Keeping Mathematics Alive in Teacher Education: Mathematics Teacher Educators Navigating Tensions

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This article reports on three cases of mathematics teacher educators (MTEs) who experienced tensions when aiming their teaching at extending and strengthening the mathematical knowledge of prospective teachers. Of particular interest in this study are tensions related to MTEs attempts to keep learners' attention on the intended mathematical ideas of lessons and activities, in addition to important emerging issues related to social justice, technology, and pedagogy. We build on literature that has considered tensions that transpire when teaching mathematics in the K–12 setting, extending the discussion to the case of mathematics teacher educators. This article contributes new and refined understanding about the interplay of tensions impacting mathematics teacher educators who, in their attempt to strengthen teachers' mathematical knowledge, are pulled away from mathematics in different directions.

Keywords · mathematics teacher education research · mathematical knowledge · societal issues · technology · tensions

Introduction

Our focus on tensions related to teaching mathematics is motivated by research that points to the importance of supporting the mathematical development of prospective teachers (e.g., Zazkis & Marmur, 2021). Tensions in teaching are unavoidable, complex, interdependent (Mason, 1988), and "reflect the complexity of teaching situations" (Sparrow & Frid, 2001, p. 453). While the majority of research on tensions has focused on K–12 teaching, we suggest that the complexities of teacher education compel attention toward tensions that can emerge for mathematics teacher educators (MTEs). With expectations to address a wide variety of issues such as cross-curricular applications, differentiated assessments, or technology integration, the scope of MTEs practice has expanded in recent years. Amongst all of these priorities, we wondered, "*How do MTEs keep mathematics alive in their practice?*"

Specifically, in the research, we focus on experiences of tension that emerged when MTEs were confronted with the realisation that their learners' attention had shifted away from the intended mathematical ideas of the activity. The premise of our study is that teachers' knowledge of mathematics can be enhanced in teacher education courses and professional development. By "enhancing knowledge" we do not mean engaging with advanced mathematical content; but rather, developing a deeper understanding of school mathematics via problem solving and building connections among different mathematical branches and concepts. As such, our focus is on experiences of MTEs attempting to engage prospective teachers in a mathematical activity, and the resulting tensions.

Attention to Tensions in the Literature

Experiences of Tensions in Teaching

Tensions in teaching can be seen as "endemic and inescapable" (Mason, 1988, p. 164), since teaching is evidently and unavoidably uncertain. Tensions lie "in the heart of the pedagogical process" (Byers, 1984, p. 36), and any choice that a teacher makes "sacrifices the advantages of the alternative" (Katz & Raths, 1992, p. 376). Tensions can ebb and flow over time (Berry, 2007) and can be unsolvable and recurring (Cuban, 1992; Lampert, 1985). As related to teaching and teachers, tension is defined as "a situation that offers a choice between at least two courses of action" (Katz & Raths, 1992, p. 376), or more descriptively as "a challenging situation with no clear-cut solution, a situation in which competing influences or forces suggest different, often opposing courses of action" (Barwell et al., 2016, p. 175). The notions of choice, and a possibility of different choices creates a tension, regardless of the action that may emerge. While our research focuses on tensions, we note that in the literature on teachers and teaching the notions of tensions and dilemmas are used interchangeably. Adler (1998) conceptualised tensions as personal or contextual. These were situations that required a teacher's choice, where either option presented advantages and disadvantages. Building on Adler's classification, tensions were described as internal or external, focusing on the sources of tensions at play (Rouleau, 2020; Liljedahl et al., 2023). Internal tensions are related to personal experiences, beliefs, pedagogy. External tensions are beyond control of individuals; they are related to, for example, imposed curriculum or assessment norms.

Navigating Tensions

According to Berry (2007), the language of tensions reflects the complexity of teaching and can serve as a framework for studying that practice. Researchers have described tensions as something that is managed rather than eliminated (e.g., Berry, 2007; Byers, 1984; Cuban, 1992). In comparison to an encountered problem, where a solution is sought, managing a tension may present a compromise or balance, rather than a definite solution. Liljedahl et al. (2023) described cases of teachers living with tensions, trying to resolve tensions, or avoiding tensions, all of which can be seen as different approaches to managing tensions. Barwell et al. (2016) mentioned mediating tensions. Both terms, "manage" and "mediate", carry with them connotations of control. Definitions provided by Oxford English Dictionary include the language "take charge of" (manage) and "intervene … in order to bring about an agreement or reconciliation" (mediate). In our view, however, an experience of tension may be beyond an individual's ability to "control," and while an individual may be able to manage or mediate their responses to experiences of tension, they cannot manage or mediate the tension itself. Rather, we suggest an individual must *navigate* tensions. Much like the captain of a ship navigates, rather than manages, the ocean, this language gives us a mechanism for conceptualising and describing experiences of tension for which an individual must plan and direct a course of action with little to no control over the imposing forces.

Tensions of Teacher Educators

Ben-Peretz and Flores (2018) acknowledged that teacher educators need to prepare teachers who will work with diverse student populations and in wide variety of school contexts. This task creates multiple tensions, such as balancing professional autonomy and creativity of a teacher with externally imposed policy and curriculum. They refer to the identified tensions as "paradoxes in teaching." Goodwin et al. (2023) identified tensions that "puzzled and plagued teacher educators." For example, in a world that is changing rapidly, teacher educators face a tension between preparing teachers for the present reality while also attempting to equip teachers with skills and knowledge for the future, which may pose unanticipated challenges and require flexibility of thought and action. Among other tensions, Goodwin et al. described the choice or balance between "place based or content fluid" in teacher education. That is, the tension raised between educating teachers with respect to their immediate environment, focusing on the local curriculum and culturally embedded initiatives (e.g., acknowledging Maori culture in New Zealand), or transcending geographical or national boundaries and focusing on universal issues.

Phillips and Park Rogers (2020) described tensions among educators' need to establish authority and credibility with prospective teachers alongside the intension to care, build relationships and rapport. Berry (2007) described tensions as binary pairs. For example, the "safety and challenge" pair described a tension between a constructive learning experience and a potentially uncomfortable learning experience, which may result in deeper learning; the "confidence and uncertainty" pair described the tension between the desire of teacher educator to build teachers' confidence while exposing them to complexities of teaching; the "planning and being responsive" pair describes a tension between responding to opportunities that arise in a particular situation and the planned activity of a lesson. We note that the tension between being responsive to learners and adhering to a plan is embedded in any teaching situation, not restricted to the work of teacher educators.

The tensions described in this section are applicable to teacher education in general, regardless of a disciplinary focus. In the next section we attend to particulars in the work of mathematics teacher educators.

Tensions of Mathematics Teacher Educators

While we found a dearth of research attending to tensions of MTEs related to mathematics, we have observed them at both a disciplinary level and at an individual level under different names. At the disciplinary level, we interpret tension as related to the long-standing debate about what kind (and how much) mathematical knowledge is necessary (or helpful) for teaching. At an individual level, we interpret tension as related to experiences of “disturbance” (Mason, 2002) that provoke moments of uncertainty and that can lead to new understandings. For instance, an experience of disturbance with respect to a student’s unanticipated mathematical solution led to a tension between expectation, novelty, and mathematical correctness (Zazkis & Mamolo, 2018). Since the teaching of mathematics content is relevant to our present study, we find it useful to consult with literature on K–12 education that has surfaced related tensions.

Tensions of Mathematics Teachers

Tensions experienced by teachers of mathematics can surface with regard to foundations of mathematics, pedagogical reform, open- versus closed-questions, task design, culture, and more. Across every grain size of experience—large scale reform to specific moments in class—tensions can emerge. For instance, Moreno-Armella (2014) described a foundational tension between intuition and formalism as essential in mathematics education, which are especially relevant to teachers’ choices in teaching calculus. Another example of a foundational tension was highlighted by Jaworski (2012), who described secondary mathematics teachers having to choose between investigative and didactic teaching approaches. In a way, the issue of “to tell or not to tell” or “when to tell” presents a common and resilient tension among mathematics teachers who value students’ engagement.

Some tensions described in the literature result from teachers’ experiencing novelty, such as learning and implementing new pedagogical approaches, attempting to change practice or being involved in reform-oriented professional development opportunities (Remillard, 1999; Ross et al., 2003). LeSage and Mamolo (2023) noted that tensions related to reform-based practices can elicit further tensions amongst competing pedagogical goals, between students and teacher, parents and teacher, administrators and teacher, and amongst colleagues. Such experiences of tension can be isolating, recurring, and layered. Herbst (2003) studied teachers’ attempts to implement a novel task in their lessons. It was observed that the actions of some teachers were in discord with the mathematical goals of the task, surfacing subject-specific tensions related to the “representation of mathematical objects, and elicitation of conceptual actions that students need to invest” (Herbst, 2003, p.198).

Tensions related to complexities of students' experience with mathematics have also emerged for teachers. For instance, Abtahi (2022) reflected on “multi-dimensional ethical tensions” related to teaching “dominant”, that is “western”, mathematics mandated by the curriculum in multi-cultural and ethnically diverse classrooms. Barwell et al. (2016) reflected on language diversity of learners and summarised language-related tensions and both centripetal and centrifugal forces: between formal and informal language to describe mathematical phenomenon, between learners’ language spoken at home and language of instruction, and tension related to the perceived status of a different languages in a society. In teaching mathematics for elementary school teachers, Oesterle (2012) described a tension between managing cognitive and affective goals, that is, between developing students’ understanding of mathematics and addressing negative attitudes towards to subject, possibly mathematical anxiety. These findings align with the observation by LeSage and Mamolo (2023) that some of the tensions felt by teachers emerge in response to their students experiencing tensions.

Outside of teaching contexts, research has also explored teachers' experiences of tension in professional development settings. While professional development is designed for teachers to acquire new knowledge related to their practice, a common tension arises when expectations do not meet the experience. For example, Nipper et al. (2011) described teachers' tensions related to both content and pedagogy. With respect to pedagogy, the instructor's use of open-ended problems created a tension for teachers who were considering the applicability of this approach in their classrooms. With respect to content, a tension arose when the mathematical content was found challenging and exposed vulnerability of participants' mathematical knowledge. Several researchers described tensions resulting from a professional development where teachers partnered with researchers or mathematicians. Widder et al. (2025) studied tensions that mathematics teachers experienced during inquiry based professional development with mathematicians. They identified epistemic tensions related to different cultural orientations of teachers and mathematicians, functional tensions and societal tensions. Johnson et al. (2016) described design tensions revealed during professional development built upon research-practice partnership. In particular, they noted tensions between teachers' and researchers' contexts, preferences and task modification. Similarly, Johnson et al. (2017) elaborated on "tensions of intentions" between designers' intentions in developing a task, teachers' intentions in implementing a task and students' experiences when engaging with the task. The tensions identified in professional development settings resonate with our study, which, in a sense, examines the issue from the "other side" by considering the experiences and perspectives of those involved.

Research Questions

In this research, we use the language of navigating tensions to frame MTEs experiences and the challenges they face in trying to "keep mathematics alive" in their teaching at a time where many competing forces are pulling prospective teachers' attention away from mathematics (Mamolo et al., 2022). In particular, we address the following research questions:

What tensions are experienced by MTEs while trying to enhance the mathematical knowledge of teachers?

How do MTEs navigate these tensions?

Methodology

To investigate experiences of tension of MTEs, we take a multiple-case study approach, which is well-suited for providing both in-depth descriptions and illustrating different perspectives (Creswell, 2007). This research is part of a broader study that focuses on ways that MTEs seek to enhance prospective teachers' mathematical knowledge. Multiple-case study research has been used in teacher education research to investigate teachers' knowledge, beliefs, and practice (e.g., Charalambous, 2015), and benefits from the collection and careful analysis of varied sources of data (e.g., Creswell, 2007; Yin, 2003). In our broad study, data collection included a written questionnaire, written reflective accounts, teaching artefacts, and semi-structured interviews. The written questionnaire collected background information about participants, including how long they have been involved in teacher education, their academic backgrounds, and professional teaching experiences. In the reflective accounts we asked MTEs to describe their teaching approaches and to identify and briefly discuss research literature that inspires or informs their practice. The collection of teaching artefacts and the semi-structured interviews were intended to provide more in-depth elaboration on teaching approaches. The interview excerpts we analysed in this study included the questions:

How do you describe your goals when working with teachers?

That is, what informs those goals?

Have you ever felt a mismatch between your goals and those of your students?

In cases where participants referred to specific teaching artefacts (e.g., learning task, assessment item) during the interviews, they were invited to share that artefact with the researchers. Interviews lasted approximately one hour and occurred at the end of semester.

At first, our interest was in how MTEs describe their work teaching teachers, including their learning goals, approaches, and reflections on their uses of mathematical knowledge. A recurring theme of tensions emerged in the interviews, and in this article, we focus on three cases in which the tensions were most visible. In each of the cases we present, we explore the experiences that the mathematics teacher educators shared during interviews and analyse them with the purpose of shedding light on MTEs respective intentions, teaching practices, and navigations of the tensions they experienced. We introduce each case by sharing background and contextual information collected from participants via the written surveys. We then turn our attention toward the interview data in which participants' experiences of tensions were evident. In the first round of analysis, the authors examined the data individually to identify emergent themes within each case. We then compared themes and addressed minor discrepancies. In our next round of analysis, we cross-analysed interview data with the teaching artefacts collected. In our final round, we sought patterns across the cases, attending to similarities and idiosyncratic differences in how the tensions were experienced and navigated.

Findings

We explore three cases in which MTEs of different professional experience encountered challenges in trying to "keep mathematics alive" in their teaching. The first case tells the story of Graeme, a mid-career teacher educator, and his experience teaching a course with a focus on problem solving in a teacher education program where students' attention was on pedagogy. The second case tells the story of Nadia, a novice teacher educator, and her experience introducing students in a teacher education program to social justice context problems in a mathematics methods course. The third case tells the story of Mikaela, an experienced teacher educator, who has always drawn upon advantages of digital technology in her work with mathematics teachers, but where availability of recent technological advances interferes with the intended mathematical activity.

In what follows, we introduce each participant (all pseudonyms, chosen by the participants) and the context of their teaching at the time of data collection. We share their stories, focusing on their negotiation of tensions that emerged when an opposing force (or forces) seemed to work against their mathematical goals. Each story is told in three parts: (i) the setting, (ii) the experience, and (iii) the navigation. The setting provides background and contextual information, including self-described orientations towards mathematics teaching and teacher preparation, and the connections to research that informed their praxis. The experience delves into the circumstances that elicited (one or more) tension, the competing forces, and the response of the MTE. The navigation offers an interpretation and analysis of the MTEs response(s) and the further, resulting, tensions that emerged.

The Story of Graeme

Graeme has been involved in teacher education for approximately 10 years, first as a sessional instructor and then as a tenure-track faculty member. Graeme taught secondary school mathematics prior to completing a PhD program in mathematics education. As a secondary school teacher, Graeme was considered "tough but good" by pupils who found his assignments and tests challenging but his explanations clear and straightforward. He was proud of this reputation and believed that problem solving and logical thinking are key transferable skills that all students need to practice. Graeme completed a Master's degree in mathematics and liked to "keep sharp by taking *Coursera* courses online and solving daily puzzles."

The setting: Graeme's orientation to mathematics teacher education

Graeme believed in the importance of strengthening teachers' experiences with mathematics and focuses his teaching on rich tasks (Flewelling & Higginson, 2001) and problem solving (Mason et al., 2001; Polya, 1945). He also liked to bring in examples of student work and have prospective teachers analyse and unpack the ideas as "a sort of didactical problem-solving task." His approaches teaching secondary school mathematics influence his approaches in teacher education, where he stresses the importance of understanding and nurturing student thinking. Graeme is familiar with research related to the teachers' knowledge of mathematics (Ball et al., 2005) and the influences that prior mathematical experiences can have on their teaching practice (Watson & Barton, 2011). Graeme described his goals when working with teachers:

Graeme:	Though I work in mathematics teacher education, where the focus is on teacher education, that is, how to become or be a better teacher, I personally like to focus on mathematics education, that is, how to be better with mathematics.
Interviewer:	What do you mean?
Graeme:	I mean that preservice teachers get enough exposure to general ideas, like how to support students, how to enact curriculum, how, like, not to leave a child behind, so to speak …

but I want to focus on mathematics. Dive deeper into mathematics, and you will be better equipped to work in your classroom.

Interviewer: You mean, take more courses? Is that what you encourage your students?

Graeme: No, absolutely not. I mean dive deeper into mathematics you know. Or you think you know.

Graeme shared several examples of "deep dive problems" he liked to use in his classes.

The experience: Mathematics Versus pedagogy

A question that emerged in the interview concerned how Graeme made use of his problem collection. He elaborated on one such problem, the *handshake problem*, and how he engaged prospective teachers with it:

Graeme: You know the handshake problem … that asks for total number of handshakes possible between a group of people where each person shakes hands with every other person exactly once. By the way, even here this rather popular phrasing is problematic. Like what does it mean "every other person"? Is it "shake with person 1, do no shake with person 2, shake with person 3, not shake with 4, etc."? So, it is important to clarify that the intention is – shake hands with everyone else in the group. So, this is my first step into introducing Polya's 4-step approach. That is, the first step is, understand the problem. So, what else is there to understand? Then we discuss different strategies. I usually start with 15 or 17 people at a party. And we discuss different approaches … It is a good problem to highlight a variety of approaches towards a solution. But the, the important part, is Polya's fourth step, looking back. I ask, "Okay, you solved the problem, what do you do next, other than, say, look up the answer at the end of the textbook?" Check the answer is the usual first response. But looking back, à la Polya, includes much more, considering different approaches, possible variations and generalizations.

Graeme then extended these problems by assigning homework tasks: suggest a possible variation and solve it. He was quite adamant about the value of having students pose their own "what if" problems, with respect to both promoting mathematical thinking (Graeme: "it's what mathematicians do after all!") and accessing mathematical thinking (Graeme: "It's like a little window into what they are thinking about, what connections.").

Interviewer: So, is this what you call "dive deeper into mathematics"?

Graeme: It is a start … Some students do it. They'll ask, like, what if people came to the party as couples, and wives shake with wives and husbands with husbands … or what if every mother comes with 2 daughters and daughters shake hands with daughters, or people come in batches … But I also had students who, instead of focusing on a mathematical problem, wrote a story on how they will "teach" the handshakes problem in their class … like what manipulatives they could use, that sort of thing. They will basically go over a subset of what we did in class, focusing on how to teach it differently from what we did. One student even explicitly criticized my "pedagogical choice" of starting with 17 people as "too confusing" … but, "confusing" for who? Obviously, I would not have started with 17 had I taught grade 5 … but with prospective teachers it was a purposeful choice to have them appreciate the heuristic of thinking of "simpler but similar" … She wanted to start with three people shaking hands, then four, five, and six … but this kills what's interesting about the problem. There's nothing to figure out with three people.

Choosing what numbers to use in an illustrative example is a teaching decision that highlights the interconnection between mathematical and pedagogical considerations. Graeme was asked about this choice, that is, if he ever discussed with his students the differences on the problem, for instance of starting with 3 instead of 17.

Graeme: It's homework, so I'll make a written comment. But no, we've usually moved on to the next topic by the time I finish going through their work. But maybe next time.

Interviewer: Yep, so students can learn about your pedagogical choices related to a particular problem

Graeme relayed to us that it was not uncommon for some students to "take the challenge out" of the problems and activities he would bring to class. However, acknowledging prospective teachers' interest in pedagogy and student learning, Graeme developed a collection of activities that require his students to interpret incorrect computations, or unconventional or incomplete problem solutions or justifications.

Graeme: Another thing we do a lot is, I give them some written work, like Johnny solved this thing, find the error, if there is one, or interpret what Johnny was thinking, if it's a correct solution but not the usual way of solving. Sometimes what I see is a deliberate choice to avoid any

	mathematical thinking or problem solving, by focusing on pedagogy, focusing on "what can I do with my students" rather than "what can my students do with math."
Interviewer:	You talk as if this is a bad focus.
Graeme:	No, not bad. Overwhelming. And Narrow. Especially when there are some explicit tasks with clear mathematical nature. There are other tasks for an explicit focus on pedagogy in my class. But what I see repeatedly is that some students, especially weaker ones, will discuss if groups in class should be random or ability based, just to avoid mathematics.

When asked about his attention to pedagogy in his course, Graeme shared the following homework task, as part of activities of interpreting students' work: "Jamie concluded that 684 is divisible by 4 because the sum of the digits is divisible by 4. How would you respond?"

Graeme:	So, while students identified that 684 is indeed divisible by 4 and that Jamie confused divisibility tests for 3 and 4, many suggested responses were not, so to speak, mathematical. The most common suggestion was to review divisibility tests, some homework submissions explicitly listed the familiar ones for review, even wanted to display these in a poster and refer Jamie to this poster. But there is a mathematical approach, which is also a good pedagogical approach, that is better. Ask about 784, or 584, or 354, or and then check. Have them revisit the claim, see what I mean?
Interviewer:	So, do you indicate your preferred approach?
Graeme:	I actually leave this to students to figure out, to discuss, like, let us discuss the possibilities, possible responses. Hopefully, some mathematical possibilities, like searching for a counterexample, will emerge, or I could sort of squeeze some in. So, the "reteaching" pedagogy becomes one of many possibilities, but also a more mathematical approach can be considered, like what would be good choice for numbers. Students are more receptive this way.

Graeme was also asked whether, given these experiences, he felt there was a mismatch between his priorities and those of his students.

Graeme:	Mismatch? Maybe. Not exactly the word I would use. We definitely have a common goal, and it is for them to become "good teachers" in general, and "good teachers of mathematics" in particular. But we seem to have different ideas on how this goal can be reached. Focusing on group work is important, but it's not enough. More mathematical awareness can be a strong support for mathematics teaching. But some of my students reject what is not immediately applicable to how they think of teaching a particular topic at a particular moment. For example, back to handshakes, one obvious solution is "n-choose-2", and here is a clear connection to combinatorics. For me, it's exciting to draw a connection between arithmetic sequences, which is a connection to Gauss' pairing method, and the combinatorial notion of combinations or permutations. Some students share my excitement, but someone will invariably comment that this is inappropriate for some grade level. OK, so what? No one is saying introduce every connection all the time … But the awareness, or further awareness, of such connections is what I would like my students to take away from my class. Then, it will be their choice later whether to bring this up in their class ...
Interviewer:	Do you tell them that?
Graeme:	As if "telling them" has any effect on what they hear …

The navigation: Adopting a compromise

Graeme's story relays experiences that he had accrued over several years of teaching. His collections of problems and activities were part of his strategy for deepening prospective teachers' mathematical thinking. A recurring tension for Graeme occurred in the avoidance of mathematical thinking by students who would focus on pedagogical approaches instead.

A strategy Graeme used to navigate this tension was to incorporate a variety of problems and tasks that explicitly attended to mathematics and did so in different ways—such as through solving and extending problems, or through interpreting student work. Graeme found that when the mathematical content was within the reach of students, some students still avoided engaging with the intended mathematics, and Graeme felt their resistance to making connections to ideas and topics that did not appear readily applicable to their teaching situations. A compromise that Graeme utilised was to "leave space" to what he referred to as "simple pedagogy", while focusing on a "more mathematical" approach, with a hope that some of his students would appreciate and later on, adopt the latter.

The Story of Nadia

Nadia moved to North America to complete a PhD in Mathematics Education. She taught middle and secondary school in her country of origin and was a strong believer in bringing real world problems into the mathematics classroom. As a teacher educator, Nadia believed in exposing teachers to a vast variety of real-world situations and contexts for which mathematics is relevant, and she was keen to push teachers' thinking beyond what she called the "typical contexts of finance and fraction-pizzas." She had a strong background in mathematics (Master's degree), and her research interests in mathematics education focused on pedagogical approaches that can potentially provide better support for disadvantaged students.

The setting: Nadia's orientation to mathematics teacher education

Nadia described herself as a social constructivist (Ernest, 1989, 1994; Vygotsky, 1978) who believes that learners feel the most engaged when they can see a concrete purpose for their learning. She is aware of a disconnect that exists between every-day and academic mathematics practices (Moschkovich, 2002) and she wanted the prospective teachers in her classes to gain experiences that are project-based, collaborative, and reflective as she believed that teacher educators need to model the practices for which they advocate.

In bringing the real-world into her classroom, Nadia emphasised the importance of helping teachers see that mathematics is relevant to more than the fields of engineering, technology, science, and finance, that mathematics is important for democracy (Raymond, 2018). Nadia was familiar with literature on teaching mathematics for social justice (Gutstein, 2006) and on the challenges that teachers face trying to balance time spent exploring the mathematics versus time spent exploring the social justice issue (Bartell, 2013; Wager & Stinson, 2012).

Nadia explained her goals when working with teachers:

Nadia:	Teachers need to know many different ways to connect real world to math class, because they will teach many, many students who will all have different interests. Maybe some will be interested in arts, maybe in nature or sports. Math is so useful, and if teachers connect it to something students care about, they [the students] will want to learn more.
Interviewer:	What connections do you think are important for teachers to make?
Nadia:	Well, everyone knows connections to budget, science, engineering, and stuff like that. Even art, though art is not as common as the other ones. I like to bring in other connections, ones that teachers didn't think about much yet, so I can help them see math is even bigger than they realize. When I say "bigger," I mean like how it is so important in the world, beyond what people might expect. For example, in my book club group, we read the *Weapons of Math Destruction* [referring to O'Neil, 2017]. This was so interesting and surprising to me! This book makes it so clear, how in many ways, math is connected to justice and to fairness in the world. I started thinking many teachers don't know about this and I felt very passionate that this is an important connection for teachers. And many young people, they care a lot about social justice. So, this is a big goal for me now ...

An example of how Nadia worked toward embracing social justice in her teaching was a semester-long project she designed as part of the requirements for a mathematics methods course. The project was called *Math for Society*, and the teachers were required to work in small groups to explore a societal issue of their choice for which incomplete or inaccurate mathematical knowledge could lead to inappropriate and socially detrimental conclusions. The objective of the project was for the teachers to create a digital learning tool that uses mathematics to educate the public about their chosen societal issue. The project included several touchpoints, where teachers shared their progress and ideas with Nadia for feedback and discussion.

The idea to engage students in projects of their choice and touchpoint discussions was part of Nadia's strategy for keeping students' attention on the mathematics, as her initial attempts to bring a social justice issue to her class mathematics were "almost side-lined." She also expected early resistance to the topic:

Nadia:	Sometimes, when I start this topic in class, I can see some teachers. Their shoulders drop, they look away, I think they stop listening. It feels like, sometimes, the air is let out of the room. But that is only some teachers. Other teachers, they sit up straighter, it's like they wake up finally.

The experience: Mathematics Versus social justice

For this research, we were interested in the tensions related to keeping teachers' attention on the mathematical ideas relevant to their projects.

Interviewer: Can you tell us more about the *Math in Society* project and why you felt the need to include what you called "touchpoints"?

Nadia: Oh, yes, sure. Touchpoints are like, for me, points in the semester where we touch on the mathematics. I would meet with each group, and they would tell me about their work so far. The first touchpoint, they talked about what was the issue they chose and what math is connected … usually, percentages, ratios, maybe some statistics. Then the second touchpoint, I ask them about examples of "mathematical misunderstandings." Like, what is the math idea that can get misunderstood by people, and what are the implications to getting the math wrong. The third touchpoint is to talk about who is the audience, and how will they understand the mathematics. That sort of thing.

Interviewer: I see. So, the touchpoints allowed you to direct the groups toward the relevant math ideas.

Nadia: Yes, sort of. Not "direct" maybe, but to push them deeper into the math. But I was unhappy with the touchpoints.

During these touchpoint discussions, where Nadia would convene with one group at a time, the conversations never stayed for long on the mathematics. Nadia found the teachers more concerned with posing questions about the upcoming parts of the project, and she struggled to maintain control of the conversation. She felt it would be unfair to her students to ignore their questions, so the touchpoint discussions turned into Q&A sessions instead. Nadia decided to use the projects as springboards for further classroom learning and began incorporating aspects of the projects into her other class activities, often pulling directly from the contexts introduced by students.

Nadia: At first, I was unhappy. Then I started thinking to share the ideas with the class, so the whole class can discuss the mathematics and maybe they will see connections to other topics too not just their own.

Interviewer: What sorts of topics did your students choose?

Nadia: So, it was actually very interesting. They chose topics like communities' access to clean water, consumer knowledge of GMOs [genetically modified organisms], diet and cancer… I liked these ones because, you know, I didn't expect these connections. Really, I learned a lot from the projects, and I don't want to say bad things about the teachers because they worked so, so hard, I can see how hard they worked for this.

Interviewer: Of course.

Nadia: Oh, yes! But still if I am honest, there is more I was hoping for. For instance, the cancer project was very interesting. They focused on percentages. Like, they said the risk of colorectal cancer increases by 18% for each 50-gram portion of processed meat, and 18% risk increase for every 100-gram portion of red meat, eaten daily. This comes from WHO website. I had no idea. The teachers thought "18% increase" could be misleading for students, maybe they see it as 18% risk instead. But then, the example they used to show this is not correct was actually showing that "18% increase" is not the same as "18 times as much," which is a different misunderstanding. They never addressed the idea of 18% risk, not even to do a calculation.

Calculating percentages came up in multiple projects, and Nadia lamented that "Most groups didn't do any calculations, even when showing this would explain what is going on, which was the point of the project!" This absence created an impetus for Nadia to address the topic in class discussion, which she began by sharing the WHO information on colorectal cancer and asking the question, "What does an 18% increase mean in this context?"

Interviewer: How did the discussion go?

Nadia: Well, [laughs] first thing they said is "oh, I had this-and-this for lunch … oh no". And then, if you can believe, they start talking about how much meat is in their sandwich; maybe it is less than 50g and one person says then the statistic is not relevant because no one puts that much in their sandwich. And then, but what about two sandwiches? And is turkey sandwich better than ham sandwich? And what about pork chop? Is it red meat or not red meat? Like, all these things they are wondering about the food, but they are not wondering what it means to have 18% increase.

Interviewer: What happened then? Did you try to push them deeper to the math?

Nadia: Yes. I told them, forget about meat. I asked them, first, what do we need to know to understand what it means to have 18% increase? I thought the answer would be obvious,

> but it took a bit of time you know. Then someone says, "You need a starting number." OK, so I say, "Let's say we start with 10%." Easy number, easy to see what will happen.

Nadia's attempt to anchor the mathematical discussion with an "easy number" was met with a mixed reception. She recalled that when circulating amongst the groups, she noticed that while some students began calculating the desired result, others were thrown off by the suggestion. This led some toward a debate on whether 10% "was really a number," with students arguing for and against.

> Nadia: They decided that it was not appropriate to call 10% a "starting number", although they acknowledged that 10% is the same as 0.1 and that 0.1 is a number, but in this context, 10% was a rate and should be called as such or it will confuse students.

For other students, the idea of a 10% cancer rate was so alarming that it derailed the conversation completely, and they spent class time researching cancer data rather than engaging with the computation. They found online sources that claimed colorectal cancer was diagnosed in men at a rate of 4.2% and in women at a rate of 3.8% and shared this information with the class. This led to a discussion around making appropriate choices when picking numerical examples so as not to cause panic or spread misinformation.

> Interviewer: Did you ever get back to the topic of percentage increase?
>
> Nadia: No. I wanted to, though. I tried to come up with a different example – like, salad dressing with low fat, 50% less fat than regular. So, I went to supermarket and took pictures of nutrition labels ... Regular dressing was one with 23% fat and 8% trans-fat, and label for 50% less fat dressing says 13% fat and 5% trans-fat. So how is it 50% less? It is not social justice, but it is interesting mathematically. But I don't know. I didn't like it as much… Anyway, what if they just start talking about ranch vs vinaigrette [laughs]? There are so many other topics to discuss …

The navigation: Tweaking tasks

At the time of this story, Nadia had been incorporating social justice issues into her teacher education courses for a few years. She was prepared for tensions related to students' resistance to the societal context and tried to mitigate some of these tensions by structuring touchpoints into her project design. Despite these deliberate interventions to keep attention on mathematics, the projects did not delve deeply enough into the content to meet Nadia's intended learning goals. In navigating this tension, Nadia made several attempts to tweak the task, first by shifting the group-based touchpoint discussions to whole class discussions. When this still resulted in minimal mathematical discussion, she introduced scaffolding ("What do we need to know?") and then prompted a computation by providing a starting figure ("Let's say we start with 10%".). These tweaks led to further experiences of tension, for her students and herself. For the students, tensions emerged with respect to the use of proper terminology (0.1 is a number, but 10% is not) and potential misinformation of a serious issue (10% is alarming and incorrect). The tension between choosing an "easy" number and an accurate number was also felt by Nadia, who responded by tweaking the task again, this time by searching for a different context. The shift away from cancer to salad dressing also surfaced a tension for Nadia, who felt that while nutrition was important, it was not as impactful as the cancer discussion.

The Story of Mikaela

Mikaela was a former secondary school teacher with "many years" experience and a background in computer programming and mathematics. She completed a Master's degree in education and technology, and doctoral degree in education. Mikaela's orientation toward teaching mathematics is influenced by her belief in the importance of investigations and problem solving. She often directed teachers towards digital technology (e.g., Dynamic geometry software, spreadsheets) to assist in problem solving as well as to pose new problems. She believed that students have truly learnt an idea when they are able to apply it. She described herself as "an old dog who loves new toys, especially tech." Mikaela had taught several courses for prospective teachers, as well as mathematics-focused professional development workshops for practicing teachers.

The setting: Mikaela's orientation to mathematics teacher education

Mikaela liked to introduce prospective mathematics teachers to a variety of teaching methods, both with and without technology, and she dedicated a portion of every class to solving low-floor/high-ceiling problems (Papert, 1980). She was very enthusiastic about the potential of dynamic and computational modelling as tools for active mathematics learning and was familiar with related research literature (e.g., Arzarello et al., 2002; Drijvers, 2015; Grover & Pea, 2013). In her teaching, Mikaela provided teachers with multiple resources related to integrating technology in their mathematics classrooms. She was, however, aware of literature that highlighted teachers' reluctance to incorporate technology into their teaching (e.g., Henderson & Corry, 2021). In her classes and workshops, Mikaela stressed the importance of having teachers critique resources and question how they can be used in the classroom. She focused mainly on resources that were freely available online (such as math games, videos, and software) since "nobody has money for anything, and people just 'google' everything anyway."

Mikaela: Active learning with technology is a huge goal for me. I think people – no matter what age, what context – people learn better when they are actually doing what they're supposed to be learning about, not just listening to someone else explain what is supposed to be done. I think too many teachers think "oh, if I just explain it really slowly and clearly then my students won't struggle." But no, that doesn't cut it. Students need to figure things out themselves, and then they understand it better. Discovery, inquiry, whatever word you want to use, they have to do it for themselves.

The experience: Mathematics Versus technology

Mikaela's goal of active learning with technology interested us, and we wanted to know more about how she connected this goal to her teaching.

Mikaela: Well, my faculty where I teach is really big on technology, so the students all expect it. Lots of it actually.
Interviewer: And how does that go?
Mikaela: Ehn. I don't know. I'm pretty deliberate about what technology I will bring to class. Dynamic Geometry Software (DGS), I love it. Coding, also cool. There are lots of decent applets and new ones all the time. Do you want a list? [*laughs*]

Having discussed various applets, the interview shifted to students' experiences.

Interviewer: And what about your students? You mentioned they expect lots of tech. Are they tech whizzes?
Mikaela: [*laughs*] They love Khan Academy videos. This is their number one, first thing they think of. On assignments, if they do lesson planning, whatever, all the time, I see "show students Khan Academy video". They don't even try to explain it themselves anymore. Just Khan Academy. Watch the video. That's it. Done.
Interviewer: Do you ever say, "No Khan Academy" to your teachers?
Mikaela: No Khan Academy? No, I never say so. It's out there. You can't stop them.
Interviewer: You mentioned DGS and coding. Can you share an example of an activity you might do with those? One that you really like?
Mikaela: Yeah. Pythagoras proof, with DGS. Actually, I don't do this one anymore. But, I loved it and used to do it all the time. Everybody knows Pythagoras, but, in my experience, surprisingly few teachers have thought about proving it.

A version of this task that Mikaela shared, asked students to construct a proof of the Pythagorean Theorem using the DGS of their choice (e.g., Geometer's Sketchpad, GeoGebra, or Desmos). They had to keep a journal documenting their attempts, successful and unsuccessful. They were required to submit their journal and a "digital proof", as well as share their constructions in a whole class discussion. The discussion often lasted over an hour.

Mikaela: You could generally count on there being at least half a dozen completely different approaches – not all of them good ones. But interesting discussion!
Interviewer: Why did you stop doing this task?
Mikaela: Well, at some point, it became pointless because all you need to do is go to Geogebra and type in "Pythagorean" to search engine, and twenty different applets that show the proof come up. Or type it into google and you get every answer, and a million videos showing just what to do. Everything is done. The teachers think it's great. Even though this stuff's been around for a couple decades now, it's still new to a lot of them. So, they get pretty excited. And because you can click and there's animation, or you drag and the shapes move, they say "oh, this is active learning". But what did they learn? Did they learn to understand the proof? Or did they learn where to find a proof?
Interviewer: Uh huh…

Mikaela:	Everything is google-able [*sic*] today, and AI will solve anything. So I changed my tasks from solving a problem, or proving something, to comparing different solutions or justifications. Like, here are two proofs for Pythagoras, or for irrationality of root-2. Which do you prefer and why? Which would you bring to your students and why? AI is not very good at such a comparison task, yet.
Interviewer:	And are you happy with this task?
Mikaela:	Sure … well, at least they have to think for themselves. Although now, instead of six or more different approaches, we only look at two.

The emergence of Generative Artificial Intelligence (GenAI) has had a sweeping and profound impact across education, and teachers (including Mikaela) are having to adapt their strategies and assessments amidst concerns over academic integrity (McGalliard & Otten, 2025). While GenAI responses currently tend to be procedural and algorithmic (Sapkota & Bondurant, 2024), there is growing research interest in how creative GenAI is or could be (Holzner et al., 2025).

Mikaela:	I can give you another example. I used to have people enact *the Monty Hall Problem* in class. We would work in small groups, act it out multiple times and record the results. Then we would collate the data from the whole class. But now there are simulations available everywhere online, and the students immediately look these up.
Interviewer:	So, do you introduce the simulations now?
Mikaela:	No. I still want them to explore it. So, I decided to rephrase the problem to talk about opening envelopes instead of doors. This way, it's not immediately google-able [*sic*].
Interviewer:	Would you say that there is a mismatch between your goals, about active learning with technology, and those of your teachers?
Mikaela:	Mismatch? No. They, my teachers, they also say active learning is important.
Interviewer:	Even given their propensity for using google, AI, Khan Academy…
Mikaela:	No. Active learning for their students is what they really want.

The navigation: Rethinking thinking (with technology)

Mikaela's story reveals several tensions in her engagement with technology. The readily available online solutions to almost any problem caused Mikaele to redesign the task from solving or proving to comparing solutions or proofs. She believed that comparing and contrasting approaches still leaves room for students' "thinking for themselves" rather than "googling." She expressed, however, a concern that this kind of task design offers less diversity in the approaches made available for her students to consider and as a result, fewer mathematical connections to explore. Another tension we identified in Mikaela's experience was related to unreliability of online information and the challenge of trying to distinguish a reliable source and an unreliable source. While the example Mikaela brought forward is troubling, we suspect that the tension highlighted is likely to be relevant for any encounter with online information. Navigating the tensions is also likely to be extremely challenging as the use of GenAI becomes more prevalent in education

Discussion

Having described how each MTE navigated their experiences of tension, we now look across all three cases and draw out common threads that weave through the different contexts and situations. We identify two key threads: tensions begetting tensions and navigating tensions through task design.

Tensions Begetting Tensions

The first research question was: *What tensions are experienced by MTEs while trying to enhance the mathematical knowledge of teachers?* Through the stories of Graeme, Nadia, and Mikaela, we identified tensions related to competing forces that pulled prospective teachers' attention away from mathematics, including pedagogical considerations, contextual details, and technological advances. Extending prior research that articulates a binary perspective of tensions as opposing forces to be mediated or managed (e.g., Berry, 2007; Barwell et al., 2016), we interpret tensions as disruptive forces to be navigated. This language draws attention to not only the emergence and experience of tensions but also the nuances involved in navigating multidimensional spaces. Like waves in a tumultuous ocean, tensions can swell and build upon each other, as evidenced in this research.

In accordance with literature on K–12 teaching tensions, the tensions experienced by participants in this research led to further experiences of tension (LeSage & Mamolo, 2023). In Graeme's case, his intention to keep students focused on mathematics created a tension with students' intentions to focus on pedagogy. This resulted in including tasks which afforded discussion of both mathematics and pedagogy, such as in considering possible responses to a student solution. Subsequently, a new tension emerged: How to guide prospective teachers to make appropriate pedagogical choices within a given mathematical context.

For Nadia, the continued derailment of her attempts to foster a discussion related to percentage increase led to a new tension regarding her goals for the societal context, which she gradually minimised. First, by removing part of the context (red meat), then by introducing false figures (10%), and finally by pivoting away from the controversial context to a "safer" but less compelling context (salad dressing).

In Mikaela's case, the availability of online solutions to her task created tension regarding her expectations for students solving problems themselves, as well as with her attempts to realise the benefits discussed in research about constructing mathematical relationships with dynamic geometry software. Subsequently, a new tension emerged regarding the relative lack of richness and diversity of mathematical ideas elicited by her attempt to navigate the tensions created by the availability of online solutions. While she was able to navigate away from a situation where students could completely side-step mathematical thinking, she was nevertheless faced with a tension in that her new tasks did not (and perhaps could not) evoke as many mathematical connections and ideas as the previous tasks. In line with Adler (1998) and Liljedahl et al. (2023), we note an intermingling of external and internal tensions in the cases; external tensions (e.g., related to student expectations or technological impositions) led to internal tensions regarding MTEs goals for their teaching.

Navigating Tensions Through Task Design

The second research question asked: *How do MTEs navigate these tensions?* A partial answer, common to the three participants, is found in task design. For Graeme, having a rich collection of problems and tasks that could be called upon, adapted, or kept in his "back pocket" was a mechanism for directing attention toward mathematics. He also included in his course, tasks that provoked discussions of both mathematical ideas and pedagogical approaches, in a way seeking a compromise between his goals and students' expectations. In Nadia's case, her familiarity with mathematics education research on teaching mathematics for social justice alerted her to possible tensions that could arise. As a result, she anticipated that there would be tension around balancing attention between the mathematics and the societal issue. In response, she designed touchpoints into her task as an attempt to keep a focus on mathematics. When these touchpoints were not sufficient to keep attention on mathematics, she adapted her lessons to accommodate. She tweaked the task repeatedly in persistent efforts to redirect her students back to mathematics.

Mikaela navigated her experience of tension with the "google-ability" of solutions in two ways, through re-phrasing and through restructuring her tasks. For instance, Mikaela came up with an isomorphic description to the Monty Hall Problem, not talking about doors but rather talking about envelopes, to engage students in mathematical activity (and not merely googling). She also abandoned the task that invited students to construct a proof in favour of a compare-and-contrast task. Mikaela's knowledge of the literature on mathematical thinking, as well as her personal mathematical awareness, contributed to her decision to pivot from one kind of mathematical activity to another. Indeed, in all of the cases, participants drew on their personal mathematical knowledge to introduce, tweak, or restructure tasks in attempts to navigate tensions related to their students' focus of attention.

Concluding Remarks

The stories of Graeme, Nadia, and Mikaela highlight the complexities and interconnection of tensions that were elicited and experienced in mathematics teacher education. The MTEs felt themselves torn between advancing their mathematics-related learning goals and adapting to their students' divergent attention. By framing tensions as experiences through which MTEs must navigate, this article contributes new and refined understanding about the interplay of tensions impacting MTEs, and the resilience and adaptability of MTE's navigations. It also provides a new language through which to interpret tensions—not as something over which individuals have "control" to manage, but rather as disruptive or imposing forces which individuals must find a way through. Further, in presenting these cases studies, we contribute new knowledge regarding some of the ways that MTEs draw on their knowledge of mathematics to navigate the "three disruptions" that pulled students' attention away from the intended mathematical experience: pedagogy (Graeme), societal issues (Nadia), and digital technology/GenAI (Mikaela).

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