"I Don't Understand the Purpose of This Experience": Tensions in Collaborative, Process-oriented Professional Development for Mathematics Teachers

Gil Schwarts
Weizmann Institute of Science

Patricio Herbst
University of Michigan

Amanda Brown
University of Michigan

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Mathematics teachers' professional learning is moving away from prescriptive professional development (PD) toward collaborative, practice-based models—such as video-clubs and lesson co-design—which provide teachers with space and time to discuss problems of practice among peers. Some models, which we term "process-oriented", exclusively focus on fostering teachers' professional discourse and reflection rather than pushing for the implementation of specific classroom practices. Although research has documented the affordances of process-oriented teacher learning, less is known about the complexities involved. This multi-perspective qualitative study describes tensions surfaced by teachers and a facilitator in an online lesson-design programme in the United States. We identify two core tensions: valuing collaboration and relating to the programme's process-oriented aims, and we trace these tensions to participants' prior PD and classroom experiences, showing how implicit expectations about PD processes and outcomes shape participants' engagement with innovative learning opportunities. These findings contribute to the field's understanding of collaborative and process-oriented teacher learning by highlighting the presence of a didactical contract within in-service teacher learning contexts, which resonates with the classroom didactical contract. We posit that recognising and addressing teachers' implicit expectations is essential for designing PD that is not only innovative in form but also responsive to practitioners.

Keywords: mathematics teacher education research • collaboration • facilitators • process-oriented professional development • lesson design • didactical contract • responsiveness toward teachers • storyboards

Prologue

> *I liked the synchronous meetings. I think that interaction pushed my thinking the most. I still feel like I don't understand the purpose of this experience.*

This quotation, from a post-program questionnaire, points to a central tension explored in this paper: Teachers may value collaboration around problems of practice yet still feel unclear about the purpose of such interactions and how they relate to learning. This study aims to enhance understanding of two key aspects of that tension: (a) how participants perceive an environment that centres their voices and interactions, emphasising responsiveness to teachers' professional needs; and (b) whether these perceptions allow them to recognise the benefits of such an environment for their professional growth. We aim to examine what counts as professional learning for both participating teachers and the facilitator. Our goal is to understand the tensions that emerge from collaborative, responsive professional development (PD) so that such designs can be refined. In this paper, "professional learning" denotes the processes through which teachers deepen their professional expertise, whereas

"professional development" refers to the organised activities, in which professional learning may happen.

Introduction

PD environments for mathematics teachers are evolving, with many models emphasising collaboration (Borko & Potari, 2024; Harvey & Teledahl, 2022) and practice-based approaches (Cohen & Ball, 1999). Although all these models aim to improve mathematics instruction through professional learning, they vary in defining this learning and its mechanisms. Notably, the desired outcomes of teacher participation in collaborative programs remain unclear not only to readers of research articles but also to the teachers themselves.

In this paper, we scrutinise this issue in the context of a collaborative PD environment—Story*Circles*—that has the following key features: (a) it takes teachers out of the solitude of their classrooms and orients them to each other as mutual sources of knowledge; (b) it encourages the development of a shared discourse about the practice of mathematics teaching; (c) it does not measure success by the implementation of specific practices in teachers' classrooms; and (d) it avoids prescriptivism and discussions of "best-practices" (see Herbst & Milewski, 2018).

These characteristics are not unique to the intervention we discuss in this paper. Other programs share similar attributes and stances, for example, Lesson Study (e.g., Druken, 2023; Fernandez & Yoshida, 2012; Lewis et al., 2009) and video-clubs (e.g., Coles, 2013; Karsenty & Arcavi, 2017; van Es & Sherin, 2010). While recently there has been progress in terms of theorising collaborative teacher learning (Horn & Garner, 2022), there are precious few empirical studies that delve into the tensions it raises from practitioners' perspectives. Moreover, previous research suggests that teachers value PD that offers concrete classroom applications (Rogers et al., 2007), which raises the question whether teachers can be expected to adopt a learning disposition in collaborative, process-oriented contexts.

Therefore, we offer a multi-perspective exploration of teacher learning through collaborative lesson development, exploring questions such as: What do teachers perceive as learning during co-design of mathematics lessons? What aspects of the learning environment enable or hinder their ability to appreciate peer-interaction? How do facilitators effectively manage such activities? Addressing these questions can deepen researchers' understanding of collaborative teacher learning environments and enhance PD designers' responsiveness to practitioners.

To better specify the problem space, next we provide background on different approaches for professional learning and locate Story*Circles* within these approaches. We then describe the challenges mathematics teachers and facilitators experience when participating in collaborative environments.

Purposes of Teacher Learning Environments

During the past 30 years, there has been increasing attention to teachers' practice-based learning both for prospective and practicing teachers. This approach emphasises the importance of situating teachers' learning experiences within the context of professional practice (Grossman et al., 2009), for example by interrogating representations of practice (e.g., Herbst & Chazan, 2011a; Ineson et al., 2015) or approximating practice (e.g., Campbell & Elliott, 2015; Kazemi & Wæge, 2015; Schwarts et al., 2024;). The application of a practice-based approach may differ when envisioned for either prospective or practicing teachers: While pre-service teacher education can be prescriptive in nature, there is an open question about whether such an approach is appropriate for practicing teachers. In particular, there is a debate (e.g., Skedsmo & Huber, 2019) based on the question of who should be responsible for initiating and directing teachers' instructional changes (simply put, top-down vs. bottom-up). Historically, these approaches have also been called, in the United States, "teacher-change" and "learning-to-teach" (Richardson, 1990). The teacher-change literature defines the goal of teacher learning as teachers adopting practices "that others are suggesting" (p. 11), namely, teachers are expected to follow the logic

of others (e.g., reform movements, program designers) and are not granted with the agency to determine if and how they want to make changes in their instruction. In contrast, the learning-to-teach approach suggests that teachers should draw on their experiences to develop their reflective skills. The major drawback of this second approach, according to Richardson (1990), is that teachers' self-driven instructional changes remain idiosyncratic, and thus fail to produce generalisable knowledge that promotes the profession. Richardson (1990) proposed combining teachers' practical knowledge with research-based empirical premises, arguing that "if we are interested in [teacher] change that is significant and worthwhile, the content of reflection should relate to standards of appropriate classroom practice" (p. 13). This argument posits that in either case, the objective of teacher learning is achieving some form of change in instructional practices.

In the years that followed, the distinctions between the two approaches have become blurred. Many programs nowadays balance implementing research-based practices and granting teachers autonomy and agency. A noteworthy difference that has emerged is the decreased emphasis on the idea of specific changes in instructional practice as the primary objective of teacher learning. In that vein, Simon (2008) distinguished between programs which have content and process goals for teacher learning (content-oriented), and programs with process goals only (process-oriented). Programs that follow a content-oriented approach promote concepts and skills, such as the pedagogical concept "understanding of classroom norms and their negotiation" (p. 16). In a process-oriented approach, however, "there are no a priori learning goals for teachers ... other than learning the processes of inquiry, reflection, etc." (ibid). In the latter model, discussions among teachers are meant to foster argumentation and professional vision—but without specifying particular practices to be changed. Process-oriented PD could be more open-ended about outcomes and seek instead the development of mathematics teaching practice as a profession. It does not seek immediate, visible, or specific changes in instructional practice but aims to develop a professional discourse in which practitioners tinker with and argue about practice. Contrasting the approaches draws attention to the difference between putting policies into practice versus fostering the growth of the teaching profession over time. This perspective is consistent with Hiebert and Morris's (2012) argument for improving "teaching, rather than teachers" (2012, p. 92), and with their methods for achieving this by promoting teachers' collaborative work on lesson design as a means of enhancing the knowledge base of teaching. Along those lines, a recent review of teacher learning (Brehmer & Ryve, 2024) uses Desimone's (2009) categories to differentiate between programs that target teacher knowledge growth and others that target changed instruction. While the review article discusses the black boxing of teachers' learning processes from a researchers' perspective, our goal in this paper is to examine how *teachers* perceive their professional learning and what constitutes learning for them. By doing so, we contribute to unpacking this black box, bringing practitioners' perspectives to the forefront of the conversation.

*Locating Story*Circles *in the Landscape of Collaborative Mathematics Teacher Learning Environments*

In the following, we describe the Story*Circles* process in relation to the distinctions above (content and process-oriented PD), positioning it among other approaches and their goals to help readers understand its contextual details.

Story*Circles* is a lesson-based approach where teachers collaboratively design a problem-based mathematics lesson represented in storyboards with cartoon characters of teachers and students (Herbst & Milewski, 2018). Inspired by Japanese Lesson Study, Story*Circles* focuses teachers' attention on the unfolding of one problem-based lesson as the primary means for engaging them with discussions of their teaching decisions publicly. Participants in Story*Circles* are introduced to these intertwined goals: (a) representing problem-based lessons for the sake of (b) eliciting and discussing the knowledge required for managing such lessons.

The teachers' iterative work on the lesson includes, for example, scripting the launching of a problem; monitoring, selecting, and sequencing student work; and responding to student ideas. While

classroom implementation is an essential source of feedback in Lesson Study (namely, teachers learn more about the lesson from teaching it), Story*Circles* replaces this feature with the perusal of the evolving storyboarded lesson to support the participants' discussions of key moments of the lesson. Collective storyboarding and annotations of the lesson allow teachers to engage in approximations of practice akin to the kind of experimentation that can happen in situ (Milewski et al., 2018). We elaborate more on the Story*Circles* context later in this paper.

Going back to the distinctions made in the previous section, Story*Circles* is a collaborative, practice-based, and process-oriented approach (with processes aimed at teacher collaboration and argumentation through lesson design). In contrast to approaches that share these characteristics, such as video-clubs, Story*Circles* is structured in what Borko et al. (2011) called a *specified*, rather than an *adaptive*, way. In adaptive programs facilitators have the freedom to shape activities according to emergent goals and needs, with no expected fidelity to specific session plans (e.g., Schwarts & Karsenty, 2020). In specified programs "goals, resources and facilitation materials are specified for a particular predetermined PD experience" (Borko et al., 2011, p. 177). Likewise, in adaptive programmes teacher learning opportunities depend greatly on the problems of practice that are surfaced during the sessions, whereas in specified programs, the responsiveness toward participants' problems of practice is realised within structured activities. For example, another approach that Story*Circles* relates to, Lesson Study, is also specified, with a set agenda for facilitators to follow in order to assist teachers in developing a lesson plan that one of them will eventually teach. Similarly, in Story*Circles* facilitators' improvisation is confined to working with teachers' responses in the scope of structured sessions.

As Figure 1 illustrates, Story*Circles* is more specified than adaptive—it relies on structured, pre-set sessions—and more process-oriented than content-oriented, since its primary aim is to foster professional discourse (while also guiding participants to engage deeply with the lesson and its mathematics). This design promotes some of Story*Circles*' goals (e.g., reaching consensus about instructional decisions) yet creates challenges, such as how to balance teachers' agency with the specified design—a challenge which we examine in this paper. Below, we discuss related challenges and tensions that characterise collaborative or process-oriented programs.

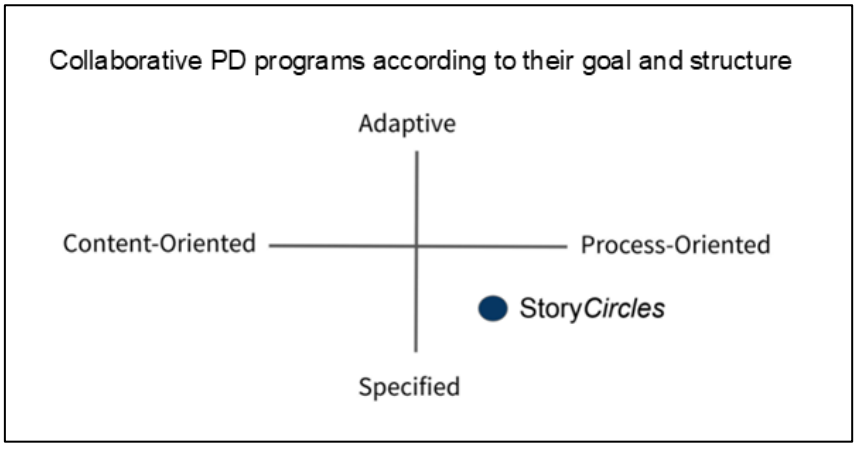

Figure 1. Locating Story*Circles* in the landscape of collaborative teacher learning programs.

Tensions Involved in Mathematics Teachers' Collaborative Learning

The literature has highlighted several tensions in collaborative professional development environments. First, the isolated nature of teachers' daily practice makes it hard for them to work together (Ghousseini et al., 2022), and concerns about their status may constrain their engagement (Eshchar-Netz et al., 2022). Furthermore, PD opportunities often fail to meet teachers' needs (Chval et al., 2010) and overlook how teachers' experiences outside PD shape their learning (Ehrenfeld, 2022). A collaborative approach that

has been studied in depth is PLC (Professional Learning Communities, see Vescio et al., 2008). Using activity theory, where *contradictions* is a key term (operationalised as dilemmas, conflicts, or double binds), Harvey and Nilsson (2022) identified 26 contradictions in the PLC activity system, such as "Teachers [...] question the usefulness of PLC" (p. 13). Importantly, they maintain that facilitators' role in PLCs as moderators of discussions is more complex than their role in traditional PD approaches. To focus on participants' voices, facilitators are implicitly required to step back during discussions, even though they themselves are experienced educators with valuable insights to share. It follows that a major challenge for facilitators is to navigate their multiple identities (Chval et al., 2010) and determine how to draw successfully on their teaching experience when facilitating (Schwarts et al., 2023). A related challenge is facilitators' positioning themselves as "liaisons between the ideal and the actual" (Rogers et al., 2007, p. 528). This often leads to tensions and double binds (Harvey & Nilsson, 2022; Lefstein et al., 2020) between research-based ideas and teachers' lived experience. These tensions are intensified by the fact that many facilitators were educated (as students and teachers) within traditional educational environments, and hence their experiences as learners may not support well their participation in innovative, learner-centered programs (Simon, 2008).

Based on this literature, we argue that two prior experiences shape—and can even constrain—productive engagement in professional development: teachers' own histories as learners in teacher-centered classrooms and PD programs, and the norms that undergird their work as classroom teachers. Similar sets of experiences may impinge on the work of facilitators, when these are former or practicing classroom teachers (Schwarts et al., 2023).

So far, few studies have investigated the interplay between facilitators and teachers' experiences when participating in the same programs. Thus, scrutinising teachers' and facilitators' expectations can help unpack the tensions that arise when these expectations are not met, which in turn can inform the design of collaborative, process-oriented environments. For these reasons, we ask:

> *What tensions did teachers and facilitators experience when participating in the collaborative environment of Story*Circles*?*
>
> *How are these tensions related to teachers' and facilitators' expectations, based on their prior PD and classroom experiences, and to their perceptions of learning?*

Theoretical Framing: Situating Tensions in the PD Triangle

This paper frames tensions of teacher collaborative learning as manifested in the interactions between a facilitator and teachers about the practice of mathematics teaching and shaped by their expectations about PD. For these purposes, we adapted the PD triangle offered by Carroll and Mumme (2007, see Figure 2). This triangle lifts and integrates the well-known instructional triangle (Cohen et al., 2003) from the classroom level into the PD level, as other frameworks that describe PD and facilitators have done (e.g., Karsenty et al., 2023; Luft & Hewson, 2014; Prediger et al. 2022; Zaslavsky & Leikin, 2004). The resulting PD triangle includes the following three vertices: the facilitator, the PD participants (teachers), and the practice of teaching mathematics (represented by the instructional triangle) in the place of content (see Figure 2). As in the case of the instructional triangle, we can hypothesise this PD triangle to be situated in environments, particularly the institutional environments that enable practitioners to participate. Of particular importance are the arrows under the "Participants" vertex, which illustrate the centrality of their interactions in collaborative contexts. This framing situates facilitators and teachers' expectations with respect to the other components of the environment. The expectations we are interested in are related to how and what participants think they should learn (participants–practice arrows), and what the facilitator's role is in relation to that learning (how the facilitator relates to practice and to participants). We hypothesise that facilitators' years-long experiences as practicing mathematics teachers and the PDs in which they had participated, inform these expectations and influence facilitators' practices in ways that are not always discernible for them and for the designers of innovative teacher learning environments. Similarly, teachers who are familiar with traditional PD approaches may also

experience discomfort when participating in innovative contexts. Using the PD triangle aims to foreground attention to the idea that the interaction of facilitator and participants is informed by what is happening in the classroom, not just as the focus of interaction (being the content vertex) but also due to the self-similarity between the two environments and the ways in which norms shaping the environments of mathematics instruction might permeate teacher learning environments.

Figure 2. The PD triangle (adapted from Carroll & Mumme, 2007, p. 11).

Our focus on expectations builds on literature on social norms (e.g., Bicchieri, 2005) and on research on instructional norms in mathematics classrooms (Herbst & Chazan, 2011b), while addressing the conspicuous gap in examining norms within teacher learning contexts. We aim at identifying unmet expectations of participants and their facilitator in the context of teacher learning, in order to reveal implicit norms of this context. This work draws on studies in sociology that used breaching experiments to identify implicit norms (Garfinkel, 1967) and builds on the idea that such norms only become apparent when violated. In other words, when people face unanticipated circumstances, they may reveal their implicit assumptions and expectations, which can be used to uncover the shared norms at play in that situation. Although the current study does not involve breaching experiments in the traditional sense, we view the Story*Circles* approach as a PD context in which some norms of professional development may be breached.

Context

This section elaborates on the Story*Circles* approach and its implementation in the two cycles analysed in this paper, each of which lasted six weeks during the 2021–22 school year.

Every Story*Circle* begins with a provided, starter version of a problem-based lesson, in the form of a storyboard consisting of frames (see Figures 3, 4), that teachers are requested to work together to improve during the cycle. Importantly, the students' work on the problem at the core of the lesson is geared toward a specific curricular goal and structured to mirror the authentic constraints of daily teaching

The starter storyboard of the lesson provides some fixed anchors: 1) the problem that opens the lesson (Figures 3 and 4, left-hand side); 2) the curricular goal that the work on the problem is intended to achieve (Figures 3 and 4, right-hand side); and 3) a representation of the arc of the lesson (e.g. the

phases of the lesson such as launching the problem, giving students time to work on it, managing whole class discussion, summarising).

The modification of the starter version of the lesson (including revising, adding, or discarding frames) is done using storyboarding software (Lesson*Depict*). In addition, planning and revisions of these modifications are done using a media annotation software (Anotemos) where participants can view the lesson frames and annotate them according to guided questions (such as: "What key mathematical ideas do you envision would have to occur to understand this problem?"). The annotation software allows participants to share their thoughts and engage with their peers' thinking, without time constraints that characterise face-to-face interactions.

The Story*Circles* approach integrates asynchronous activities (such as scripting and annotating frames) with synchronous online meetings (facilitated via videoconference). In these meetings, participants discuss how to refine the lesson, trying to reach a consensus. Each week of the cycle includes one synchronous meeting that builds on the ideas participants shared in the asynchronous activities. This hybrid design accommodates teachers' busy schedules and, despite its brevity, has proven useful at eliciting professional knowledge in prior iterations (Milewski et al., 2020). It also builds on efforts to simulate practice in online PD by presenting realistic contingencies that support pedagogical decision-making in virtual contexts (Brown et al., 2021).

Guided by a process-oriented approach, facilitators of Story*Circles* are expected not to direct participants to make certain decisions, but to help them identify and unpack the knowledge they use while making any decisions they wish to make. The *specified* nature of the program means that each synchronous meeting is structured to focus on particular phases of the lesson (e.g., launching).

This paper studies two consecutive cycles of Story*Circles*, both focused on geometry lessons and led by the same facilitator (details below). Hereafter, we refer to these cycles as Cycles 21 and 22. The first cycle we analysed for this article, Cycle 21, used a problem where students are requested to find a circle tangent to two intersecting lines at two given points, in order to arrive at the tangent segment theorem (see Figure 3). The second cycle, Cycle 22, introduced a problem involving the location of a prize that is equidistant from three swimmers in a rectangular pool. This problem is used to arrive at the midpoint of the hypotenuse theorem (see Figure 4).

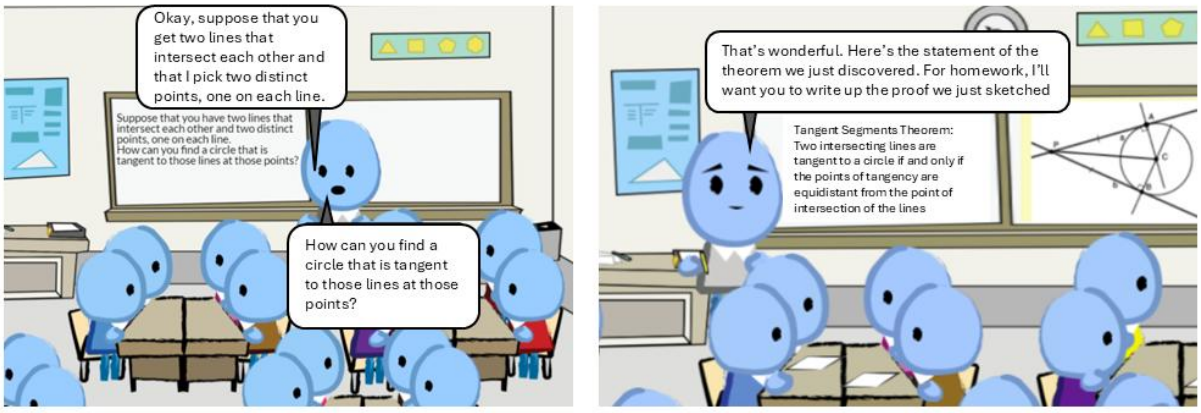

Figure 3. Scenes at the beginning and the end of the Tangent Circle lesson (cycle 21). Graphics are © 2021, The Regents of the University of Michigan, used with permission.

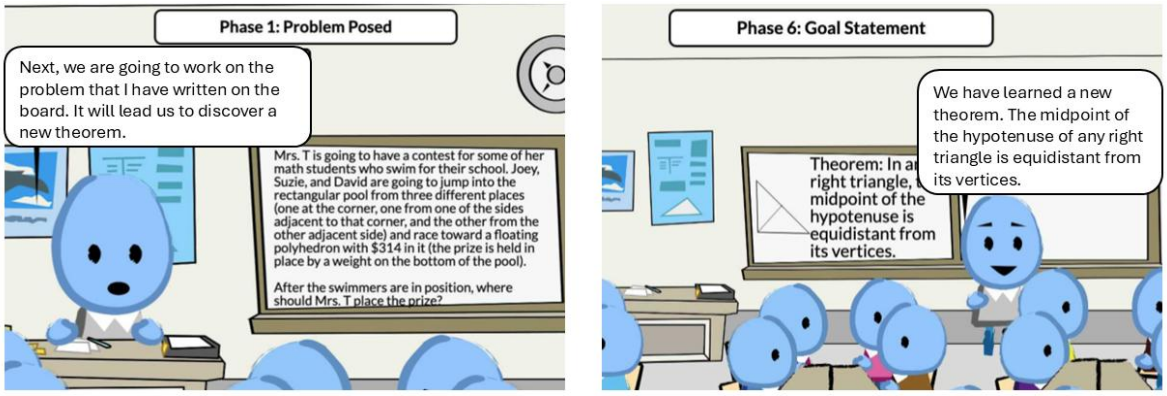

Figure 4. Scenes at the beginning and the end of the Pool Problem lesson (Cycle 22). Graphics are © 2021, The Regents of the University of Michigan, used with permission.

A lesson-based approach that prioritises process over content carries inherent friction between its two intertwined goals of (a) representing a lesson; and (b) fostering professional discourse. To distinguish between the two, we call the *avowed goal* of each cycle that of co-designing a problem-based lesson, whereas the *expected outcome* is to cultivate an environment in which arguments about the practice of teaching mathematics can develop as evidence of an emergent professional discourse. As mentioned, participants are made aware of the two goals; but most of their work and attention is focused on the avowed goal.

Although the particulars of the entire Story*Circles* design have been described elsewhere (e.g., Herbst & Milewski, 2018), one element is important to note. During synchronous meetings, participants script lesson episodes, but the facilitator does not use the storyboarding software to document their proposals. Instead, two assistants (skilled in the software but not in teaching mathematics), hereafter referred to as *storyboarders*, are present in every meeting to represent participants' suggestions in real time. The resulting frames serve as collective artifacts for participants to discuss and argue about.

Methods

Participants

The corpus of data for this analysis includes documentation of two Story*Circles*, including videos of all synchronous meetings, log files from all asynchronous activities, planning and summary documents, and various forms of feedback collected from participants (see Table 1). Both cycles were facilitated by Quincy (pseudonym), an experienced mathematics teacher who participated (as a teacher) in a previous iteration of Story*Circles.* This was her first time facilitating the work. Quincy was recruited as a facilitator because of her robust understanding of the mathematical content and her enthusiasm for the program. A diverse group (in terms of gender, ethnicity, and experience) of 17 secondary mathematics teachers participated in the cycles (13 in Cycle 21, and seven in Cycle 22, three participated in both). Participants were recruited via email invitations distributed through relevant professional networks. Ethical approval for the research was obtained from the University of Michigan's Institutional Review Board, and informed consent was secured from all participants prior to data collection. Participants received professional development credit for participation.

Data Collection

To understand participants' experiences and expectations, we draw on the following sources of data: (1) video recordings of online synchronous meetings (11 sessions, 1 hour each); (2) recordings of video debriefs with the facilitator that took place immediately after the meetings, with 2–3 project team

members, lasting about 20 minutes (11 in total); (3) participants' reflections that were collected after each cycle had ended, using one or more of the following means, depending on participants' preferences: i) recordings of individual semi-structured video interviews (n = 6); ii) written questionnaires (n = 6); and iii) questionnaires in which teachers were given written questions and were asked to record their responses via video (n = 4). All sources used the same open-ended questions (for example: "Could you identify things that you learned during this Story*Circles* cycle that might influence your practice?"; "Compared to your previous PD activities, what stood out to you?"). One teacher engaged in two reflection formats, while five opted out of all post-cycle reflections (see Table 1).

Table 1
Participating Teachers and Data Collected

Participant	Interview	Written questionnaire	Recorded questionnaire	Participation in synchronous meetings
		Cycle 21		
Clader			✓	✓
Hamza		✓		✓
Ira			✓	✓
Lea		✓		✓
Lisa				✓
Llara			✓	✓
Quill				✓
Quintin	✓		✓	✓
Rachel		✓		✓
River	✓			✓
Zara		✓		✓
Zena				✓
Zoey				✓
		Cycle 22		
Clader		✓		✓
Ira	✓			✓
Labronna	✓			✓
Lazar				✓
Rachel	✓			✓
Ran	✓			✓
Roberta		✓		✓

Data Analysis

All recordings were transcribed. The main sources for finding evidence were the facilitator's and participants' reflections (Sources (2) and (3)). Meeting recordings (Source (1)) were used as secondary sources in which we could find illustrations of tensions identified in the reflections. Guided by the research questions stated above, we conducted an iterative content analysis (Braun & Clarke, 2006), focusing on expectations, surprises, and tensions. The focus on these aspects builds on the idea that individuals' tacit expectations can be revealed through their reactions to deviations from habitual practices (Garfinkel, 1967; Herbst & Chazan, 2011b). Initially, we analysed the facilitator's data. We coded the debrief sessions and the moments in the meetings to which she referred in the debriefs, according to our focal questions. Then we grouped all the coded segments into larger themes, which were named "tensions." In the following stage we focused on participants' reflections, and coded segments that alluded to expectations, surprises, and tensions. At this stage we already identified many similarities between the facilitator's and the participants' stances, which led us to try to group the participants'

coded segments into the themes identified. This stage led us to modify the initial themes, which resulted in two themes that reflect both the facilitator's and the participants' tensions.

After identifying the major tensions, we scrutinised their origins. In each coded segment we sought the source of the unfulfilled expectation/surprise/tension (which was not always evident). In particular, we examined how the practitioners related to previous PD or classroom experiences when expressing some sort of confusion or tension. When searching for the sources of expectations we sometimes inferred implicit references to contexts (e.g., when the facilitator said, "It's a different mindset here, having help," we inferred that her expectation to work individually, rather than having storyboarders help her, is related to her classroom teaching experience in which she does not have assistants). We then mapped the connections between tensions, expectations, and the Story*Circles* design and its learning goals.

Note that the data collection tools we used with participants were designed to elicit their reflections (yet only by the end of the cycle), while the debrief sessions with the facilitator primarily served to support her professionally. Consequently, the analysis of the two sets of data differs slightly. Through an examination of the facilitator's debrief data, we were able to zoom in on specific instances of tension that emerged during synchronous meetings. In contrast, the participants' reflection data provided us with a more comprehensive view of the overall process. Despite differences in the parties' perspectives and the granularity of the data, we identified shared themes that enrich the description of the phenomenon.

Findings

Below, we describe two tensions that emerged from our analysis in relation to: 1) Valuing collaboration, and 2) Relating to the program's process-oriented goal. Pseudonyms are used for all names mentioned, and notations such as "21M2" refer to "Cycle 21, Meeting 2" for brevity. The notation T242 represents turn numbers.

Tension I: Valuing Collaboration

Isolation and solitude are two terms that are often used to characterise teachers' work. We found that the attempts to break these patterns and bring participants to work together do not bear fruit so easily. Furthermore, we found that the challenge to collaborate was evident not only at the participants' level, but also for the facilitator while she was interacting with storyboarders during synchronous meetings. Below, we parse this tension from the two perspectives.

The facilitator's perspective

To illustrate the facilitator's difficulty in adjusting herself to work in collaboration, we provide some more details on the work of storyboarders during synchronous meetings. These meetings usually opened with a question that the facilitator posed about a scene in the storyboarded lesson (e.g., "Are there things that the teacher might say in their introduction of that student work?" [21M2, T242] When participants raised alternatives, the storyboarders depicted their suggestions in storyboard frames (see example in Figure 5). These depictions of participants' suggestions are essential to Story*Circles*' argumentative process: They create shared artifacts that participants can contest (e.g., "that's not quite what I meant," revealing points of divergence), and give facilitators something to build on—asking, "Which option fits best?" or "Is this what you had in mind?"—thereby surfacing nuances that drive deeper discussion. It follows that the resulting depictions serve as indispensable facilitation resources. Yet Quincy reflected that, in practice, the abundance of storyboard frames in the first meeting proved overwhelming:

> *So, I guess the part that I felt most awkward about was like transitioning between that, and then like jumping down to the depictions* [...]. *Partly I was worried like I ... did I give them [the storyboarders] enough time to depict before jumping down? So, I was stalling on that a little bit.* [21M1 debrief, T546–T547]

This quotation revealed the facilitator's difficulties in managing a discussion while simultaneously handling the new frames created by the storyboarders. Furthermore, she mentioned that moving between storyboard frames and referring to ad-hoc ideas was "awkward." She expressed similar feelings two weeks later, when she debriefed after the third meeting:

> *Not sure how to—like really utilise what you guys [storyboarders] are doing on the depicting, like I can see you're going crazy, I don't know if I need to just dive into the depiction earlier.* [21M3 debrief, T601]

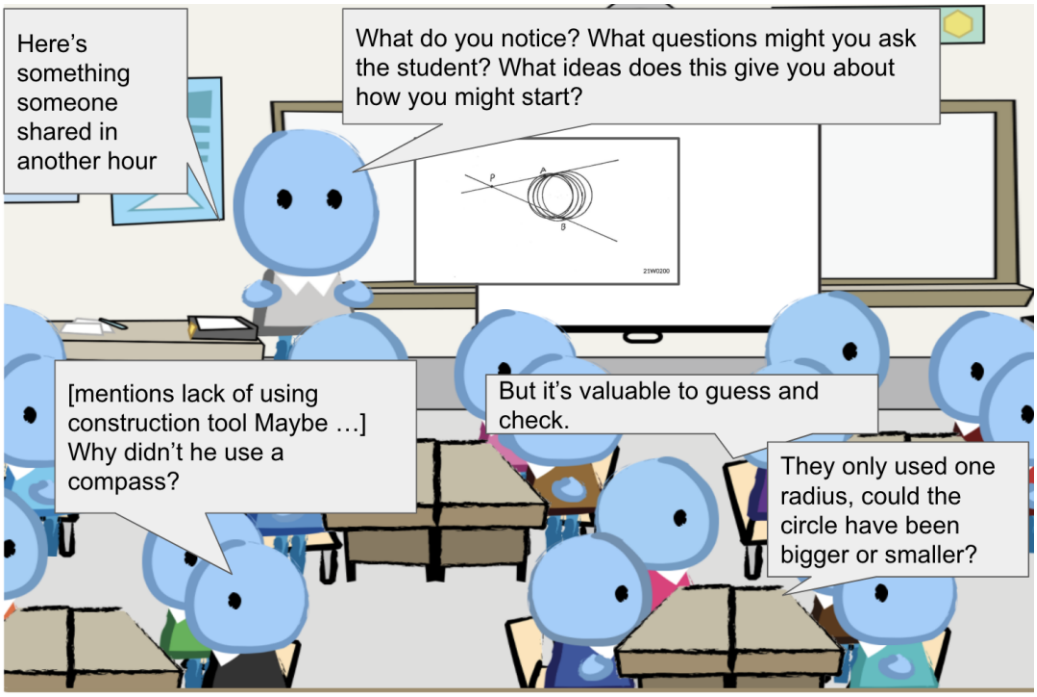

Figure 5. A depiction created by storyboarders in 21M3, representing participants' suggestions. Graphics are © 2021, The Regents of the University of Michigan, used with permission.

Beyond knowing how to manage the conversation related to the depictions, another source of tension for Quincy was how to use the depictions in these conversations: In this meeting, the storyboarders represented almost every comment made by participants. Doing so resulted in many new depictions, illustrating different possible pathways in the lesson, that Quincy felt obliged to "utilise". All of the above evidence suggests that Quincy had an expectation of having control on timing while being responsive to all of the participants' contributions. This expectation is informed by her habits as a teacher, as was revealed when she compared facilitation to teaching after the third meeting:

> *When I'm teaching off of the Google slides right now [during the COVID-19 pandemic], like every day, I'm in control of them or I'm having the students interact with them, so it's very* [...] *It's a different mindset here, having help.* [21M3 debrief, T681]

These moments suggest that Quincy's expectation of having full control was informed by her experiences as a mathematics teacher, used to working individually. Namely, "having help" is something she was unaccustomed to, and perhaps, at that moment, she felt that the work of the storyboarders did not help her at all. This challenge points to a disruption in the traditional interaction between instructors and learners that Quincy sources to the typical teacher–student interactions (see Figure 7). In the classroom, the teacher is charged with being responsible for the interpretations of the students' contributions and their mediation with the content. In Story*Circles*, however, the participants' ideas are also interpreted and picked up by the storyboarders. Situating this tension in the PD triangle, the interaction between facilitator and participants is disrupted, and looks different than teacher-student interaction. The expectation that her facilitation role would be similar to that of a schoolteacher seemed

to rouse in Quincy the expectation that her job included mediating and being responsible and responsive for all interactions, namely, to ensure that: (1) participants' ideas are explicit enough for the storyboarders to depict, (2) the storyboarders have accurately captured participants' ideas. Moreover, as described above, she appeared to take responsibility for pacing the discussion according to the amount of time it took to depict participants' suggestions. It follows that even for a facilitator who is well-familiar with managing classroom discussions, facilitating a teacher discussion about emerging representations of practice could be a challenging task as it might involve a decreased sense of control and a need to collaborate with peers in real-time.

The participants' perspective

Though the facilitator quotes point to specific events because they were collected in debrief sessions, the participants' reflections relate to the entire cycle as they were collected at the end of each cycle. Nonetheless, participants' reflections resonate with the tension described above. The analysis showed that overall, teachers were enthusiastic about conversing with peers:

> *My favorite thing about StoryCircles compared to a lot of the other PD is how we work together as a team.* [...] *I really liked getting feedback from professionals who have been in the classroom, who've done the job.* [Ira, 21, recorded questionnaire]

Although other participants shared similar sentiments, most of them also reflected on the challenges of collaborative decision-making, which is required for designing a lesson together:

> [...] *The challenge is just taking such a diverse group of teachers with a lot of different opinions and trying to figure out one way of doing things. I think it is complicated and I don't necessarily know if, like, the goal is to figure out what would work best for every single teacher and every single context? Having the disagreement is good, because I think, in reality, if you were to teach that lesson, everyone's [lessons] would look a little bit different in different spots. I think sometimes it made it hard, though, because everyone kind of had their own train of thought* [Labronna, 22, interview, T54–T58]

Labronna suggested that there was something artificial in the collaborative decision-making that the group was asked to do, since it did not quite approximate their individual practice. Although it seems that she understood the rationale behind "having the disagreement," her words suggest that she was not sure if participants could depart from their own contexts for the sake of envisioning the lesson together. Another participant expressed a different aspect of difficulty in collaborative asynchronous work on the lesson, which made her want to be the first to respond in asynchronous collaborative activities:

> *I have a certain vision* [...] *I would have led the lesson that way. But then [if someone else started working on the lesson], it was already going that way, and then, how do I get it back to the way that I was thinking? I would have to kind of shift and think "Okay, where is this person going? What was their thinking?" versus if I'm at the beginning and I'm starting to lead.* [Rachel, 22, interview, T140-T14]

Rachel's response to others' ideas indicated that she found it difficult to engage with the decisions in a lesson that she was not developing alone. Her description suggests that the activity of subjecting individual decision-making processes to public professional scrutiny was a core challenge, which contradicted her "certain vision." Interestingly, even though the participants were not aiming at the implementation of the planned lesson—thus were not required to execute decisions they were uncomfortable with—they were still irritated if the lesson evolved in ways they did not endorse. Rachel shared another orientation that seem to hamper her appreciation of the collaboration:

> *I do not feel like I learned anything. I am used to PD being more reading scholarly articles and helping me think about my practice and how I can improve upon it. Possible warrant: I guess I must be a research-based instructor. I value others' opinions, but struggle when it is just an opinion rather than a research-based evidence of best-practice.* [Rachel, 21, written questionnaire]

This comment shows how prior PD experiences, and their related expectations, deeply informed Rachel's Story*Circles* experiences and impeded her engagement since she was expecting to receive

recommendations from some kind of authority. Positioning herself as a "research-based instructor," she was not receptive to the model of collaborative learning and therefore did not consider Story*Circles* as a learning experience. A similar phenomenon was identified in another reflection:

> *Usually with PD it's like "here's a folder" and we watch somebody model it for us, and then you go back to your classroom with all these good ideas and I love this.* [Ira, 22, interview, T179]

These comments show that some teachers would rather learn from an authority than from each other's experiences. Ira loved that "somebody models it for us" and appreciated the ready-made materials, a perception which contrasts with the idea of generative peer-learning. Interpreting these stances with the PD triangle presents a contrast between teachers' daily isolated work, and the collaborative nature of Story*Circles*, suggesting that this collaboration does not have an equivalent source in the instructional triangle (see Figure 6).

In summary, this section demonstrates how for some participants, individual decision-making was prioritised over collaboration. The facilitator's desire for coherence and responsiveness hindered her ability to work alongside peers, while some participants' inclination to make decisions alone and their expectation to receive knowledge from some kind of authority impeded their willingness to develop a shared lesson vision and learn from peers. Although participants expressed enjoyment in interacting with colleagues, some did not consider these interactions as learning experiences, as is also elaborated in the next section.

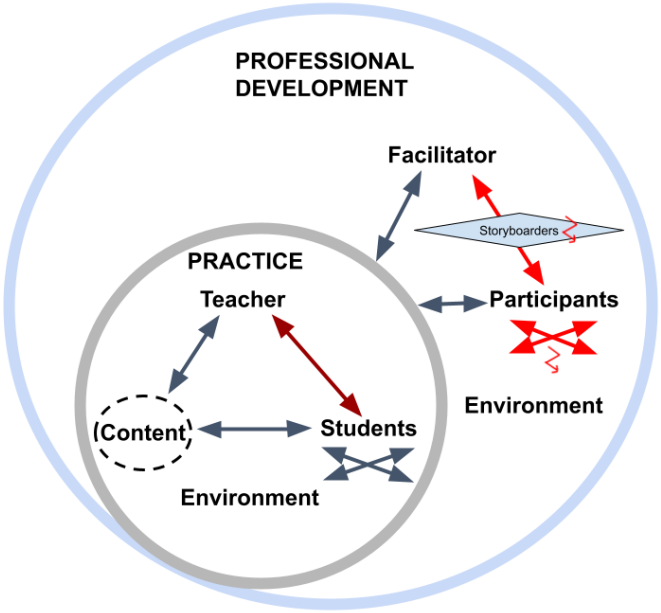

Figure 6. The tension of valuing collaboration mapped into the PD triangle, and its source in the instructional triangle.

Tension II: Relating to the Program's Process-oriented Goal

Below, we explore each cycle separately to examine the subtle distinctions between them in terms of how participants related to Story*Circles*' avowed goals and expected outcomes.

The facilitator's perspective

The analysis of Cycle 21 identified that in terms of goals, Quincy's facilitation moves were sometimes at odds with the Story*Circles* design. According to the latter, the focal lesson is meant to serve as a resource that enables participants to communicate on their practice, under the premise that the lesson itself can evolve in multiple ways, each having its own merits, and participants can ponder on their decisions

without being encumbered by the expectation to enact best-practices. Quincy, however, sometimes facilitated discussions in ways that tried to direct participants to make decisions about the lesson that she thought were worthwhile. For example, she envisioned that the Tangent Circle lesson must end with proof of the tangent segments theorem. When other participants raised doubts about ending the lesson with proof, on the 4th meeting of Cycle 21, she was puzzled. She shared later in the meeting debrief:

> Project leader: So you started saying that you were worried at the beginning, what do you mean? [T553]
> Quincy: Well, when they were like, "We don't do proofs".[...] Oh well, that's the premise of this lesson! If we're not gonna play that game, I don't know where we're going. But they came around, I think it was nice that Quintin [a participant] could help bring everybody in. So that was the part that I got a little scared (laughs). [21M4 debrief, T554-T558]

Here, Quincy was disrupted by the participants' divergence from the path she had envisioned (i.e., proving). She was worried that they wanted to modify aspects of the lesson which were introduced for them as anchors. This highlights a tension present in Story*Circles* and related to the role of the lesson as a motif for the work (representing the lesson is the *avowed goal*) but not the *expected outcome* of the activity (which is to enhance professional discourse by eliciting teachers' tacit knowledge). The facilitator's discomfort with participants' resistance to formal proof reveals her expectation—rooted in the typical mathematics lesson didactical contract (Brousseau, 1997)—that the avowed goal and the expected outcomes would align. Using the PD triangle to interpret this moment, we suggest the facilitator assumed that the avowed goal—representing the lesson—and the expected process-oriented outcome were aligned. This assumption was challenged when participants pushed to revise the lesson's ending in ways that disrupted that alignment.

The analysis of Cycle 22 showed an emerging change in how Quincy picked up on participants' suggestions. On 22M3, participants discussed how the virtual teacher should redirect the whole-class discussion after the students had worked on the Pool Problem for some time (see the problem in Figure 4). The participants coalesced around one option out of five that were offered on the slideshow (See Figure 7). The facilitator presumably thought that they reached a consensus too soon, and urged them to re-think their choice. One participant questioned this facilitation move:

> Ran: So if I didn't know that the objective [of the lesson] is the [theorem about the] hypotenuse [...] if the question was "what can we teach with this problem?", then fine, we can go in many different directions [...], but being [that] the objective to talk about the midpoint of the hypotenuse [...] Why would you be interested in getting us to change our mind? [T38-T48]
> Quincy: Oh, I'm not necessarily encouraging you to change your mind. I want you to be able to make the connection between the part that came before and then this one… Does it make sense, based on what came before it, to have this be the redirect, given that the goal is to have the students discover that theorem about the midpoint of the hypotenuse? [...] you don't have to change your mind here. But I want to push at you a little bit, to see why you're thinking that this is the right move [...] I'm interested in knowing if what came before the redirect informs the way that you want to proceed [...] I want to push on that a little bit, hear about why. [T49-T57]

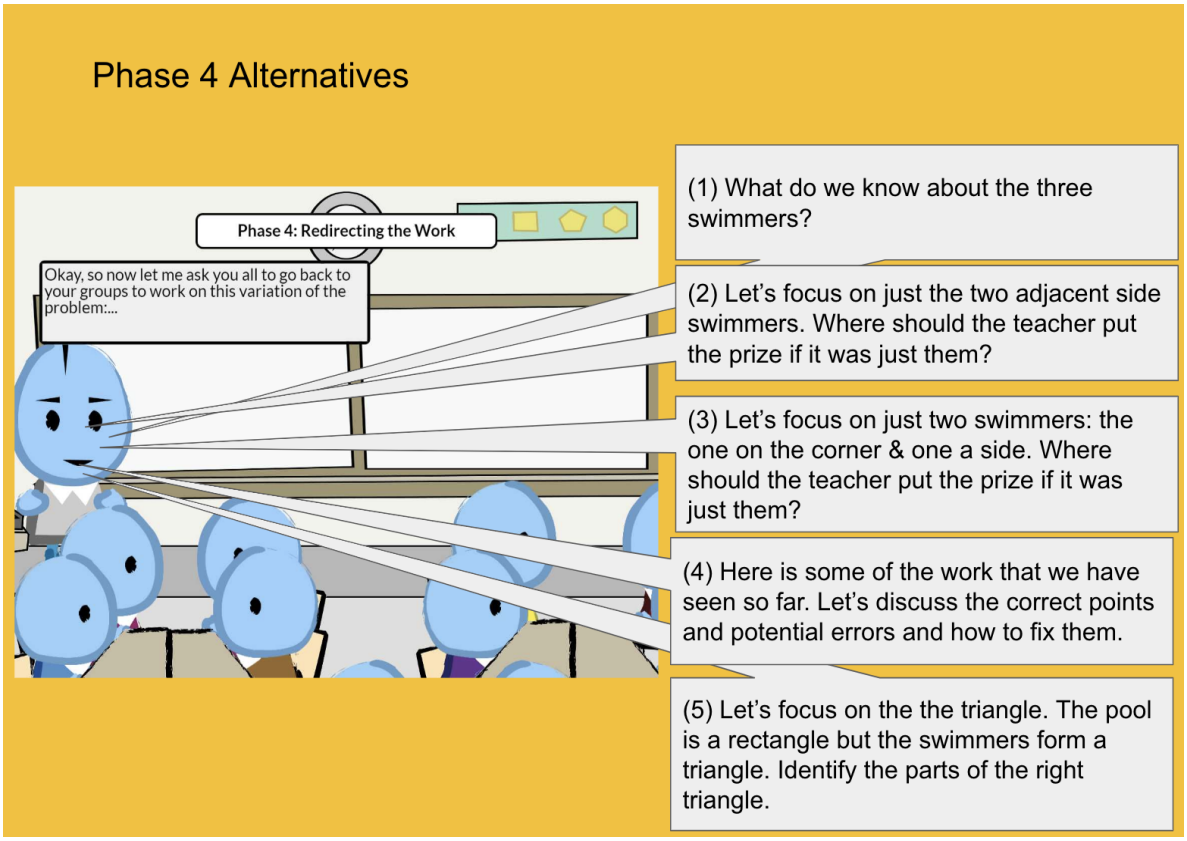

Figure 7. An image with five options for redirecting teacher moves, discussed in 22M3. Graphics are © 2021, The Regents of the University of Michigan, used with permission.

Here, the facilitator described her goal to encourage teachers to re-think their choices, justify them, and ensure they were coherent with other phases of the lesson. This exchange demonstrates the facilitator's emerging adoption of the process-oriented goal of Story*Circles*. In particular, we imply that Quincy aimed at conveying two ideas: 1) Even if a problem and an instructional goal are given as anchors, there is more than one way to enact the lesson; and 2) such decisions can and should be publicly justified. When answering Ran, Quincy modeled how the latter idea is true also for the PD triangle context, justifying her facilitation decision. Moreover, she made the process-oriented goal explicit.

The participants' perspective

Reflections from Cycle 21 showed that many participants were keen to gain a clearer sense of the rationale behind Story*Circles*. Zara expressed her confusion in writing: "It would be nice to know the larger purpose of the project and how things are structured." Similarly, some participants were unable to identify the specific knowledge they had gained from their experiences, as highlighted by Lea who wrote, "I often didn't understand the objective(s) of our discussions or tasks to know if we were making progress or not." The phrase "making progress" suggests that participants were anticipating a prespecified learning path that they were expected to follow. Interestingly, another participant used the word "progress" positively in their reflection:

> *In all of the zoom live discussions we've had, I always walked away with that feeling like we made progress on something and that kind of made me rethink some things or just try to re-emphasise some parts of my teaching style or philosophy.* [Quintin, written questionnaire]

The phrase "progress on something" likely refers to advancing the work on the lesson. Although this progress presumably refers to the avowed goal, Quintin's reflection suggests he also progressed toward the expected outcome of Story*Circles*: developing a deeper understanding of his own teaching

philosophy. By stating "that kind of made me think", he connects between the collaborative lesson work and his reflective decision-making. His reflection illustrates the kind of learning stance that Story*Circles* aims to cultivate, illustrating how the avowed goal and the expected outcome can reinforce one another. However, other participants expressed confusion, as the Story*Circles* experience diverged sharply from the forms of PD they were accustomed to:

> *It wasn't what I was expecting. I guess I walked in thinking of it as PD. And I was thinking that and I would like, read an article and discuss how we can incorporate those objectives in our classroom or I was expecting it to be... I don't know. I wanted to learn something new, and maybe I did, but it wasn't presented in that way.* [River, interview, T6]

The tension in a process-oriented approach is vividly captured in River's remark "I wanted to learn something new, [...] and maybe I did, but it wasn't presented in that way". Her expectations about professional learning are shaped by her previous PD experiences where external authorities (facilitator, articles) recommended normative practices, while participants only discuss how they "can incorporate those objectives." Being accustomed to this approach, River did not identify her discussions with peers as part of a learning process. Her words "it wasn't presented that way" suggest she expected explicitness from the facilitator. Yet, as the interview progressed, River described growing awareness of her teaching decisions—changes that, in fact, align with the expected outcomes of the PD.

> *As a math teacher, you make so many instructional decisions in an hour that it's insane. [After participating in StoryCircles] I found myself looking things up, trying to figure out—why I would make that instructional decision? Why would that happen? Why is that important?* [River, interview, T31]

River's account of how her practice had evolved, despite her uncertainty about what she learned in StoryCircles, suggests that individual change resulting from a process-oriented model may not always be recognised as learning. This tension underscores the need to unpack how participants themselves define learning, beyond gaining explicit knowledge or skills.

We now turn to a brief account of reflections from Cycle 22. Although Quincy showed growing awareness of the process-oriented goal (as discussed above), some participants continued to struggle with the purpose of Story*Circles*. This suggests that such tensions may not be resolved through responsive facilitation alone. The following two examples illustrate this point:

> *Is like the goal to sort of see how teachers are coming together to come up with a lesson, or kind of the goal is to think about what they actually came up with in the end? Is it more that you're interested in the process or more interested in the outcome?* [Labronna, interview, T218-T219]

> *I liked the synchronous meetings. I think that interaction pushed my thinking the most. I still feel like I don't understand the intention and purpose of this experience.* (Clader, written questionnaire]

In summary, the analysis of both cycles revealed that most participants had positive experiences with the main design features of Story*Circles*, such as interacting with peers and engaging in processes of exploring practice. However, not all participants viewed these interactions and processes as goals in and of themselves. This tension can be better understood by being mapped to the teachers-practice arrow in the PD triangle: When participants questioned how their interactions with each other about practice were informing their learning, their implicit assumption, sourced in the instructional triangle, seemed to be that interactions about practice would eventually result in learning something concrete (See Figure 8). The self-similarity between the *practice* and *content* vertices in the small and big triangles suggests that how teachers relate to practice in PD is similar, to some extent, with how students relate to content in the classroom. However, as the analysis illustrates, the PD environment can manifest a diffraction between the avowed goal and the expected outcome. This diffraction has been found to disrupt participants' expectations.

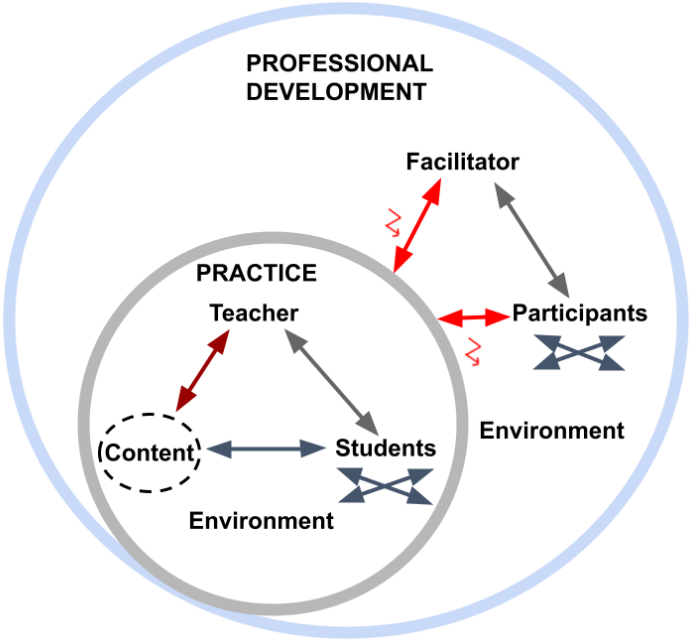

Figure 8. The tension of relating to the process-oriented goal mapped into the PD triangle, and its source in the instructional triangle.

Discussion

This study explored tensions involved in an innovative model for teacher learning under the lens of participants' and facilitators' expectations. The analysis identified two main tensions: 1) valuing collaboration, and 2) relating to the program's process-oriented goal. Moreover, the analysis tied these tensions to the participants' expectations based on their prior PD and classroom experiences. This analysis, framed through the lens of normative expectations, suggests that the very disruptiveness driving innovation can undermine responsiveness to participants. Below, we delve into this issue and then propose strategies for designing process-oriented and disruptive PD that stays attuned to practitioners' expectations. Before doing so, we briefly summarise our key findings and discuss how they support and extend current literature.

We demonstrated that the isolated and individualistic nature of mathematics teaching (in the United States), coupled with the emphasis on explicit and implementable goals, challenge participants in appreciating the value of their Story*Circles* experience, that is, valuing collaborative, process-oriented PD. In accordance with previous research on collaborative teacher learning (Eshchar-Netz et al., 2022; Harvey & Nilsson, 2022; Rogers et al., 2007), we have found evidence that a collaborative, process-oriented approach to teacher learning can be challenging and even feel unproductive for teachers, despite its responsive design. Ironically, the very fact that the program was designed to be responsive to participants' voices, without imposing instrumental objectives, ultimately made it challenging for participants to make sense of what they learned from it. As Story*Circles* continues to evolve into formats that support asynchronous, cross-cultural collaboration (Brown et al., 2024), questions of how to maintain responsiveness while working across differing normative expectations become even more pressing.

We suspect that even though Story*Circles* experience is similar in nature to lesson-based programs such as Lesson Study, the fact that it does not include an implementation component made it much harder for participants to make sense of its contribution. Although the component of classroom implementation in Lesson Study is a source of feedback for lesson design, rather than a test to see if teachers have adopted new practices, it might feel for some participants similar enough to a PD where

teachers are asked to implement specific practices, since some learning is demonstrated in practice. We posit, then, that the absence of any form of implementation in Story*Circles* breaches a PD norm.

Our analysis extends prior work (e.g., Brehmer & Ryve, 2024; Harvey & Nilsson, 2022) by highlighting persistent complexity in collaborative PD. Teachers may participate actively and articulate insights yet still question the overall value of the experience. We noticed participants who were active and engaged, later described several insights they had gained in Story*Circles,* yet their holistic view of the process included feelings of confusion, bewilderment, or skepticism when asked questions about their own learning. This suggests that teachers may have had difficulty reconciling their positive experiences in the program with their broader perceptions of its effectiveness and their definition of learning. While feelings such as puzzlement and surprise could fuel reflection (Schön, 1983), our analysis showed that for some participants, their perception of Story*Circles* as a non-normative environment for teacher learning was an obstacle.

Our use of the non-normative term relates to the implicit expectations the facilitator and participants held, which point to the existence of implicit norms that shape professional learning. The analysis suggests that teachers (including the facilitator) draw on the norms of the didactical contract as they strive to make sense of the activity of participating in PD. The expectations to work alone and have full control, and to act according to explicit implementable goals, seem to align to the expectations instruction imposes on classroom teachers and that have been described using the theory of the didactical contract (Brousseau, 1997). And so, just as there are regularities that constrain improvements or changes in teaching (Herbst, 2003), there are also implicit expectations that restrict the practices of facilitators and the learning opportunities perceived by participants, as they are in odds with the program's design. Therefore, we suggest that there also exists a didactical contract for professional development. This variant of the didactical contract includes teachers' expectation that the PD environment has specific predetermined knowledge at stake, just as they have in their teaching. It also includes an expectation from the facilitator to provide them with explicit guidelines. Our analysis delineates tensions sourced in the vagueness of this contract.

Future research could involve a more systematic approach for examining the didactical contract in professional development by using targeted questionnaires and scenario-based vignettes. Targeted questionnaires (such as those described by Nieman et al., 2023) could be designed to capture teachers' and facilitators' expectations and perceptions of the PD environment, focusing on specific aspects like anticipated knowledge and guideline clarity. Scenario-based vignettes (such as those used by Ko et al., 2024) could be employed to simulate various PD situations and gather feedback on how these expectations play out in practice. This approach would provide a structured way to analyse tensions and inform the refinement of process-oriented PD programs.

Limitations and Future Directions

One limitation of this study is the use of different data sources in the analysis of the facilitator and the participants, which made us consider their perspectives at different grain sizes. Notwithstanding this limitation, this multiple-perspective study was able to offer a richer picture of the interrelations between design, facilitation, and participants' experiences. Future research might use a multi-perspective approach to consider different viewpoints on the same PD events or artifacts.

This study underscores the importance of attending to the expectations that teachers and facilitators bring into professional learning environments—and the need to create structures that explicitly support shifts in those expectations. We are, however, only at the beginning of formulating what expectations and norms mean in the study of PD contexts for mathematics teachers, in a sense that will be theoretically coherent with previous works on classroom norms. The differences in learning goals, institutions, and audience between the classroom and PD environments should be considered during such endeavors, including considering the inherent epistemological and authority shifts between the environments (Schwarts, 2022).

Epilogue

This paper began with an excerpt from a teacher that prompted us to ask: How could environments be designed in ways that support teachers in valuing their collaborative learning? What kinds of outcomes, if any, can be expected from teachers' participation in these environments? Based on the analysis, we propose several ways to address these questions, acknowledging that they might not be fully answerable: 1) Narrowing the gap between avowed goal and expected outcome by creating shared artifacts (e.g., in the form of storyboards) that participants perceive as resources that can inform their practice, as was endorsed by Hiebert and Morris (2012) and others (e.g., Wake et al., 2016); 2) Being attuned to how participants experience the program at various points in time (see Nieman et al., 2023), ensuring that what was designed to be disruptive is also productive; and 3) Designing alternative PD models that are responsive for teachers' diverse professional needs and preferences (e.g., online self-paced PD, Schwarts et al., 2024).

In conclusion, we reiterate that our aim was not to dismiss the value of process-oriented environments but to highlight the tensions involved in order to inform and refine future design and research. In doing so, we underscored the importance of incorporating teachers' perspectives in the design, implementation, and evaluation of professional learning.

Corresponding author

Gil Schwarts
Weizmann Institution of Science
Hertzl 234, Rehovot, 7610000
Gil.Schwarts@weizmann.ac.Il

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Ethical approval

Ethical approval was granted by the University of Michigan Ethics Research Committee, and informed consent was given by all participants for their data to be published.

Competing interests

The authors declare there are no competing interests.

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