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Can Minimal Support for Teachers Make a Difference to Students' Understanding of Decimals?

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This study investigated whether new teaching resources and minimal support could help teachers make a difference to children's understanding of decimal numeration. Three schools were offered resources and a professional development session specifically to address the common misconceptions children have about decimal numbers. One school agreed to participate, and four teachers were supplied with the resources. Use of the resources was unexpectedly low, but teachers who used them achieved an educationally and statistically significant improvement in decimal understanding, indicating that a small amount of deliberate attention to decimal concepts can make a difference.

This paper reports on the outcome of a study of a small, low-cost intervention in which resources for teaching and learning about decimals were offered to three schools to address a long-standing difficulty. The schools were selected because they were known to have considerable scope for improvement in this topic. The support offered to the schools and the involvement required of them was minimal: complimentary copies of resources for teaching, an initial professional development meeting for the teachers to demonstrate the materials and diagnostic testing of all students by the researchers. We were concerned to limit the requirement of teacher time and the researchers' investment in professional development, in order to establish whether a small amount of input and targetted teaching can reduce the incidence of decimal misconceptions. It is well known that many students experience difficulty with decimal numeration, but it is not known how difficult it is to alleviate this situation.

The effects of larger scale professional development projects, such as Count Me In Too (Bobis & Gould, 1999), the Numeracy Strategy Project (Gervasoni, 1999) are frequently reported in the literature. The intervention reported here differs in several ways: the investment of time by teachers is very low, there is no financial support (e.g., for release time) and the focus is on a highly specific, although very central, aspect of the curriculum (decimal numeration). In presenting a study of an intervention offering minimal support for teachers, we do not claim that this approach is preferable to larger scale projects that offer greater opportunity for trialling and reflection. Rather, we are exploring what can be achieved when resources are limited.

The study explored two major issues:

1. Can a small intervention help teachers to overcome children's misconceptions about decimals?

We investigated the likelihood that teachers, given minimal support, could change their teaching so that more children developed their knowledge of decimal numeration. How much additional teaching effort is required for measurable changes in children's understanding of decimal numeration to be achieved?

2. How do schools and teachers respond to the new resources?

Answering this question included documenting the extent to which schools and teachers took advantage of the opportunity to try something new which addressed an identified problem; how teachers made use of the resources in their classrooms; and teachers' views on their effectiveness. The resources offered to the schools had been collected and/or developed by the Learning Decimals Project (Steinle & Stacey, 1998) over a period of several years and have been used successfully for teaching about decimals, and for addressing particular misconceptions. Thus it was considered timely to conduct a more formal investigation.

The Incidence and Persistence of Difficulties with Decimal Numeration

It is well documented that many students throughout schooling, and indeed many adults, including teachers, have difficulties with decimals (Hiebert & Wearne, 1986; Putt, 1995; Resnick et al., 1989; Sackur-Grisvard & Leonard, 1985; Thipkong & Davis, 1991). A central problem (and probable underlying cause of many other difficulties) is that many people lack understanding of decimal numeration. This is a long-standing and international problem. For example, Grossman (1983) reported that, even though over 50% U.S. students entering tertiary education could add, subtract or multiply decimals, less than 30% could select the smallest decimal from five possibilities.

Recent Australian data from the Learning Decimals Project confirm that Australian students have similar difficulties and that misconceptions are widespread and persistent. Using a carefully designed Decimal Comparison Test we have found that the proportion of students able to compare decimals reliably is relatively stable from year 6 (about half) to year 10 (less than two thirds). These results suggest that normal teaching is making little difference to the way students think about decimals. Longitudinal data also indicates that about half of the students who have a decimal misconception have the same misconception a year later (Stacey & Steinle, 1999a, 1999b).

The data above demonstrate that understanding decimal numeration is a complex task. Students have to coordinate their ideas of place value learned in the whole number setting with their understandings of fractions. For example, to compare 0.3 and 0.34, a student who knows that the first is 3 tenths and the second is 34 hundredths has to be able to mentally coordinate the effect of the varying numerators (3 and 34) and the varying implicit denominators (tenths and hundredths). There are other ways of comparing these two numbers, for example by using knowledge of equivalent fractions so that 0.3 can be read as 0.30 and therefore as 30 hundredths, making the comparison easier. Another way is to think of 0.34 as 3 tenths + 4 hundredths. However, all of these methods require coordination of place value knowledge and fraction knowledge.

Some schools and teachers seem to teach decimal numeration well and others not so well. Steinle and Stacey (1998) found that although 52% of grade 6 students were classified as experts on the Decimal Comparison Test, the results by class varied from 0% to 82%. This variability is only partly explained by socioeconomic factors. For this reason, it seems likely that attention to the meaning of decimal numbers in teaching is a major factor in determining success. It is likely that, given just a small impetus, the understanding of children in this topic could be markedly improved. This study set out to investigate whether this is the case.

Resources for Learning about Decimals

The Learning Decimals Project has developed a collection of classroom resources for teaching about decimal numeration, including activities to address common misconceptions. The resources include classroom activities using simple equipment such as cards and the chalkboard (see Appendix for examples), computer games, and a concrete model of decimals which we call Linear Arithmetic Blocks (LAB) (see Figures 1 and 2).

This model was first shown to us by Heather McCarthy, a local teacher, who saw it at an in-service training session given by a person whose name we do not know. LAB is similar to Dienes' Multibase Arithmetic Blocks (MAB) in that it can be used as a hands-on model of decimal numbers. Units, tenths, hundredths and thousandths are represented by hollow tubes of decreasing, and proportionally accurate, length. The unit piece we use is just over one metre long, so that thousandths are just over a millimetre long.

Figure 1. LAB pieces laid linearly to illustrate and compare the numbers 0.2, 0.27 and 0.3.

Figure 2. Placing pieces on the organiser to illustrate place value and the base 10 structure of decimals

LAB pieces can be arranged in two ways to represent a number, either end-to-end or on what we call an organiser. When placed end-to-end the pieces form a linear representation of decimal numbers, as in Figure 1. This facilitates direct comparison of decimal numbers. The second possibility is to place the pieces on the organiser, as in Figure 2. The organiser consists of upright rods attached to a wooden base, which provide a concrete model for the place value columns. The height of the rods is such that only nine of the relevant pieces can be placed on it, providing an in-built constraint on column overflow. Thus if there are ten or more pieces of any one size, they will not fit on the appropriate rod of the organiser and so ten of them must be exchanged for a single component of the next highest place value.

In this study, we followed the advice of Judah Schwartz who observed that "in the case of education reform, there need to be new curricular artifacts that allow the users to mark the newness of their undertaking" (1994, p. 4). In this study, LAB, something that teachers and children had not seen before, served as the marker of this new undertaking. It was offered as an alternative to MAB, which is well known to teachers and used widely to model decimal place-value relationships, as well as whole-number place value relationships and operations.

Beyond its role as signifying increased attention to the topic, several arguments can be put forward in favour of LAB over MAB as a tool for supporting the learning of decimal concepts:

1. Reduced confusion. The values of MAB pieces used for decimals are often confused with whole number values that they previously represented. Since LAB is probably new to teachers and children, this confusion is less likely.
2. LAB is a simpler model than MAB. The underlying representation of the size of number by MAB is by volume (or mass if the density of material used is constant). One number is larger than another if the volume of the pieces assembled to represent it is larger. LAB, however, represents numbers by the length of the pieces assembled, which in view of children's difficulties with volume concepts (Battista & Clements, 1996) may be more developmentally appropriate. MAB material is also complicated by what users may perceive as a switch from the 3 dimensional block, to the 2 dimensional flat to the 1 dimensional long to the 0 dimensional unit.
3. LAB prevents column overflow. As noted above, no more than 9 pieces of any one denomination fit on a rod of the organiser, forcing trading more than nine of one unit for the next.
4. LAB is structurally similar to the number line. Because LAB can have structural similarity to the number line, it has important advantages over MAB in being able to demonstrate the density property of decimals (the property that between any two decimal numbers, there is another). It also provides an excellent concrete basis for rounding decimals.

These arguments can be seen collectively as claiming that LAB is more transparent than MAB. Transparency is traditionally conceived as the extent to which the tangible features of a model correspond to a target knowledge domain, so that users can see through the model to the underlying principles and relations, without being confused or distracted by features of the model itself (Lesh, Behr & Post, 1987). More recent investigations argue that transparency is more than a feature of the materials themselves, and develops as students work with them in the classroom situation (Meira, 1998). This conceptualisation of transparency transcends the limits of formal correspondences between a model and the knowledge domain it represents, so that transparency emerges through learners' activities and participation.

Two small studies (Archer, 1999; Condon, 1999) compared MAB and LAB in practice. LAB was found to be a more effective mediator of student understandings, based on analyses of lesson transcripts. These revealed significantly more discussion, conjecture and explanation by groups of children using LAB than by groups using MAB. Some children reported being confused by MAB but there were no such reports for LAB. Children were also more likely to visualise LAB than MAB when the model was not available. According to Meira's (1998) definition of transparency, there is a strong case for arguing that LAB is a more transparent model than MAB and therefore worthy of consideration as an alternative to MAB. These studies are currently being prepared for publication.

Method

Six primary schools had participated in the longitudinal study of the Learning Decimals Project (Stacey & Steinle, 1999a, 1999b). For these schools, the project had three years of data showing the numbers of students in each school who tested as expert and the numbers who held misconceptions of various types about decimal numeration. Two of the primary schools had high percentages of experts and so were not suitable for targetted instruction.

After eliminating a distant school, we approached the three remaining schools and invited them to participate in an intervention designed to improve children's understanding. They were each presented with summaries of the data over three years on their own students' understanding of decimals, held by the Learning Decimals Project. (Prompt feedback on every child had also been given to all teachers at every stage of the project.) At these schools, a high percentage of children held the most common decimal misconception for primary school children: the tendency to judge longer decimals to be larger. Many children with this misconception read the "decimal" part of a number like a whole number, believing for instance that 4.63 is larger than 4.8 because 63 is larger than 8. One school, through 'M' their Grade 6 coordinator, willingly agreed to be involved. The other two schools did not accept the offer.

Two members of the research team visited M's school, a coeducational primary school in an outer suburb of a major city, and conducted a one hour professional development session with the four grade six teachers (M, S, X and Z) all of whom agreed to participate. M, S and X taught grade six, and Z taught a combined grade five and six class. The session included an explanation of common misconceptions, and each teacher received two booklets and a complimentary set of LAB that was demonstrated in the session. One booklet contained lesson plans for using LAB (Archer & Condon, 1999) and the other contained general lesson ideas for teaching about decimals (Condon & Archer, 1999). Some of the contents are described below, and two activities are provided in the Appendix.

Teachers were asked to use as many activities as they wished, but record their experiences of five activities from each booklet on simple feedback sheets supplied by the project team. They were asked to report on how they used LAB, and given space to record their observations of children, as well as general comments. They were also asked to provide a written account of their thoughts on teaching decimals during the time of the intervention. Teachers were asked to send back their responses at the end of the following school term (about three months later).

The teachers offered to test their students prior to and after the intervention, and the researchers marked these and forwarded the results. This gave the teachers data on their current students' misconceptions to supplement the longitudinal data about the school already in the Learning Decimals database. Ninety-eight children completed the Decimal Comparison Test, a one-page 5-minute test which presents

30 pairs of decimals and asks students to nominate which of each pair is the larger. (Refer to Steinle & Stacey, 1998 for details). The test results were returned to the teachers, with a description of what the test indicated about each child's thinking.

At the end of the intervention period (four months later) a member of the research team visited the school to finalise the post-testing and meet with teachers again to discuss their participation in the project. Ninety-six children were tested on the second occasion, making 87 children who were tested twice. Three teachers attended the follow-up meeting, two of the original group (M and S) and teacher Y, who had been on leave at the time of the initial training session.

Results and Discussion

Although all four teachers had agreed to participate, only M (the Grade 6 coordinator who had volunteered the school for the study) trialled the activities and provided a record of what she did. She commented that other curriculum priorities prevented her and the other teachers from completing the task. Thus her reports were limited to a record of three activities from the booklet using the LAB model, and four activities from the Lesson Ideas Booklet.

At the follow-up meeting, teacher S reported that LAB had been used once in her class. Class Y had not used them at all. Their teacher, Y, was on leave at the time of the training session and there was no record of her temporary replacement X, who had attended this session, having used the materials. Despite repeated requests for information after the end of the study, no record ever became available of use of the activities by Z and it seems reasonable to assume there was no use in this class either. Thus we loosely defined three categories of use: Most use (Class M), Some use (Class S) and No use (Class X/Y and Class Z).

Pre- and Post-test Results

A total of 87 children from four different classes were tested twice and the data below is only for these students. All were grade six classes except for Class Z, which was a combined grade 5 and 6 class. Data collected earlier by the Learning Decimals Project over a period of three years (1996-1998) indicated that, for this school, 41% of grade 6 students tested as expert in decimal notation. In the cohort of 1999, 46 children (53%) tested as experts on the pre-test, a slightly higher proportion than previously.

Table 1
Numbers and Percentages of Experts by Class on Pre- and Post- tests.

	Class M		Class S		Class X/Y		Class Z		Total	
	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
Numbers of students	24	24	23	23	20	20	20	20	87	87
Numbers of experts	8	15	19	23	10	10	9	9	46	57
Percentages of experts	33	63	83	100	50	50	45	45	53	66
Percentage gain	30		17		0		0		13	

Table 1 shows the results of the Decimal Comparison Test before and after the intervention for each of the four classes. Improvement occurred for the two classes that used the resources, particularly the improvement from 33% to 63% expert for Class M. No improvement was evident for the two teachers for whom there was no record of any intervention. A chi-squared test showed that the improvement was statistically significant both for class M considered alone (c^2 (1, N = 24) = 4.09, p = .043) and for class S and M considered together (c^2 (1, N = 47) = 6.03, p = .014). This very clean result indicates that deliberate attention to this topic is likely to make a substantial difference to students' understanding at Grade 6, which will in turn put them in a much better position to understand secondary school mathematics.

In an attempt to identify any trends in how students' knowledge changed, the data for classes M and S were combined. The Decimal Comparison Test classifies students according to four major categories of understanding of decimal numeration, details of which are given by Steinle and Stacey (1998):

(a) experts (i.e. students who can reliably compare decimals on this test – they may not have a full understanding of all aspects of decimals);
(b) students who exhibit various of the longer-is-larger misconceptions;
(c) students who exhibit various of the shorter-is-larger misconceptions and
(d) unclassified students who do not show any known pattern in their responses.

Table 2 reports the changes in classification that occurred over the intervention period. All 27 students who were initially classified as experts retested similarly, suggesting that once students reach expertise, they generally retain it. This finding is consistent with other studies (e.g., Stacey & Steinle, 1999a).

Over half of the non-expert students (55%) in Classes M and S tested as experts after the intervention period. This is an encouraging result given that Stacey and Steinle (1999a) found that over a period of about six months only about one third of non-expert students in a very large sample moved to expertise. It is also markedly different to the static nature of the results in Classes X/Y and Z. Another encouraging result is that none of the students with misconceptions retained them, either moving to expertise or into the unclassified category. Examination of individual test papers for the three students who moved into the unclassified category revealed small improvements in understanding for each.

Table 2
Numbers of Students by Classification from Pre-test to Post-test for Classes M and S (N=47)

Pre-test	Post-test classification			
classification	Experts	Longer-is-larger	Shorter-is-larger	Unclassified
Experts	27	0	0	0
Longer-is-larger	2	0	0	1
Shorter-is-larger	1	0	0	2
Unclassified	8	3	0	3

Teacher M's use of LAB and the Lesson Plans

As discussed above, Teacher M was the only teacher who provided a written record of how she used the resources. Since she achieved significant gains in performance, we examined more closely what she did.

M selected the initial three activities from the LAB activities book, and the first four activities from the Lesson Ideas book. The three LAB activities included familiarisation with the pieces and the organiser; creating decimal numbers with the pieces; placing LAB pieces on the organiser and writing down the number; and comparing decimals of different length using LAB. She commented that "the children reacted favourably to the material" and that "most children found this work to be very valuable for their understanding of decimals."

The four activities selected from the Lesson Ideas book were Number Trails, Decimal Skip Counting, Number Between and Stickers. (Number Trails and Stickers are included in the Appendix.)

Number Trails and Decimal Skip Counting. These both involve starting with a particular number and adding or subtracting a constant number, preferably mentally. For example, a beginning class might start at 0.3 and successively add 0.1, producing the sequence 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, and then erroneously 0.10. An advanced class might start with 0 and successively add 0.125, obtaining the sequence 0, 0.125, 0.25, 0.375, 0.5 etc. which contains decimals of lengths 0, 1, 2 and 3. Both of these activities can be used to expose misconceptions, including difficulties with column overflow (eg. the number after 0.9 is 1.0 and not 0.10).

Number Between. This starts with the teacher nominating two endpoints of a segment of the number line and children have to nominate any number between the endpoints. The nominated number and one of the previous endpoints become the endpoints of a new segment. As the activity continues, the number line is divided into smaller and smaller segments. Thus children gain an understanding of the value of numbers from their relative position on the number line and an appreciation of the density of decimal numbers.

Stickers. This activity (see Appendix) involves students arranging themselves in order from smallest to largest according to the decimal number they have been given on a sticker. Depending on the numbers chosen, the common misconceptions about the size of decimal numbers can be addressed by these activities, as well as issues such as the function of zeros in different positions in a numeral.

M reported that students participated enthusiastically in all activities. She noted the effectiveness of Number Trails for addressing column overflow problems, and used LAB to demonstrate for example, why 0.9 plus 0.1 is 1.0 and not 0.10. She commented that LAB was useful for helping children to visualise the numbers. She also reported that column overflow problems became less evident as children continued to play the game. She commented that children responded with enthusiasm: "Children loved this simple activity." She reported that children "learnt heaps" from Number Between, with children of all abilities participating enthusiastically.

It was evident from M's feedback that the resources were not always used in the way intended, nor to their full potential. For example, the Stickers activity is designed to address misconceptions about the size of decimal numbers. Because it can incorporate numbers of different length, or zeros in different places, it has great potential for developing understanding of decimal numeration, depending on the numbers chosen by the teacher each time the game is played. The activity as written (see Appendix) provides ten examples of numbers with different features so that issues of length and placement of zero are drawn to the teacher's attention. In this instance, M used the ten numbers provided as the set of numbers for children to arrange in order from smallest to largest. These numbers were not intended to be selected as a group, but to stimulate teachers to produce a set of numbers that are similar in size to each other. For example, we expected that 34.8 would be used along with numbers such as 34, 34.08, 34.88, 034.0 and 34.800. It was hoped that teachers would devise such examples for themselves, but the instructions were clearly not specific enough and a learning opportunity was lost. This is not intended as a criticism of the teacher. Rather, it points to the need for providing more detail on the rationale for the activity, making the instructions more specific, and including some actual sets of numbers that could be used more productively.

Follow-up Meeting with Teachers

The follow-up meeting was conducted at lunchtime (the only available time) and due to heavy commitments, only three teachers could attend (M, S and Y). The general impression was that of a school that expected a lot from the teachers, who had been very busy with extra work, such as orientation programs for secondary school.

The group was asked to discuss the sorts of problems children have with decimals and the strategies they found effective in overcoming them. The problem first mentioned was column overflow (that 17 tenths, for example, is not 0.17) and M repeated what she wrote on her feedback sheet about the effectiveness of Number Trails (see above). S mentioned that she also did this activity and used the LAB, which appeared to trigger conceptual change:

> Actually, it was good to show what happens [when there are more than nine of any denomination] because there's nowhere to go so you've got to take it to the left concretely. And I remember doing that at the time because some of them said, "Oh yeah!"

Another activity that appeared to trigger conceptual change was Number Between. M described how effectively the activity, when used in the interval between 0 and 1, demonstrated that small decimals (i.e., zero point something) could never be less than zero:

> I just remember putting the number line up on the board and showing them where nought was, and one, and then getting them to halve. Remember one of the activities was getting them to write a fraction in between each time, and they were getting smaller and smaller and smaller and they suddenly realised they could never get smaller than nought.

This discussion brought up a related issue; that pressures of competing tasks and responsibilities interfere with good teaching:

S: You need to talk to them don't you, and say, well, why do you do this?

M: That was coming out with that maths conference we went to. They said that teachers are too busy giving work and looking at results.

M: What we need to do is talk to them.

S: Talk to them.

M: —and get them to think about, and even getting them to write why they got a certain answer.

These pressures may partly explain why the other two schools initially contacted did not participate, and why teachers who initially agreed to participate did not undertake any activities. As discussed earlier, the only teacher who made a significant effort to use the materials in this school was the initial contact M. She clearly felt an obligation to complete the task, whereas the other teachers, faced with competing commitments and responsibilities, discontinued their involvement. M reported that, having already taught decimals earlier in the year, they found it difficult to justify spending more time on the topic when they were expected to be covering other areas of the curriculum. An intervention in the period when the teacher planned to teach decimals may have increased participation in this school.

Finally, it emerged in the discussion that teachers did not make use of the pre-test results which the project team supplied to learn about children's difficulties or target children with particular misconceptions. This was the case even though an explanatory sheet was sent along with the pre-test results, and the project team also invited teachers to ask for clarification.

Conclusion

This study investigated several issues associated with intervention. We explored the extent to which schools and teachers would take advantage of the opportunity to make use of resources designed to address difficulties their students were known to have with understanding decimals. We investigated the impact of the resources, once taken up, on children's understanding of decimals, particularly their effectiveness in overcoming decimal misconceptions. Finally, we investigated whether it is difficult to make changes to children's understanding of decimals with a small amount of deliberate attention.

A small amount of teaching targetted at decimal numeration made an impressive change to students' learning outcomes. In the class taught by the fully participating teacher, the percentage of students who tested as experts almost doubled from 33% to 63%, demonstrating that a small amount of deliberate attention can make a difference to children's understanding of decimals. In this class, it seems that one third of children had achieved a reasonably good understanding of decimal numeration from the standard instruction in the class. The new materials and a few targetted lessons were sufficient to achieve good understanding for the middle third of the class, and the final third needed more intervention than was provided. The progress in the classes that used the materials contrasted with the static situation in the other classes.

A thought-provoking outcome of this study is the low participation by schools and teachers. An early outcome was the decision by two of the three invited schools not to take part in the study. Further reduction then occurred within the school that took part. The initial group of four participating teachers was reduced to less than two: one who fully participated and one who was partially involved (although with excellent results). There are many possible reasons for low participation, but it genuinely seemed to be due to other commitments rather than disappointment with the materials. For us, this highlights the necessity of teaching topics very well at every stage. These teachers had "done decimals" earlier in the year but knew that large numbers of children in their classes had not mastered this central topic. Despite the good ideas we supplied and their own good intentions, they could not find the time to attend to this, amongst many other demands.

The lack of use of diagnostic data on individual children is also thought provoking. We believe that the feedback was easy to read, well presented and reasonably simple to understand. The lack of use is consistent with a personal communication from Kathleen Hart, the director of the large CSMS project in Britain around 1980. Very large numbers of children were interviewed and good descriptions of children's thinking were carefully prepared for their teachers. Hart believes that there was no evidence that any teacher ever used any of this information. For us, this observation emphasises the need for teacher education (especially pre-service) to markedly increase pedagogical content knowledge so that teachers are better able to identify students' difficulties, plan teaching which addresses what children do not know, and identify conceptual change when it occurs.

Decimal numeration is a difficult topic for some students to learn. This study shows that it can be taught much better.

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References

Archer, S. (1999). The comparative effectiveness of Linear Arithmetic Blocks and Multibase Arithmetic Blocks in addressing students' misconceptions about decimals. Unpublished Honours Project, University of Melbourne.

Archer, S., & Condon, C. (1999). Linear Arithmetic Blocks: A concrete model for teaching decimals. Melbourne: DSME, University of Melbourne.

Battista, M., & Clements, D. (1996). Students' understanding of three-dimensional rectangular arrays of cubes. Journal For Research in Mathematics Education, 27(3), 258-292.

Bobis, J., & Gould, P. (1999). The mathematical achievement of children in the Count Me In Too program. In J. & K. Truran (Eds.), Making the difference (Proceedings of the 22nd annual conference of the Mathematics Education Research Group of Australasia, pp. 84-90). Adelaide: MERGA.

Condon, C., & Archer, S. (1999) Lesson ideas and activities for teaching decimals. Melbourne: DSME, University of Melbourne.

Condon, C. (1999). The comparison of two decimal models - The MAB and LAB. Unpublished Honours Project, University of Melbourne.

Gervasoni, A. (1999). Teachers exploring numeracy learning and teaching in the early years of schooling. In J. & K. Truran (Eds.), Making the difference (Proceedings of the 22nd annual conference of the Mathematics Education Research Group of Australasia, pp. 84-90). Adelaide: MERGA.

Grossman, A. S. (1983). Decimal notation: An important research finding. *Arithmetic Teacher*, 30, 32-33.

Hiebert, J., & Wearne, D. (1986). Procedures over concepts: The acquisition of decimal number knowledge. In J. Hiebert (Ed.), *Conceptual and procedural knowledge: The case of mathematics* (pp. 199-223). Hillsdale, NJ: Lawrence Erlbaum.

Lesh, R., Behr, M., & Post, T. (1987). Rational number relations and proportions. In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics* (pp. 41-58). Hillsdale, NJ: Erlbaum.

Meira, L. (1998). Making sense of instructional devices: The emergence of transparency in mathematical activity. *Journal for Research in Mathematics Education*, 29, 121-142.

Putt, I. J. (1995). Pre-service teachers ordering of decimal Numbers: When more is smaller and less is larger! *Focus on Learning Problems in Mathematics*, 17(3), 1-15.

Resnick, L. B., Nesher, P., Leonard, F., Magone, M., Omanson, S., & Peled, I. (1989). Conceptual bases of arithmetic errors: The case of decimal fractions. *Journal for Research in Mathematics Education*, 20(1), 8-27.

Sackur-Grisvard, C., & Leonard, F. (1985). Intermediate cognitive organization in the process of learning a mathematical concept: The order of positive decimal numbers. *Cognition and Instruction*, 2, 157-174.

Schwartz, J. (1994). The role of research in reforming mathematics education: A different approach. In A. Schoenfeld (Ed.), *Mathematical thinking and problem solving* (pp. 1-7). Hillsdale, N J: Lawrence Erlbaum.

Stacey, K., & Steinle, V. (1999a). Understanding decimals: The path to expertise. In J. & K. Truran (Eds.), *Making the difference* (Proceedings of the 22nd annual conference of the Mathematics Education Research Group of Australasia, pp. 446-453). Adelaide: MERGA.

Stacey, K., & Steinle, V. (1999b). A longitudinal study of children's thinking about decimals: A preliminary analysis. In O. Zaslavsky (Ed.), *Proceedings of the 23rd Conference of the International Group for the Psychology of Mathematics Education* (pp. 233-240). Haifa, Israel: PME.

Steinle, V., & Stacey, K. (1998). The incidence of misconceptions of decimal notation amongst students in grades 5 to 10. In C. Kanes, M. Goos & E. Warren (Eds.), *Teaching mathematics in new times* (Proceedings of the 21st annual conference of the Mathematics Education Research Group of Australasia, pp. 548-555). Brisbane: MERGA.

Thipkong, S., & Davis, E. (1991). Preservice elementary teachers' misconceptions in interpreting and applying decimals. *School Science and Mathematics* 91(3), 93-99.

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Appendix

Number Trails

Goals:

- to help students incorporate decimals into their existing number sense,
- to explore the notation of the base 10 number system.

Year level: Grade 2 to Year 10, depending on the choice of numbers
Number of players: Whole class
Equipment:

1. Strips of paper for all students
2. A blackboard
3. Stopwatch
4. Calculators (optional)

Time: 5 minutes
Activity Instructions:

- All students are given 60 seconds to write a sequence of numbers starting from a given number and using a certain increment, without using a calculator. For example, Grade 2 may start at 25 and increment by 10. Grade 4 may start at 3 and increment by 0.1.
- After the time has elapsed, all students stand. One is chosen to start reading out their sequence. The teacher or a selected student writes the numbers on the board so that all can see.
- Other students may challenge as the sequence is read out. If the person is incorrect, they sit down and the challenger takes over reading out the numbers in the sequence. The class, the teacher, or a calculator can adjudicate. Students remain standing if they still have numbers to offer to the sequence on the board.
- A student can continue reading the numbers of their sequence until they are successfully challenged or the teacher calls on someone else. The last person standing is the winner.

Variations:
All four operations could be used with the same basic game format. Some sequences to try include adding or subtracting 2, 4, 5, 9 and multiples of 10 or multiplying/dividing by 10 or 2. The starting number, the operation and the increment or ratio could all be chosen by students who might seek to provide unusual/challenging possibilities.

Comments:

- Students who win too often can be handicapped by 5 seconds or more to allow others to become winners.

- The teacher can start the sequences on the board if there are some students who need this assistance to start the game.
- Number Trails has different challenges for different ages. It is a useful game format that can be used within the domain of whole numbers or extended to fractions and decimals and negative numbers. It can be used as an integral part of a lesson or to fill in a few minutes spare at any time of the day.
- This is a good opportunity to let students know that there are more numbers than those they have studied formally without the pressure of being required to assimilate all the details. Longer decimals, or negative numbers may arise naturally. Discussion about various methods for predicting answers could draw attention to number patterns as well as basic features of notation. Students will look for patterns so that they can write numbers without calculating. As the numbers are written on the board, the students who sit down early get time to observe the resulting patterns, and use the constant addition facility of their calculator to check answers. Negative numbers might be 'discovered' by those using their calculators.
- When decimals are used, there will probably be disputes over correctness. For example, a sequence such as 4, 4.125, 4.25 (or 4.250) includes decimals of varying length, and those using a calculator may be able to offer alternative notation.
- This simple game, a variation of skip counting, comes from Maggie Marriott of Sunshine East Primary School. The students love it.

Note: This activity is from Condon & Archer (1999), pp. 3 - 4.

Stickers Game

Goals:

- to order numbers including decimals
- to connect whole numbers and decimals
- to reveal difficulties in understanding place value.

Year level: Grade 3 to Year 8 (depending on the numbers chosen)
Equipment: One sticker per child with a (decimal) number written on each.
Time: 5 - 10 minutes (warm up activity)
Activity Instructions:

1. Before class, write a number on each sticker, appropriate to the grade level. This game format is very flexible and so the actual numbers can be adjusted to most grades.
2. Consider using whole numbers (positive integers) of varying length (e.g., 53 879, 2 003 689), decimals of varying length (e.g., 34.8, 0.007, 4.79883) as well as numbers which combine these (e.g., 359 278.2, 5 443.229, 30.000056). Include zero (0) in the list and 'unnecessary' zeros (e.g., 054 and 3.200). Make the decimal points large, to be easily read at a distance.

3. Place stickers on students' foreheads. Tell them to arrange themselves in a line (smallest on the left, largest on the right) across the room, without speaking to each other.
4. Stop the activity when interest lags (do not expect the correct solution).
5. Discuss.

Variations:

6. Trial another round where students cannot see their own numbers.
7. Include negative numbers (e.g., -2, -5, -35.6) and mixed numbers (e.g., $2^2/_3$)

Comments:
This activity can expose students to numbers that they may not often encounter, yet should be able to deal with conceptually. It may not be a 'stand-alone' activity, but may be used as an introduction to further activities if required.
Note: This activity is from Condon & Archer (1999), pp. 10 – 11.