Mathematics Teacher Educator Use of Structured Learning Routines and Curriculum Materials to Support Teacher Learning

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This article explores how Mathematics Teacher Educators (MTEs) can effectively engage classroom teachers in professional learning. Rooted in empirical data, we hypothesise that MTEs can foster productive teacher conversations by leveraging ambitious and educative curriculum materials in conjunction with a structured set of professional learning routines. The study details the implementation of a "lesson analysis cycle", which includes collective planning, lesson enactment, and reflection. Analysing interactions between MTEs and groups of same-grade level teachers, the findings reveal that this approach facilitates conversations characterised by their integration of student thinking, mathematical content, and pedagogy. These discussions delve into deep instructional issues with high levels of specificity, evident in instances of negotiating learning goals, identifying critical lesson moments, and debriefing teaching enactments. We interpret the findings as confirming our hypothesis and propose that the curriculum materials plus the structured learning routines offer a replicable model for MTEs to cultivate rich professional learning.

Keywords · mathematics teacher education research · mathematics teacher educator · teacher professional learning · educative curriculum

Introduction

Perhaps the biggest challenge facing Mathematics Teacher Educators (MTEs) who support teachers' efforts to improve their practice is to engage with teachers in ways that matter, in ways that have a direct, positive impact on teaching practice. What kinds of professional learning opportunities should they provide, what kinds of activities should they pose, what kinds of conversations should they facilitate? The literature is filled with advice for MTEs (e.g., Charalambous & Delaney, 2019; Gibbons & Cobb, 2017; Lampert & Graziani, 2009). Research has identified features of productive instructional conversations—those that integrate content, pedagogy, and student thinking while addressing deep issues with specificity—and linked these conversations to improvements in teaching practice (Russell et al., 2020; Stein et al., 2022). Less is known, however, about specific tools and structures MTEs can use to reliably facilitate such conversations, particularly when working with multiple teachers simultaneously. Although we know what productive conversations look like, we have limited models that MTEs can replicate to consistently generate these conversations (Borko et al., 2014; Karsenty et al., 2023; Prediger et al., 2021).

This study addresses the question:

> *How do MTEs use structured professional learning routines and ambitious curriculum materials to facilitate productive instructional conversations with groups of classroom teachers?*

Specifically, we examine how MTEs use these resources to facilitate conversations that integrate mathematical content, student thinking, and pedagogy—the features prior research has linked to improvements in teaching practice. Our goal is to document the nature of these conversations and to develop a replicable model of how routines and curriculum work together to support facilitation.

In this article, we describe a structured set of learning routines that encourage MTEs to engage groups of teachers in planning, enacting, and reflecting on individual lessons from their curriculum. The routines help MTEs plan learning activities that facilitate conversations about the lessons in extraordinary detail focusing on the mathematical content, students' thinking, and appropriate pedagogical moves (Cohen et al., 2003). The evidence we share suggests that these routines support MTEs to engage teachers in the kinds of conversations about teaching their curriculum that make a difference for teachers' practices (Russell et al., 2020; Stein et al., 2022). The contribution outlines a replicable model that MTEs can use to reliably generate these kinds of productive instructional conversations. This model builds on research on MTE facilitation practices (Borko et al., 2014; Gibbons et al., 2021; Schwarts et al., 2021) and considers how MTEs work within a professional learning routine to engage with, and leverage the promise of, ambitious and educative curricula as resources for teacher learning (Davis et al., 2017; Horn et al., 2025; Lampert et al., 2013).

Conceptual Framework and Literature Review

Conversations That Matter for Teaching

Teachers learn through interacting and collaborating with others (Lave, 2019; Wenger, 1998). Rich conversations about teaching allow teachers to collectively build their capacity through working with, talking to, and learning from colleagues (Lipscombe et al., 2023; Mac & Albertsen, 2020). When teachers work collaboratively to combine and examine their knowledge and understandings (Kauffeld, 2006; Little, 2002), they can initiate new patterns of action, shared understanding, and increased knowledge (McCotter, 2001; Van den Bossche et al., 2006).

We draw on work at the University of Pittsburgh's Learning Research and Development Center to identify key ingredients in conversations among teachers and MTEs that lead to more ambitious forms of teaching (Coburn et al., 2012; Coburn & Russell, 2008; Russell et al., 2020; Stein et al., 2022). "Ambitious teaching" has been referred to as teaching that focuses on students' deep conceptual understanding, responds to student thinking, and involves students in the co-construction of knowledge (Lampert et al., 2011). To learn this form of teaching, Russell and colleagues (2020) found that the most productive conversations are deep and specific conversations that "focus on the interactions between teachers, students, and mathematics" rather than on 'what the teacher will do'" (p. 443). References to "teachers, students, and mathematics" point to Cohen's and colleagues' (2003) description of the "instructional triangle," a visual depiction of three primary ingredients that shape the learning opportunities experienced in the classroom. When teachers plan for the interaction among these ingredients, they are more likely to engage in high-quality instruction that yields richer learning opportunities for students (Herbst & Chazan, 2012; Russell et al., 2020; Stein et al., 2022).

The findings from the Pittsburgh researchers went beyond claiming that productive teacher conversations are situated within the instructional triangle. They also identified particular features of these conversations that "predicted teaching improvement, specifically increased opportunities for students to engage in conceptual thinking" (Russell et al., 2020, p. 439). The researchers extracted these features from conversations in which a teacher and MTE were planning a lesson. Three features were found to have been especially significant in identifying conversations that matter for teacher learning (Stein et al., 2022). First, the conversations were *deep conversations*, meaning they went beyond surface level considerations, like pacing or logistics, and tackled substantive questions about why particular mathematical ideas matter, how students are likely to reason about them, and what pedagogical decisions will best support that reasoning. Second, the conversations were *specific*. They exhibited high levels of specificity, such as naming particular students, describing specific tasks, anticipating student responses, or planning precise pedagogical moves—rather than discussing general pedagogical strategies (e.g., use turn-and-talk more often). For example, the MTE and teacher might have predicted how particular students would be thinking about the key mathematical idea before the teacher planned to ask a pivotal question. Third, the conversations regularly integrated the three vertices of the instructional triangle rather than treating each aspect separately. The previous example—anticipating students' thinking about an important mathematical idea before the teacher asks a pivotal question—integrates attributes of students, content, and pedagogy.

The goal in this article is to report on an approach MTEs can take to facilitate conversations with teachers that possess three critical features: (1) tackling deep issues of teaching (2) with high levels of specificity (3) by integrating the three dimensions of the instructional triangle—students, content, and teaching. Central to the approach we describe are MTEs using a professional learning routine that uses an ambitious and educative curriculum to guide the conversations.

Resources that Support Productive Conversations

Role of curriculum

Teachers in the United States often use curriculum materials to plan their lessons. These materials usually come in the form of textbooks, with one version for the students and another, elaborated version, for the teacher referred to as the teachers' guide. Conversations that focus on the content of the textbook are relevant and immediately useful to teachers. In addition, if the curriculum is both ambitious and educative, it serves as a rich resource for studying content, planning instruction, and anticipating student thinking (Davis et al., 2017). Ambitious curricula contain mathematical tasks that could foster conceptual understanding, attend to student thinking, and position students as co-constructors of knowledge (Horn et al., 2025; Lampert et al., 2013). Educative curricula provide information that teachers use to learn about the mathematical and pedagogical objectives of the lesson, to implement the lesson consistent with intent of the authors (Davis & Krajcik, 2005; Drake et al., 2014). In these ways, the curriculum supports rich conversations among teachers when examined collaboratively.

A curriculum, however, is not enough. No matter how ambitious and educative a curriculum, it is not enough for MTEs to hand the materials to teachers and ask them to have productive conversations about teaching (Drake et al., 2014). Teachers benefit from structured opportunities to engage in these conversations. We argue that a structured set of learning routines–organised around a sequence of questions and tasks–offers one effective tool for facilitating these conversations. When anchored in the curriculum, we believe these conversations are most productive among teachers teaching the same grade level using the same curriculum.

Role of professional learning structures

Facilitating teacher conversations that engage deeply with teaching, students' thinking, and mathematical content is complex and demanding work (Gibbons & Nieman, 2024). This work can be facilitated by a structured set of learning routines. Routines can be defined as "repetitive, recognisable patterns of interdependent actions, carried out by multiple actors" (Feldman & Pentland, 2003, p. 94). Researchers have described different learning routines for teachers, including those involved in "lesson study" and "coaching cycle." Both structures have been shown to enable conversations between MTEs and teachers that support instructional improvement (Fernandez & Yoshida, 2012; Lewis et al., 2012; Russell et al., 2020; Stein et al., 2022) and position curricular lessons as a central resource for teacher learning.

Lesson study involves educators collectively planning for teaching, observing a teacher enact the planned lesson while other teachers take careful notes on students' reasoning. Afterwards they collectively analyse the teaching in order to re-examine the decisions made while planning and annotate the plan for future enactments (Fernandez & Yoshida, 2012; Stigler & Hiebert, 1999). Instructional conversations often focus on anticipating students' thinking in response to instructional tasks and, during the reflection, correcting the predictions to better plan for future lessons.

In the coaching cycle described by Bean and colleagues (2010), and Russell and colleagues (2020), an MTE engages one teacher in a three-part cycle: the MTE plans with the teacher for an observation, observes, and then discusses the observation with the teacher. The discussions before and after the lesson is taught afford MTEs the chance to talk with teachers about content, students' thinking, and pedagogy.

In both lesson study and the coaching cycle, the activities within the planning phase include setting learning goals, selecting tasks aligned with those goals, anticipating how students will approach the task, and identifying moves the teacher can make to advance student thinking toward the goals of the lesson (Fernandez & Yoshida, 2012; Stein et al., 2022). Both routines also include post analysis of teaching, where MTEs can use carefully selected student responses to reflect on gaps between learning goals and student thinking (Hiebert et al., 2007; Stein et al., 2022). MTEs play a critical role in implementing these routines, routines that structure and support teachers' professional conversations. Their work extends beyond merely enacting the steps of a routine or ensuring that teachers remain on task. As knowledgeable others, they: monitor participation (Karsenty et al., 2023; van Es et al., 2014); press for explanation, evidence, and specificity (Borko et al., 2014; Schwarts et al., 2021); and contribute their own perspectives and expertise (Gibbons et al., 2021).

Research Context

The Professional Learning Project

This study is nested within a four-year project called *Transforming and Understanding from Professional Learning*, which sought to understand how teachers translate experiences during professional learning into teaching that ultimately improves student learning. The Blinded project, which emerged from a partnership between MTEs at a statewide mathematics coalition (project facilitators) and researchers at a state university, employed lesson analysis cycles guided by MTEs using ambitious and educative curricula to engage Grades 4–7 teachers (n = 36) in a video-based professional development (PD) program. The study reported in this article focused on the conversations of Grades 4–5 teachers and the MTEs who supported them. The curriculum materials used by the teachers in this study was *The Bridges in Mathematics* curriculum (Math Learning Center, 2019), an educative curriculum, aligned with mathematical content and practice standards, emphasising mathematical discourse, fluency, and critical thinking to prepare students for future success (Lunde, 2021).

The Bridges in Mathematics Curriculum

The Bridges in Mathematics curriculum materials are comprised of a teacher guide, assessment guide, student handouts and workbooks, home resources, and answer keys (Math Learning Center, 2019). Our analysis focused on the conversations among the Grade 5 teachers. Their curriculum includes eight units, each with four modules. Each module is composed of four to six lessons. The teacher's guide provides teachers with an extensive overview of each unit, module, and lesson including descriptions of mathematical concepts, models used, algebraic connections, teaching tips, a trajectory of skills across grades, and observational and written assessments.

Professional Learning Routine: Lesson Analysis Cycle

The professional learning routine designed by project facilitators, called the lesson analysis cycle, involves of three phases of implementation: (1) collectively planning for teaching, (2) enacting the lesson plan, and (3) reflecting on how the teaching and learning unfolded. The lesson analysis cycle is typically organised around lessons that are rich enough to warrant teachers' professional inquiry.

The planning process begins with teachers spending 15–20 minutes independently reviewing the unit overview to understand key concepts, models, and strategies, and to situate the lesson within the unit's trajectory. They then work through the lesson as learners to experience the mathematical ideas firsthand. MTEs lead the unpacking of the lesson, drawing on a set of structured questions and tasks to help MTEs support teachers in identifying and wrestling with the key mathematical understandings in the lesson. This step helps teachers focus their discussions and instructional decisions on the mathematics of the lesson rather than general pedagogical moves (Figure 1).

Routine 1: Define learning goal of lesson

Routine 2: Identify key mathematical understandings teachers need

Routine 3: Describe mathematical connections students should make

Routine 4: Locate the critical moment where the connections most likely occur

Routine 5: Describe the mathematical claims student could make at the end of the lesson

Figure 1. Set of structured routines for MTEs

First, the MTE guides teachers in identifying and articulating the student learning goal for the lesson, not just by refining wording but by clarifying its instructional focus. This learning goal, recorded in the written record (a place to make public the decisions the group made, often recorded by the MTE), anchors subsequent planning and is elaborated throughout the unpacking process. See Figure 2 for an example of the written record from the focal example, the image of the clock and money model come from the curriculum's teacher guide. Once the learning goal is identified, the MTE prompts teachers to describe key mathematical ideas that they must understand to teach the lesson effectively. This prompt is intended to focus the interactions on the mathematical concepts of the lesson, including how students' prior engagement with related content informs the current lesson.

Our Written Record/Shared Understandings

➔ **Come to consensus on a shared learning goal and capture your agreement in the written record.**

Students will consider denominators of the fractions when determining whether a clock model or a money model will work and justify which model is most helpful when adding or subtracting fractions with unlike denominators.

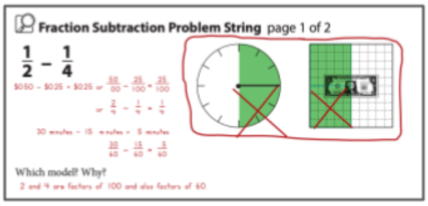

Figure 2. Example of learning goal as documented in the written record from focal example.

Next, MTEs prompt teachers to discuss the mathematical connections they want students to make. Teachers are encouraged to identify connections among important mathematical ideas by examining relationships among different representations and comparing different solutions to mathematics problems. Identifying these connections informs the questions posed to students.

The MTE then supports the team to identify the lesson's critical moment—the point of the lesson when students will engage most deeply with key mathematical ideas and make meaningful connections related to the lesson's learning goal. Identifying the critical moment prompts teachers to anticipate when significant mathematical thinking may occur and consider how they might respond to students' thinking. The MTE's role is to prompt a discussion about how the critical moment can provide opportunities for students to articulate mathematical claims–general statements about mathematical relationships or operations that emerge from patterns students notice. Importantly, the critical moment also serves as a focal point for reflection during the final phase of the lesson analysis routine.

As a final component, the MTE returns to the beginning of the lesson and supports teachers to analyse and plan for each step in the lesson in detail. They discuss the timing and pacing of each section of the lesson. Following the collaborative planning process, each teacher enacts the lesson developed in their classroom while the MTE video records the lesson. Each teacher is committed to following the lesson plan as closely as possible.

Following the enactment, teachers reflect on the lesson with the MTE, paying particular attention to the part of the lesson identified as the "critical moment." The MTE asks the teacher how the critical moment unfolded in practice. The MTE poses questions like: "Did students meet our identified learning goal? What evidence shows that students grappled with the big ideas? What partial conceptions emerged—both expected and unexpected?" The interplay between the video and the lesson planned is intended to deepen insight into the instructional decisions made before and during enactment, while also supporting a more analytical stance toward how students responded. The MTE asks the teacher to consider how they might adjust the lesson plan so that when they teach it the following year, they create richer learning opportunities for their students.

Methods

Participants

The project included 15 MTEs, four based in local education agencies and 11 hired by the project. All MTEs were experienced mathematics teachers with an average of 20 years teaching experience (range from 5 to 30 years) and 11 years' experience as MTEs who worked with pre- or in-service teachers (range from 1 to 23 years). For this study, we shadowed three MTEs working with Grade 4 and Grade 5 teachers. All three were considered expert MTEs with an average of 30 years teaching experience and a range of facilitating experience from 4 to 20 years. To examine how MTEs used the professional learning routine and curriculum to facilitate conversations, we analysed data from all phases of the routine across the three MTEs. To illustrate our findings, we present excerpts from one cycle led by Nina—a former elementary teacher with 30 years of classroom experience and 4 years as an MTE—working with three fifth-grade teachers, Carla, Diana, and Tracy, whose teaching experience ranged from 2 to 17 years. Pseudonyms are used to identify the participants to maintain anonymity and confidentiality.

Data Collection

The authors collected audio recordings, observational notes, and artefacts (e.g., curriculum documents, the group's written record of the planning process, and the final annotated lesson plan) of seven lesson analysis cycles across three MTEs, which involved planning sessions and post enactment debriefs for each cycle. Recordings were collected using audio recorders or conferencing technology. The data set includes 42:39:58 hours of recorded data.

Data Analysis

All audio recordings were uploaded, transcribed, and marked using the video analysis software, *Vosaic* (www. vosaic.com). We began data analysis by listening to audio recordings of the planning sessions and post enactment debriefs. Each of the authors carried out direct content analysis (Hsieh & Shannon, 2005) by marking three-minute segments of the audio for the presence of interactions about mathematics content, pedagogy, and/or student thinking.

Because segmentations in discourse analysis should be driven by what is being investigated (Degand & Simon, 2009), we selected three-minute segments to capture how teachers' discussions with the MTE connected the vertices of the instructional triangle. Shorter units fragmented coherent episodes in which participants were actively integrating all three elements of the triangle, whereas longer units tended to merge multiple distinct topics. Three minutes provided the appropriate timeframe needed to capture conversational episodes (Crookes, 1990; Passonneau & Litman, 1997). If any of the three topics appeared at any time during a three-minute period, the entire segment would be marked with that topic (see Figure 3). Because the unit of analysis was the conversational episode rather than individual speakers, codes reflected the topics addressed within the discussion.

We then compared selections and came to consensus on the final coding for each segment. Forty six percent of the data was coded in this way. Next, one researcher completed the remainder of the coding. To ensure the consistency and validity of the analysis an independent researcher coded 19% of that data. The two coders agreed on 96% of their coding decisions, discrepancies between the two coders were resolved through discussion.

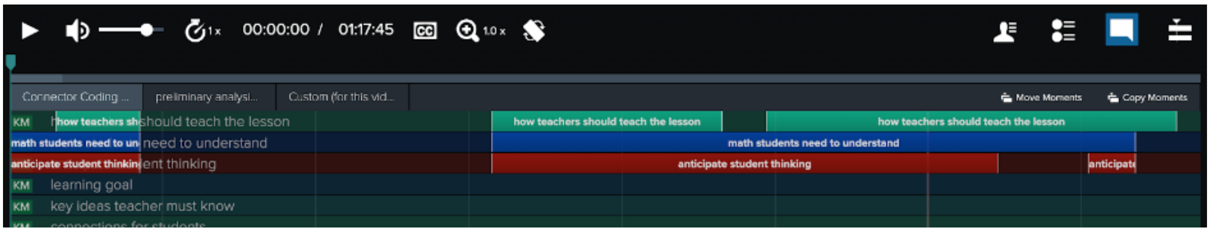

Figure 3. Example of coding data in three-minute segments.

The image in Figure 3 is the result of the coding from the planning phase of the focal cycle, which lasted 77 mins 45 secs. Only times when the MTE and the teachers were engaged in small group discussions were coded (from 00:08:31–00:31:27 the teachers were engaged in a different activity). The coloured bars indicate when pedagogy (green), mathematics (blue), and students' thinking (red) were discussed. The bar lengths reflect multiple three-minute segments in *Vosaic*, except for final segments which contained the remaining conversation time. Each segment was coded for whether discussions addressed one, two, or all three elements of the instructional triangle.

Descriptive analyses using frequency counts (time-on-task) provided an overview of the time spent engaged in instructional conversations. The analysis also showed the percentage of these conversations containing interactions among components of the instructional triangle.

Analytic memos were used to examine how the learning routine and curriculum materials shaped the instructional conversations. We noted the moments when discussions moved beyond logistics to address mathematical ideas, student reasoning, and pedagogy, and when participants spoke with high levels of specificity about students, mathematics tasks, or supportive pedagogical moves. From this broader analysis, we selected episodes that revealed how the learning routine and/or the curriculum materials supported the MTE in facilitating instructional conversations characterised by these features.

Findings

We present our findings in two parts. First, we document the extent to which the instructional conversations among one coach (Nina) and a group of teachers possessed the features shown by others to support professional learning. Second, we examine three episodes from the planning conversation in depth to illustrate how the learning routine and the curriculum materials enabled participants to address deep issues of teaching with high levels of specificity. We interpret the findings as evidence for a more informed hypothesis: a replicable model can be described using a structured learning routine and ambitious curriculum materials that MTEs can employ to foster the kinds of conversations among teachers shown to yield rich professional learning.

Did Conversations Integrate Content, Students' Thinking, and Pedagogy?

We documented the extent to which MTE-facilitated small group planning conversations integrated content, students' thinking, and pedagogy by calculating the total time, and the percentage of time, during which teachers' conversations focused on only one of these elements, exactly two elements, and all three elements. We did this for the two phases the MTE engaged in conversations with teachers--the planning and the debrief phases of the cycle. In Table 1, we address the question: When MTEs facilitated conversations among teachers about instruction, to what extent were these conversations embedded in the instructional triangle? To what extent did they integrate content, students' thinking, and pedagogy?

Table 1
Time Spent on Instructional Triangle Elements During Nina's Planning and Debriefing Phases

	Total Time	Only one element		Exactly two elements		All three elements	
		Time	%	Time	%	Time	%
Planning	02:06:01	00:04:51	3.76%	00:46:20	35.83%	01:18:00	60.41%
Debriefing	00:33:21	00:02:15	6.96%	00:11:58	37.03%	00:18:06	56.01%

Note. Instructional triangle elements include content, students' thinking, and pedagogy. Data are split by conversations focusing on one, two, or all three elements.

As Table 1 shows, at least 96% of the planning conversations and at least 93% of the debriefing conversations connected at least two of the instructional triangle vertices. This suggests the conversations met a key criterion for productive professional learning.

How did the Mathematics Teacher Educators use the Curriculum and the Professional Learning Routine to Facilitate Conversations?

In the example that we explored, Nina (the MTE) led a group of 5th-grade teachers through the lesson analysis cycle routine while preparing to teach a lesson from *The Bridges in Mathematics* curriculum (The Math Learning Center, 2019). The lesson titled, *Which Model Works Best?* is part of a larger module that focuses on adding and subtracting fractions with unlike denominators using a variety of common denominators. The lesson examined is the fifth of five that were designed to use money and clocks to "help students develop intuitions about finding common denominators in order to compare, add, and subtract fractions" (The Math Learning Center, 2019, p. 309). During the first part of the lesson, the teacher is to lead the class through a series of fraction subtraction problems, referred to as a problem string (Figure 4). For each problem, teachers are to ask students to share their thoughts publicly. The textbook authors intend for the students to reason through the model—money or clock face—that would best help them solve the problem (see Figure 2). Students then complete four practice problems to demonstrate their understanding of how to use the models to add and subtract fractions. For example, one problem with three parts says:

> Anthony's teacher gave the class this problem: 7/10 + 5/100. a) Anthony says it's better to think about money rather than a clock face to help solve this problem. Do you agree? Explain your answer. b) Solve 7/10 + 5/100. Show your work. c) Express the answer to 7/10 + 5/100 as a fraction: _____" [The Math Learning Center, 2019]

The team of fifth-grade teachers and the MTE spent a total of two hours and six minutes planning the lesson and creating a written record. See Table 2 for a time spent on each of the key components of the routine. The timestamps are provided to show the amount of time that was spent on each aspect of the routine, as well as a description of each aspect of the routine. As shown in Table 2, substantial time was devoted to negotiating the learning goal and identifying the critical moment, highlighting the deliberate sequencing of the routine and the emphasis on in-depth analysis of key instructional decisions.

1/2 - 1/4
3/4 - 1/5
3/4 - 2/5
2/3 - 5/12
4/5 - 1/10
4/5 - 10/100

Figure 4. Fraction subtraction problem string (The Math Learning Center, 2019).

Table 2
Timeline for the Grade 5 Small Grade Group Planning Led by Nina

Timestamp	Routine	Description
00:00:00 - 00:27:33	Read lesson overview	The facilitators outlined the day's agenda for all grade 5 teachers, then gave them 20 minutes to read the lesson overview and work through the mathematics.
00:27:33 - 00:38:45	Identify key learning goals for students	The MTE led their small groups to negotiate and agree upon a learning goal for the lesson.
00:38:45 - 00:49:32	Identify key mathematical ideas teachers must know	The group shared ideas about what key mathematical understandings they should understand in order to teach this lesson well.
00:49:32 - 00:57:12	Identify mathematical connections for students to make	The group discussed the mathematical connections they want students to make.
00:57:12 - 01:14:51	Identify critical moment and claims	The group identified the critical moment where the learning goal can be realised and what claims they want students to make.
01:14:51 - 01:32:27	Annotate lesson plan	Teachers were provided time to individually annotate points in the lesson they think the group should analyse in more detail.
01:32:27 - 02:18:45	Discuss the number string with greater specificity	The MTE guided small groups through each expression in the number string, focusing on timing, how to record student thinking, anticipating responses, and planning teacher moves.

To demonstrate how the professional learning routine and curriculum materials enabled the MTE to support robust teacher conversations, we present three episodes from the focal cycle. The first episode illustrates how the MTE prompted teachers to identify the learning goal of the lesson. The second episode illustrates how the MTE prompted teachers to identify the critical moment of the lesson. The final episode presents the post-enactment reflection between the MTE and one of the fifth-grade teachers. To make the transcripts easier to follow, we left out words like "um" and "like," but otherwise kept participants' language intact. For brevity, we use timestamps and […] to indicate that a period of time has passed.

Episode 1: Negotiating the Learning Goal

As part of the professional learning routine, Nina prompted the group to reach consensus on the lesson's learning goal. Prior to this episode, while individually reading the lesson and engaging with the mathematics, each teacher had identified a possible learning goal. Nina then facilitated a discussion in which they shared their ideas and collectively agreed on the learning goal.

27:33
Nina (MTE): So, we always start off first with "What's the learning goal?" and trying to get a handle on that because that's going to drive the rest of our conversation. [...]

28:18
Carla: This is my rough draft idea of [the learning goal] and then I'm hoping that we can…refine it together… To me, the learning goal is for the students to be able to look at both of the denominators and to use that…when they're adding or subtracting fractions to justify which model is the best or most efficient model for them to use. [...]

28:53
Diana: I had highlighted in the summary where it said, 'which model works best for each problem and why', and then how to use models to add or subtract fractions. So similar [to Carla, I chose] why they're using the model they're using.

As specified in the routine, Nina began by reminding the team that identifying the learning goal would shape their subsequent instructional decisions about how to enact the lesson. Carla offered a suggestion for what the learning goal could be, inviting others on the team to help refine her initial idea. Diana responded by reading text that she had highlighted in the curriculum and indicated that she agreed with Carla. Nina then revoiced Carla's idea while recording the evolving learning goal in the written record.

29:12
Nina: One thing I heard Carla say that may be just a little different that I put in [the written record] was… looking at the denominator and that may push away from "what's my favourite [model]"… I think we can prove both [models] work, but one was more helpful than the other. …that whole idea of not just what's my favourite. [...]

30:26
Carla: I'd like to add something about their ability to justify [which model]. Because both in the problem string and in the fraction work sample, it's not just picking a model, but it says, why is that model the best. [...]

Evident in Nina's interaction is that she listened carefully to Carla and Diana and highlighted their focus on having students explain and justify why different models are better suited to different expressions in the number string. Nina's comment prompted Carla to emphasise the importance of students being able to justify their choice of model. Anticipating later parts of the lesson, she reminded the group that students will be asked to explain why a particular model works better for certain fraction addition and subtraction problems. Prompted by Nina's questions and drawing on the curriculum, Carla connected the tasks students would engage in and the mathematics the teachers wanted students to understand. This pushed the team to consider a more nuanced learning goal that emphasised justification. Returning to the expressions in the curriculum's number string, Nina posed a question to the group about how students would justify which model works best.

30:51
Nina: … I was thinking, would you first look at the denominators and determine whether the clock or money will work, and then justify which is most helpful?

31:10
Carla: Yeah, because a lot of the problems like you said, do lend to both [models], especially when you first start off. Like ½ - ¼, both of those are like accessible to a fifth grader because they understand half an hour, they understand 50 cents. And then as it progresses, it does come down to, there are ones like the ⅕ that you could use either model, which one is more efficient. But then when they get to the 2/3 minus the 5/12, well you can't use money because… three is not a factor of 100. [...]

32:30
Tracy: Money [model] might be the easier one. But if you don't understand money and you understand clocks, then you can easily use clocks. But I think that the key part is that for that child it needs to be the most efficient and they need to be able to use the mathematical language. Not just, it's my favourite, but because with a denominator of five, denominator of four, …I can use the clock model, I can use the money model. [...]

33:41
Nina: So really articulating specifically like "the clock [model is] hard". "Well, what was hard about it?" Maybe the subtraction was harder because 45 minus 12 [¾-⅕] felt harder than 75 minus 20. So specific justification sounds different to me than, you know, "I'm not good at clocks" or "I don't know clocks". Because some kids, as you say, that division or knowing five times what is 60 is easy. Other kids, it's hard for them. They're not as fluent in those facts which make it harder. And I think that's a great justification.

In the conversation above, Nina recorded the group's current draft of the learning goal in the written record and asked how students might justify which model works best for each expression. This wondering prompted teachers to anticipate how students' reasoning would unfold. Clarifying how students might justify their choice of model supports later conversations when teachers considered the pedagogical decisions needed to foster students' reasoning. Notice that Carla's response highlighted how the number string is designed by the curriculum authors to first present expressions where either model could work, followed by expressions where one model works better. Tracy then added that she wanted students to move beyond simply stating which model they prefer and instead use mathematical language to justify why the clock model or the money model can be used. Nina agreed with Tracy and added depth to the conversation by anticipating that students might comment on the clock model, noting its difficulty of use. Nina then added a question that could possibly press students to explain what is hard about the clock model—suggesting a pedagogical move teachers could make in the moment. Collectively, the team was working on the type of justification they would like to hear students' give about determining which model is best.

Next, Carla introduced the notion of efficiency—considering which model may be more effective—to prompt justification of the model choice. Tracy, however, worried that emphasising efficiency could complicate the discussion and indicated that she preferred, instead, to focus on having students use mathematical terminology.

00:34:20
Carla: Like, if you are trying to get to this idea of efficiency. [...]

00:34:37
Tracy: But I don't get it. I don't think that's the point is to have them [use] their most efficient [model]. [...]

00:35:16
Carla: I guess, and my thought was that not necessarily forcing your most efficient by creating a scenario where they're thinking, "What is efficient?" Just for those particular students … Maybe making them think deeper about what is efficient. [...]

00:35:36
Tracy: … And I think that if they … just [use] the math terminology to explain.

00:35:45
Diana: And that's what I just wrote down. I just wrote down the factors of whatever it is if they can differentiate that.

00:36:05

Nina: So, do we agree then? That the goal is for students to be able to … consider both denominators and determine whether a clock or money model will work. And then that's adding a little more; you're going to push them to … justify which model is most helpful when adding or subtracting fractions with unlike denominators (see Figure 2).

In this exchange, Carla shared with the group a potential move they could make to steer the conversation toward asking students about efficiency. Tracy was concerned that the conversation about efficiency might deter students from the learning goal of comparing the models. Carla then became more specific about "creating a scenario" in which students would use efficiency to think "more deeply" about which model works best. Tracy and Diana emphasised that they want students to use mathematical terminology, including factors, in their explanations. Nina concluded this part of the routine by recording the group's negotiated description of the learning goal in the written record, incorporating the refined wording developed through their conversation.

Summary Episode 1

In this episode, the structured learning routine created an opportunity for Nina to support teachers to negotiate the lesson's learning goal while elements of the curriculum materials—the problem string and representations in the textbook's lesson—provided a shared mathematical context for the discussion. The conversation centred on how students might justify which model works best. Within this structure, Nina facilitated a conversation that pushed teachers to articulate the mathematical understanding they wanted students to develop and to anticipate how students might justify their reasoning. In doing so, she pressed the group to examine the learning goal with greater depth and specificity. Anchored in the curriculum tasks, the discussion moved beyond a general learning goal of helping students better understand subtraction of fractions to more specific considerations about how they wanted students to reason about which model would be most helpful to use.

Episode 2: Identifying the Critical Moment

The next excerpt took place 20 minutes after the discussion about the learning goal. The teachers had just discussed the mathematics they needed to understand to teach the lesson well and the mathematical connections they wanted students to make. This second excerpt began with Nina posing the next question in the learning routine: What is the critical moment of the lesson? That is, what is the moment students are most likely to engage directly with the learning goal? The teachers had already considered possible critical moments when they individually read the lesson.

00:58:24
Nina: One idea that we haven't talked about is identifying what we think is a critical moment.

00:58:56
Carla: My initial thought was the 2/3 minus the 5/12 because that was the first time in the string where there is only one [model] option. But now I'm thinking if our learning goal is truly about focusing on the denominators and the factors, then could the critical moment happen sooner? Where you are kind of pushing them to [consider] that both models can be used. I could be swayed, but my initial gut was 2/3 minus 5/12 because I was like: yes, this is the moment where you really do have to make a choice. And you have to understand it's because three and 12 are not factors of 100. So, you cannot do the money model. And I think also it's because of my experience … my students tend to be most comfortable with money. So that was my initial reaction.

00:59:57
Nina: Tracy, did you say the same thing?

01:00:01
Tracy: Because it's the first time that the option is not an option anymore. Like all those other ones, the kid could go, 'well, but I liked it' or '[I] know how to do this quicker because I know the factors of this.' But this is the first time you can't use the money model. This is the first time that that denominator really drives into a particular model.

Carla was the first to respond to Nina's question, offering the expression of 2/3 minus 5/12 as the point in the number string where students examine the denominators to justify that the clock model is the best model to use. Carla connected her rationale for the critical moment back to the learning goal the team had collectively negotiated. The team's prior discussion prompted Carla to think more critically about the mathematics they want students to understand. She also noted that, in her experience, students tended to gravitate toward the money model, and this problem was the first time they needed to use the clock model.

Tracy agreed with Carla and added that students cannot simply choose the model they are most comfortable with; they must attend to the denominator to see why the clock model will work best. Evident in this negotiation is the focus on what teachers want students to learn about the mathematics and how the curriculum is written to support students to dwell on those ideas. Their work as teachers is to determine where to linger in order to give students the time to justify and explain their thinking. Next, Carla invited the group to consider how the critical moment connects to the entire lesson – the number string as well as the fraction work samples students will be asked to complete.

01:00:25

Carla: Then, the other lens I'm looking through is that we do this problem string and have this discussion, and then the next action is the fraction work sample. So, the teacher in me is like, I really want to make sure I'm clarifying any … misconceptions or ideas. Because … I want them to walk away from this discussion more confident to show what they know in the work sample.

Here, Carla stepped back and looked across the whole lesson, beyond the number string. She confirmed that the chosen critical moment can support students in developing criteria for selecting the best model, which allows them to engage meaningfully with the following tasks in the lesson and achieve the learning goal. Carla's highlighting of a pedagogical move she wanted to make during the critical moment firmly embedded the conversation within the instructional triangle, drawing on all three vertices.

Next, Diana was invited by the MTE to share what she had identified as the critical moment. She agreed with Carla and Tracy and reiterated the point that the expression supports students to justify the model they choose.

01:02:21

Diana: … I agree because I like that point, of, in order to make the determination that we have to use the clock for this one [expression], that means that you are thinking about those different things, like looking at the factors and justifying.

01:02:35t

Carla: … we'll be looking at both of the denominators and that factor relationship because [students] could argue [that] the first three [expressions], all of those are factors of both 60 [and] 100 […]

After Diana shared her agreement and rationale, Carla added that the number string is intentionally designed by the curriculum authors so the denominators in the first three expressions are factors of 60 and 100, giving students an entry point to argue that either model could work. In what follows, Nina shares what she had identified as the critical moment. Evident in her remarks is how her thinking has been influenced by the team's conversation. She referred to her earlier remark that students might struggle to use the clock model for expressions with fifths.

01:05:03

Nina: I was torn [about which moment] too. Like I thought this [expression] 3/4 minus 1/5 was pretty important. In my thinking, no one was going to use the clock [model]. Now, maybe I shouldn't say no one, that's too strong. But [in my] experience with 1/5, [students] wouldn't use that. So … they're forced to look at the denominator and say, 'Wow, it did work, but I didn't like it.' And then I agree that in my thinking too…if we don't get to the 2/3 minus 5/12 it definitely needs to be a summary question of 'will it work?' We're not going to solve it. I just want to know, based on this claim we made [about the 2nd expression, 3/4 – 1/5]. So, I think no matter what, you're going to have to plan a great conversation around 3/4 minus 1/5 and then the 2/5 should flow based on that conversation of how did you use it? And then if

> you can get to that next one, that's great because we have to justify why it won't work. And if you don't, I would definitely get it up there and say, yeah, only the clock [model] will work.

Nina shared that she was torn between identifying the second or fourth expression as the critical moment. Based on her experience, she anticipated that many students would not be comfortable using the clock model to subtract 1/5 from 3/4. She felt it was an important moment to support students to make the connection that 1/5 of a clock is equivalent to 12/60, supporting the justification that when the denominator is a factor of 60, the clock model context can support finding equivalent fractions to subtract. She agreed with the group that 2/3 – 5/12 was an important expression, because it could support students to extend earlier reasoning to justify the clock model as the best context for subtraction. Nina also addressed pacing, noting that if the teacher is running short on time, they should focus on discussing which model would help solve the expression rather than spending time on the computation. The team spent three additional minutes negotiating the critical moment and ultimately concluded that there were two within the number string (see Figure 5): the first at 3/4 – 1/5 and the second at 2/3 – 5/12. Nina recorded the team's rationale in the written record (see Fig 5).

Our Written Record

➔ **What is the critical moment of the lesson. Why did you select this specific part of the lesson to focus on most closely?**

¾ - ⅕	Students may not be comfortable with finding ⅕ on clock. When asked if possible, they will reason 60 is divisible by 5 therefore a clock model must work. They may make the connection that the 12 sections on a clock are 5 minutes each or 1/12 of 60 is 5 therefore ⅕ of 60 is 12. It's an opportunity to make the claim that when a denominator is a factor of 60, the clock model will work. If the denominators are factors of 100, the money model will work.
⅔ - 5⁄12	This is the first problem when both models do not work. It's an opportunity to make the claim that when a denominator is a factor of 60, the clock model will work. If the denominators are factors of 100, the money model will work. Since 3 is not a factor of 100, the money model will not work.

Figure 5. The critical moment as documented in the written record from focal example.

Nina's contributions to the critical moment show that she is an important member of the group. While as the MTE she is responsible for orchestrating the discussion and documenting the team's decisions, she also has a role to think critically about and engage with the different aspects of the routine as they analyse the curriculum. The routine creates space for her to influence the grade 5 teachers by sharing her own experiences, while also learning from their insights.

Summary Episode 2

In this episode, the structured routine prompted Nina to guide the team in identifying the lesson's critical moment—the point in the lesson when students would need to engage most deeply with the mathematical ideas. This step built on earlier parts of the routine, when the team negotiated the learning goal and examined the mathematical ideas students were expected to develop. Like in Episode 1, the rich instructional conversation was enabled by the interaction between the specific questions and learning tasks posed by Nina and the mathematical tasks and representations provided by the curriculum.

Episode 3: Debriefing the Teaching Enactment

Several days after the planning session Carla enacted the agreed upon lesson and Nina video-recorded it. The next day, Nina and Carla met online to debrief Carla’s enactment of the lesson. The learning routine began with Nina sharing on the screen the Written Record the team had created. Nina had selected a seven-minute video clip, where Carla posed a question about whether students used the money or clock model to add fractions for the expression 1/4 + 1/3. Before watching the clip, Nina asked Carla to pay attention to what students were saying in order to “assess where [students] are [in relation to] the learning goal” Carla described an interaction she had with a student, Cody, who was explaining why a money model would not work.

00:25:58

Carla: I did see a moment to think of the next time [I teach this lesson]. It's … when Cody was saying, “You can't add something three times.” I feel like that could have been an opportunity to bring it back to division, or factors. ... I wish that I would have had the opportunity in that moment to use a question or something—to lead him away from saying, “I can't add something three times” to bring it back to [the idea] that we're dividing the 100 [into three equal parts]. Because that's what he was talking about, he was saying … “There's nothing that I can evenly break. I can't break a hundred evenly into three groups.” But he kept saying, “add” and using the word “something”, and so I think maybe that could have been just like an opportunity… to clarify … for other kids, [because] they might have been sitting there like, “Well, yeah, I can add … three other numbers [together to get 100]” ...

00:27:03

Nina: And that connection with repeated addition is like multiplication. And where's the connection between multiplication and division? And I don't think in our planning, as I thought back, we were focused a lot on factors, which was fine. But we didn't think enough about: well, what about kids who don't have access to the word factor and are focusing a bit more on that division. Like that finding one third of something is the same as dividing it by three, and we didn't talk as much about that. And I think kids who didn't have access to the word factor, I think they were understanding that you're breaking it up. We're dividing it. So really getting that kids understand one half is divided by two, but they don't necessarily apply that to one third is dividing by three, into three groups and one fourth into [groups of] four.

Carla identified an interaction where a student reasoned about why a money model would not work for solving 1/4 + 1/3. She recognised a missed opportunity to connect Cody’s language about adding to the learning goal. Nina built on Carla’s reflection by highlighting the importance of linking repeated addition to multiplication and division, noting that the team did not anticipate how students without a firm understanding of the term ‘factors’ might reason about equally partitioning a clock or money model.

Summary Episode 3

The debrief conversation is a crucial component of the structured learning routine, with the MTE responsible for selecting specific clips to centre the discussion. Nina’s choice of the 7-minute clip was strategic–it highlighted student thinking that had not been anticipated but directly aligned with the learning goal of justifying which model works best. The clip created space for Carla to reflect on student reasoning and consider alternative pedagogical moves she might have used in response to Cody’s ideas. Rather than evaluating Carla’s teaching, Nina facilitated the discussion by building on Carla’s noticing and pressing her to consider the mathematical ideas underlying students’ reasoning, helping Cody and others to create a more nuanced justification. Although the curriculum is not as visible in this episode, the mathematical content of their conversation is directly rooted in ideas developed in the textbook lesson.

Discussion

This study addressed the question of how MTEs can engage teachers in professional learning that makes a difference for teaching. We began by hypothesising that structured professional learning routines alongside ambitious and educative curriculum materials would facilitate productive instructional conversations with teachers. We interpret our findings as providing initial confirmatory evidence for our hypothesis. We believe the data are compelling enough to propose a model, consisting of these two elements, that could be repeated by MTEs in other settings with similar results. Such a model would allow MTEs to define their work in clearer, more structured, and more specific ways.

To review, we built our argument on findings that showed richer, more conceptual teaching in mathematics was facilitated by professional learning opportunities that engaged teachers in particular kinds of instructional conversations (Russell et al., 2020; Stein et al., 2022). The critical features of the conversations were (a) they tackled deep issues of teaching; (b) they examined teaching with high levels of specificity; and (c) they integrated attention to students' thinking, content, and pedagogy;. We then asked what resources or tools MTEs could use to facilitate these kinds of conversations. Based on previous research, we identified structured learning routines and curriculum materials as two key resources (Davis & Krajcik, 2005; Davis et al., 2017; Drake et al., 2014; Horn et al., 2025; Lampert et al., 2013). We used these findings to shift our dependent variable from observations of teaching to the nature of instructional conversations themselves, treating conversation quality as a meaningful proxy for professional learning with the potential to improve practice.

To enact the model, the MTE is not merely a moderator moving through routine questions, but an active pedagogical agent whose judgment and expertise shape what the routine makes possible. The three episodes illustrate how Nina's facilitation, structured by the routine and anchored in the curriculum, enabled conversations that would have been difficult to sustain without her. Similar to other research on facilitator practice, Nina strategically enacted moves to facilitate productive instructional conversations (Borko et al., 2014; Gibbons et al., 2021; Karsenty et al., 2023; Schwarts et al., 2021; van Es et al., 2014). Across the episodes Nina listened carefully to teachers' contributions and revoiced them in ways that sharpened their mathematical focus (Episode 1); introduced new questions about student justification (Episode 1); drew on her own mathematical knowledge to identify when students' thinking would be most consequential and in most need of pedagogical support (Episode 2), and strategically selected video clips to analyse that captured unanticipated student reasoning at precisely the critical moment the team had identified (Episode 3). In each case, the routine provided the structure and the curriculum provided the shared object of inquiry, but it was Nina's active facilitation that transformed these resources into productive professional learning conversations. This points to an important implication: the routine and curriculum together do not make MTE facilitation easy, but they make it more learnable and replicable.

While prior research has established the value of lesson study and coaching cycles for supporting instructional improvement (e.g., Druken, 2023; Stein et al., 2022), the model we describe offers three features that extend this work. First, rather than focusing on one-on-one coaching, the Lesson Analysis Cycle builds a community of inquiry among grade-alike teachers, an approach that we argue is more sustainable and scalable than individual coaching. Second, by anchoring the routine in an ambitious and educative curriculum, the model gives the MTE and teachers a shared object of inquiry that constrains and focuses conversation productively, without requiring the MTE to generate rich discussion content from scratch. Third, the specific sequence of routine steps—negotiating the learning goal, identifying the critical moment, and debriefing the enactment—constitutes a replicable structure that has the potential to support novice MTEs to learn to facilitate strong conversations. Together, these features suggest that the lesson analysis cycle offers a practical and replicable model for MTEs who want to move beyond one-on-one work and support groups of teachers in sustained, curriculum-grounded professional learning.

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Competing interests

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