

Promoting Content-related Facilitator Expertise: Facilitator Professional Development Approach in the QuaMath Program

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Received: September 2025 || Accepted: May 2026

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In scaled-up professional development programs for teachers, the sessions are mainly conducted by facilitators who were not part of the original design team. The quality of teacher professional development (TPD), therefore, relies on facilitator preparation and ongoing support, enabling them to develop expertise with respect to the content and concepts included. Whereas relevant components of facilitator expertise have been unpacked theoretically and empirically from different perspectives, reports on the designs of facilitator professional development (FPD) programs aimed at promoting such expertise are still rare. In this article, we report on one FPD program which was designed and enacted within the nationwide ten-year implementation program QuaMath (Developing Quality of Mathematics Classrooms and Teacher PD) for 515 mathematics facilitators in different age groups (K–1, 1–4, 5–10, 11–13). We present the FPD approach, including its overall goals for mathematics content-related facilitator expertise, its macro design in terms of structural and institutional characteristics, and its micro design through typical FPD activities. We take this program as a case to argue that fostering pedagogical content knowledge for professional development requires a systematic combination of work at both levels: the classroom and the TPD level.

Keywords • professional development programs • mathematics facilitator preparation • pedagogical content knowledge for PD • PCK-PD • scaling up program

Describing Facilitator Professional Development Approaches

When scaling up teacher professional development (TPD) programs, facilitators play a crucial role in facilitating the sessions and supporting teachers' professional learning communities (Beisiegel et al., 2018; Borko et al., 2014a, 2014b; Lesseig et al., 2017). For mathematics teacher professional development in Germany and many other countries, these facilitators are typically experienced mathematics teachers who are partially released from school duties to conduct their TPD work (Tekkumru-Kisa & Stein, 2017), for which further research has been called to develop empirically grounded, research-based facilitator professional development (FPD) programs (Jacobs et al., 2017; Karsenty et al., 2025).

The work of facilitators is consistently described as demanding (Borko et al., 2014b; Lesseig et al., 2017; Pöhler-Friedrich et al., 2026). Thus, various professional development (PD) designers and researchers have specified the necessary components of facilitator expertise (Borko et al., 2014a, 2014b; Elliott et al., 2009; Lesseig et al., 2017; Prediger et al., 2022b) and designed and enacted systematic FPD

programs to promote the specified components of facilitator expertise (Jacobs et al., 2017; Karsenty et al., 2025). Although various FPD programs have been developed in different contexts, most of these programs have only been examined with respect to specific aspects. For instance, some studies elaborated on how facilitators learn and adapt key elements of a PD model over three school years (e.g., Borko et al., 2021), while others have investigated whether facilitators' noticing of teacher learning improves through the use of video cases (e.g., Lesseig et al., 2017). Other lines of research scrutinised facilitators' growth in moderating activities with a strong simultaneous focus on goals and participants (e.g., Pöhler-Friedrich et al., 2026) and on facilitator practices during high-quality conversations specific to PD activities (e.g., González et al., 2016). Taken together, these studies and many others help paint the picture of facilitator expertise, but only selectively regarding the design of targeted learning opportunities for acquiring it. Consequently, various authors have called for more comprehensive reports on the different structural and institutional characteristics of FPD programs (Karsenty et al., 2025), as well as on different approaches with respect to the chosen FPD goals and associated FPD activities (Jacobs et al., 2017; Borko et al., 2025).

In this research-informed conceptual design article, we follow these calls by outlining the research-informed background and implementation of an FPD approach in mathematics education as adopted for the nationwide program QuaMath (Prediger & Selter, 2024), which involves 515 facilitators at the German Centre for Mathematics Teacher Education (DZLM). Before presenting the macro design and the micro design of this FPD program, the international state of academic discourse on facilitators' expertise in mathematics education and the theoretical background on which the program was developed are outlined. This research will help to elaborate the background of types of FPD activity series that combine facilitators' learning opportunities at the classroom level and the TPD level.

Theoretical Background: Facilitators' Content-related Expertise and Approaches for its Promotion

Complementary Specifications of Facilitators' Expertise: Knowledge and Practices

Following the initial description of the complex work of TPD facilitators (also called PD leaders, multipliers, teacher educators, teacher leaders, etc.) by Zaslavsky and Leikin (1999), research began to focus on what facilitators should know and/or do when leading TPD in mathematics education (Borko et al., 2014a, 2014b, 2021; Lesseig et al., 2017; Peters-Dasdemir et al., 2023; Tekkumru-Kisa & Stein, 2017). These studies emphasised that facilitators need knowledge and practices, not only at the classroom level but also at the TPD level (Zaslavsky & Leikin, 2004).

Practices were often disentangled in content-independent perspectives (e.g., by specifying facilitator moves in video-based discussions; Coles, 2019). In some studies, authors emphasised the structural analogy between teachers' and facilitators' work to elaborate, for example, on productive discussion-leading practices by lifting teacher frameworks for productive moves to the facilitators' level (Borko et al., 2014a; Lesseig et al., 2017; Van Es et al., 2014). Other studies unpacked facilitators' *practices* while preparing, conducting, and reflecting on their TPD sessions, analysing the commonalities and differences with the classroom level (Lesseig et al., 2017; Zaslavsky & Leikin, 1999).

With respect to facilitators' knowledge, teachers' knowledge for mathematics teaching continues to be relevant, but at two levels (Borko et al., 2014b; Elliott et al., 2009; Prediger et al., 2022b; Wilhelm et al., 2019). First, facilitators need flexible and extensive knowledge of everything that teachers learn in TPD at the classroom level. This *content knowledge on the PD level (CK-PD)* can comprise elements of teachers' mathematical content knowledge for classrooms (CK-C), pedagogical content knowledge (PCK-C), and/or generic pedagogical content knowledge (GPK-C) (Shulman, 1986). Second, facilitators need knowledge about how teachers learn in content-independent ways; this is *generic pedagogical knowledge at the PD level (GPK-PD)*. The TPD level, however, also includes content-specific knowledge: namely, what teachers need to learn with regard to particular elements of CK-PD. This is referred to as

pedagogical content knowledge at the PD level (PCK-PD). PCK-PD encompasses all aspects of teacher thinking and teacher learning concerning the specific PD content (Wilhelm et al., 2019) and can be characterised as the knowledge that underpins “the ability to engage teachers in purposeful activities and conversations about those mathematical concepts and relationships and to help teachers gain a better understanding of how students are likely to approach related tasks” (Jacobs et al., 2017, p. 3). Figure 1 shows how the different knowledge components are lifted and how the classroom level remains nested within the PD level (Prediger et al., 2022b).

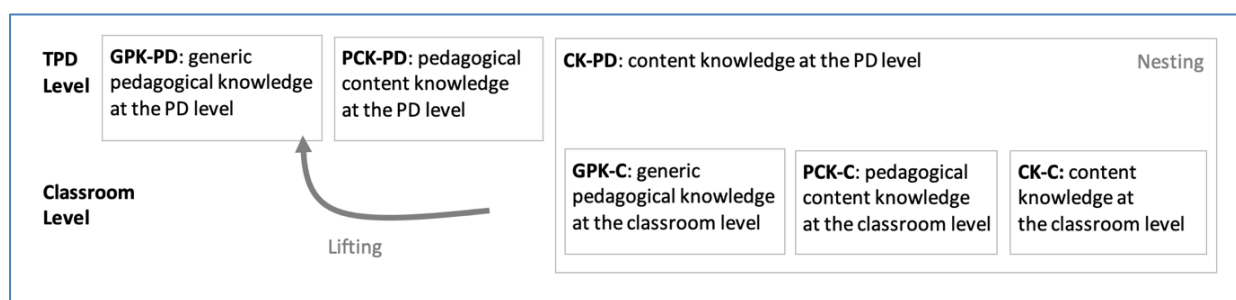


Figure 1. Components of facilitator knowledge categories at the TPD level and the classroom level (Prediger et al., 2022b, p. 408).

However, the nested structure of facilitator knowledge (i.e., the fact that elements of GPK-C, PCK-C, and CK-C are nested in CK-PD, Figure 1) has received less attention than the structural analogies that allow lifting (i.e., the approach used to characterise GPK-PD, PCK-PD, and CK-PD in structural analogy to GPK-C, PCK-C, and CK-C) when describing FPD programs. For example, in Borko et al.’s (2014b) seminal article on preparing facilitators to conduct TPD sessions using the problem-solving cycle as TPD content, the lifting strategies for enacting and reflecting discussion-leading practices were explicated insightfully, yet the knowledge elements activated by the facilitators were treated only implicitly and require further unpacking.

While some publications treat knowledge and practices separately, several frameworks have also emphasised that they are complementary and should therefore be combined (Borko et al., 2014a; Karsenty et al., 2023). In our work and in the current article, we draw on the *framework for content-related facilitator expertise* developed in the DZLM (for details see Prediger et al., 2022b) to systematically link knowledge and practices. The framework relates to that of content-related teacher expertise (Prediger, 2019), while bearing in mind the specifics of facilitating TPD.

The framework starts from typical situational demands (facilitators’ “jobs”) that facilitators need to master with respect to the goals and content of mathematics teaching addressed in the TPD. Typical jobs include: (I) launching and moderating an FPD activity, including the reflection part to leverage teachers’ ideas; (II) adapting TPD materials for one’s teacher group and context; (III) harnessing teachers’ experiences from the classroom experiments, with a focus on students’ mathematical thinking and opportunities for conceptual learning; and (IV) promoting school-internal cooperation on TPD content (Prediger et al., 2022b).

These jobs support the designers to specify relevant facilitators’ *practices*. In this context, practices are defined as the recurrent patterns of their utterances and actions for managing the jobs in ways that support high-quality mathematics teaching. These practices can be characterised by (a) the underlying knowledge categories for perceiving, interpreting, and decision-making; (b) pedagogical tools; and (c) orientations from which the facilitator implicitly or explicitly draws.

Within this framework of content-related facilitator expertise (Prediger et al., 2022b), facilitators’ expertise has been shown to be enhanced in facilitator *preparation* sessions and ongoing support

meetings by (a) restructuring and extending knowledge categories for perceiving, interpreting, and decision-making for particular TPD content; (b) offering new pedagogical tools (e.g., curriculum materials or other moves for leading discussions); and (c) transforming or strengthening orientations (Pöhler-Friedrich et al., 2026; Prediger et al., 2022b).

Despite these insights, there remains a lack of systematically structured arrangements for preparing facilitators to manage these diverse jobs in a coordinated manner within TPD. For designing FPD opportunities, however, it is also necessary to clarify the influence of the structural and institutional characteristics of an FPD program, as well as how the specific FPD goals and activities selected within the program can contribute to preparing facilitators for these jobs. Adopting this integrated perspective at both the macro and micro design levels is crucial, as facilitators' jobs cannot be fully understood in isolation from the overarching program structures in which they are embedded.

Macro Design of Facilitator Professional Development Programs with Varying Structural and Institutional Characteristics

Alongside the growing international discourse on what facilitators should learn, reports and case studies of FPD programs in mathematics education have increased as well (e.g., Borko et al., 2014b; Borko et al., 2021; Bruns et al., 2022; Jacobs et al., 2017; Lesseig et al., 2017; Valoyes-Chávez & Felmer, 2021). These programs use facilitator preparation sessions and ongoing support meetings to introduce and reflect on the TPD approaches while combining them with material support in the form of elaborated TPD materials (including, e.g., videos, inputs, and activities from TPD sessions; cf. Borko et al., 2021). While some programs conducted within larger implementation research studies are highly specified and aim to optimise implementation fidelity, others are highly flexible and encourage facilitators to adapt the materials to their contexts while maintaining fidelity to the main program goals (Jacobs et al., 2017; Jarry-Shore et al., 2023).

By comparing and contrasting different international programs, Karsenty et al. (2025) documented a notable diversity of macro design characteristics. They suggested that the following structural and institutional characteristics might substantially affect the macro design of the programs: the target group of facilitators; the duration of the preparation and support period; the certification and attendance; the nature of the TPD program; the facilitator's initial involvement with TPD materials; the form of integration of facilitation experience in TPD; and the mathematical focus of the FPD content.

One characteristic particularly shapes the conditions for the micro design: the *form of integration of facilitation experience in TPD*. Some FPD programs (such as Video-LM, Karsenty et al., 2023) selected their facilitators from among teachers who had already completed the one-year TPD program and conducted the FPD program successively after the TPD program, together with facilitators who already knew the TPD program well. In contrast, other programs (such as Borko et al., 2021) worked with facilitators for whom the TPD content was also (partially) new, meaning that they had to simultaneously implement the TPD and FPD programs. In this simultaneous constellation, the activities in the FPD program need to account for both facilitators' learning needs at the classroom level (e.g., productive classroom practices and their underlying GPK-C, CK-C, or PCK-C elements) and at the TPD level (e.g., productive facilitation practices and their underlying GPK-PD or PCK-PD elements).

Micro Design of Facilitator Professional Development Programs: Goals and Activities

In addition to the variety of structural and institutional characteristics that shape the macro design of FPD programs, reports on different FPD programs in published case studies also highlight substantial differences in the micro design (e.g., Borko et al., 2014b; Borko et al., 2021; Bruns et al., 2022; Jacobs et al., 2017; Lesseig et al., 2017; Prediger et al., 2022b; Valoyes-Chávez & Felmer, 2021). These differences primarily concern the FPD goals (i.e., which aspects of content-related facilitator expertise are foregrounded in the program: content-independent practices or specific CK-PD, GPK, or PCK-PD elements?) and the FPD activities through which these goals are pursued. For instance, numerous

video-based TPD programs focus their FPD on the discursive quality of discussions about these videos (e.g., Borko et al., 2014b; Coles, 2019), while others empower facilitators to support mathematics teachers through modest classroom experiments with new curriculum materials and one-to-one coaching (Cobb & Jackson, 2015).

Especially for the micro design of FPD activities, Borko et al. (2025) proposed the Learning to Facilitate Framework (Figure 2). This framework was adapted from a teacher education framework developed for preparing teachers based on three pedagogies (Grossman et al., 2009): teachers learn to enact practices that meet core situational demands of teaching (briefly called tasks of teaching) when these practices are *represented*, *decomposed*, and *approximated* in teacher education. The Learning to Facilitate Framework (Figure 2) has lifted these pedagogies (i.e., adapted from the TPD level to the FPD level) so that they can be used to enhance facilitators' professional learning. It comprises four pedagogies: *selecting* a TPD situational demand for facilitators (i.e., a TPD activity to be enacted) and the practices to address this TPD demand; *representing* productive practices for mastering this demand; *approximating* these practices by the participating facilitators; and *unpacking* the represented and approximated practices with respect to the underlying knowledge elements and orientations.

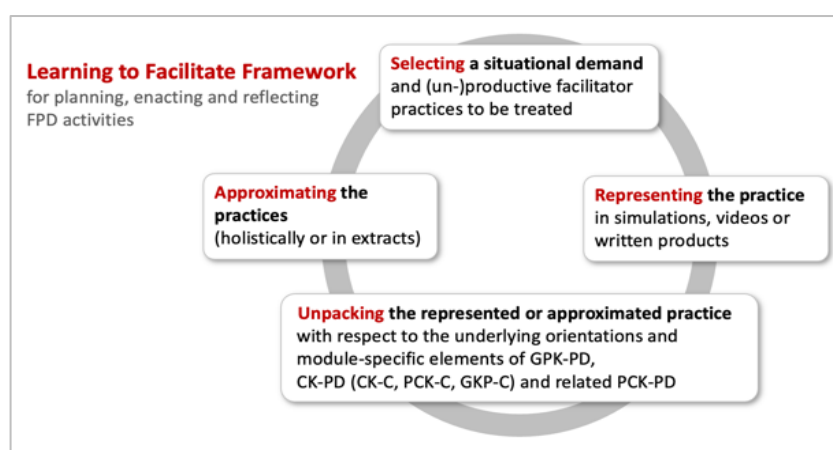


Figure 2. The Learning to Facilitate Framework for designing FPD activities (adapted from Borko et al., 2025).

Not all four pedagogies need to be included in each FPD activity, and the pedagogies can be combined differently and in different orders (Borko et al., 2025). FPD designers are advised to first *select* the focal situational demand, which can vary in level of specificity—from engaging teachers to analyse a particular student product with specified PCK categories to leading a discussion with extensive participation. *Representing* productive practices for meeting the selected demand can be designed in different modalities (e.g., simulation, video, or physical artefact). *Unpacking* the represented practices often entails explicating the underlying knowledge elements or decisions made by the facilitator educator. Prospective facilitators may have the opportunity to *approximate* the entire situational demand or a particular aspect of it, and again, the participants are then asked to *unpack* the decisions they made when planning or enacting the approximation.

Focus Question

While these characteristics and pedagogies are helpful for the micro design of FPD activities, we see a further need to conceptualise how the different knowledge elements are addressed precisely with respect to high-quality mathematics teaching. In particular, it is necessary to consider how the nested structure of practices and knowledge elements can be taken into account in structural constellations of

simultaneous TPD and FPD programs in which facilitators' learning needs at both the classroom level and TPD level are addressed.

So far, accounts of existing approaches mainly used the lifting strategy, in which facilitators' learning needs at both the classroom and TPD levels are addressed. Therefore, our report on the QuaMath FPD program will address the following focus question:

How can facilitator professional development activities be designed so that facilitators' learning needs at both the classroom and the teacher professional development level are addressed productively with respect to high-quality mathematics teaching?

We first introduce the context of the QuaMath FPD program in the next section, before addressing the focus question in the section after that.

Context of the QuaMath Facilitator Professional Development Program

Systemic Context and Goals of the Nationwide QuaMath Implementation Program

Before the DZLM was founded in 2011, the responsibility of TPD for German schools was completely left to decentralised TPD authorities in the 16 federal states. These authorities only occasionally cooperated with universities and mostly conducted one-shot PD sessions without a research base (Richter et al., 2019). The PD facilitators in the states sometimes had no explicit facilitator qualification beyond being experienced teachers. Most of them participated in a generic adult education facilitator preparation course (GPK-PD), but professional learning opportunities on PCK-PD for particular TPD content were scarce. The main aim of the DZLM is to establish long-term cooperation with the TPD authorities of the federal states, as well as to develop a sound theoretical and empirical basis for TPD design that accounts for practical demands and to implement systematic FPD programs for the facilitators in the states' TPD systems.

Together with the state authorities, the DZLM currently conducts the QuaMath program (titled Developing Quality in Mathematics Classrooms and Teacher PD) as a nationwide TPD project spanning from 2022 to 2033 (Prediger & Selter, 2024), led by the authors of this article. The project involves a consortium of 28 mathematics education professors, 40 academic project team members, 60 state project leaders, and 515 facilitators and aims to reach 10,000 schools in these 10 years. Each year, the facilitators work with 1,250 new schools within 200 school networks, offering a selection of 26+1 different TPD modules catering to all age groups (26 school modules from Grade 1 to Grade 13 and 1 for Kindergarten teachers). In their Year 1, participating primary or secondary schools start with a basic module in which the principles of teaching and related tasks are introduced. In Year 2, the schools specialise in two to four further modules, for example, on stochastics education in Grades 1–4, algebra education in Grades 8/9, using digital media in Grades 5/6, or language-responsive teaching in Grades 1–4. From Year 3 onwards, the schools continue with collaborative work autonomously in established between-school networks and within-school teams using self-learning TPD online modules.

The program's overall aim is to increase the quality of mathematics teaching in the participating schools and kindergartens. Since empirical studies have consistently shown that deep subject-specific teaching quality has a much stronger impact on students' learning gains than non-subject-specific factors or aspects of school organisation (Kunter et al., 2013; Seidel & Shavelson, 2007), QuaMath focuses on fostering deep subject-specific quality in line with a shared vision of high-quality mathematics teaching (Cobb & Jackson, 2021; Newmann et al., 2001; Schoenfeld, 2014). To this end, the TPD content of all 26 school modules promotes coherently the same five principles of high-quality mathematics teaching (see Figure 3 from Prediger et al., 2022a).

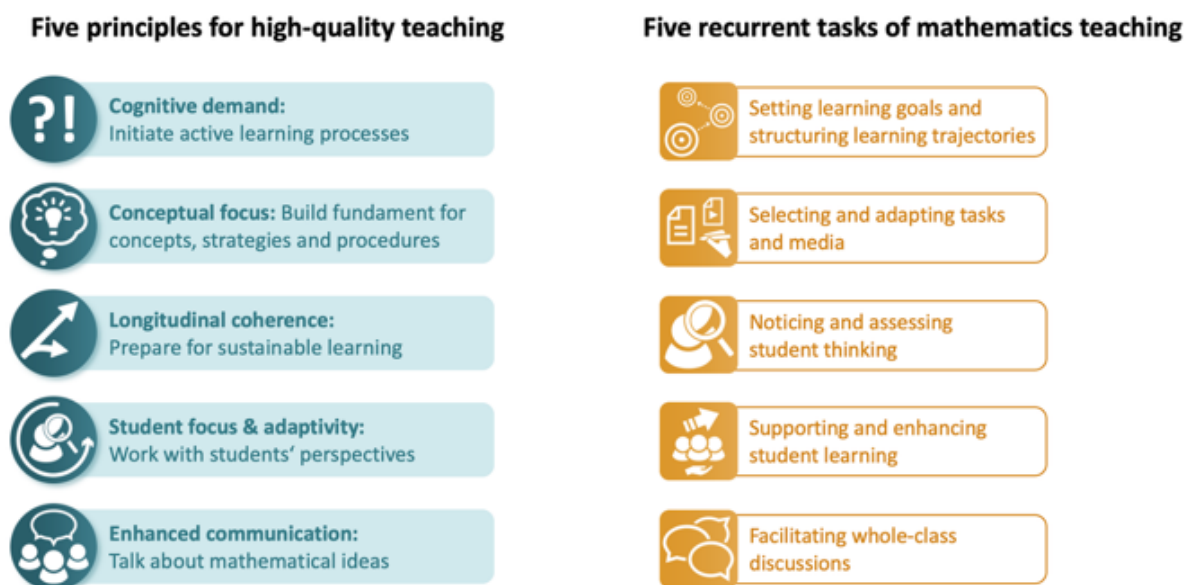


Figure 3. Coherent framework for the TPD content of all 26 QuaMath TPD modules for primary and secondary schools.

The TPD content in each module aims to promote module-specific productive practices for mastering tasks of teaching at the classroom level (see the content-related teacher expertise framework by Prediger, 2019, in which tasks of teaching play the analogous role to facilitator jobs). For the QuaMath TPD, we specified five recurrent tasks of teaching (depicted in Figure 3). While a current survey has revealed that these five tasks of teaching are also targeted in many other practice-based TPD programs (Shure et al., 2025), what is unique about the QuaMath TPD program is that the same set of five tasks of teaching are related coherently to the same set of principles for high-quality teaching across the 26 modules.

Structural and Institutional Characteristics of the QuaMath FPD Program

For each of the 26 QuaMath TPD modules (spanning six to twelve months each), an FPD module is designed and conducted. All 515 QuaMath facilitators begin with the basic FPD module for primary or secondary education before specialising in two to four further modules. The QuaMath FPD program lasts two to three years (each conducted in groups of 30-50 facilitators) and continues with ongoing support and exchange meetings. We briefly present the structural and institutional characteristics of the QuaMath FPD program outlined by Karsenty et al. (2025) that are relevant for explaining the context.

The target group for the FPD modules comprises experienced facilitators (two-thirds) and novice facilitators (one-third). All of them are experienced teachers (for mathematics and at least one other subject) with permanent contracts as teachers and partial buy-outs for their TPD responsibilities (up to 20% of their working time). The QuaMath FPD program is organised as a long-term program over 4 years in a combined model of preparation and support. Participants are expected to conduct the classroom experiments and to document and reflect on their TPD experience.

With respect to facilitators' initial involvement with the TPD materials, it should be noted that each TPD design team involves one or two experienced facilitators in the design process. However, this co-constructive design process cannot include the other (up to 300) facilitators who will later conduct the TPD module as part of the scaled-up nationwide project, so alternative approaches are required. Although facilitators are supported with highly specified TPD materials, they are encouraged to adapt the program to the needs of their TPD contexts and the participating teachers. The program pursues

clearly specified TPD contents for each module and aims to establish particular processes, some of which are unfamiliar in the German school context, e.g., teachers' collaboration. The TPD materials include videos, inputs, and activities for the TPD sessions, outlining explicit options for adapting the TPD session to teachers' potential diverse starting points. This structure of the TPD program is depicted in the upper part of Figure 4. It combines the pedagogies of practice-based TPD (Grossman et al., 2009), here realised by representing and approximating practices for the five tasks of teaching (Figure 3), and unpacking them with respect to the underlying module-specific CK-C and PCK-C elements. Following empirical findings on the significance of teachers' classroom experiences with new practices and curriculum materials (Clarke & Hollingsworth, 2002), teachers' classroom experiments with the provided curriculum materials form a key component for deepening and extending teachers' classroom practices. These experiments are prepared jointly in a TPD session, and teachers' classroom experiences are reflected on and analysed in the subsequent TPD session.

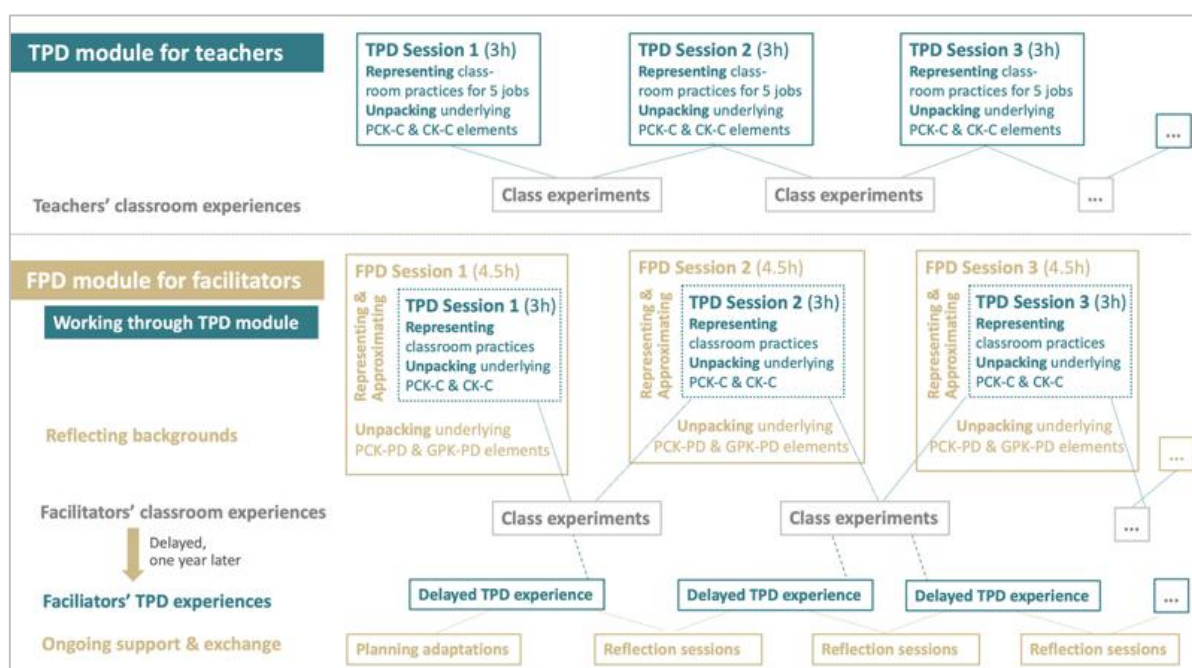


Figure 4. Structure of the QuaMath TPD module for teachers and the QuaMath FPD module for facilitators.

While some elements of the TPD modules are familiar to some experienced facilitators, others are new, and none of the facilitators have experienced the entire TPD module. Therefore, facilitators' experience of TPD is deliberately integrated into their FPD. The lower part of Figure 4 presents the structure of the FPD program: For each 3-hour-lasting TPD session in any six-month TPD module, we organise 4.5 hours of FPD, in which the TPD session is worked through by representing facilitator practices and unpacking them with respect to the underlying module-specific PCK-PD and GPK-PD elements, as well as the module-overarching principles of high-quality teaching. When facilitators conduct the module in the succeeding year, they are offered ongoing support and opportunities to share experiences and discuss potential adaptations.

The focus of the FPD goals is on both the classroom level and the TPD level: CK-PD includes some elements of CK-C (when it is assumed that facilitators do not have sufficient knowledge of the mathematical content), and the PCK-C underlies the classroom teaching practices for the five tasks of teaching following the five QuaMath principles (Figure 3). At the TPD level, the FPD focuses also on the content-independent processes that teachers need to engage in, thus some generic GPK-PD elements for content-independent facilitation. Most prominently, the FPD goals are focused on the

content-related and module-specific practices, and the underlying PCK-PD elements needed to initiate and support teacher learning in a targeted manner.

Micro Design of the QuaMath Facilitator Professional Development Activities

For the QuaMath FPD program, the simultaneous provision of TPD and FPD experience means that the micro design requires facilitators to move flexibly between two levels in relation to high-quality mathematics teaching. Following the Learning to Facilitate Framework (see Figure 2), we plan a sequence of FPD activities for each task of teaching that the facilitators will later implement with their participating teachers (orange parts in the second column in Figure 5). We then decide how the prospective facilitators will work through the TPD activity (e.g., through live presentation of the task and discussion of productive practices as part of their engagement with the TPD activity, through video representation of a TPD session, or through teachers' products from real TPD) and how this can be unpacked with respect to productive classroom practices and the underlying CK-C and PCK-C elements (petrol-coloured parts in Figure 5). In this way, prospective facilitators can develop or extend their classroom practices as mathematics teachers and the underlying knowledge at the classroom level. The FPD activity series then lifts the reflection from the classroom level to the TPD level, unpacking the represented, experienced or approximated TPD activity with respect to planning and facilitation practices, as well as the underlying PCK-PD and GPK-PD (outer ochre-coloured parts in Figure 5). In addition, there are, of course, individual familiarisation and preparation phases for the facilitators.

FPD level: Selecting a facilitator job and substantiating it for a particular task of teaching		Representing/approximating/experimenting with productive facilitator practices		Unpacking facilitator practices with respect to underlying PCK-PD/GPK-PD
TPD level: Selecting a typical task for classroom teaching and module-specific productive practices		Representing/approximating/experimenting with a TDP activity	Unpacking classroom practice with respect to underlying CK-C/PCK-C elements & principles	
Facilitator Job I: Launching and moderating a TPD activity including the reflection part to leverage teachers' ideas				
Example FPD Activity Series 0 (from Borko et al, 2025)	 Moderate classroom discussions on arguing	Work on TPD Activity 0a: Analysis of a classroom video on constructing arguments and critiquing the reasoning of others (Live represented and then approximated in rehearsals)	TPD Activity 0b: Unpack teacher moves (PCK-C) with focus on arguing in a particular topic (CK-C)  Enhanced communication	FPD Input & Activity 0c: Analyse facilitators' planning decisions (re particular PCK-PD elements) and discussion moves that focus on these challenges (GPK-PD)
Example FPD Activity Series 1	 Identify meaning-related phrases for language support for mathematical explanations	Work on TPD Activity 1a: Select, then compare selections (Live represented)	TPD Activity 1b: Systematise differences between formal and meaning-related phrases (CK-C)  Conceptual focus  Enhanced communication	FPD Activity 1c: Inform about research findings on typical teacher challenges and analyse teachers' typical answers with respect to resources and difficulties (explicating PCK-PD)
Facilitator Job II: Harness teachers' experiences from their class experiments for reflecting the principles				
Example FPD Activity Series 2	 Support and enhance students' learning processes  Monitor students learning progress	Between-TPD Activity 2a: Own class experiment with given curriculum materials for particular math content Focus on principle xxx... (Experimented)	TPD Activity 2b: Reflect in the next session: What did students understand (CK-C)? How to enhance that in the next lesson? What worked (not so) well regarding principle x? What will you keep for your next classes?  Conceptual focus	FPD Activity 2c: What would be most important for your teachers to reflect on regarding principle x? FPD Activity 2d: Plan the preparation of the experiment in your TPD
Facilitator Job III: Adapt TPD materials for the facilitator's own teacher group and context				
Example FPD Activity Series 3	 Sequence of activities on different tasks of teaching	In a (usually self-organised) planning meeting to prepare for the facilitator's own TPD	FPD Activity 3b: Unpack the components of the TPD material with respect to addressed CK-PD (principles and CK-C, PCK-C) and their relevance for teacher learning in PCK-PD	FPD Activity 3a: Discuss facilitators' ideas for adaptations of the TPD materials (GPK-PD and PCK-PD) (to be implemented in later class experiments)
Facilitator Job IV: Promote school-internal cooperation in school team meetings and mutual peer observation				
Example FPD Activity Series 4	Intended teacher process: Module-specific cooperation, e.g., on  	Between-FPD Activity 4c: Accompany the teacher team in their team meeting and mutual peer observation (Experimented at TPD level, potentially represented in video of a team meeting)	FPD Activity 4b: Which PCK-C elements should be reflected in the team meeting and peer observation?	FPD Input 4a: Cooperation as a challenge in German school culture, quality degrees of content-related cooperation and enablers (GPK-PD) FPD Activity 6d: Reflect on focused elements (PCK-PD) and productive moves for enhancing teamwork (GPK-PD)

Figure 5. FPD activities integrating TPD activities in the *Learning to Facilitate Framework*: Five examples for four typical facilitator jobs.

The Example FPD Activity Series 0 in Figure 5, taken from Borko et al. (2025), exemplifies the Learning to Facilitate Framework in a video-based TPD activity focusing on the mathematical practice of arguing (CK-C in light of the principle of enhanced communication). In a video-based TPD Activity 0a (enacted live with the facilitators, who take on the roles of teachers), one teacher's practice of moderating classroom discussions is presented, and in TPD Activity 0b, it is unpacked with respect to

the teacher moves (CK-PD), focusing on arguing within a particular mathematical topic that is slightly unpacked (CK-C). Then, in FPC Activity 0c, prospective facilitators are asked to change from “wearing the math teacher hat” to “wearing the facilitator’s hat” (Borko et al., 2025, p. 5) to unpack how the TPD discussion was planned and enacted. While analysing facilitators’ planning decisions and discussion moves that focus on these challenges, important PCK-PD and GPK-PD elements are explicated, reflected upon, and connected to facilitators’ visible actions.

In the following sub-sections, we present four other example FPD activity series for the same facilitator job and other facilitator jobs from the QuaMath FPD program, each using the same overarching structure in different ways. This illustrates the variety of micro designs under the same general idea of nesting TPD activities within FPD activities with the pedagogies of representing, approximating, experimenting, and unpacking at two levels. By presenting and analysing these cases of FPD activity series, we aim to further elaborate the design framework developed. Systematic empirical research on facilitators’ learning within these activities will be published in subsequent publications, as the focus of the current article is on the research-informed design framework.

Facilitator Job I: Launch and Moderate a Teacher Professional Development Activity Including Reflection to Leverage Teachers’ Ideas

To prepare facilitators to moderate a TPD activity, we have chosen an approach in which the activity first strengthens the facilitators’ extended CK-PD (FPD Activities 1a and 1b in Figure 5) before making the learning experiences explicit with respect to underlying PCK-PD elements (FPD Activity 1c in Figure 5). Activities 1a and 1b play a significant role in ensuring that facilitators first undergo their learning processes as teachers first. These processes are not always straightforward or linear; yet it is precisely through them that CK-PD is strengthened with respect to the addressed CK-C, PCK-C, or GPK-C elements. After engaging in these learning processes themselves, facilitators can reflect on the kinds of difficulties that teachers may encounter with the respective content. This enables them to relate more closely to teachers’ challenges and learning trajectories, thereby fostering the development of their PCK-PD.

The example of FPD Activity Series 1 in Figure 5 is taken from the QuaMath basic module for primary schools (Grades 1 to 4). The PD materials are grounded in the fundamental principle of enhanced communication, particularly with respect to the development of shared mathematical language in primary school. They provide empirical evidence of the central role of language in children’s mathematical learning processes and include specific recommendations for fostering a shared and meaningful mathematical language. This primarily concerns the language through which children can conceptualise mathematical content. It focuses on meaning-related phrases—linguistic expressions that children produce when addressing questions such as “How should I set up or draw the task?” or “How can I describe it clearly?”—which support them in interpreting and making sense of mathematical representations. Thus, the goal of the TPD is to identify such meaning-related phrases for specific mathematical content that children need in order to engage in demanding discourse practices such as explaining meanings and arguing as well as developing conceptual understanding (Götze, 2019; Pöhler & Prediger, 2015). The TPD recommends visualising and recording these meaning-related phrases in a language frame (right of Figure 6; a central task of teaching, as shown in PD research on enhancing communication by Prediger, 2019). Many teachers, however, tend to focus primarily on technical vocabulary, even though research shows that this has only a limited effect on the development of conceptual understanding (Götze, 2019). In doing so, teachers often overlook representations and meaning-related phrases.

In order to prepare the facilitators for this content (which is usually unfamiliar to them), they are first asked to develop their conceptual language frame while wearing the teachers’ hat (TPD Activity 1a in Figure 5). This then serves as a starting point for discussion in TPD Activity 1b (Figure 5) what distinguishes a conceptual language frame from a technical vocabulary frame (a crucial CK-C element).



The differences between formal and meaning-related phrases can then be negotiated and systematised with respect to providing language support (a crucial PCK-C element).

Figure 6 illustrates these processes through products generated in a QuaMath FPD session in February 2024. As the reproduction (and translation) of a first draft by one facilitator (left in Figure 6) shows, many facilitators focused predominantly on technical vocabulary, while references to concrete representations remained abstract and did not sufficiently support students' use or communication of mathematical ideas. In Activity 1b, facilitators collaboratively discussed the limitations of such drafts and identified the need for visual representations accompanied by meaning-related phrases—linguistic responses that make the structures in the representations explicit (e.g., “replace one counter, total number remains”). A later version of the language frame, emphasising meaning-related phrases and representations, was jointly developed (right in Figure 6).

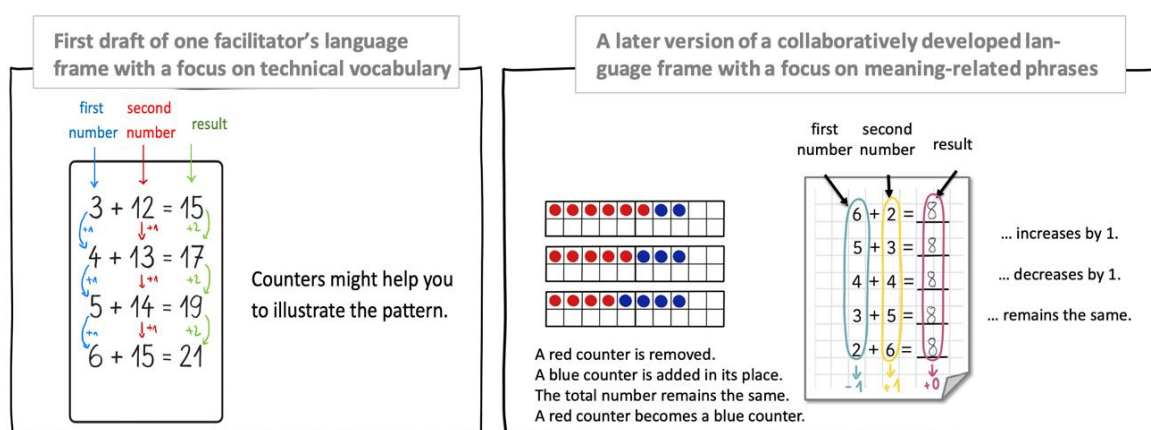


Figure 6. PCK-C discussed in the FPD Activity Series 1: First drafts and later versions of identifying phrases for language support for mathematical explanations.

In FPD Activity 1c (Figure 5), facilitators begin by reflecting on their initial drafts and their limitations and relating them to the information received about research findings on typical teacher challenges. They analyse teachers' typical responses with respect to resources and difficulties (explicating PCK-PD). In this way, the activity series exemplifies how facilitators' engagement with concrete teaching challenges can be transformed into opportunities for deepening both their CK-PD and their PCK-PD as a basis for their capacity to design effective TPD.

FPD activities such as 1c aim to strengthen facilitators' capacity to address teachers' diverse practices and underlying orientations within TPD discussions. Beyond familiarising facilitators with anticipated teacher practices, these activities focus on developing and collectively refining responsive strategies—particularly for situations in which teachers' contributions are not immediately conducive to students' learning progress. The strategies generated are documented and shared across facilitators to inform the preparation of subsequent TPD sessions, thereby fostering the development of facilitators' PCK-PD and moderation practices.

Accordingly, FPD activities like 1c are systematically integrated into the program, especially in contexts characterised by heterogeneous teacher practices and high demands on facilitation. Engaging in these activities requires facilitators to activate their CK-PD, drawing simultaneously on elements of CK-C, PCK-C, and GPK-C in order to anticipate teacher responses and orchestrate productive discussions. In this way, Activity 1c functions as a vehicle for the further development of facilitators' PCK-PD.

Facilitator Job II: Harness Teachers' Experiences from Their Classroom Experiments for Reflecting on the Principles

Classroom experiments are a central design element in all QuaMath modules and serve to implement TPD content in mathematics teaching (Figure 4). However, simply preparing these experiments at the end of a TPD network meeting and hoping that teachers will carry them out is not sufficient. It is equally important to reflect on these experiences in subsequent TPD sessions and in discussions with other teachers (Clarke & Hollingsworth, 2002; Timperley et al., 2007). Facilitators play a central role in supporting the preparation and reflection phases of classroom experiments, helping to explicitly link teachers' plans and experiences to the underlying principles and knowledge elements (Prediger & Selter, 2024, p. 36) of the TPD content. Preparing facilitators for this job is therefore a central component of the QuaMath FPD (Baumanns et al., 2025). Accordingly, after the first FPD session, the facilitators are asked to conduct a classroom experiment themselves, using the curriculum materials provided in their mathematics teaching (facilitator's classroom experiments, see Figure 4). The results of these experiments are documented and form the basis for the first phase in the next FPD session.

The advantage of this approach is that facilitators' experiences with classroom experiments provide the facilitator educators with in-depth insights into their CK-C and PCK-C. Of particular importance are insights into how facilitators identify and address the underlying principles or core messages in their mathematics teaching practices and how they interpret and internalise these principles or core messages in the classroom experiment when wearing their teacher hats.

One classroom experiment for facilitators (and teachers) involved working with a conceptual language frame (following Activity Series 1a-c, Figure 5). This should be designed to fit the current mathematical content learning goals, enhancing communication and building a conceptual understanding as a foundation for concepts, strategies, and procedures under consideration. Facilitators were asked to monitor the extent to which the children's engagement in ambitious discourse practices developed and whether they adopted the offered meaning-related phrase. To illustrate the facilitators' reflections on their classroom experiments while wearing teachers' hats, we give three quotations from TPD activity 2a (Figure 5):

Facilitator Aline: The children only gained knowledge through the use of concrete representations! In class, it was important to visualise the pattern and to talk about the patterns in a meaningful way. This supports the epistemic function of language.

Facilitator Beat: I used to give the children a [technical] vocabulary frame that I had designed by myself. Now, I still prepare parts of it in advance, such as visual representations, but I formulate the meaning-related phrases together with the children ...

Facilitator Chris: As I write this reflection, I realise that this material causes me the most concern and that I still have a lot of work to do. I hope that I will have internalised this more by the time of the first network meeting on this TPD session (...) by asking myself what an enormous role language plays in meaning-making processes.

[Data from the QuaMath FPD basic module]

The lessons learned by the facilitators show variation: Facilitators Aline and Beat appear to have recognised changes in their teaching practices. Facilitator Chris additionally begins reflections on how she might effectively address this topic within the TPD session she is planning to conduct one year later, as she does not yet seem confident in her PCK-C. These in-depth personal experiences were discussed in the next session as TPD Activity 2b (Figure 5), with explicit connection to the principles (in this case, conceptual focus and enhanced communication) and particular CK-C elements that students understood or still need further support with.

Chris' reflection served as a productive starting point for FPD Activity 2c (Figure 5), signalling what would be most important for the teachers to reflect on regarding the design of language supports aimed at enhancing students' conceptual understanding. In these reflections, facilitators re-discovered that many of their future teachers might also initially tend to design technical vocabulary frames, as

exemplified by Beat. In contrast, Aline's experiences were jointly identified as particularly valuable and should be highlighted as an important "gem" to be cherished together. In this way, the PCK-PD elements concerning typical teacher challenges were discussed during FPD Activity 2c (Figure 5).

In FPD Activity 2d (Figure 5), the group then started the phase of preparing the classroom experiment in the future TPD and anticipated what kind of gems and challenges might occur. The facilitators' in-depth personal experiences were now reconsidered at the second level to foster the development of PCK-PD among facilitators. In the case of the language frames, these reflections mitigate the uncertainties articulated by Chris, and the preparation phase of the classroom experiment was carried out jointly in the FPD, which proved beneficial also in other contexts. The reflection of the classroom experiments can also be simulated in the FPD.

Facilitator Job III: Adapt Materials for the Facilitator's Use in Teacher Professional Development

While the FPD Activity Series 0 to 2 relies on the facilitators' content-related expertise to conduct the TPD sessions as intended in the TPD material design, the QuaMath program does not seek to achieve highly specified standardisation and implementation fidelity in the narrow sense (as problematised, e.g., by Jarry-Shore et al., 2023). Instead, our aim is to promote facilitators' content-related expertise in adapting the TPD materials to better suit the specific needs of their teachers and regional contexts, while still maintaining the core of the modules with integrity (Prediger & Selter, 2024). These adaptations should be implemented in a way that preserves the core ideas of the five principles and the tasks of teaching in the TPD modules.

To promote facilitators' design capacities (Brown, 2009) for making adaptations with integrity, the typical FPD activity series for this facilitator job includes a collective discussion of facilitators' ideas for adapting the TPD materials (FPD Activity 3a, last column of Figure 5). This allows unpacking ideas on adaptations not only with respect to organisational aspects of GPK-PD (e.g., "I will use a digital board to collect teachers' ideas"), but mainly in view of the underlying PCK-PD elements (e.g., "My teacher group has so many experts on the CK-C element xxx that we can directly go to PCK-C element yyyy"). PCK-PD-related discussions often involve unpacking connections between addressed CK-PD components (principles and CK-C, PCK-C) and their relevance for teacher learning (FPD Activity 3b).

Figure 7 provides an FPD slide in a generic form that is to be substantiated for each TPD with different core messages and underlying module-specific CK-C and PCK-C elements. Facilitators are asked to work through the slides once more, this time looking at the background information they are directed to. Only after analysing how the specific slides contribute to the TPD goals of the respective phases are they asked to discuss adaptations with respect to teachers' resources, learning needs, and systemic constraints in schools.

Goals of activity sequences and their adaptation

Group Activity on analysis and adaption of Phase 3 (30 min)

Slides 31-54 belong to Phase 3 of the TPD session works towards the summary slide printed on the right.

1. Work through the slides and their comments again to analyze:
 - What does each of the slides / activities contribute to the elements on the summary slide?
 - What are the aspects that will be familiar to most teachers? What those that need most attention?
2. How do you plan to adapt the slides (omit/add/change) so that
 - your teachers' resources are elicited and used?
 - your teachers' learning needs are even more focused?
 - you respect the constraints of the participating schools?

Summary of Phase 3

((Core message on productive practice for a certain tasks of teaching in light of certain principles)))

Underlying module-specific CK-C elements

Description of practices with underlying PCK-C elements

Figure 7. FPD activity 3a and 3b with a generic form of summary slide.

Facilitator Job IV: Promote School-internal Cooperation with “Team Experiment Tasks”

Facilitator Job IV extends beyond the immediate enactment of TPD sessions. It promotes internal school cooperation during in-school team meetings and mutual peer observations (inspired by light forms of lesson study, Yoshida et al., 2021). It refers to the intended teacher processes of module-specific cooperation (Borko & Potari, 2024; Clarke & Hollingsworth, 2002) regarding different tasks of teaching, for example, formatively assessing student thinking and fostering and enhancing student learning. The FPD Activity Series 4 begins in the last column, with FPD Input 4a, which provides facilitators with GPK-PD knowledge on content-specific cooperation—an approach not yet widely established within German school culture, where content-related cooperation often remains at a low-quality level—and on enablers for higher-quality collaboration. This is followed by FPD Activity 4b, which addresses which PCK-C elements should be focused on during teachers' team meetings and peer observations. Between two FPD sessions, facilitators are asked to accompany one school team in their team meetings and peer observations. Although staff resources in states do not allow for the regular inclusion of individual coaching of all school teams in the QuaMath program, we include this experiment at the TPD level to provide facilitators with FPD learning opportunities for conducting such team meetings. Another possibility would be to analyse a video of a team meeting instead. During the next FPD session, Activity 4d includes reflecting on these team meetings with respect to the elements focused on by the team (PCK-PD) and productive facilitation strategies to enhance teamwork (GPK-PD). In this FPD activity series, facilitators' experimentation occurs at the TPD level rather than the classroom level, as in Activity Series 0 to 3.

Discussion and Conclusion

In this article, we have presented the structural and institutional characteristics of the QuaMath FPD program (Karsenty et al., 2025), highlighting the high relevance of intense cooperation with the state educational authorities to co-constructively shape these structural and institutional characteristics.

Facilitators' expertise for planning and facilitation practices and underlying knowledge (Borko et al., 2014b; Prediger et al., 2022b) is crucial for the success of scaled-up TPD programs (Zaslavsky & Leikin, 1999). This is why the facilitators' preparation for each TPD module and the ongoing support

are important aspects of the design duty for program designers (Borko et al., 2021; Karsenty et al., 2025; Jacobs et al., 2017; Lesseig et al., 2017).

With respect to the micro design, we have focused on one of those characteristics: the need to prepare facilitators simultaneously for the CK-PD, i.e., the TPD content (specific classroom practices and underlying elements of CK-C, PC-C and GPK-C at the classroom level) and for the facilitator jobs of planning and moderating the TPD (including productive facilitators' practices and underlying elements of GPK-PD and PCK-PD at the TPD level; Wilhelm et al., 2019). The FPD activities presented in this article were systematically structured in relation to the challenges that facilitators encounter in their work. These activity series were not only refined but newly developed and implemented for the first time within this design-based research cycle. By aligning each series with specific facilitator jobs, we aimed to make explicit how theory-informed and research-based design principles can be operationalised in concrete FPD formats.

We have described four important facilitator jobs and exemplified FPD activity series that prepare for these jobs with regard to specific TPD activities (Figure 5): (I) launching and moderating an FPD activity, including the reflection part to leverage teachers' ideas; (II) adapting TPD materials for one's teacher group and context; (III) harnessing teachers' experiences from the classroom experiments; and (IV) promoting school-internal cooperation on TPD content. While this list is by no means exhaustive, it illustrates that, in addition to the pedagogies of representing–unpacking–rehearsing of TPD activities, which have already been described by others (Borko et al., 2021; Borko et al., 2025), there can also be other types of FPD activity series involving nested TPD activities. The examples allude to the complexity involved in designing FPD that promotes facilitators' expertise in supporting teachers' professional learning, indicating that each task of teaching may require a slightly different activity series—with carefully designed representations, approximations, or experimentations—followed by unpacking activities addressing underlying principles and knowledge at both classroom and TPD levels.

The primary aim of this article is therefore not to provide evidence of facilitators' learning outcomes or to demonstrate measurable impact on their expertise. Rather, we seek to show how facilitators' learning can be deliberately supported through carefully designed, theory-guided, and research-based FPD activities. In unpacking the micro-design cases of four facilitator jobs, we occasionally refer to short accounts of facilitators' learning processes. These references, however, do not constitute a systematic qualitative data analysis. A comprehensive empirical investigation of facilitators' learning trajectories would require further data collection and in-depth analysis, which is currently being prepared as part of our follow-up research.

Future research should therefore empirically investigate these FPD activity series to identify the conditions under which they succeed, the mechanisms through which they support facilitators' professional growth, as well as the obstacles that hinder realising their intended potential. Such analyses will be necessary to complement the present design-based contribution with robust empirical evidence on facilitators' learning and development.

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Funding

The QuaMath program is financially funded by the Standing Conference of State Educational Departments (KMK). Fifteen federal states engage by providing 450 facilitators and approximately 60 state coordinators to conduct the program. The article was developed for the Transfer Centre of the Competence Consortium learning: digital (funded by the German Ministry of Education and Research, BMBF Grant No. 01JA23T01I to S. Prediger).

Ethical approval

Ethical approval was granted by the praesidium of the Standing Conference of the Ministers of Education and Cultural Affairs of Germany (Kultusministerkonferenz) by resolution of July 19, 2023.

Competing interests

The authors declare there are no competing interests.

References

- Baumanns, L., Eichholz, L., Götze, D., & Gutscher, A. (2025). Teaching experiments as a central design element of facilitators' PD. In M. Bosch Casabò, S. Carreira, G. Bolondi, M. Gaidoschik, & C. Spagnolo (Eds.), *Proceedings of the 14th Congress of the European Society for Research in Mathematics Education (CERME14)*. ERME / HAL. <https://hal.science/hal-05305117v1>
- Beisiegel, M., Mitchell, R., & Hill, H. C. (2018). The design of video-based professional development: An exploratory experiment intended to identify effective features. *Journal of Teacher Education*, 69(1), 69–89. <https://doi.org/10.1177/0022487117705096>
- Borko, H., & Potari, D. (2024). *Teachers of mathematics working and learning in collaborative groups: ICMI Study 25*. Springer.
- Borko, H., Carlson, J., Deutscher, R., Boles, K. L., Delaney, V., Fong, A., Jarry-Shore, M., Malamut, J., Million, S., Mozenter, S., & Villa, A. M. (2021). Learning to lead: An approach to mathematics teacher leader development. *International Journal of Science and Mathematics Education*, 19(1 Suppl), 121–143. <https://doi.org/10.1007/s10763-021-10157-2>
- Borko, H., Jacobs, J., Seago, N., & Mangram, C. (2014a). Facilitating video-based professional development: planning and orchestrating productive discussions. In Y. Li, E. A. Silver, & S. Li (Eds.), *Transforming mathematics instruction* (pp. 259–281). Springer International.
- Borko, H., Jarry-Shore, M., Schwartz, G., Karsenty, R., Prediger, S., & Rösike, K.-A. (2025). The learning to facilitate framework. In M. Bosch Casabò, S. Carreira, G. Bolondi, M. Gaidoschik, & C. Spagnolo (Eds.), *Proceedings of the 14th Congress of the European Society for Research in Mathematics Education (CERME14)*. ERME/HAL. <https://hal.science/hal-05305128v1>
- Borko, H., Koellner, K., & Jacobs, J. (2014b). Examining novice teacher leaders' facilitation of mathematics professional development. *Journal of Mathematical Behavior*, 33, 149–167. <https://doi.org/10.1016/j.jmathb.2013.11.003>
- Brown, M. W. (2009). The teacher-tool relationship. Theorizing the design and use of curriculum materials. In J. T. Remillard, B. A. Herbel-Eisenmann, & G. M. Loyd (Eds.), *Mathematics teachers at work: Connecting curriculum materials and classroom instruction* (pp. 17–36). Routledge.
- Bruns, J., Hagena, M., & Gasteiger, H. (2022). Effects of facilitator professional development on teachers' learning: An intervention study in the context of early mathematics education. In J. Hodgen, E. Geraniou, G. Bolondi, & F. Ferretti (Eds.), *Proceedings of 12th Congress of the European Society for Research in Mathematics Education (CERME12)*. University of Bolzano and ERME. <https://hal.science/hal-03746235v1>



- Clarke, D. J., & Hollingsworth, H. (2002). Elaborating a model of teacher professional growth. *Teaching and Teacher Education*, 18(8), 947–967. [https://doi.org/10.1016/S0742-051X\(02\)00053-7](https://doi.org/10.1016/S0742-051X(02)00053-7)
- Cobb, P., & Jackson, K. (2015). Supporting teachers' use of research-based instructional sequences. *ZDM—Mathematics Education*, 47(6), 1027–1038. <https://doi.org/10.1007/s11858-015-0692-5>
- Cobb, P., & Jackson, K. (2021). An empirically grounded system of supports for improving the quality of mathematics teaching on a large scale. *Implementation and Replication Studies in Mathematics Education*, 1(1), 77–110. <https://doi.org/10.1163/26670127-01010004>
- Coles, A. (2019). Facilitating the use of video with teachers of mathematics: Learning from staying with the detail. *International Journal of STEM Education*, 6(1), 1–12. <https://doi.org/10.1186/s40594-018-0155-y>
- Elliott, R., Kazemi, E., Lesseig, K., Mumme, J., Carroll, C., & Kelley-Petersen, M. (2009). Conceptualizing the work of leading mathematical tasks in professional development. *Journal of Teacher Education*, 60(4), 364–379. <https://doi.org/10.1177/0022487109341150>
- Götze, D. (2019). Language-sensitive support of multiplication concepts among at-risk children: A qualitative didactical design research case study. *Learning Disabilities*, 17(2), 165–182.
- González, G., Deal, J. T., & Skultety, L. (2016). Facilitating teacher learning when using different representations of practice. *Journal of Teacher Education*, 67(5), 447–466. <https://doi.org/10.1177/0022487116669573>
- Grossman, P., Compton, C., Igra, D., Ronfeldt, M., Shahan, E., & Williamson, P. W. (2009). Teaching practice: A cross-professional perspective. *Teachers College Record*, 111(9), 2055–2100. <https://doi.org/10.1177/016146810911100905>
- Jacobs, J., Seago, N., & Koellner, K. (2017). Preparing facilitators to use and adapt mathematics professional development materials productively. *International Journal of STEM Education*, 4(1). <https://doi.org/doi.org/10.1186/s40594-017-0089-9>
- Jarry-Shore, M., Delaney, V., & Borko, H. (2023). Sustaining at scale: District mathematics specialists' adaptations to a teacher leadership preparation program. *Investigations in Mathematics Learning*, 15(1), 67–84. <https://doi.org/10.1080/19477503.2022.2140553>
- Karsenty, R., Borko, H., & Prediger, S. (2025). Mapping structural characteristics of preparation and support programs for mathematics PD facilitators. In M. Bosch Casabò, S. Carreira, G. Bolondi, M. Gaidoschik, & C. Spagnolo. (Eds.), *Proceedings of the 14th Congress of the European Society for Research in Mathematics Education (CERME14)*. ERME / HAL. <https://hal.science/hal-05305163v1>
- Karsenty, R., Pöhler, B., Schwartz, G., Prediger, S., & Arcavi, A. (2023). Processes of decision-making by mathematics PD facilitators: the role of resources, orientations, goals, and identities. *Journal of Mathematics Teacher Education*, 26(1), 27–51. <https://doi.org/10.1007/s10857-021-09518-z>
- Kunter, M., Klusmann, U., Baumert, Richter, D., Voss, T., & Hachfeld, A. (2013). Professional competence of teachers. *Journal of Educational Psychology*, 105(3), 805–820. <https://doi.org/10.1037/a0032583>
- Lesseig, K., Elliott, R., Kazemi, E., Kelley-Petersen, M., Campbell, M., Mumme, J., & Carroll, C. (2017). Leader noticing of facilitation in videocases of mathematics professional development. *Journal of Mathematics Teacher Education*, 20(6), 591–619. <https://doi.org/10.1007/s10857-016-9346-y>
- Newmann, F. M., Smith, B., Allensworth, E., & Bryk, A. S. (2001). Instructional program coherence. *Educational Evaluation and Policy Analysis*, 23(4), 297–321. <https://doi.org/10.3102/01623737023004297>
- Peters-Dasdemir, J., Holzäpfel, L., Barzel, B., & Leuders, T. (2023). Competency framework for the qualification of facilitators of mathematics. In A. Manizade, N. Buchholtz, & K. Beswick (Eds.), *The evolution of research on teaching mathematics* (pp. 311–338). Springer. https://doi.org/10.1007/978-3-031-31193-2_11
- Pöhler-Friedrich, B., Prediger, S., Schwartz, G., & Karsenty, R. (2026). Learning to leverage teachers' ideas: Facilitators' growth in moderating activities with simultaneously strong goal focus and participant focus. *Journal of Mathematics Teacher Education*, 29(1), 107–137. <https://doi.org/10.1007/s10857-024-09660-4>
- Pöhler, B., & Prediger, S. (2015). Intertwining lexical and conceptual learning trajectories. *Eurasia Journal of Mathematics, Science & Technology Education*, 11(6), 1697–1722. <https://doi.org/10.12973/eurasia.2015.1497a>
- Prediger, S. (2019). Investigating and promoting teachers' expertise for language-responsive mathematics teaching. *Mathematics Education Research Journal*, 31(4), 367–392. <https://doi.org/10.1007/s13394-019-00258-1>
- Prediger, S., & Selter, C. (2024). Establish shared visions and support productive adaptations on all levels. *Implementation and Replication Studies in Mathematics Education*, 4(1), 1–35. <https://doi.org/10.1163/26670127-bja10020>
- Prediger, S., Götze, D., Holzäpfel, L., Rösken-Winter, B., & Selter, C. (2022a). Five principles for high-quality mathematics teaching. *Frontiers in Education*, 7(969212), 1–15. <https://doi.org/10.3389/feduc.2022.969212>
- Prediger, S., Roesken-Winter, B., Stahnke, R. & Pöhler, B. (2022b). Conceptualizing content-related PD facilitator expertise. *Journal of Mathematics Teacher Education*, 25(4), 403–428. <https://doi.org/10.1007/s10857-021-09497-1>



- Richter, D., Becker, B., Hoffmann, L., Busse, J., & Stanat, P. (2019). Aspekte der Aus- und Fortbildung von Lehrkräften im Fach Mathematik und in den naturwissenschaftlichen Fächern. [Aspects of preservice and in-service teacher education in mathematics and science] In P. Stanat, S. Schipolowski, N. Mahler, S. Weirich, & S. Henschel (Eds.), *IQB-Bildungstrend 2018 [IQB-Educational trend 2018]* (pp. 385–410). Waxmann.
- Schoenfeld, A. H. (2014). What makes for powerful classrooms, and how can we support teachers in creating them? A story of research and practice, productively intertwined. *Educational Researcher*, 43(8), 404–412. <https://doi.org/10.3102/0013189X14554450>
- Seidel, T., & Shavelson, R. J. (2007). Teaching effectiveness research in the past decade. *Review of Educational Research*, 77(4), 454–499. <https://doi.org/10.3102/0034654307310317>
- Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15(2), 4–14. <https://doi.org/10.3102/0013189X01500200>
- Shure, V., Lehmann, M., Friesen, M., Roesken-Winter, B., & Prediger, S. (2025). Professional development research on promoting productive practices for teachers' handling of typical tasks of mathematics teaching: A systematic review. *ZDM–Mathematics Education*, 57(4), 795–813. <https://doi.org/10.1007/s11858-025-01659-2>
- Tekkumru-Kisa, M., & Stein, M. K. (Ed.) (2017). Design and facilitation of video-based professional development programs. *International Journal of STEM Education*, 4(1). <https://doi.org/10.1186/s40594-017-0086-z>
- Timperley, H., Wilson, A., Barrar, H., & Fung, I. (2007). *Teacher professional learning and development: Best evidence synthesis iteration*. Ministry of Education, New Zealand.
- Valoyes-Chávez, L., & Felmer, P. (2021). “She was probing me to see if I knew”: Becoming a credible and confident PD facilitator. *ZDM–Mathematics Education*, 53(5), 1097–1108. <https://doi.org/10.1007/s11858-021-01283-w>
- Van Es, E. A., Tunney, J., Goldsmith, L. T., & Seago, N. (2014). A framework for the facilitation of teachers' analysis of video. *Journal of Teacher Education*, 65(4), 340–356. <https://doi.org/10.1007/s10857-013-9245-4>
- Wilhelm, N., Zwetzschler, L., Selter, C., & Barzel, B. (2019). Vertiefung, Erweiterung und Verbindung von Wissensbereichen im Kontext der Planung einer Fortbildungsveranstaltung zum Thema Rechenschwierigkeiten. [Deepening, extending and connecting knowledge domains in the context of planning teacher professional development on mathematical difficulties]. *Journal für Mathematik-Didaktik*, 40(2), 227–253. <https://doi.org/10.1007/s13138-019-00143-1>
- Yoshida, N., Matsuda, M., & Miyamoto, Y. (2021). The landscape of Lesson Study: A methodology for teachers' professional development and educational research. In J. Kim, N. Yoshida, S. Iwata & H. Kawaguchi (Ed.), *Lesson study-based teacher education* (pp. 29–50). Routledge.
- Zaslavsky, O., & Leikin, R. (1999). Interweaving the training of mathematics teacher educators and the professional development of mathematics teachers. In O. Zaslavsky (Ed.), *Proceedings of the 23rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 143–158). Technion/PME.
- Zaslavsky, O., & Leikin, R. (2004). Professional development of mathematics teacher educators: Growth through practice. *Journal of Mathematics Teacher Education*, 7(1), 5–32. <https://doi.org/10.1023/B:JMTE.0000009971.13834.e1>

