Cultivating Elementary Mathematics Specialist Expertise

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This study explored elementary teachers' (N = 26) development as novice Elementary Mathematics Specialists (EMSs). They worked in 21 urban-situated schools in the United States that served historically marginalised students. Professional learning experiences aimed to prepare them as effective and equitable mathematics teachers and teacher leaders equipped to serve as a *more knowledgeable other* in their school communities. Their role as EMSs was distinctive because their primary responsibility was teaching students, and their work as a teacher leader was in an informal capacity. A case study design was used, with quantitative and qualitative data forming the descriptive findings. Across two years of professional learning, participants had meaningful improvements in their implementation of learner-centred, equitable mathematics instruction; mathematical knowledge for teaching; and productive mathematics beliefs. Results from interview data illuminated these shifts, with themes related to: *centring of children, elevating equity and access, deepening knowledge for teaching*, and *navigating constraints in shifting instruction.*

Keywords: mathematics teacher education research • elementary mathematics specialists • mathematics teachers • professional development

In most countries, elementary teachers are prepared as generalists who teach all subjects, with a few exceptions (e.g., China, cf. Norton & Zhang, 2018). This all-purpose preparation has led to a multitude of elementary teachers needing improved content knowledge and confidence for effective teaching of mathematics (Australian Institute for Teaching and School Leadership [AITSL], 2017; Wu, 2009). Accordingly, there has been much interest in elementary educators who specialise in mathematics, with initiatives occurring in countries such as Australia, the United Kingdom, and the United States (Mills et al., 2020). Research into the use of subject area specialists evidence variability and complexity, including who should serve in the role, their ways of working, and their purposes in schools, which hold implications for their preparation. For example, national policy in Australia (AITSL, 2015; 2017) has primarily driven efforts of initial teacher education programs directed at the development of teacher candidates. This has caused scholars to question the positioning of new teachers as both novice and expert and their readiness as a specialist upon graduation (McMaster et al., 2018; 2024). Although there is general agreement in the United States that those prepared to serve as specialists should be experienced teachers, programs that develop these professionals evidence considerable differences, with each program operating according to local theories about the best way to produce desired learning outcomes (Rigelman, 2017). This variability is similarly evident across initial teacher education programs in Australia (Main et al., 2023). Whatever the range of roles and responsibilities these professionals may assume, their preparation should equip them to serve the needs of local educational agencies effectively and responsively.

Elementary Mathematics Specialists (EMSs) are mathematics teachers and teacher leaders responsible for supporting ambitious and equitable mathematics instruction at the classroom, school, district, or state levels (Association of Mathematics Teacher Educators [AMTE] et al., 2022). In the United States, there is renewed interest in EMSs, including an updated call by several mathematics education organisations that reiterated every elementary school should have access to an EMS (AMTE et al., 2022), and newly revised and extended guidelines for EMS preparation and on-going support (AMTE, 2024). Unfortunately, research on the role, preparation, and support of EMSs lags the advocacy efforts for EMSs (Baker et al., 2023). Needed are studies into the preparation for the various specialised roles EMSs may assume. Given the substantive variability of EMS preparation programs across the United States, understanding differing program models and their effectiveness in supporting these professionals' development is essential in building a strong knowledge base on their preparation (Reys et al., 2017). There is a particular need for inquiry on the development of EMSs who are boundary crossers leading from within and beyond their classrooms (Rigelman & Lewis, 2023). Accordingly, this study focused on the first two years of a 5-year professional development program aimed at preparing EMSs (N = 26) whose chief responsibility was teaching students, with their work as teacher leaders in an informal capacity. Adding to an earlier inquiry that focused largely on participants' growth as teacher leaders (Swars Auslander et al., 2023, 2024), this study explored the development of EMSs in the role of mathematics teacher.

Related Perspectives and Literature

EMSs should serve as a *more knowledgeable other* with highly developed instructional practice and mathematical knowledge within communities of practice in their schools (Campbell & Malkus, 2014). They are considered colleagues with expertise to support students' learning of mathematics and to serve as an on-site resource and leader for teachers. Although only some EMSs work directly with students, all should serve as teacher leaders, whether in informal capacities (e.g., grade level chair, school mathematics committee chair) or as part of their workload in formal capacities (e.g., instructional coach) (Rigelman et al., 2024). The variability of EMSs' roles, responsibilities, and titles is evident in the literature, which is warranted so these professionals may adapt practices to serve the needs of their local educational contexts.

In EMSs' growth from novices to qualified experts, they should be readied for a variety of child- and adult-facing responsibilities, thus their preparation is multi-faceted and complex. Recommendations of professional organisations and policy broadly contend that preparation of elementary educators who specialise in mathematics should focus on their development of expert content knowledge, pedagogical content knowledge, and highly effective classroom teaching (AITSL, 2017; AMTE, 2024; AMTE et al., 2022). The extent to which preparation should cultivate leadership varies, but in the United States there has been much emphasis on developing EMSs' leadership knowledge and skills (AMTE, 2024). Further, it is asserted that preparation programs should foster EMSs' abilities for navigating productively vested interests of others (e.g., school leaders, teachers, parents/caregivers) and restrictive and conflicting school conditions when implementing instructional change. EMS learning experiences should deeply connect with practices in classrooms and schools (Reys et al., 2017), including supervised opportunities that support both teachers and students (AMTE, 2024). There have been similar calls in Australia for supportive opportunities to be provided for practicing mathematics teaching in school settings as part of initial teacher education, including during preparation programs and subsequent induction experiences (Main et al., 2023; McMaster et al., 2024).

A critical outcome of EMS preparation programs is the development of capabilities for ambitious and equitable mathematics instruction (AMTE, 2024; AMTE et al., 2022). As a *more knowledgeable other* in their school communities, EMSs must be able to both implement this pedagogy and support others in doing so. This learner-centred, equity-based mathematics instruction (NCTM, 2024, 2020) emphasises the use of cognitively demanding problems and dialogic discourse, and conceptual understandings as the foundation for procedural fluency. Lessons should intentionally provide opportunities for all students to learn rigorous mathematics, with teachers leveraging children's mathematical, cultural, and linguistic strengths and cultivating their mathematics identity (Aguirre et al., 2013/2024; NCTM, 2020). Programs should dually foster this pedagogy and promote effective ways of supporting others in its enactment, which could occur one-on-one (e.g., coaching, mentoring, co-teaching, modelling) or in groups, such as grade- or school-level teams, through facilitation of ongoing and collegial professional learning experiences (Gibbons & Cobb, 2017; McGatha et al., 2015). Challenges abound as this pedagogy diverges from the norm in many elementary schools in the United States. Top-down requirements of prescribed curricula and instructional delivery, inflexible pacing guidelines for concepts, and instruction that is teacher-centred and focused on fragments of information and skills on standardised assessments, constrain teacher implementation (Bartell et al., 2019; Rich, 2021).

Other salient outcomes of EMS preparation programs are content knowledge for teaching elementary mathematics and productive mathematics beliefs (AMTE, 2024; AMTE et al., 2022). EMSs need a deep and broad knowledge of elementary mathematics. The mathematical knowledge for teaching (MKT) framework (Ball et al., 2008) incorporates *specialised content knowledge* that includes: (a) analysing and interpreting students' mathematical thinking and ideas, (b) using multiple representations of mathematical concepts, and (c) defining terms in mathematically correct and accessible ways. Beliefs, in general, have been defined as mental constructs subjectively true for an individual, value-laden, relatively stable, and influential upon one's interpretations and practices (Skott, 2015). Mathematics beliefs relevant to this study centre on teaching and learning (pedagogical beliefs) and on one's capabilities for teaching effectively and influencing student learning (teaching efficacy).

Advocacy for EMSs has resulted in creation of preparation programs that vary substantially (e.g., delivery, duration, internship experiences, course content) (AMTE, 2024; Spangler & Orvick, 2017) and link to diverging goals of programs as well as to who is offering them (e.g., universities, school districts, professional development companies). Some scholars assert that while there is general agreement on what EMS preparation programs should promote, there is no consensus on what it takes to develop those competencies (Reys et al., 2017). Collectively considered, there is a compelling impetus for the study of such programs.

Within the context of EMS preparation programs, researchers have examined participants' content knowledge and beliefs and to a relatively lesser extent instructional practices (e.g., Campbell & Malkus, 2014; Kutaka et al., 2018; Rigelman & Lewis, 2023). One inquiry investigated the effects of *Primarily Math*, an EMS preparation program designed to develop the mathematical knowledge of elementary teachers (Kutaka et al., 2018). *Primarily Math* was offered over 13 months in three content and three pedagogy courses. Participants were three cohorts of K–3 teachers (n = 126) who completed the program and a matched comparison group (n = 92). Those completing the program endorsed more student-centred mathematics pedagogical beliefs. Although those completing the program demonstrated greater gains in MKT relative to the comparison group, scores on this measure showed variability between cohorts and by subscale (e.g., the program did not significantly impact any cohorts' algebra scores).

Another inquiry examined the influences of a 15-month preparation program, which included five mathematics courses team-taught by a mathematician and mathematics educator, a leadership-coaching course, plus a second leadership-coaching course during the first year as a specialist-coach (Campbell & Malkus, 2014). Findings showed significant positive changes in participants' (n = 24) beliefs about mathematics pedagogy as sensemaking. Changes in participants' scores on the MKT measure were mixed. Whereas one cohort (n = 12) evidenced significant increases across the 15-month courses and the specialist-coach year, the other cohort (n = 12) only experienced a significant increase during the specialist-coach year. Recently, Rigelman and Lewis (2023) explored the effects of job-embedded professional development with optional university courses, offered over three years. The 30 mathematics specialist participants demonstrated significant increases in their MKT on the subscale assessed: number and operations. Participants' use of research-proven instructional practices also increased, doing so in all 10 areas evaluated with three areas evidencing significant changes (questioning, students' linking, students' providing).

When broadly considering the preparation of elementary educators who specialise in mathematics, other contexts (e.g., Australia) similarly evidence variability in their training with limited associated research (Lomas, 2022; Main et al., 2023; McMaster et al., 2024). In response to national policy requiring all elementary education graduates to have a subject specialisation by 2019 with priority given to mathematics, science, or languages (AISTL, 2015), efforts have focused on developing teacher candidates. There is seemingly less emphasis on practicing teachers, though some state-level policy is evident in relation to recognising primary mathematics specialists who have completed an in-service professional learning program (Lomas, 2022; McMaster et al., 2024). A case study conducted by Lomas (2022) explored a 2-year program for in-service teachers (N = 14), which included 20 days of professional learning focused on curriculum and contextual experiences that were grounded in a theoretical framework of leadership. Data from a reflective survey collected one year into the program illuminated growth in the areas of: knowledge of mathematics and its teaching, approaches for mentoring and supporting fellow teachers, and building school capacity. Within the context of an initial teacher education program, McMaster and colleagues' (2024) inquiry illuminated challenges of adding required courses and staffing of instructors in response to policy and the new mathematics specialisation. They explicated the need for communication and coordination of policymakers and university and school personnel, including the support of teacher candidates after graduation.

Research Question

This question guided the study:

> *How does a professional development program support elementary teachers' development as Elementary Mathematics Specialists across two years?*

Methodology

The design of this study includes a descriptive, holistic singular-case approach (Yin, 2014), focusing on in-depth investigation of a case within a real-world context. Multiple sources of qualitative and quantitative data were collected to form the descriptive findings. The two types of data were given equal priority, with the qualitative data illuminating the findings of the quantitative data.

Participants and Context

Participants were 26 elementary teachers who had completed the first two years of a 5-year professional development program focused on their preparation as EMSs. Additional program aims included bolstering: equity and access in mathematics education, teacher retention, and a hiring pipeline of teacher candidates. Partners included a university, school district, and non-profit organisation within an urban context in the southeastern United States. Participants were chosen for the program based on a history of success in teaching mathematics and potential for teacher leadership. They self-identified as 23 females, 3 males, and 70% persons of colour (38% Black, 8% Hispanic, 8% Asian, 8% Hispanic/White, 4% Hispanic/Black, and 4% Black/White). All had a master's degree, and 34% had an educational specialist degree. They were experienced teachers with 12.5 years, on average, in the role. They taught in 21 elementary schools in varying roles, including generalist teachers (e.g., K-5 classrooms, including dual language immersion), specialist teachers (e.g., mathematics and STEM specials), and interventionists (e.g., special education, English language learners). While some taught only mathematics, others taught mathematics plus other subjects. As EMSs, the participants' primary responsibility was teaching students, with them doing so at least 4.5 days per week, thus their role as a teacher leader was in an informal capacity.

Student demographics across the participants' schools included 69% eligible for the federally-funded free and reduced lunch program and 91% non-White (e.g., 46% Hispanic, 34% Black). For approximately half of the first year of the program, the schools had a mix of students attending in-person and remotely due to COVID-19. To promote consistency across schools, the district provided an instructional model in mathematics, specifically, gradual release. This model involves direct instruction, where teachers demonstrate strategies for solving problems followed by students practicing them. The district also provided pacing guidelines for standards with assessments for each grade level that were administered every nine weeks, along with supportive curricular materials and resources.

The professional development program included: Professional Learning Communities (PLCs); individual mentoring; and two university endorsement programs, specifically K–5 Mathematics (K–5 ME) and Teacher Support & Coaching (TSCE) (see Table 1). Goals were to develop: (a) ambitious and equitable mathematics instructional practices (Carpenter et al., 2015; NCTM, 2020, 2024); (b) deep and broad knowledge of elementary mathematics, including *specialised content knowledge* (Ball et al., 2008); (c) productive mathematical beliefs; and (d) teacher leader capabilities, including coaching skills (AMTE, 2024). The endorsement programs included four elementary mathematics content courses integrating pedagogy, one course focusing on teacher leadership and coaching, and two internship courses, with one emphasising mathematics and the other coaching. Across this study's first two years implementation, the participants completed all seven courses of the endorsement programs, participated in 16 PLCs meetings, and engaged in individual mentoring. All class sessions and meetings occurred virtually and synchronously.

Table 1
Program Timeline and Elements Aimed at Preparing and Supporting EMSs

Year 1			Year 2		Years 3–5
Fall	Spring	Summer	Fall	Spring	
One TSCE course (Teacher Leadership & Coaching)	One K–5 ME course (Number & Operations)	One K–5 ME course (Data Analysis & Probability, 2-week summer institute)	One K–5 ME course (Algebra & Rational Number) and one TSCE course (Internship)	Two K–5 ME courses (Geometry & Measurement and Internship)	
PLCs and Mentoring	PLCs and Mentoring		PLCs and Mentoring	PLCs and Mentoring	PLCs and Mentoring

Professional learning communities and individual mentoring

Support for the participants in their roles as EMSs was provided through PLCs and individual mentoring, both facilitated by a program director. The PLCs met monthly eight times across the school year and included eight to nine participants clustered around grade level/teaching focus. The meetings focused on: building a community of learners; augmented support for developing ambitious and equitable mathematics instruction; and collaborative planning of teacher leader efforts that involved leading instructional change in their schools, district, communities, and other contexts. Required teacher leadership included coaching a teacher candidate each year and supporting the non-profit's after-school tutoring program for at least one year, with the participants engaging in other efforts responsive to their schools' needs. PLCs included reading, analysis, and discussion of a book purposefully chosen for its focus on developing student identity in mathematics (*Impact of Identity in K–8 Mathematics Teaching: Rethinking Equity-based Practices* [Aguirre et al., 2013/2024]), as well as engagement in cognitively demanding mathematics tasks. To support improved mathematics instruction, individual mentoring included the program director conducting yearly observations of each participant's classroom teaching, using an observation protocol (see *Data Collection*) as a tool for improvement through reflection and feedback.

Teacher support and coaching endorsement

The TSCE program focused on developing teacher leaders, including knowledge and capabilities for coaching, mentoring, leading professional development, and facilitating professional learning communities. It aimed to develop understandings of adult learning and teacher development across the career span and skills for: coaching others; facilitating ongoing, collegial professional learning; and leading teacher development activities focused on problems of practice and school improvement. There was immersion in cognitive coaching (Costa & Garmston, 2016), which is a facilitative approach, as well as transformational coaching (Aguilar, 2020) that centres on equity with continual analyses of behaviours, beliefs, and ways of being. Content also focused on leading teacher development activities responsive to the differentiated needs of mentees, such as curriculum and lesson plan support, data analysis focused on student learning, modelling, co-planning, co-teaching, and video self-study. These understandings and capabilities were emphasised during the first course, which the participants were expected to apply in a second internship course through their coaching of a teacher candidate or novice teacher.

K–5 mathematics endorsement

In the K–5 ME program, the development of ambitious and equitable pedagogy focused on learner-centred, responsive instruction that honours student thinking, positions them as capable of making sense of mathematical ideas, and supports them to use their understandings to solve complex problems (Carpenter et al., 2015; Jacobs & Empson, 2016). Participants were immersed in NCTM's (2024; 2020) eight mathematics teaching practices, especially: (a) selection/modification/creation and implementation of cognitively demanding instructional tasks; (b) use of multiple representations and tools; (c) promotion of problem solving and reasoning, explanation and justification, and connections and applications; and (d) use of children's thinking and understanding to guide instruction. They learned about planning for and enacting equity-based instruction that: accounts for and leverages children's varying mathematical competencies and cultural and linguistic backgrounds; uses the subject as a lens for understanding, critiquing, and creating change in the world; and cultivates student identity in mathematics (Aguirre et al., 2013/2024; NCTM, 2020).

Learning during class sessions occurred through: (a) active engagement in and analysis of the mathematics in the elementary curriculum, especially through cognitively-demanding instructional tasks; (b) study of children's thinking and learning via video clips and written work samples; (c) examination of classroom practice via video clips and written teaching cases; and (d) scrutiny of the research base on elementary mathematics education and of critical aspects of equity with connections to classroom practice and schools (e.g., culturally responsive teaching, instruction for multilingual learners, and teaching for social justice). In addition to assigned equity-focused readings, there was substantial immersion in the professional development materials from Cognitively Guided Instruction (CGI) (e.g., Carpenter et al., 2015) and *Developing Mathematical Ideas* (e.g., Shifter et al., 2018). Grounded in CGI's guiding principles for teaching and learning, there was much exposure to Number Story lessons as a model for learner-centred, equitable instruction, with assignments in the internship course requiring enactment and documentation of this pedagogy. Assignments across the content courses included clinical-style interviews of children's understandings of concepts with significant analysis, as well as creating, analysing, and solving cognitively demanding K–5 instructional tasks with a portion explicitly evidencing cultural responsiveness and connections to social justice. Additional assignments involved an in-depth data design, collection, and analysis project, as well as written syntheses and oral presentations of research.

Data Collection and Instruments

Quantitative data (see Table 2) were collected from all participants via: observations of classroom instruction (Standards-based Learning Environment Observation Protocol, SBLEOP); an assessment of MKT (Learning Mathematics for Teaching, LMT); and surveys of mathematics beliefs (Mathematics Beliefs Instrument, MBI; Mathematics Teaching Efficacy Beliefs Instrument, MTEBI). Baseline SBLEOP, LMT, MBI, and MTEBI data were collected at the beginning of the program. Post data were gathered via the: MBI and MTEBI at the end of Year 1 and Year 2; SBLEOP at the conclusion of Year 2; and LMT by subscale at the end of the related content course. Qualitative data were gathered through individual interviews of eight randomly selected participants at the end of Year 2, as well as three focus group interviews of the other participants.

Table 2
Data Collection Timeline Across Program's First Two Years

Data Source	Baseline/Pre	Post Year 1	Post Year 2	Post Related Content Course
SBLEOP	✓		✓	
LMT	✓			✓
MBI	✓	✓	✓	
MTEBI	✓	✓	✓	
Interviews			✓	

The SBLEOP documents the degree to which the teacher facilitates a standards-based learning environment (SBLE), and the extent to which students experience the SBLE (Tarr et al., 2008). The protocol was slightly modified for use with elementary teachers. The protocol evaluates specific mathematics classroom learning events using a scale of 1-3. Scores on five classroom events were included as data, with a higher score indicating more evidence of a SBLE. A mean score of 2 or higher across these classroom events has shown to correlate with higher student achievement. These five classroom events are grounded in learner-centred, equitable mathematics instruction (NCTM, 2020; 2024), with a focus on developing conceptual understanding by emphasising students who are: (a) problem solving, reasoning, and connecting concepts; (b) explaining and justifying; and (c) conjecturing and generalising. Classroom events address equity by attending to and valuing students' mathematical thinking and positioning students as capable by encouraging varied, invented solution strategies. Each participant was observed teaching one mathematics lesson at the two data collection timepoints. These observations were conducted by one rater, who had previously developed scoring reliability through extensive use of the protocol and inter-rater comparisons (e.g., Myers et al., 2021).

The LMT instrument assesses an individual's MKT, including *specialised content knowledge,* common content knowledge, and knowledge of content and students (Ball et al., 2008; Hill et al., 2004). IRT-scaled (Item Response Theory) scores for the LMT range from -3 to +3, with an expected value of zero. Subscales exhibit good reliability, including number and operations (α = .79), algebra (α = .75), and geometry (α = .85). Content validity was established by mapping items for congruence with the NCTM Standards.

The MBI measures teachers' beliefs about the teaching and learning of mathematics and the degree to which those beliefs are cognitively aligned (Peterson et al., 1989, as modified by the CGI project). The 48 items have five Likert-type scale response categories ranging from 'strongly agree' to 'strongly disagree', with higher scores indicating beliefs that are more cognitively oriented. The three subscales, Learner, Curriculum, and Teacher, have high reliability (α = .89, .80, and .90, respectively) and represent independent constructs based on confirmatory factor analysis.

The MTEBI measures teachers' beliefs in their individual capabilities to be effective mathematics teachers and influence student learning (Enochs et al., 2000). The 21 items use a Likert-type scale with five response categories, ranging from "strongly agree" to "strongly disagree", with higher scores indicating greater teaching efficacy. The two subscales, Personal Mathematics Teaching Efficacy and Mathematics Teaching Outcome Expectancy, have good reliability (α = .88 and .81, respectively) and represent independent constructs based on confirmatory factor analysis.

All individual and focus group interviews were approximately 1-hour in length. Interview protocols are in the Appendix, and the prepared questions provided a starting point with the interviewer asking questions for elaboration. The interviews were audiotaped and transcribed for analysis.

Data Analysis

Descriptive and inferential statistics were used for analysis of the quantitative data. For data from the SBLEOP, an overall code was determined (High, Medium, or Low) on each of the five classrooms events for each participant. Specifically, individual scores (3, 2, or 1) from observations were used to determine whether each event should be characterised as either High (3), Medium (2), or Low (1). These categorical codes were then assigned the numerical codes of: High = 2, Medium = 1, and Low = 0. The five numerical codes were summed to yield a composite score for each participant ranging from 0 to 10. These composite scores indicate participants' overall demonstrations of SBLE and were grouped into intervals of High (7–10), Medium (3–6), and Low (0–2).

The research team included five university professors and two doctoral students, collectively holding expertise in a variety of methodologies. Three team members analysed all individual and focus group interview transcripts, supported and documented via the qualitative data analysis software program, NVivo. They first, individually, used line-by-line open coding, focusing on segments of data aimed at addressing the research questions. This coding generated numerous meaning units (i.e., embedded coherent and distinct meanings). Applying constant comparative methods (Corbin & Strauss, 2008), those meaning units were then compared across participants as the researchers reduced the data by comparing and refining the units. As the team reached consensus, they collapsed and renamed coded meaning units until they collectively determined final themes. For example, implementation constraints, variability in encountering constraints, and managing of tensions were collapsed to form *navigating constraints in shifting instruction*.

Trustworthiness

Trustworthiness of the study was established in several ways. The study involved: (a) prolonged engagement of multiple researchers who had in-depth knowledge of the context; (b) multiple data sources both quantitative and qualitative in nature; (c) an audit trail involving careful recording of research activities in a researcher log with supportive documentation; and (d) consensus-building procedures around interpretation of the data, for example, multiple examinations and discussions of interview transcripts by the researchers.

Results

Quantitative Findings

To examine how the program supported participants' development as EMSs, data from the SBLEOP, LMT, MBI, and MTEBI were analysed. Table 3 displays data focused on the extent to which participants facilitated SBLEs characterised by learner-centred, equitable mathematics instruction during their classroom observations (SBLEOP). Shown are the percentages of participants who were scored as Low, Medium, or High by classroom event, as well as the percentages of participants who fell within each range of composite scores, at the two observation timepoints. A paired sample t-test revealed a significant difference in the SBLEOP composite scores between pre- and post-Year 2 (t_{25} = 13.60, p < 0.001) with a large effect size (d = 2.67). At post-Year 2, 92% implemented SBLEs at a high level with 8% doing so at a medium level, while at the inception of the program, 8% implemented SLBEs as a high level with 31% doing so at a medium level and 61% doing so at a low level. Some variability was evident across classroom events and at the end of two years. The classroom events most evident at a high level (88%) included: fostering conceptual understanding, explaining of solution strategies by students, and supporting multiple perspectives for solving problems. All in all, across the two years of the program, there was substantial growth in participants' facilitation of learner-centred, equitable mathematics instruction.

Table 3
Percentage of Participants by Score for SBLEOP Observed Classroom Events and Composite Scores

Classroom Event	Scores with Numerical Codes of 0, 1, 2					
	Pre-Year 2			Post-Year 2		
	Low (0)	Med (1)	High (2)	Low (0)	Med (1)	High (2)
Making Conjectures	69%	31%	0%	0%	69%	31%
Conceptual Understanding	42%	50%	8%	0%	12%	88%
Explaining Strategies	46%	46%	8%	0%	12%	88%
Multiple Perspectives	69%	23%	8%	0%	12%	88%
Using Student Statements	58%	42%	0%	0%	50%	50%
	Sum of Event Scores (0–10)					
	Low (0–2)	Med (3–6)	High (7–10)	Low (0–2)	Med (3–6)	High (7–10)
Composite Scores	61%	31%	8%	0%	8%	92%

Participants' pre- and post- (after the related content course) MKT (LMT) is presented in Table 4 as IRT scores ranging from -3 to +3 across the three subscales. At the beginning of the program, the participants collectively evidenced average algebra knowledge, slightly below average geometry knowledge, and slightly above average number and operations knowledge. Paired sample *t*-tests of subscale scores revealed significant increases across all subscales, including large effect sizes for algebra (t_{25} = 4.5, p < 0.001; d = .89) and geometry (t_{25} = 5.46, p < 0.001; d = 1.07), and a moderate effect size evident for number and operations (t_{25} = 2.69, p = 0.01; d = 0.53).

Table 4
IRT Means and Ranges for MKT (LMT)

	Pre-IRT Scores		Post-IRT Scores	
LMT	Mean	Range	Mean	Range
Algebra	-0.03	-1.94 to 1.53	0.57	-1.04 to 2.06
Geometry	-0.20	-1.81 to 1.22	0.46	-0.60 to 1.97
Number and Operations	0.44	-1.32 to 2.18	0.85	-0.51 to 2.27

Repeated measures ANOVAs showed significant changes in participants' mathematics pedagogical (MBI) and teaching efficacy beliefs (MTEBI) (see Table 5). MBI scores showed a positive shift in pedagogical beliefs toward cognitive orientation with a post hoc analysis, using a Bonferroni adjustment, which revealed significant differences between all measurement timepoints: pre- and post-Year 1 (p < .001), post-Year 1 and Year 2 (p = .031), and pre- and post-Year 2 (p < .001). MTEBI scores showed a positive increase in beliefs about their abilities to teach mathematics effectively and influence student learning. A post hoc analysis, using a Bonferroni adjustment, revealed significant differences between post-Year 1 and Year 2 (p = .038) as well as pre- and post-Year 2 (p < .001), with an insignificant increase between pre- and post-Year 1 (p = .055). The timing of changes implies the second year of the program was more influential upon their mathematics teaching efficacy. Altogether, over the two years of the program, participants experienced substantial growth in productive beliefs.

Table 5
Means, Standard Deviations, and Repeated Measures ANOVAs for MBI and MTEBI

Measure	Pre		Post-Year 1		Post-Year 2		$F(2,24)$	η_p^2
	Mean	SD	Mean	SD	Mean	SD		
MBI	3.72	0.40	4.13	0.37	4.26	0.45	31.61*	.56
MTEBI	3.98	0.24	4.09	0.31	4.26	0.37	11.74*	.32

Note: MBI measures mathematics pedagogical beliefs and MTEBI measures mathematics teaching efficacy beliefs. Both use a Likert-type scale of 1-5. *$p < 0.001$.

Qualitative Findings

Findings from the interview data illuminated participants' development as EMSs and provided much insight into the results of the quantitative data. Illuminative quotes are provided, with the interview source (Individual Interview = II or Focus Group = FG) and ID (either participant or FG number) in brackets. Themes included: *centring of children*, *elevating equity and access, deepening knowledge for teaching*, and *navigating constraints in shifting instruction.* Cultivated by program experiences, these themes illuminated shifts in beliefs and knowledge and a concurrent reshaping of their mathematics teaching. Immersion in the ideas and concepts in the K–5 ME courses and PLC generated a sense of responsibility, compelling them to act:

> *When you know better, you do better. But you have to know, right? There are levels to that knowing. I [now] know that I'm supposed to teach math this way* [II 17].

Shifts in instructional practices varied across participants, with some describing incremental changes and others a major overhaul. During these shifting instructional practices, they were encountering and navigating conflicting contextual conditions.

Centring of children

Centring of children was evident in participants' shifting beliefs about the powerful capabilities of children to think and reason about mathematics, including those linked to children's informal understandings (e.g., "They come with such a wealth of knowledge." [FG 2]). This was coupled with reshaping of participants' instructional practices, especially centring on responsiveness to students and elevation of their mathematical ideas. Notably, when asked about how the program experiences influenced them, the notion of children-centredness was the most prevalent response.

Participants' instructional practices were shifting from those marked with teacher-centredness, including modelling, demonstrating, and explaining, to posing problems and allowing children to solve them using their own strategies, followed by discussion and debate of these strategies where children were learning from one another. One participant described this change as:

> *Instead of me telling students how to do stuff [mathematics] and modelling every single thing for them, they have started listening to the problem and thinking more critically about how they would solve it and coming up with their own strategies. And then learning from each other.* [FG 2]

In this valuing and honouring of students' ideas and solution strategies, students were deeply engaged with making sense of mathematics, and building improved confidence and motivation, thus developing their mathematics identity. One participant explained,

> *Putting that trust and telling them [students], "You're going to show me* [how to solve a problem] *instead of me showing it" has helped build their confidence in that identity of, "I am a mathematician."* [FG 2]

Another participant shared:

> *Having the kids present their work afterward* [after solving a mathematical task], *they become the teachers, and they want to be the teachers. I can't tell you how many times kids have asked, "Can I share my work today? Do you see my strategy? Have you looked at my strategy yet? Can I share?" So, it's taking that workload off the teacher from being the person who has all of the information and giving it to the students and really validating their knowledge as mathematicians.* [FG 3]

Participants described their struggles with these shifting instructional practices, particularly stepping back and scaffolding rather than explaining or demonstrating when students experienced productive struggle. One succinctly explained this engrained pattern as:

> *I'm so quick to help them fix it, instead of letting them figure it out for themselves.* [II 14]

Another shared his process of managing this tension as:

> *It was difficult because I knew I had to watch some of the students really struggle with some of the work. At the same time, I grew comfortable over time in seeing certain kids who had not performed previously, show up and perform .... It's kind of like the old saying, you give a man a fish, he eats for a day. Teach a man to fish, he'll eat for forever. And I know that the mathematical stuff was teaching children to fish, and so that's what allowed me to stick with it. But initially, there were times when I was like, "Oh, I just need to stop and help and guide and support." And sometimes it's better if you don't do that.* [II 17]

Specific program learning experiences were described as salient in cultivating children-centred practices. These included the study of children's thinking and reasoning and classroom instruction through videos, work samples, cases, and the student interview assignments, much of which was grounded in the framing of CGI, as well as supports for implementation via individual mentoring and the internship course. However, while viewing the videos during class sessions piqued curiosity and interest, some participants simultaneously held scepticism and disbelief about the capabilities of their own students to engage in similar ways. The process of implementing the newly learned pedagogy with their own students and seeing and experiencing these students' capabilities was critical. One participant shared about this initial uncertainty and subsequent shifts as:

> *Something that helped me a lot were all the videos that we watched of other teachers teaching. It was amazing to see how the students were teaching each other or helping each other to understand the concepts. At the beginning with the first videos, [I] was like, "Wow, that's not going to work in my class. Yeah, isn't going to happen." But, when I tried the first time, it wasn't beautiful, it wasn't perfect, but I was amazed at how much they [students] were able to produce by themselves. And I'm still working on it, but it works really well.* [FG 1]

Elevating equity and access

Many participants developed stronger beliefs about the criticality of equity and access, especially marginalisation and privilege based on students' race/ethnicity, culture, language, gender, income status, and ability. This held strong relevance for their school contexts and students. Prevalent across the interview data was marked valuing and celebrating of their students' cultural diversity. With these beliefs and understandings, for many there was a reshaping of their instructional practices. They especially realised the importance of having high expectations for every student, coupled with providing access to cognitively demanding mathematics. One participant said when reflecting on the differing abilities of his students,

> *How do I make sure I provide the same level of access to me as a teacher and the same level of rigor and really high expectations for students?* [II 17]

Further, participants recognized the importance of fostering students' mathematical identity through humanising the subject and establishing cultural affirmation. This compelled them to critically consider contexts and participants in their story problems and mathematical tasks with the aim of inclusivity and positively mirroring of their students. One participant shared:

> *With the word problems that I give to the students, I really made a point this year to make it more culturally relevant for them, so that they can see themselves in the problems. So that they really can understand the context of the problem, and it will help them to solve it.* [FG 1]

In this, they spoke of the importance of knowing their learners, including their cultural backgrounds, communities, homes, and families. They mostly did this through "asking them [students] questions" [FG 2], and to a lesser extent via surveys, with one sharing:

> *I gave my parents a survey this year .... They got to explain what their child likes, what it's like at home, and I use that information to come up with different activities and different word problems for them.* [FG 3]

In these shifting beliefs and understandings and reshaping of instructional practices, the participants spoke of influential program learning experiences. Immersion in aspects of equity and access in both the courses and PLC through intentional readings, with thoughtful analysis and discussion focused on application, was salient. They also described the requirement of creating mathematical tasks that are concurrently cognitively demanding, culturally responsive, and social justice oriented. In this, they described a broadened and deepened awareness of what equity and access entail, including existing practices, structures, and outcomes related to marginalisation and privilege of learners, coupled with a strong sense of responsibility. One participant reflected on this as:

> *The access and equity part, specifically tied to content where we don't give our students assignments because we think they can't do it. But we [should] give them an opportunity to do that because all kids deserve the opportunity to work on worthwhile mathematical tasks. That's mind blowing to me because I've never seen it [equity and access] as the way it was presented to us in class. Almost made me feel like I was discriminating against students if I didn't provide them with that work, or the opportunity to do that work. And so that alone made me think, "I have to do something different... If I don't think about this differently, I will be doing a disservice to the students I serve. And then, what's the point?" The same is true about special needs students. We read an article about the access that we provide for special needs students. And again … the way I've been perhaps trained to think about some of this stuff, I had to reevaluate. And, I had to think about, "Am I really providing equity for all students?" Because, regardless of what the assessment said, they should have an opportunity to do this work because these things lead to greater things and greater opportunities. Or they don't, right? ... So essentially you promote the achievement gap by not providing access. And that was mind blowing to me. So, for me that is perhaps one of the greater takeaways that I have. I have not learned to fight the system. I'm a part of the system.* [II 17]

While some participants provided strong examples of integrating mathematics and social justice, others expressed hesitancy and uncertainty. When it comes to this tentativeness, one reported she was "…at the beginner level for all of that." [FG 2] Another described this as:

> *I will start off by saying that probably is my biggest struggle. I would say in the whole 2 years, figuring how to implement those ideas within my math content. In all of our courses we had mathematical tasks that we had to create that encompassed either diversity or social justice. So, that was a small step I think in that direction of trying to implement it into the mathematics content …. Honestly, I've always worked in a diverse school, [and I] had never thought about it. So that's something that I plan to continue to work toward.* [II 6]

Deepening knowledge for teaching

Newly developed MKT was evident, especially connections of in-depth elementary mathematics content with children's thinking and reasoning. One participant shared:

> *[The program] increased my content knowledge and my conceptual understanding. The longer I'm a part of the program, the better I become at math, [and] the more confident I become at math. I think it can be hard for teachers to reflect on their own content knowledge because we're always like, "I know this, we know this", especially when you've been teaching the same thing [content] for a while. But, really understanding what the students know about it [content], and what they think about it, and how they're going to go about it, has really helped me grow. Because it's not about me understanding, it's about me understanding in the way that's going to help them understand what they're thinking, and how they're going about it. So, that's been really valuable for me.* [FG 2]

Although the participants were chosen for the program based on established success as mathematics teachers, they were developing a newfound understanding of the depth and complexity of elementary mathematics, with one describing this as:

> *Content depth that even if you thought you might have known some stuff …* [there was] *peeling back the layers of it to keep deepening, deepening, and deepening it … there was more to it [content] than I even realised that still existed.* [II 22]

The participants especially valued their new understandings of vertical progressions of the mathematical content across the elementary grades. Said one:

> *Since I am third grade, right there* [in] *the middle, just seeing what every student can do on every level all the way from kindergarten to fifth has just been so valuable … I love knowing where they can be coming from and where they should be going.* [FG 2]

The participants deepening MKT translated into newfound confidence and reshaping of their classroom instruction, especially the use of cognitive-demanding instructional tasks, as they better understood the mathematical concepts. They particularly connected this to planning, as they were better able to anticipate children's thinking for mathematical tasks and story problems, which contributed to improved responsiveness during instruction, specifically decision-making around teaching moves (e.g., purposeful questioning). One shared this new knowledge:

> *Gets you to look at how students choose to solve problems and their strategies, and the anticipatory sets of trying a problem. It has helped me become more responsive, because I am now pre-determining what they might have, the responses that you might get back. Also, just understanding the math more deeply makes me feel more confident when delivering [it].* [FG 2]

When considering influential program learning experiences and MKT growth, participants again spoke about immersion in children's thinking and reasoning through videos, work samples, and cases, which allowed them to see mathematics through children's eyes. Further, their engagement in cognitively demanding tasks during class sessions was especially meaningful, which involved both individual and collective productive struggle. They spoke of becoming learners themselves in ways that generated empathy for their students, and as they made sense of the mathematics, they leaned on one another within an environment of safety and risk taking. One participant shared:

> *It was a productive struggle together and independently. I think that really helped because we got to feel what the kids feel sometimes. I mean, it was really quiet in some of those breakout rooms at times, and then we'd be like, "Does anybody else get this?" And then we had to work it out together. But I think that really helps us see the importance of collaborative work with other teachers and as kids, to see what somebody else did because then when we had one of our other people in our class presenting or showing, it's like, oh okay, I got it. Now, let me figure this out. So, I think those experiences every week definitely helped build that productive struggle and seeing the importance of it.* [FG 1]

Navigating constraints in shifting instruction

As the participants were reshaping their mathematics instruction, they encountered *constraints within their school and district contexts*. Pervading challenges were grounded in the differences between the district's prescribed instructional model and the pedagogy learned in the program. One participant characterised this divergence as:

> *We're so used to having been told, you [teacher] show, you model the strategy. Then, you have students practice, and then they practice independently. But it's kind of like the opposite of what we have learned [in the program] and seen the benefits of.* [FG 1]

Given this, they frequently mentioned those in leadership positions in their schools who sought adherence with the district model, including administrators and mathematics coaches. Sometimes the differences were pronounced based on particular learners, including those receiving special services due to learning struggles or identified as English language learners. One participant shared:

> *That has been a challenge, because a lot of people feel like CGI is more for advanced kids or those who already have the basics. So, if I say I want to try it with my special education population, it was like, "Well, they're not going to be able to do it, because they don't have the basics. So, they can't just get a problem, you need to show [how to solve] it for them.* [FG 2]

There *was variability related to encountering constraints* with those in leadership positions, particularly related to two aspects: (a) the participant's role as a teacher of students; and (b) the degree to which administrators provided instructional autonomy. A number of the participants are "specialised" teachers, meaning they only teach mathematics, with classes of students rotating to them for instruction throughout the day. For these participants, they described having tremendous instructional autonomy, as long as they were addressing the standards. One shared:

> *All I do is teach math, my full day is math specials, so I teach K-5 math. I really have full autonomy on what I do. What they [administrators] want to see when they come is that the students are engaged in the mathematical task, and they are participating. They are working together, working independently. It can look any different way.* [II 13]

Other participants similarly spoke of increased instructional choice based on their position, such as those in dual language immersion classrooms. Also, for some, they shared that because they were seasoned teachers of mathematics with a proven history of successfully supporting students' learning, their administrators were hands-off:

> *My admin*[istration] *does not watch my classroom like they do others* [(FG 2)]; and *My admin was like, "You know what you are doing. You do you."* [II 19]

Participants *navigated* school and district-based constraints in a variety of ways, including communicating and collaborating with those in leadership positions and looking for spaces to include the learned pedagogy. Notably, in their managing of tensions they leaned heavily on one another, drawing support from the community of like-minded learners in the program. They valued their collaboration with other seasoned educators across the district with whom they otherwise would not interact, and spaces for open, honest conversations, especially in the PLCs, were critical. One characterised this as:

> *There was lots of stress, lots of venting, especially among the cohort. I don't know how any of us would have gotten through without each other. I have never been more grateful to have a group of teachers going through this with me. Because while you love your staff and you love your co-workers, I mean I do mine, it was just different. We felt we're in a different boat. We were all operating from the same place of purpose, but not knowing how to balance that with the district's expectations, our boss' expectations, our coworkers' expectations. We all had each other to kind of bounce suggestions, things to try, off of each other and moral support.* [II 24]

Ways of being strategic in managing pedagogical differences included communicating with administrators and mathematics coaches and sharing resources such as supportive videos, research, and student data, with the aim of understanding and endorsement. Participants had varying degrees of success in these efforts. One participant who taught multiple grades and garnered support shared:

> *My principal was on board, and she let us roll it out to my entire first grade. Then, we rolled it out kindergarten. And our math coach is on board, so everyone has embraced it. I wasn't expecting it [to] be as simple, and I don't think everyone has had that opportunity.* [FG 3]

For participants who did not have support from those in leadership positions, they navigated constraints by looking for spaces in their instruction to include the new pedagogy, with differences in scale. One described this as switching up the gradual release model "in a different cycle" from starting with,

> *"I do" to "you do": Day-to-day, I've gotten really good at kind of sneaking and letting the kids just try to solve it [mathematics problem] first.* [FG 1]

Another, who was a language interventionist supporting multiple classes, described how she adjusted her practices according to the context and openness of school leaders and teachers. For some classes, she taught the entire group using the new pedagogy, while for other classes where,

> *[When] they are not as open and willing, you have to compromise. So, maybe I won't do it whole group but when I pull my small group of my ELs specifically. I may not follow the whole CGI framework per se, but maybe I'll include pieces.* [FG 1]

Discussion

There is much complexity and variability related to the use and preparation of elementary educators who specialise in mathematics (Mills et al., 2020). Further, widespread advocacy for these subject area specialists has outpaced research (Baker et al., 2023), including that on their preparation. This study's context includes a sustained, intensive, and practice- and content-focused professional development program with intentional learning experiences aimed at developing in-service teachers' mathematics content, pedagogy, and leadership knowledge and skills (AMTE, 2024), providing a model for potential use by others. A recent call to action from several mathematics education organisations in the United States asserted the need for "further EMS-related research designed to inform preparation" (AMTE et al., 2022, p. 4), specifically that focusing on programs, courses, and learning tasks. The findings from this study revealed significant positive changes in participants' instructional and content knowledge expertise and beliefs as a result of program experiences, which contribute to the knowledge base on these professionals' preparation (e.g., Reys et al., 2017).

This inquiry's quantitative and qualitative findings illuminate positive changes having some similarities with other studies (Campbell & Malkus, 2014; Kutaka et al., 2017; Rigelman & Lewis, 2023) as well as show complexity and nuance in the novice EMSs' development. EMSs experienced substantial increases in MKT and productive changes in beliefs. Notably, previous studies indicated mixed MKT findings during EMS preparation programs (Campbell & Malkus, 2014; Kutaka et al., 2017), and it is optimistic that all three subscales evidence significant increases. When considering shifts across outcomes, the interview data provide insights into program elements that were especially influential. Collectively, results reveal marked changes toward implementing learner-centred, equitable mathematics instruction. Importantly, there is a new intentionality and urgency around equity and access in their shifting instruction and the development of students' mathematics identity. This is especially evident through: centring and elevating students' own and varied mathematical reasoning when solving problems; providing opportunities for every student to learn cognitively demanding mathematics; and efforts directed toward culturally affirming instruction and humanising of the subject. Yet, there was uncertainty around some of the expressions of equitable mathematics teaching, especially the integration of social justice and mathematics. All in all, these mathematics teaching and learning shifts are critical to disrupting the opportunity gap perpetuated in and through mathematics education (NCTM, 2020) and should be foremost aims of EMS preparation programs.

Complexity was evident as the participants strove to implement new pedagogy in their classrooms. They were navigating constraining and conflicting contextual conditions, largely emanating from the district's prescribed instructional model. Findings illuminate the degree to which they encountered resistance based on their role and context and their ways of being strategic, including looking for spaces for scaled inclusion of the new pedagogy. In fact, this noticing and leveraging of spaces and opportunities in teaching, curriculum, and assessments to make adaptations and incremental shifts, supports progress toward instructional change (Bartell et al., 2019). Program learning experiences must support EMSs in using their agency and drawing from their situated understandings, developed expertise, and professional judgement to productively navigate conditions such as increased standardisation of education and inequitable practices and structures (AMTE, 2024). Since EMSs are the ones who have in-depth knowledge of their students, learner-centred instruction requires them to have agency over their own classroom instruction (Rich, 2021).

Like the findings of others (cf. Campbell & Malkus, 2014; Kutaka et al., 2017) the EMS program experiences cultivated more productive mathematics beliefs, yet the quantitative data reveal some variability in timing of significant changes. When considering shifts in beliefs over time, all scores significantly increased across all measurement timepoints with one exception. While their mathematics teaching efficacy grew across the program, the second year was especially impactful. This may be linked to their high scores on the MTEBI at the inception of the program, which could be connected to their desire to participate in the professional development program and existing success as a mathematics teacher. We also posit that as participants were immersed in new ways of teaching and learning that likely diverged from their own experiences and the norm within their schools and district, they needed time to explore the new pedagogy and navigate school cultures before enacting it.

This study's findings illuminate the complexities of EMSs' shifting instruction as teachers of students, and their more developed MKT, instructional practices, and beliefs, which are essential for their growth from novices to qualified and capable experts and aspirations for positioning as a *more knowledgeable other.* This study's findings serve to illuminate what is possible and effective when it comes to EMS preparation, helping to build the base of knowledge on the preparation of elementary educators who specialise in mathematics. It must be noted that the development and use of EMSs in schools serving historically marginalised students is a critical step toward improving the teaching and learning of every student. Across the subsequent three years of the program, which involves shifting from the EMSs' preparation to on-going support for them, research findings have the potential to reveal the sustainability of the changes reported here as well as the participants' continued development of ambitious and equitable mathematics instruction.

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Ethical approval

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Competing interests

The authors declare there are no competing interests.

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Appendix

Individual Interview Protocol

Thank you for participating in this interview. You have been a Master Teaching Fellow in the NSF program for 2 years now. I am going to ask about your experiences across the past 2 years as a mathematics teacher as well as a mathematics teacher leader. Know that before analysis of the interview data occurs, your identity will be removed and combined with the data of others. We appreciate your honest responses.

1. How has your participation in the NSF program impacted you as a **mathematics teacher?**
    a. What key experiences in the program contributed to these new understandings and practices (*or whatever they describe as being impacted*)?
    b. What have you learned in the program that has been challenging to apply as a mathematics teacher? Why?
        b.1 How have you navigated those challenges?
    c. How has this program impacted your students' learning?
    d. Are there challenges for your teaching of mathematics that are particular to your school context? If so, describe.
        d.1 How have you navigated these challenges?

2. During your time in the NSF program, you've learned about leveraging students' cultural and linguistic strengths during mathematics instruction, along with the importance of connecting with parents and the community. Your courses have focused on social justice during mathematics instruction, and PLCs have focused on developing positive mathematics identities. How are you using any of these ideas in your mathematics instruction?

3. How has your participation in the NSF project impacted you as a **mathematics teacher leader?**
    a. What key experiences in the program contributed to these new understandings and practices (*or whatever they describe as being impacted*)?
    b. What have been some challenges for your service as a teacher leader? Why?
        b.1 How have you navigated those challenges?
    c. Are there challenges for your service as a teacher leader that are particular to your school context? If so, describe.
        c.1 How have you navigated these specific challenges?

4. Briefly tell us about your experiences in the project of coaching teacher candidates and/or novice teachers.
    a. What specific coaching strategies did you use?
    b. What do you think are the most important practices for a coach to be effective? (*prompt for several*)
        b.1 Which of these do you think you do really well?
        b.2 Which are the most difficult for you?

5. In your work as a mathematics teacher leader, you are serving in an <u>INFORMAL</u> teacher leader role, in contrast to someone in a formal teacher leader role such as a mathematics instructional coach. Tell me about your current work as an informal teacher leader.
    a. Are there advantages to you serving in this role in an <u>informal</u> way?
    b. Are there drawbacks to you serving in this role in an <u>informal</u> way?

6. You have shared so many great thoughts about your insights and experiences, and now I have just a couple final questions.
    a. What is the most exciting thing you have learned in the program that you want to share with others?
    b. You have finished your endorsement programs and next year you will focus on teacher leadership and continue to meet in your PLCs and individual mentoring with the program director. What do you need this next year to support your growth?

Focus Group Interview Protocol

Thank you for participating in this interview. You have been a Master Teaching Fellow in the NSF program for two years now. I am going to ask about your experiences across the past two years as a mathematics teacher as well as a mathematics teacher leader. Know that before analysis of the interview data occurs, your identity will be removed and combined with the data of others. We appreciate your honest responses.

1. How has your participation in the NSF program impacted you as a **mathematics teacher?**
    a. What key experiences in the program contributed to these new understandings and practices (*or whatever they describe as being impacted*)?
    b. What have you learned in the program that has been challenging to apply as a mathematics teacher? How have you navigated those challenges?

2. During your time in the NSF program, you've learned about leveraging students' cultural and linguistic strengths during mathematics instruction, along with the importance of connecting with parents and the community. Your courses have focused on social justice during mathematics instruction, and PLCs have focused on developing positive mathematics identities. How are you using any of these ideas in your mathematics instruction?

3. How has your participation in the NSF project impacted you as a **mathematics teacher leader?**
    a. What key experiences in the program contributed to these new understandings and practices (*or whatever they describe as being impacted*)?
    b. What have been some challenges for your service as a teacher leader? How have you navigated those challenges?

4. Now I want to ask about your school context and your students. How are your students currently learning mathematics in your classroom? How is this different than before your participation in the NSF project?
    a. Are there challenges for your teaching of mathematics that are particular to your school context? How have you navigated these challenges?
    b. Are there challenges for your service as a teacher leader that are particular to your school context? How have you navigated these challenges?

5. As a part of the project, you have coached teacher candidates and/or novice teachers.
    a. What do you think are the most important practices for a coach to be effective?
    b. Which of these have been difficult to implement? Easy to implement?

6. In your work as a mathematics teacher leader, you are serving in an INFORMAL teacher leader role, in contrast to someone in a formal teacher leader role such as a mathematics instructional coach. What are some advantages and drawbacks to you serving in an informal way?