTEs engaged with the VSRC versus the CPDS.

While we acknowledge differences between the two cases—including context, project aims, and the MTEs' roles—we view these differences as productive for comparison, given our shared focus: understanding how each tool was used to support inquiry into MTE practice.

Findings

Problems of Practice that Emerged When Using the Tools

In the VSRC context, a central problem of practice concerned managing time. Naomi repeatedly paused the video during moments requiring real-time decisions about whether to preserve opportunities for PTs' sense-making or to intervene in order to stay on schedule. For example, she described the dilemma



of skipping a ten-minute segment of a videotaped lesson in geometry: such a choice risked overlooking a potentially critical mathematical insight but allowed her to maintain pacing. When this occurred, Naomi often compensated by weaving the skipped idea into later commentary, for instance by saying, "If we had had time to watch further, we would have seen that the teacher is"

In another instance, in the later recorded session, Naomi adjusted her facilitation plan on the spot due to time constraints. She skipped a planned hands-on task (constructing a comparison table of probability representations) and instead directed PTs to watch a video clip she deemed more valuable. Later, she chose to show a pre-prepared solution from the video-club website rather than allowing participants to derive the method themselves. These shifts illustrate how time pressure pushed Naomi toward more efficient but less participatory approaches, marking departures from her intended design.

Closely intertwined with time management was a problem of practice regarding balancing facilitator input with creating space for PTs' contributions. The moments in which Naomi stepped in—either by summarising a point herself or providing a pre-made solution—were typically linked to pacing concerns. Thus, time management and the regulation of facilitator input were mutually reinforcing problems of practice.

A third problem of practice related to the learning environment. Naomi reflected on the importance of sustaining respectful and inquiry-oriented discourse, in alignment with the judgment-free video-club norms. She recognised that subtle shifts in her language shaped whether PTs framed their interpretations of classroom episodes judgmentally (e.g., critiquing a teacher's move) or inquiry-and reflection-oriented (e.g., asking why a teacher might have acted in a certain way). She concluded that upholding norms of professional discourse required her continuous attention.

Finally, Naomi identified problems of practice tied to the content of professional learning. She deliberated over how much mathematical detail to foreground (e.g. when watching a geometry lesson, to what extent she should discuss the focal proof) and how to support PTs in linking mathematical goals and classroom tasks. For example, she grappled with helping PTs interpret classroom interactions non-judgmentally, surfacing geometry-specific issues such as how students write and justify proofs.

In the CPDS context, several problems of teacher education practice emerged as the three MTEs reflected on their work. A recurring theme concerned cultivating a professional learning environment where participants (PTs and mentor teachers) could ask authentic questions and share their experiences. For example, in their Session 1 debrief, the facilitators noted that participants seemed more willing to speak in breakout rooms than in the whole group. Brita recalled one participant directly naming their vulnerability and hoped this would encourage others to do the same. Building on these impressions, Adam proposed adding time at the end of sessions "to let people speak freely." The facilitators framed these observations as early indicators of how participants were beginning to ask questions and share experiences, and their subsequent conversation identified decisions they would make in future sessions to support this goal.

Time management surfaced explicitly in the MTEs' post-session "download", when they shared their recollections from the session. After reviewing participants' open-ended responses to the CPDS item, "*Today's session was relevant to my work as a teacher — Yes/No. If yes, what did you find relevant? If no, why not?*" Adam noted that the responses "alleviated" worries about having "moved too fast." The participants' reflections suggested alignment with the MTEs' learning goals for the session, which reassured the team that the pace had been appropriate. Here, time management was less about in-the-moment trade-offs and more about interpreting feedback after the fact and recalibrating expectations for pacing in future sessions.

Another central problem of practice in the CPDS context involved balancing MTE input with creating space for participants' contributions. Neil reflected on the difficulty of "holding back" to let participants' ideas emerge organically, while Brita elaborated on the tension between holding back and "jump[ing] in." She wondered, "Am I jumping in too much, too soon? ... There's always that balance, right, if we wanna hear everyone's voice and encourage dialogue. But sometimes one of us does need to jump in and spark it." She then praised her colleagues for their skill in "jumping in at appropriate



times." These reflections illustrate how MTEs continuously negotiated whether to intervene or step aside to foster participants' participation.

Shared Problems of Practice

Across both contexts, two related problems of practice emerged: managing time and balancing facilitator support with creating space for participants' contributions. In the VSRC context, Naomi reflected on how she made on-the-spot adjustments such as skipping tasks or providing pre-prepared solutions. In the CPDS context, facilitators addressed pacing retrospectively, drawing on CPDS data to reassure themselves that the session had not "moved too fast" and to recalibrate for the future. In both cases, time and facilitator input were tightly intertwined: for Naomi, pacing concerns often drove her to step in with direct input, while for the CPDS facilitators, worries about pacing informed their reflections on when to "hold back" or "jump in." Together, these cases illustrate how time management and the regulation of facilitator input emerged as shared and interconnected problems of practice, although they were experienced and addressed through different means and in distinct teacher education contexts.

What is backgrounded in both contexts?

While distinct problems of practice surfaced in each setting, both RoPs also backgrounded critical dimensions of MTEs' work—particularly those related to PTs' learning. Neither the VSRC nor the CPDS provided MTEs with artefacts of PTs' learning processes to examine during reflection. In the VSRC, facilitators can observe what PTs say during discussions, and in the CPDS, they gain access to what PTs report as relevant through survey responses. Yet in both cases, the RoPs did not include artefacts of PTs' actual work, comparable to student work in classroom settings, that MTEs could inquire into as evidence of PTs' learning. Just as teachers learn by examining student work to make sense of learners' thinking and inform instructional decisions (e.g., Kazemi & Franke, 2004), MTEs would benefit from artefacts that make PTs' reasoning visible in similarly concrete ways. The absence of such artefacts highlight an additional problem of practice: MTEs must make inferences about PTs' learning indirectly, relying on traces of participation or self-report rather than on tangible records of mathematical reasoning. We return to this shared limitation in the Discussion section, where we identify key considerations for selecting RoPs.

How the Tools Made Problems of Practice Visible

In this section, we address the second research question: *How do the tools afford and limit MTEs' opportunities for sensemaking about their practice?* Drawing on key constructs from the SIP framework, and after describing the main problems of practice which emerged in both contexts, we examine the semiotic means of each RoP and how they functioned in the MTEs' interaction with the RoP. In other words, we focus on moments where MTEs engaged with problems of practice, articulated insights, or considered possible next steps, and analyse how the focal RoPs mediated what was foregrounded or backgrounded in those moments, by using the SIP systems visibility, temporality, and individuality. We first illustrate this with Naomi's reflections in the VSRC, then turn to Adam, Brita and Neil's use of the CPDS, and conclude with a comparative discussion.

How the video stimulated recall conversation made problems of practice visible

In analysing Naomi's interaction with the video during the VSRC, we noticed several ways in which the video, alongside the focal questions in the conversation, supported her in noticing problems of practice and reflecting on them. Three categories emerged in this analysis: passing of time (which relates to temporality), reference to exact words and gestures (*visibility*), and attention to PTs' actions (*visibility and individuality*).

Passing of time (temporality) Both the session graph and the video supported reflection on issues related to time. At the beginning of Session 2, using the session graph as a reference, Naomi pointed to



the page and reflected on her time management, noting: *"I'm starting to recall that from this point on, I actually needed twice as much time to get through everything I had planned."*

When viewing Session 6, Naomi repeatedly paused the video to comment on moments when she had cut or altered her plan due to time pressure. For instance, after only a few seconds of watching, she recalled: *"Here, when I tell them, 'I've achieved my goal, I want to stop the video', that wasn't part of the original plan ... There's always this dilemma about whether to stop now, because I might be missing something"*. Later in the same session, she reflected on skipping a hands-on task and instead showing a ready-made solution from the website: *"I had hoped they would construct the method on their own, but time pressure pushed me to rely on the resource."* These moments illustrate how Naomi experienced time management as an ongoing tension. In particular, they highlight how RoPs that make time visible, for example, through time stamps on the session graph or the running clock on a video, foreground this dimension of facilitation. And because episodes from the session video are played in real time, they evoke the urgency of pacing and the felt passage of time more vividly than still representations such as screenshots from the recorded session.

Reference to exact words and gestures (visibility) The video also foregrounded Naomi's reflection on the precise words and gestures she and the participants used, and how these linguistic and embodied choices related to her facilitation goals. Given the video-club program's emphasis on non-judgmental talk about others' practice, the session video became a resource for Naomi to reflect on how both she and the PTs positioned the filmed teacher through language.

For example, in Session 6 Naomi paused to analyse her discourse: *"My role as a facilitator here was by saying to them, 'And what else [happened in the lesson video]? And what else?' Kind of drawing it out in order to get to that point,"* and she expressed satisfaction when a participant eventually articulated the intended idea. When watching another episode, she pointed to the screen and declared: *"Right or wrong is not part of our lexicon."* Naomi stressed how much she works to suppress evaluative tones and instead redirect talk toward inquiry: *"I don't let the judgmental tone surface—I kind of squash it ... I said, 'right or wrong'—it's not how we speak in the video-club."* Later, she described how difficult it was to discuss a mathematical mistake made by the filmed teacher, in a neutral fashion: *"Emotionally, that was really hard for me—on a personal level. Because I think this is something that simply should not happen in the classroom. But here I'm coming in as a facilitator ... trying to examine what the teacher's goals were"*. Watching herself navigate this clash of identities (as teacher vs. facilitator) underscored how she used specific language to uphold norm of inquiry over judgment.

These moments also offered Naomi an opportunity to take pride in her developing practice. Watching herself, she remarked: *"It's amazing to look at it ... suddenly when you look from the side and you hear what you said, it feels like the words make sense"*. The construct of visibility clarifies how video affords a return to the exact phrasing, tone, and gestures of facilitation, enabling reflections that are not only tied to these details but also deepened by their precision—making it possible for MTEs to scrutinise the subtle moves through which facilitation norms are enacted. Finally, the opportunity to hear and analyse her own phrasing also supported Naomi's recognition of how her teaching expertise could inform her MTE identity. Thus, the VSRC afforded her both a mirror for refining discourse practices and a bridge for reconstructing how she is carrying forward strengths from her teaching repertoire into facilitation.

Attention to prospective teachers' actions (visibility and individuality) Whereas the previous category highlighted Naomi's discourse, here the focus shifts to what the video revealed about PTs. Although the VSRC protocol is structured around MTE goals and decisions, the video inevitably brought PTs' responses into view. Their words and gestures become visible, allowing Naomi to notice dimensions of participant engagement that might otherwise have escaped her attention in real time (and that were not included in the VSRC protocol).

For example, in Session 2 Naomi watched herself facilitating a lively discussion, but her attention shifted when she noticed on screen that two PTs were still working intently on the mathematical task even as she spoke (Figure 1, left-hand annotated screenshot). Pointing to the video, she remarked: *"Look how they keep trying, they can't seem to break away from ... from succeeding at it"* (Figure 1,



right-hand screenshot). In this moment, the video served as a window into participants' persistence, something she had not fully realised while managing the discussion live.

Later, Naomi reflected on how individual PTs' identities shaped their contributions. She paused the video after one PTs' contribution in a discussion about the importance of planning and recounted that the PT, a career-changer, was drawing on his prior professional experience to argue for postponing decisions until the last possible moment. For Naomi, this was a vivid example of how career-changers bring frameworks from other domains into teacher education, how these imported assumptions may or may not translate into educational practice, and how she should address those moments, especially when they happen five minutes before the session ends. Indeed, SIP's construct of *individuality* helps articulate how video expands what counts as relevant to a problem of practice: not just the facilitator's moves, but also the persistence, identities, and value systems of the PTs themselves, which are made somewhat visible through the video.



Figure 1. Left: Screenshot from session 2 showing two teachers engaged in the mathematical task during the discussion. Right: Naomi points to the screen to highlight the teachers' persistence, which she did not capture in real time.

Taken together, these examples show how the VSRC made Naomi's practice visible in ways that were deeply shaped by the affordances of video. Temporality allowed her to re-experience the pacing and sequencing of events, visibility gave her access to the exactness of discourse and gesture, and individuality highlighted the distinctive voices and commitments of the PTs. In contrast to the CPDS, the video foregrounded how facilitation unfolded in real time and in interaction with participants while still leaving PTs' experiences less accessible as objects of inquiry within the video-based RoP itself.

How the Collaborative Professional Development Survey Made Problems of Practice Visible

In analysing Adam, Brita, and Neil's interaction with the CPDS, we noticed several ways in which the CPDS, alongside key components of their conversation, supported them in attending to problems of their practice and reflecting on them. We identified two categories: (1) *foregrounding PTs' experiences (visibility and individuality)* and (2) *foregrounding change over time (temporality)*.

Foregrounding participants' experiences (visibility and individuality)

As a PT-facing survey, one affordance of the CPDS is that it makes visible PTs' experiences of the professional learning environment. Following Session 1, Adam, Brita, and Neil administered the CPDS to the eight participants (six PTs and 2 mentor teachers) in the session. They began their debrief by sharing recollections centred on goals for the learning environment, particularly building community and fostering the group's comfort in asking questions and sharing experiences. For example, Adam noted a contrast between settings, observing that PTs seemed more comfortable in breakout rooms than in the larger group. Brita recalled a PT introducing a comment with, "*I'm going to be vulnerable here ...*", and reasoned that this explicit statement might encourage others to take similar risks. Together, these

reflections identified a focal problem of practice: how to support the group's comfort in sharing ideas with one another.

The facilitators then turned to participants' CPDS responses. On the item, *"In today's session, I felt like I could share something I'm wondering about my own teaching,"* seven of eight participants responded, "Yes." Brita drew attention to one participant's elaboration, *"I have severe anxiety,"* and connected this detail to facilitation: *"It's ... important for us to know and recognise that and be thinking about how to support that person."* This prompted Adam to suggest creating an explicit option not to share in whole-group conversations, while Brita proposed offering "safer" modalities, such as adding ideas to a shared document or using the chat function.

This exchange illustrates how the CPDS foregrounded participants' experiences in ways that complemented MTEs' impressions from the session, creating a richer evidence base for inquiry into practice. Because the survey data were anonymous rather than individualised, participants could disclose sensitive challenges—such as anxiety—without fear of judgment, and the MTEs could identify collective patterns of experience that might otherwise remain hidden. In terms of *individuality* within the SIP framework, this anonymity thus carried significant affordances, protecting participants' sense of safety and encouraging candid feedback. At the same time, it also introduced limitations, as the MTEs could not link particular responses to specific individuals or trace how their perspectives evolved across sessions.

Foregrounding change over time (temporality)

One distinctive affordance of the CPDS lies in its capacity to reveal patterns of change across sessions. The CPDS itself does not capture the real-time passage of events within a session; instead, its design makes change visible by enabling comparisons across administrations. By administering the survey in both Session 1 and Session 5, Adam, Brita, and Neil were able to trace trends and consider whether shifts in their facilitation led to desired improvements.

Following Session 5, the MTEs first reviewed participants' responses from that session and then examined them alongside responses from Session 1 to identify patterns of change (Figure 2). More participants reported being able to share mathematical and pedagogical ideas they were unsure about, ask for elaboration, and push back on an idea in both whole-group and small-group settings. More also indicated that they could share a problem of practice. The data confirmed the MTEs' sense that their adjustments had improved the learning environment. Adam remarked, *"This is good to see,"* while Brita conjectured that participants' growth would continue as they gained more shared experience. In summary, although the CPDS does not depict the flow of time within a single session, it foregrounds evidence of change across sessions, making it a valuable tool for iterative improvement in facilitation practice.

Comparative Summary

The analyses of the VSRC and CPDS show that each tool foregrounded and backgrounded different aspects of facilitation practice. Table 1 brings these findings together, summarising the distinct representational affordances and limitations of each tool in terms of what became available—or remained less accessible—for MTEs' inquiry into problems of practice.



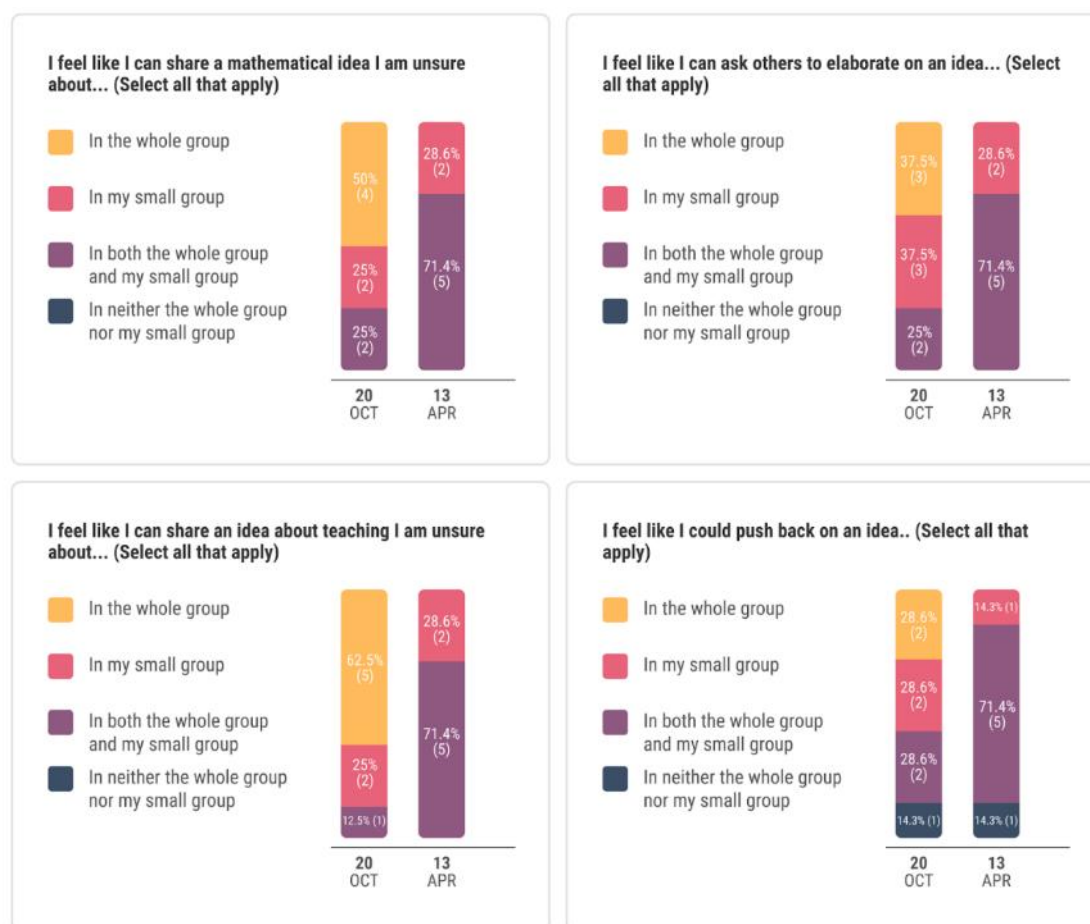


Figure 2. Responses to Items 1–4 in the Collaborative Professional Development Survey protocol, Session 1 ($n = 8$) and Session 5 ($n = 7$).

Table 1

Comparative Summary: What Each Tool Foregrounds and Backgrounds

Dimension	VSRC	CPDS
What is foregrounded?	Pacing and sequencing of facilitation (temporality) Precise words and gestures (visibility) PTs' specific contributions (individuality) Lived contingency of facilitation	PTs' perspectives on the learning environment High-leverage aspects of facilitation (e.g., discussion practices, relevance) Patterns of change over time (temporality) Anonymity that supports candid feedback and disclosure of sensitive experiences
What is backgrounded?	PTs' perspectives on the learning environment are not directly visible Change over time is difficult to capture Attention to high-leverage problems of practice depends on facilitator interpretation rather than being structured by the tool	Mathematical content of PD sessions Fine-grained interactional flow; Linking responses to individuals or tracing perspectives across sessions (individuality)



Discussion

This study examined how two tools—the VSRC and the CPDS—supported MTEs in reflecting on their facilitation practice. We organised the findings around two guiding questions. The first asked which problems of practice emerged in each context, and we identified recurring challenges around managing time, balancing facilitator authority with responsiveness, and fostering norms of inquiry-oriented discourse. The second asked how the tools afforded and limited opportunities for sensemaking about these problems of practice. Using the SIP framework, we showed how each representation made certain problems of practice visible while backgrounding others.

Shared Affordances of the Tools: Making Connections Between Content, Participants, and Facilitation

A central contribution of the analysis lies in showing how different representations of practice (namely, a video recording coupled with a conversation protocol) may shape what MTEs could see and say about their work. Despite their differences, both the VSRC and the CPDS enabled MTEs to access session particulars and link those particulars with broader problems of practice.

The VSRC, built around video, foregrounded one-to-one temporality, precise discourse moves, and participants' individual contributions. It allowed Naomi to revisit facilitation in its lived contingency, to recognise adaptive decisions made under time pressure, and to notice how PTs' identities informed their responses. However, it risked privileging what was most visible on screen.

The CPDS, in contrast, foregrounded PTs' perspectives on the learning environment and patterns of change across sessions. Its survey items highlighted dimensions of facilitation linked to research on teacher learning, and its longitudinal structure allowed facilitators to trace growth in PTs' experiences of participation and safety. Yet the CPDS backgrounded the mathematical content of the professional learning and the fine-grained interactional flow, limiting facilitators' ability to connect PTs' reported experiences to specific facilitation moves.

Together, these findings illustrate the complementarity and limitations of the two tools, adding to the limited literature on how MTEs learn from representations of practice (Borko et al., 2025; Lesseig et al., 2017; Shure et al., 2022) by showing how different representational forms afford distinct opportunities for inquiry into facilitation practice. The VSRC anchors reflection in the immediacy of video evidence, while the CPDS provides structured access to participants' perspectives over time. Each representation thus opens different windows onto problems of practice, with inherent blind spots: the VSRC's reliance on retrospective reconstruction emphasises what is visible and audible, while the CPDS limits access to embodied and interactional dynamics. This is in line with the understanding that every representation is necessarily partial (Little, 2002).

Using semiotic infrastructure for practice to analyse various representations

Our analysis demonstrates how the SIP framework (Herbst et al., 2024) applies beyond storyboard-based RoPs (Schwartz et al., 2025) to video and survey data. The SIP categories of temporality, visibility, and individuality enabled us to articulate how the VSRC and CPDS each foregrounded certain problems of practice while backgrounding others. This analysis does not extend SIP conceptually but shows its utility for understanding the affordances and limits of diverse RoPs. In this way, SIP becomes a tool not only for characterising representations themselves, but also for articulating what they make possible for professional inquiry into practice.

This focus on representational affordances aligns with prior scholarship that underscores the importance of particulars in teacher inquiry. Horn (2010), for example, argued that access to details of practice provides educators with an evidence base for instructional inquiry. In our case, the particulars of video in the VSRC and of survey responses in the CPDS offered distinct evidence bases for MTE inquiry. Both the VSRC and CPDS were grounded in what research suggests matters for facilitation: in



the CPDS, dimensions of productive professional learning environments; in the VSRC, the study of observable facilitation moves in relation to underlying goals and orientations (Schoenfeld, 2010).

Lifting representations of practice to the mathematics teacher educator level

In addition, the research contributes to a growing body of research that considers how RoPs can be “lifted” (Prediger et al., 2019) from the teacher level to the MTE level (e.g., Borko et al., 2025; Jackson et al., 2025). While prior literature focuses on RoPs for teachers’ learning (e.g., Weingarden & Ehrenfeld, 2025), we examined how similar tools can support MTEs’ reflection on facilitation. This lifting acknowledges the nested nature of practice: just as teachers need opportunities to reflect on classroom practice, MTEs require opportunities to reflect on their facilitation practice in professional learning settings. By examining how video and survey-based RoPs supported such reflection, the analysis shows how the design logics of RoPs can be adapted across levels of professional practice.

Supporting mathematics teacher educators’ professional growth: Key considerations

To sum, supporting MTEs in developing expertise is essential but often insufficiently structured. Mechanisms such as mentored facilitation in communities of practice, collegial reflection, and collaborative research teams can provide critical support, yet MTEs frequently face professional isolation, lack of institutional structures, and limited attention to their own growth (Even, 2014). Representations of practice can serve as one way to structure opportunities for reflection, but their design and use matter. Thus, a main aim of this article is to provide guidance toward selecting RoP for reflection. From our comparative analysis, we identified several key considerations for selecting and designing tools for MTEs:

- *Foregrounding problems of practice.* The RoP and its associated conversation should orient reflection toward concrete problems of facilitation, creating an evidence base for inquiry.
- *Connecting facilitation, content, and participants.* Tools should enable MTEs to see connections among facilitation moves, participants’ experiences, and the mathematical and pedagogical content of professional learning.
- *Capturing participants’ learning.* Professional learning settings often lack artefacts analogous to student work – analysable evidence of participants’ learning. Reconceptualising exit tickets or teachers’ written reflections as RoPs may therefore expand what can be made visible for MTE inquiry.
- *Using multiple tools in tandem.* Because each RoP foregrounds only some dimensions of practice, MTEs’ professional growth is best supported when multiple tools are used together, creating a richer evidence base.

Conclusion and Future Directions

This analysis lays the ground for future lines of inquiry, such as: What other RoPs could be adapted from the teacher level to the facilitator level? What differences emerge when the same RoPs are used with teachers compared to MTEs? Which artefacts produced in professional learning can most effectively make visible teachers’ learning and development of practice? And how might the subject-matter specificity of mathematics be represented more fully?

Overall, the study maintains that, similar to teachers, MTEs need opportunities to develop their practice. Carefully designed representations of practice, including generative conversation protocols, can provide valuable entry points for inquiry and growth. By analysing the affordances and limits of the VSRC and CPDS, we identified how different RoPs make distinct problems of practice visible while backgrounding others, and we offered key considerations for designing and selecting such tools. In particular, we showed how both tools supported MTEs in making connections among content, participants, and facilitation—a shared affordance that underscores the promise of representations of practice for structuring inquiry into facilitation. Our hope is that this work encourages further exploration of how representations can support the ongoing development of MTEs’ professional practice.



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Competing interests

The authors declare there are no competing interests.

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Appendix 1: Protocol for Using the VSRC

This protocol is meant to support MTEs who are facilitating a video club, in learning from watching recorded video of their practice.

Opening statement:

The conversation is meant to open a window into your thoughts about yourself as a mathematics teacher educator. You, as a mathematics teacher educator, are the focus.

The conversation will start with a general question, and then we will look together at the session graph. Due to time constraints, several sections of the session have been selected to focus on. During the conversation, in some sections you will be asked to stop the video whenever you see something that interests you or that you want to comment on. In other sections, the fellow MTE will stop the video and ask a question, for example: What do you see in this scene? What do you think about what you see? There may be other questions, but this is the general guiding question. Feel free to say whatever comes to your mind. There are no incorrect answers.

- **Session graph:**

The conversation starts with reviewing the session graph, to help the MTE recall what generally happened in the session.

- **Video-based questions:**

The expert MTE will display the video sections they chose beforehand. Before each section, they will ask a question or give instructions for that specific part. Examples of video-based questions include:

- What do you see in this section? What do you think about what you see? You can stop the video whenever you see an interesting issue for you as an MTE.
- Watch this 12-minute section, in which you conduct the discussion. Stop each time you identify an MTE's role in the discussion, and if you can, characterise this role.
- In the next section, stop the video whenever you identify an important question that you asked, and explain what made you ask this question and what happened as a result.
- Please stop the video whenever you identify a decision you made and say what you think about it.

Closing question (without video):

This question is meant to evoke comparisons between sessions, to help MTEs notice and reflect on changes in their practice.

For a conversation early in the year, the question might be:

We have now watched the video of your second session as an MTE. Can you say something about this session in comparison to your first session? Do you recognise any differences? If so, what do you think caused these differences?

For a conversation toward the end of the year, the expert MTE will ask the MTE to compare their practice at the beginning of the year with a later session.



Appendix 2: CPDS, Annotated with Key Aspects of Professional Learning

Key aspects	Survey items
Discussion practices employed in the group	I feel like I can share a mathematical idea I am unsure about ... (Select all that apply.) ▪ in the whole group ▪ in my small group ▪ in neither the whole group nor my small group I feel like I can share an idea about teaching I am unsure about ... (Select all that apply.) ▪ in the whole group ▪ in my small group ▪ in neither the whole group nor my small group I feel like I can ask others to elaborate on an idea ... (Select all that apply.) ▪ in the whole group ▪ in my small group ▪ in neither the whole group nor my small group I feel like I can push back on an idea ... (Select all that apply.) ▪ in the whole group ▪ in my small group ▪ in neither the whole group nor my small group
Opportunities participants have to make their practice public and thus available for inquiry	I would be open to sharing the following with this group of participants and leaders: (Select all that apply.) ▪ an anecdote about what my students said or did ▪ an anecdote about something I said or did when teaching ▪ samples of my students' written work (examples: exit tickets; photos of students' work) ▪ a math task or activity ▪ video of my students solving problems ▪ video of my teaching ▪ I would not be open to sharing any of the above. I would be open to inviting members of this group of participants and leaders to join a lesson of mine. ☐ Yes ☐ No In today's session, I felt like I could share something I'm wondering about my own teaching (examples: a question, a dilemma, a challenge). ☐ Yes ☐ No If no, why not?
The extent to which participants see themselves as valued members of the group	In today's session, I felt like my ideas were valued. (Select all that apply.) ▪ in the whole group ▪ in my small group ▪ in neither the whole group nor my small group
The extent to which participants experience the PD focus as relevant to and possible to try in their own instructional contexts	Today's session was relevant to my work as a teacher. ☐ Yes ☐ No If yes, what did you find relevant? If no, why not?

Note: This is one version of the CPDS, specifically for use by MTEs with PTs. See <https://pmr2.org> for other versions of the survey for facilitators working with in-service teachers in professional learning settings.



Appendix 3: CPDS Protocol

This protocol is meant to support facilitators who are investigating one set of participants' responses to the CPDS. When analysed alongside other types of data from a session, participants' survey responses can help facilitators¹ (a) decide whether a facilitation change they have made improves participants' learning opportunities and (b) identify changes to make in facilitation of future sessions.

After administering the CPDS in a professional learning session:

- **Revisit norms** — Discuss norms around analysing practice and data.
- **Revisit the facilitation goal and goals for participants' learning and experiences** — Remind yourselves of the goals for the session.
- **Reflect on participants' learning and experiences during the session** — Analyse written work notes on discussions or any other data collected to document what they learned in the session.
 - How do participants' learning and experiences compare to what you hoped for?
 - What initial, tentative explanations do you have for *how the PD learning environment influenced participants' learning and experiences*?
- **Predict participants' survey responses based on the session** — Predict how participants responded to the focal survey items *before* analysing data. This will help you understand whether progress was made towards the facilitation goal.
- **Analyse participants' survey responses** — Analyse participants' survey responses to elaborate and/or revise your tentative explanations regarding *how the professional learning environment, including your facilitation, shaped participants' engagement and learning in the session*.
 - Did participants respond to focal items in the ways you anticipated? Is there anything surprising?
 - What happened in the PD that might explain participants' learning and survey responses? What else happened in the PD that might explain participants' learning and survey responses?
 - Look across participants' responses to the remaining items. Do participants' responses to any of the other items surprise you? Why?
- **Assess progress towards the facilitation goal**—Drawing on the above analyses, assess progress towards the facilitation goal and identify current facilitation strengths and potential areas for improvement.
- **Identify a facilitation goal and goals for participants' learning and PD experiences for the next PD session**—Based on your analyses of participants' learning in relation to your facilitation, identify goals for the next session.
 - What is a next step for participants' learning?
 - What is a next step for your facilitation?
- **Decide when it would be useful to administer the practical measure again in relation to the next steps and goal(s).**

Note: See www.pmr2.org/ for other protocols for data use.

¹ The CPDS was developed for facilitators of professional learning, including MTEs.

