Preservice Teachers' Perspectives on Connections Within Mathematical Concepts: A Qualitative Study

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This study explores how preservice secondary mathematics teachers comprehend and apply advanced mathematical concepts to the content they will teach in K–12 classrooms. Responding to student inquiries and guided by the *Mathematical Education of Teachers II* recommendations, an innovative capstone course was developed and integrated into the mathematics education program to bridge the gap between college level mathematics and secondary instruction. The course was designed to foster meaningful mathematical connections through assignments and guided reflection, enhancing preservice teachers' ability to connect advanced concepts to secondary teaching. Employing a generic qualitative research design with thematic analysis, written reflections from five preservice teachers in their final semester of a teacher preparation program at Northwest Missouri State University were analysed. The purpose of the written reflection was to help these teachers examine their evolving mathematical thinking and instructional planning. The findings indicate that participants transitioned from procedural to conceptual reasoning, developed heightened awareness of mathematical interconnectivity, and gained confidence in designing lessons that integrate advanced mathematics. These results suggest that purposefully designed capstone courses can strengthen preservice teachers' pedagogical effectiveness and promote curricular coherence. Further, it is recommended teacher preparation programs prioritise connecting advanced mathematics to school level instruction.

Keywords: ⬩mathematics curriculum integration⬩common content knowledge⬩specialised content knowledge⬩horizon content knowledge capstone experiences⬩

Introduction

Improving preservice teachers' mathematical content knowledge and pedagogical skills remains a key focus in mathematics education (Wu, 2011). Research underscores the impact of well-designed coursework on deepening conceptual understanding and enhancing instructional effectiveness (Lachance & Confrey, 2003; Olson, 2016). For example, Gonzales and Gonzales (2021) highlighted that integrating pedagogical content knowledge (PCK) frameworks enables preservice teachers to deliver mathematics instruction with greater relevance. Despite advancements made, challenges endure. Misaligned curricula often hinder the ability to form coherent connections across mathematical topics and grade levels (Nguyen & Munter, 2024). Additionally, Sapkota and Hayes (2024) indicated that horizon content knowledge (HCK) is often underdeveloped when there is no intentional instructional support.

To address these challenges, a mathematics education program was restructured to meet state certification requirements and align with the *Mathematical Education of Teachers II* (Conference Board of the Mathematical Sciences [CBMS], 2012) recommendations. Central to this revision is a capstone course that connects advanced mathematical content to K–12 curricula to address shortage of qualified mathematics educators by improving preservice teacher preparation (Ma, 1999; Olson, 2016; Wu, 2011). This course equips preservice teachers with the necessary skills to translate complex mathematical concepts into accessible instruction for diverse learners.

Positioned as the cornerstone of the revised program, the capstone course emphasises the integration of college-level mathematics with K–12 teaching practices. Aligned with the *Standards for Preparing Teachers of Mathematics* (Association of Mathematics Teacher Educators [AMTE], 2017), particularly Standard C.1, it emphasises the ability to link mathematical concepts across grade levels. Developed through collaboration among mathematics educators and content specialists, the course highlights mathematical progressions through topics from foundational topics (e.g., number operations) to more advanced concepts such as calculus and abstract algebra. For example, illustrated within the course is how the concept of slope connects to derivatives, how understanding area as additive leads to the development of the Riemann sum, and how long division connects to polynomial division. These progressions help preservice teachers understand mathematics as an interconnected discipline, enabling them to create coherent and engaging lesson plans.

The course also integrates theoretical frameworks with practical teaching strategies to prepare preservice teachers for classroom realities. Grounded in MET II (CBMS, 2012) and AMTE (2017) standards, it focuses on PCK, blending deep mathematical understanding with effective instructional methods (Gonzales & Gonzales, 2021). Preservice teachers participate in activities that simulate K–12 teaching scenarios, such as crafting lesson plans that integrate algebraic and geometric concepts. Reflective practices allow preservice teachers to evaluate their teaching demonstrations, refining strategies to meet diverse student needs and ensuring effective application of advanced mathematics in secondary instruction.

The restructured mathematics education program, anchored by its innovative capstone course, bridges gaps in preservice teacher preparation by connecting advanced mathematical content to K-12 curricula, enabling teachers to make mathematics engaging and accessible and enhancing student understanding. Developed through interdisciplinary collaboration and aligned with research-based standards (AMTE, 2017; CBMS, 2012), the program prepares preservice teachers to deliver effective mathematics instruction in diverse classrooms. This approach equips preservice teachers with strong content knowledge and practical skills, thereby addressing the shortage of qualified mathematics educators (Ball et al., 2008; Hill et al., 2005; Sears et al., 2019). Further research is essential to assess the long-term impact of this program on teacher effectiveness and student outcomes, ensuring continuous improvement in mathematics education.

Research Questions

The aim of this study is to explore the impact of the capstone course developed on preservice teachers' understanding of mathematical content connections through engagement in lesson planning, group discussions, and presentations. Specifically, it addresses the following research questions:

1. *How do mathematics methods courses enhance preservice teachers' mathematical reasoning abilities?*
2. *Do preservice teachers regard mathematics as a set of interrelated concepts, and what is the mechanism through which preservice teachers' understanding of mathematical reasoning develop?*
3. *What are the qualitative outcomes of the capstone course in terms of preservice teachers' understanding, engagement, and ability to apply Horizon Content Knowledge in future classrooms?*

Literature Review

The critical role of teacher knowledge in shaping instructional quality is well established in mathematics education research. Alexander et al. (1991) conceptualised knowledge as individuals' awareness or beliefs about a subject, regardless of accuracy, laying the foundation for understanding teacher expertise. Berliner (2001) further emphasised that the depth and quality of teacher knowledge significantly influence classroom outcomes, prompting inquiries into the types of mathematical knowledge most essential for effective teaching (Thames & Ball, 2010) and the forms that best enhance instructional practice (Ball, 2003). Recent work by Weingarden (2025) supports this view, highlighting how teachers' beliefs about mathematics shape their ability to foster student engagement and conceptual understanding in secondary mathematics classrooms.

A robust body of research underscores the necessity of strong mathematical content knowledge for fostering students' conceptual understanding. Several studies (Hill et al., 2005; Philipp et al., 2007; Reid & Reid, 2017; Tucker et al., 2015; Tutak & Adams, 2015) have demonstrated a strong correlation between teachers' mathematical proficiency and the quality of their instruction. Hill et al. (2008) and Olson (2016), however, have argued that content knowledge alone is insufficient; it must be integrated with pedagogical practices to support effective teaching. PCK involves transforming theoretical understanding into practical instructional strategies, as highlighted by Shulman (2013) and Eraut (2002). PCK enables teachers to make informed instructional decisions by combining deep content knowledge with an understanding of student learning processes. Further, Copur-Gencturk et al. (2023) have recently demonstrated that teachers with strong PCK design lessons address effectively students' mathematical misconceptions.

Building on these ideas, Ball et al. (2008) introduced the Mathematical Knowledge for teaching (MKT) framework, which was later refined by Patterson et al. (2020) to emphasise its application in preservice teacher education. This framework, which categorises teacher knowledge into three domains: Common Content Knowledge (CCK), Specialised Content Knowledge (SCK), and Horizon Content Knowledge (HCK). CCK refers to general mathematical understanding applicable beyond teaching contexts (Ball et al., 2008; Hill et al., 2008). Meanwhile, SCK involves a deeper, teaching-specific understanding that supports analysing student thinking and explaining concepts clearly (Hoover et al., 2016). HCK, critical to this study, refers to an awareness of how mathematical concepts connect across topics and grade levels, enabling teachers to contextualise content within the broader mathematical landscape (Ball & Bass, 2009; Patterson et al., 2020). The capstone course examined in this study targets all three domains, with a particular emphasis on developing HCK through activities that trace mathematical progressions, such as connecting slope to derivatives, area to Riemann sums, and long division to polynomial division. Recent work by Nieto et al. (2021) validated the MKT framework, demonstrating that HCK enables teachers to create coherent lesson sequences that connect mathematical concepts across grade levels.

Despite the relevance of these frameworks, challenges in preservice teacher preparation persist. Olson (2016) noted that many preservice teachers struggle to align their mathematical training with secondary curricula, often questioning the relevance of advanced topics such as calculus or abstract algebra. Nguyen and Munter (2024) identified misaligned curricula in teacher education programs as a barrier to developing coherent content knowledge, limiting preservice teachers' ability to form meaningful mathematical connections. Similarly, Sapkota and Hayes (2024) found that HCK development requires explicit instructional support, as preservice teachers often lack the ability to independently recognise conceptual connections across grade levels. Moreover, Fitriati et al. (2023) found that preservice teachers benefit from structured coursework that explicitly links advanced mathematics to classroom practice.

Recent scholarship highlights the efficacy of integrative coursework, such as capstone experiences, in addressing these challenges. Sapkota and Max (2024) demonstrated that structured reflective practices paired with content-focused learning enhance preservice teachers' mathematical reasoning and instructional application. Meanwhile, Chavira-Quintero and Olais-Govea (2023) found that capstone courses foster PCK by promoting collaborative inquiry and active learning, enabling preservice teachers to design conceptually rich lessons. Henriques and Silva Bueno (2022) showed that technology-supported environments aid in crafting lessons that integrate digital tools, strengthening content connections. Additionally, Sears et al. (2019) emphasised that collaborative learning environments deepen conceptual understanding and student engagement, while Graham and McDuffie (2023) highlighted the role of shared lesson design and content mapping in fostering a nuanced understanding of mathematics as an interconnected system. Recent works by Buchbinder et al. (2024) and Sohdi (2025) underscore the critical role of capstone courses, showing that they enhance preservice teachers' ability to integrate content and pedagogy through collaborative lesson design. Moreover, Sapkota (2025) underscored how capstone courses foster preservice teachers' ability to create standards-aligned lessons with strong conceptual coherence. Collectively, these findings suggest that integrative, collaborative, and technology-supported capstone experiences equip preservice teachers with the skills to design coherent, engaging, and conceptually robust mathematics lessons, ultimately fostering a deeper understanding of mathematical connections and improving instructional effectiveness across diverse educational contexts.

Further supporting these findings, Bruna (2025) explored academic portfolios in advanced mathematics courses, revealing that reflective documentation deepens engagement and strengthens preservice teachers' ability to bridge theory and practice. García Lázaro and Reyes de Cózar (2025) demonstrated that structured reflection during field experiences enhances content knowledge and professional growth. Maher et al. (2022) applied the Knowledge Quartet framework to senior secondary mathematics teaching, underscoring the importance of HCK in enabling teachers to enact connected mathematical knowledge across domains and grade levels. These studies collectively affirm the need for teacher education programs to deliberately cultivate conceptual connectedness, a core objective of the capstone course in this study. Additionally, Hubbard and Livy (2021) found that reflective practices in teacher education improve preservice teachers' ability to design lessons that emphasise mathematical connections.

Comparative analyses from international contexts further enrich this discussion. For instance, studies in education systems, such as Ma (1999), highlighted how cohesive curricula and intensive content-focused training enable teachers to develop deep HCK, resulting in more effective instruction. By contrast, the systems often face challenges with fragmented curricula, as noted by Nguyen and Munter (2024), which can hinder HCK development. These cross-cultural insights underscore the importance of aligned, integrative coursework, as implemented in the capstone course under study. Recent research by Leong et al. (2024) supports this view, showing that cohesive curricula in East Asian contexts enhance teachers' HCK and instructional effectiveness.

Efforts to integrate advanced mathematics with pedagogical training in teacher education programs, course observations indicate that some preservice teachers struggle to identify essential mathematical connections, often due to unfamiliarity with advanced mathematics or an overemphasis on isolated standards. This observation aligns with Sapkota and Hayes (2024), who advocate for explicit instructional strategies to support HCK development. This evidence exposes a critical gap in current teacher preparation practices, necessitating a qualitative inquiry into how capstone experiences shape preservice teachers' mathematical reasoning, instructional planning, and understanding of curricular coherence. The present study addresses this gap by examining how an integrated capstone course, grounded in the MKT framework with a focus on HCK, influences preservice teachers' ability to forge mathematical connections and apply these insights through lesson planning, discussions, and presentations in the United States secondary mathematics context, contributing to more cohesive teacher preparation. By fostering a deeper understanding of mathematics as an interconnected system, this approach promises to produce more effective, conceptually grounded educators capable of designing coherent and engaging lessons.

Methods

Research Design

This study utilised a generic qualitative research design (Tisdell et al., 2025) to explore how a capstone mathematics methods course shaped preservice secondary mathematics teachers' ability to make meaningful content connections and apply pedagogically aligned reasoning. A qualitative approach was selected to capture the depth of lived experiences, perceptions, and interpretations of participants within the natural context of the classroom, as qualitative inquiry excels at uncovering how individuals construct meaning in specific educational settings (Maxwell, 2008). This design facilitated an in-depth examination of how course activities, such as lesson planning, peer discussions, and instructional presentations, influenced participants' views of mathematics as an interconnected discipline. The focus was on generating rich, contextual insights into participants' reasoning and instructional decision-making rather than pursuing broad generalisability.

Participants and Sampling

The study involved five undergraduate preservice secondary mathematics teachers in their final semester of a teacher preparation program at Northwest Missouri State University in Missouri. Participants were enrolled in a semester-long capstone course, *Connections to Teaching Mathematics*, offered in Spring 2024. Among the eight students in the course, five voluntarily provided informed consent for their reflections and coursework to be analysed. The participants were selected through purposive sampling to ensure they were actively engaged in the capstone experience and these participants were representative of preservice teachers in the program. A representative preservice teacher in this program is an undergraduate mathematics education major who has completed core coursework blending advanced mathematical concepts with pedagogical strategies and is immersed in field experiences, honing skills in designing coherent, conceptually rich lessons for secondary mathematics classrooms.

Although the small sample size limits the generalisability of findings, it is consistent with the goals of qualitative research, which emphasise depth over breadth in exploring participants' experiences (Maxwell, 2008). Small, purposively selected samples are common in qualitative inquiry because they allow researchers to focus on meaning-making, context, and the richness of participants' perspectives (Patton, 2014). This approach supports the development of a detailed and context-specific understanding rather than statistical representativeness (Creswell & Poth, 2016). Furthermore, to address concerns about potential bias given the course instructor's role in the study, several steps were taken to enhance trustworthiness, including maintaining reflexive documentation and having a mathematics education faculty member independently review coding decisions.

Study Context

Preservice mathematics education students at the university complete a rigorous sequence of mathematics courses, including Calculus I and II, Transition to Proofs, College Geometry, Linear Algebra, Modern Algebra, and two elective upper-level mathematics courses, totalling at least 19 credit hours. Upon completion, they are eligible to enrol in the capstone course, *Connections to Teaching Mathematics*. The course embodies the department's vision of contextualised mathematics, emphasising real-world applications and connections across mathematical concepts (see Figure 1).

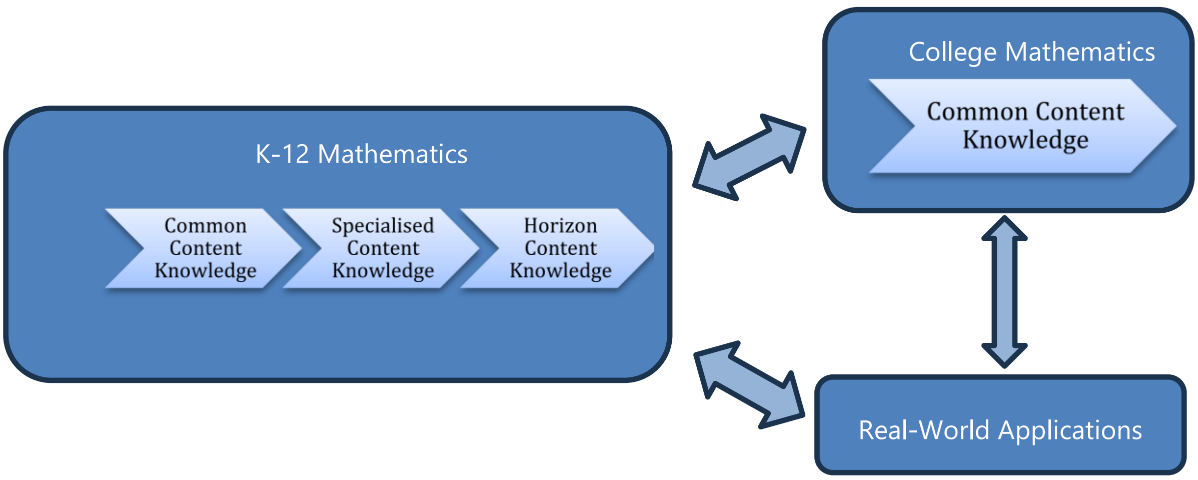

Figure 1. Vision of the Connections to Teaching Mathematics course.

The capstone course focuses on advanced mathematical topics relevant to K–12 teaching, such as derivatives, matrix inverses, polynomial division, composite functions, exponent and logarithm laws, and the law of cosines. In the first three weeks, preservice teachers explore connections between college-level and K–12 content. For example, they examine how the concept of area, introduced in secondary education, progresses to Riemann sums in calculus. These connections are analysed collaboratively through online discussions, where each preservice teacher is assigned a mathematical concept. Over the semester, they develop and teach an introductory mini lesson focusing on the foundational understanding of their assigned topic, either individually or in pairs, over a two-week period. Following each lesson, they present a progression of standards from middle to high school mathematics, culminating in the development of their assigned topic, thereby, fostering HCK.

Data Collection

Data were gathered through open-ended surveys administered two times: the first and final weeks of the semester. The initial survey prompted participants to reflect on their prior mathematical experiences and their initial understanding of mathematical connections. The final survey asked them to articulate how their perspectives had evolved, focusing on their ability to recognise conceptual relationships, apply mathematical ideas across grade levels, and justify pedagogical decisions (see Appendix). Surveys were administered via the university's Learning Management System to ensure accessibility and confidentiality. Open-ended questions were prioritised to encourage diverse, authentic reflections, capturing the complexity of participants' evolving understanding.

Data Analysis

Thematic analysis, guided by Braun and Clarke's (2006) six-phase framework, was employed to analyse survey responses, aligning with the study's aim of exploring how preservice teachers developed mathematical connections across K–12 and college-level content. This approach supported inductive identification of patterns, remaining flexible to participants' perspectives without being constrained by a specific epistemological framework. The analysis process involved:

1. Repeated readings of the dataset to gain familiarity,
2. Open coding to identify meaningful segments of participants' reflections, and
3. Iterative refinement of codes to group them into broad themes.

Themes were developed inductively from the data, focusing on three domains of mathematical knowledge: Understanding of Mathematical Concepts (CCK), Developing Instructionally Relevant Reasoning (SCK), and Connecting Concepts Across the Curriculum (HCK). These themes emerged directly from participants' responses, reflecting their evolving understanding and instructional approaches. To ensure trustworthiness, a mathematics education faculty member independently reviewed the codes and confirmed the thematic structure. Researcher bias was further minimised through reflexive note taking and ongoing comparison of themes with raw data. This rigorous process ensured a robust interpretation of participants' reasoning and instructional decision-making, directly addressing the study's research questions.

Results

Presented within this section are the findings from the thematic analysis of the five participants' survey responses, organised into three domains of mathematical knowledge: CCK, SCK, and HCK. These domains, derived inductively from the data, align with the study's aim to explore how a capstone course influenced participants' ability to forge mathematical connections and apply pedagogically aligned reasoning. The themes, representative codes, descriptions, and guiding questions used in the analysis are presented in Table 1, providing a cohesive overview of the findings.

Table 1.
Themes, Codes, Descriptions, and Guidance Questions from Thematic Analysis

Theme	Representative Codes	Description	Guidance Questions
Common Content Knowledge (CCK)	Emphasis on procedural steps; Listing of related concepts; Surface-level understanding; Misconceptions in fundamental concepts	Reflects general mathematical knowledge expected of any mathematically literate adult, often limited to isolated facts or procedures.	Does the response demonstrate a basic understanding of core mathematical ideas? Does the participant list related concepts without describing their relationships?
Specialised Content Knowledge (SCK)	Deeper conceptual explanations; Use of precise mathematical language; Examples of conceptual continua; Awareness of student errors	Demonstrates teaching-specific knowledge, including the ability to unpack student thinking and articulate connections clearly.	Does the participant provide teaching-specific explanations that demonstrate deep conceptual understanding? Does the response address student misconceptions or instructional strategies?
Horizon Content Knowledge (HCK)	Curriculum-wide conceptual links (K-12 to college); Future-oriented thinking about student learning; Real-world applications; Confidence in teaching impact	Reflects an understanding of how mathematical ideas interconnect across grade levels and build conceptually over time.	Does the participant show awareness of how mathematical ideas connect across grade levels? Do they reflect on how these connections influence student understanding or real-world applications?

Common Content Knowledge

At the start of the capstone course, participants' survey responses revealed a reliance on procedural approaches shaped by their prior mathematical experiences in middle and high school. For instance, one participant noted:

> *I only understood math because I could memorise formulas … I didn't understand why I needed to use certain formulas, but I could do the work.*

Such reflections, consistent with prior research on procedural learning (Skemp, 2006; Wade et al., 2017), indicated a focus on memorisation over conceptual understanding. Early responses often lacked depth or justification, highlighting fragmented knowledge of mathematical concepts. By the end of the course, participants demonstrated a shift toward more flexible, conceptually grounded thinking. One participant reflected:

> *In middle school, I understood math as a straightforward path of steps and formulas …. Now, I see it as a structure of interrelated concepts.*

This transition suggests that the capstone course fostered a deeper understanding of mathematical structures, moving participants beyond isolated procedures to a more cohesive grasp of content.

Specialised Content Knowledge

Midway through the semester, participants began articulating how conceptual understanding enhanced their ability to explain mathematical ideas effectively. For example, one participant described:

> *When doing the chain rule in calculus, I used to just think about the pattern. Now I think about the pattern and how it relates."*

This progression reflects the development of SCK, characterised by the ability to unpack mathematical concepts and address student thinking (Ball et al., 2008). Participants increasingly used precise mathematical language and provided examples that bridged related concepts, demonstrating teaching-specific knowledge essential for effective instruction.

Horizon Content Knowledge

By the end of the course, participants exhibited a broader understanding of mathematics as an interconnected discipline, recognising how concepts progress across grade levels. One participant illustrated this awareness:

> *We took slope as rise over run… then moved to slope of nonlinear functions, finding the derivative. The connection helped me understand limits better.*

Such reflections indicate the development of HCK, enabling participants to connect foundational and advanced topics, such as area to Riemann sums or long division to polynomial division (Ball & Bass, 2009; Patterson et al., 2020). Participants also reported increased confidence in instructional planning, with one stating:

> *This class helped me become more knowledgeable about why and how to integrate connections …. I now feel equipped to defend my instructional choices.*

This comment highlights how HCK supported the design of conceptually coherent lessons aligned with student learning trajectories.

The thematic analysis revealed a clear developmental trajectory across the three knowledge domains. Initially, participants' understanding was rooted in procedural knowledge, reflecting limited conceptual depth. Through the capstone course's emphasis on collaborative discussions, lesson planning, and reflective practices, they shifted toward more integrated, instructionally relevant reasoning. The course's design, particularly its focus on tracing mathematical progressions, played a pivotal role in fostering this shift. Participants moved from recognising isolated concepts (CCK) to explaining them with teaching-specific insights (SCK) and, ultimately, situating them within a broader curricular framework (HCK).

Discussion

This study advances the understanding of how preservice secondary mathematics teachers develop conceptual coherence and pedagogical reasoning through a capstone course designed to bridge K–12 and college-level mathematics. The findings align with prior research indicating that many preservice teachers enter teacher education programs with fragmented, procedure-driven mathematical knowledge shaped by memorisation-focused experiences (Olson, 2016; Papick et al., 2010; Wu, 2011). Such approaches, as noted by Al-Mutawah et al. (2019), Hattie and Yates (2014) and Stokke (2015), often limit conceptual understanding and flexible reasoning, a challenge evident in participants' initial survey responses describing mathematics as a collection of isolated procedures.

This emphasis on memorisation is persistent despite curriculum reforms, likely due to deeply rooted instructional practices in K–12 settings and focus on conceptual connections in earlier teacher preparation coursework (Appova & Taylor, 2019; Fan & Bokhove, 2014). Unlike studies such as Lestari et al. (2018), which found persistent procedural thinking even after targeted interventions, the participants demonstrated a marked shift toward viewing mathematics as an interconnected discipline. This transition, facilitated by the capstone course's reflective and integrative structure, aligns with Ball et al.'s (2008) MKT framework, particularly in the development of CCK, SCK, and HCK. The course's focus on tracing mathematical progressions, such as slope to derivatives or area to Riemann sums, mirrors conceptual models proposed by Findell et al. (2001), Ma (1999), and Ponte and Chapman (2008), who emphasised interconnectedness as a cornerstone of effective mathematics teaching.

The capstone course's design fostered critical reasoning over rote learning, supporting arguments by Dixon and Brown (2012), Gainsburg (2008), and Jia et al. (2018). Participants' ability to articulate connections across topics and grade levels reflects a deeper engagement with HCK, a finding consistent with Sapkota and Hayes (2024) and Bruna (2025), who highlighted the role of explicit content coherence in building long-term teaching capacity. Compared to prior studies, such as Ball and Bass (2000), which showed that preservice teachers struggled to integrate advanced content, participants in this study demonstrated greater success, likely due to the course's structured emphasis on collaborative lesson design and reflective practice, as advocated by Graham and McDuffie (2023).

A noteworthy outcome was participants' increased confidence in making and defending instructional decisions, a factor underexplored in prior research. Wang et al. (2023) and Li et al. (2024) suggested that reflective practices and dialogic feedback enhance situated PCK, enabling preservice teachers to adapt instruction to diverse student needs. This study extends these findings by showing that confidence in instructional decision-making, grounded in HCK, prepares teachers for leadership roles and licensure assessments, indicating broader implications for teacher readiness. However, unlike Wang et al. (2023), who investigated in-service teachers, this study's focus on preservice teachers highlights the formative role of early, structured interventions in building such confidence.

Despite these advancements, participants' reflections revealed gaps in their prior mathematical preparation, reflecting concerns raised by Fan and Bokhove (2014) and Stokke (2015) about curricular coherence. Some expressed surprise that conceptual connections were not emphasised earlier, suggesting that the persistence of procedural approaches may stem from misaligned K–12 and undergraduate curricula, as noted by Nguyen and Munter (2024). This finding contrasts with studies in East Asian contexts, such as Ma (1999), where cohesive curricula foster early HCK development, highlighting a need for systemic alignment in Western teacher education programs.

Preservice teachers' emotional and cognitive contributions, as revealed in this study, enrich discussions on the affective dimensions of teacher preparation. Participants' reconceptualisation of mathematics as a coherent, interconnected discipline reduced their reliance on rote strategies and increased problem-solving confidence, aligning with Powell (2022) and Soysal et al. (2022a; 2022b), who link conceptual understanding to reduced math anxiety and enhanced student engagement (Sony, 2024). Unlike Lorenzen and Lipscomb (2021), who focused on student outcomes, this study suggests that such affective benefits extend to teachers, potentially enhancing long-term instructional effectiveness by fostering resilience and adaptability in classroom settings.

Building on these affective gains, the study provides evidence for advocating for integrative capstone courses in teacher preparation programs, particularly for fostering HCK and pedagogical confidence. Unlike prior studies that often address content or pedagogy in isolation, this study demonstrates the efficacy of combining reflective practices, collaborative inquiry, and explicit content connections to prepare preservice teachers for secondary instruction. There is the potential for other teacher education programs to address curricular fragmentation and to promote conceptual coherence by implementing similar course designs.

The results highlight that a strategically designed capstone course significantly enhances preservice secondary mathematics teachers' ability to forge meaningful connections between K–12 and college-level mathematical content. Through structured reflection, collaborative inquiry, and targeted instructional tasks, participants shifted from viewing mathematics as a collection of isolated procedures to recognising it as a cohesive, interconnected system, aligning with the findings from Ball et al. (2008), Hsieh et al. (2011), and Stokke (2015). The course emphasis on curricular coherence not only strengthened participants' mathematical understanding but also enhanced their instructional self-efficacy, as evidenced by their reported ability to justify teaching decisions and address diverse student needs. This aligns with prior research highlighting the role of reflective practices in building pedagogical confidence (Li et al., 2024; Wang et al., 2023).

Limitations

Despite its contributions, the study has notable limitations. First, self-reported qualitative data may introduce potential biases, such as selective recall or social desirability, potentially undermining trustworthiness of findings. This was addressed by cross-referencing self-reports, instructor observations, and lesson plan documents for reliability. Second, the small sample size and single-course context limit generalizability of the findings to broader populations or settings. A diverse sample and detailed context were included to aid transferability. Third, external factors, like instructor facilitation or group dynamics, were underexplored due to scope of the study. To address these factors, peer feedback analysis offered insights into these influences, but further investigation is needed. These limitations warrant careful interpretation of the findings and highlight the need for further validation.

Future Directions

Future research could adopt longitudinal designs to examine how these conceptual and affective gains translate into classroom performance, incorporating classroom observations and student feedback for a comprehensive perspective. Mixed-methods approaches could further examine the interplay between cognitive and affective outcomes. For example, building on suggestions by Henriques and Silva Bueno (2022), researchers can explore technology-enhanced tools, such as dynamic geometry software or collaborative platforms, in strengthening conceptual connections. Comparative studies across diverse educational contexts could also clarify how program design influences HCK development, particularly in systems with varying curricular alignment.

Conclusion and Implications

In summary, this study highlights the transformative potential of integrative capstone courses in equipping preservice mathematics teachers with the knowledge, skills, and confidence to provide conceptual teaching and connect ideas across grade levels. By addressing curricular fragmentation and fostering instructional self-efficacy, such courses improve mathematics instruction and enhance student learning outcomes, offering a model for teacher preparation programs worldwide.

In terms of theoretical contributions This study advances the application of the Mathematical Knowledge for Teaching (MKT) framework by embedding Horizon Content Knowledge (HCK) within a newly designed capstone course, *Connections to Teaching Mathematics*, which integrates advanced mathematical concepts with K–12 pedagogical strategies to foster coherent lesson design. Specifically, it builds on Buchbinder et al. (2024), who showed that preservice teachers' knowledge for teaching proof enhances their ability to connect mathematical concepts, by embedding HCK-focused activities, such as tracing mathematical progressions, within a capstone context. This approach enriches teacher knowledge development by enabling preservice teachers to contextualize mathematical concepts within a broader curriculum, fostering deeper conceptual coherence. Practically, the study aligns with Sohdi (2025), who emphasized content-pedagogy integration, providing a framework for teacher education programs to design curricula and professional development initiatives that cultivate conceptually grounded educators. These insights highlight the need for systemic reforms to align K–12 and undergraduate curricula, ensuring preservice teachers are equipped from the outset to design lessons that bridge mathematical ideas effectively.

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Ethical approval

Ethical approval for the research was granted by Institutional Review Board, and informed consent was given by all participants for their data to be published. Individual student contributions to twice research surveys, on the other hand, are not made public due to privacy concerns.

Competing interests

The authors declare there are no competing interests.

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Appendix

The purpose of this initial reflection is not to grade your current understanding of these concepts or how many connections can be listed. It is to obtain you to think about connections in math and how regularly you think about connections versus how regularly you learn math procedures and formulas as isolated information.

Answers to the following questions:

What are some mathematical concepts (choose anything from elementary through college math) that are directly related to the law of cosines $a^2 = b^2 + c^2 - 2bccos\theta$?

What mathematical concepts are directly related to polynomial division? For example: $(x^3 + 5x^2 - 2x + 7) \div (2x + 3)$

What mathematical concepts are directly related to the composite functions? For example: If $f(x) = |x + 2|$ and $g(x) = x^2 - 5$ find $f(g(x))$.

What mathematical concepts are directly related to determining the inverse of a matrix?

Which mathematical concepts are directly related to the laws of exponents and logarithms? $\log_a(mn) = \log_a m + \log_a n$ and $\log_a(\frac{m}{n}) = \log_a m - \log_a n$ and $\log_a(m^r) = r\log_a m$

What mathematical concepts are directly related to derivatives?

Think of a continuum about how we view math. One end of the continuum is thinking about math as a set of steps to follow in a specific order and lists of formulas to memorize to pass a test. Another end of the continuum is thinking about math as a concept for understanding and applying connections to other concepts to develop a deeper understanding and appreciation of math.

Thinking about your approach to math classes.

Where did you fall on this continuum of middle school?

Where did you fall on this continuum in high school? Explain any shifts from middle school to high school and what might have led to these shifts.

Where do you fall on this continuum in high school? Explain any shifts from high school to college and what might have led to them.

Choose a “connection” for which another classmate presented lessons. Do not choose one that you are involved in.

Derivatives
Inverses of Matrices
Polynomial Division
Composite Functions
Laws of Exponents and Logarithms
Composite Functions
Law of Cosines

Once you have chosen a connection, answer the first two questions:
Write an explanation of how the topic relates to other mathematical concepts. This explanation is mathematical in nature.

Explain how your understanding of the concept or the link between the concept and others increased this semester.

Think of a continuum about how we view math. One end of the continuum is thinking about math as a set of steps to follow in a specific order and lists of formulas to memorize to pass a test. Another end of the continuum is thinking about math as concepts to understand and apply connections to other concepts to develop deeper understanding and appreciation of math.

Where do you fall on the continuum? Explain.

Return and read your responses and explanations from your initial reflections. Compare and contrast the two and reflect on their similarities and differences.