Preservice Teachers' Readiness to Teach Reform-based Mathematics Standards: The Role of Beliefs

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Research has shown that teaching reform-based mathematics standards as intended often requires knowing how teachers' knowledge and beliefs correspond to the principles informing standards' development. We examined survey (N = 244) and interview responses (n = 14) of preservice primary and secondary mathematics teachers to assess the strength of their beliefs regarding mathematics and mathematics instruction and their knowledge of reform-based mathematics standards in the United States. Results showed most preservice teachers did not have a thorough knowledge of reform-based mathematics standards. Additionally, no substantial correlations were found among preservice primary teachers' beliefs whereas some moderate correlations were found among preservice secondary teachers' beliefs. This study highlights that teacher preparation programs working to improve mathematics instruction through reform-based mathematics standards might need to implement additional opportunities for preservice teachers to become acquainted with standards and to ascertain their beliefs in view of mathematics education reform.

Keywords: mathematics teacher education • reform-based mathematics standards • preservice teacher beliefs • primary teachers • secondary teachers

The level of attention on educating children in mathematics has increased worldwide over the years as the sustainability of many socio-economic, military, and political infrastructures progressively depends on high levels of mathematical knowledge (de Castro, 2019). Also, there is a growing concern among experts that the mathematics education students receive might be ill-suited for the mathematics needed after graduation (Goodman, 2019; Richland et al., 2012; Stigler et al., 2010). One solution advanced by supporters of mathematics education reform is using mathematics standards that put greater emphasis on students' mathematical sense-making, hereafter named reform-based mathematics standards, to raise the quality of instruction and increase students' conceptual understanding of mathematics (McCallum, 2015; National Academy of Sciences, 2013).

In the United States, the introduction of the Common Core State Standards—Mathematics (CCSSM) and the Mathematical Practices (MPs) standards signalled the first time a more coherent, student-focused, and mathematically sound set of standards resembling those of high-performing countries was available at the national level (McCallum, 2015; Zimba, 2014). The expectation was that teaching that takes advantage of these features would be more likely to develop students' mathematical thinking than practices that do not (Schmidt & Houang, 2012; Wu, 2011). Unfortunately, these reform-based mathematics standards' potential could remain unrealisable without teachers' full cooperation (Farfan et al., 2020; Schoenfeld, 2014). Successful implementation of mathematics standards often entails determining how teachers' content knowledge, beliefs, and practices align with the principles that underpin the standards' development (Olson et al., 2014; Opfer et al., 2017; Wu, 2011). Furthermore, implementing mathematics standards in the United States is the responsibility of individual states, not the federal government, and it is thus subject to regional and local politics that cannot be overlooked,

a similar but more pronounced situation than that of other federal countries such as Australia and Germany (Cortina & Thames, 2013; Eacott & Holmes, 2010).

The literature further suggests instructional changes stemming from implementing standards that focus more on students' mathematical sense-making may call into question teachers' prior understanding of the subject and the mathematical practices that go with it (Bartell et al., 2017; Hennessey, 2007; Spillane et al., 2018), including their beliefs about what mathematical knowledge is and how it should be taught to others (Fives & Buehl, 2016; Muis & Foy, 2010). Yet, little is known about whether preservice teachers (PTs), during their teacher preparation, are aware of their role in instructing with reform-based mathematics standards or of the epistemological and pedagogical adjustments that may be required (Paolucci, 2015; Shilling, 2010). As the use of standards to incentivise mathematics education reform increases globally (Ruiz et al., 2023), studying PTs' understandings and beliefs regarding reform-based mathematics standards may help mathematics teacher educators and others evaluate the impact similar reform-based standards initiatives have on the professional education of future teachers. Thus, the present study contributes to this research area by examining how PTs enrolled at a major research university in the Southeastern United States responded to questions concerning reform-based mathematics standards and related beliefs.

Conceptual Rationale

We define a belief as a weaker claim of knowing whose meaning is part of a cognitive state, held with varying degrees of conviction, and not necessarily consensual (Gilbert, 1991; Nilsson, 2014; Wilson & Cooney, 2003). A belief is implicit if it develops and resides largely in the subconscious, or explicit if it is deliberately formed and known to the beholder (Wilcox-Herzog et al., 2015). For simplicity, and based on prior research (Muis, 2004; Rott, 2020; Schoenfeld, 1992, 2002), we conceptualised two sets of beliefs:

Mathematics instruction beliefs are implicit and explicit beliefs about teaching mathematics to students. These beliefs range from "traditional" instruction beliefs, focusing disproportionally on procedures, memorisation and quick solutions, to "reform-based" instruction beliefs, focusing on instruction that aligns closer with the CCSSM and MPs, such as students discussing and explaining multiple solutions, and their ability to connect and make sense of mathematical concepts (Ross et al., 2002; Schoenfeld, 2014).

Mathematical epistemic beliefs are implicit and explicit beliefs regarding the nature of mathematical knowledge, including how we know something is true in mathematics (Hofer, 1999; Muis et al., 2016). These beliefs range from "steady" epistemic beliefs, where mathematical knowledge is largely seen as fixed and infallible, to "dynamic" epistemic beliefs, where mathematical knowledge is seen as evolving and potentially subject to revision, though core parts might remain fixed. It is the possibility of innovation and failure during knowledge construction that is key in dynamic epistemic beliefs, as is the case for creative mathematicians (Hennessey, 2007; Muis, 2004). Moreover, past research suggests instruction beliefs and epistemic beliefs are somewhat correlated (Fives & Buehl, 2016; Hofer & Bendixen, 2012; Schraw & Olafson, 2003), and that those who hold "traditional" instruction beliefs and "steady" epistemic beliefs may have the most difficulty in adapting to instructional changes accompanying mathematics education reform (Depaepe et al., 2016; Muis & Foy, 2010).

Beliefs in Mathematics and Mathematics Instruction

The literature highlights the connection between primary and secondary students' beliefs in mathematics and teachers' mathematics instruction (Depaepe et al., 2016; Muis & Foy, 2010). Because teachers direct, structure, and control the presentation of content students are required to learn, they play an important—though not exclusive—role in the kind of learning and corresponding beliefs students are supposed to attain (Ernest, 1994; Handal, 2003). For instance, a belief such as "mathematics is just computation" could become reinforced by classroom experiences where computing takes precedence over everything else. This belief becomes internalised over time and may turn problematic

later when faced with mathematical situations where computational skills are of little help (Schoenfeld, 2013; Wijaya et al., 2014).

Research conducted in the last thirty years suggests many school-aged children believe mathematics is a fixed discipline that requires little more than memorising procedures and formulas (Depaepe et al., 2016; Muis, 2004; Op't Eynde et al., 2003). These beliefs are thought to have negative effects on students' performance and behaviours (Garofalo, 1989; Li et al., 2021; Schoenfeld, 1992) and may lead to adverse emotions such as mathematics anxiety that carries over after graduation (Caniglia & Duranczyk, 1999; Henschel & Roick, 2017; McLeod, 1988). Research has also shown a likelihood among teachers to pass on their beliefs to students (Buehl & Fives, 2016; Lortie, 1975). If those beliefs happen to conflict with the principles informing reform-based mathematics standards, they may interfere with the correct implementation of said standards as teachers misinterpret or teach them superficially. Correct implementation potentially requires a conceptual change of those beliefs (Vosniadou et al., 2008).

Similarly, PTs' beliefs regarding mathematics and mathematics instruction are initially drawn from prior school experiences (De Corte et al., 2010; O'Meara et al., 2017; Wu, 2011), and PTs would reasonably start their teacher preparation at different positions in the traditional/reform-based and steady/dynamic belief spectra. These beliefs, however, may change over time, moving across the spectrum as PTs acquire expertise and advance in teacher preparation programs (De Corte et al., 2002; Marbach-Ad & McGinnis, 2009; Tanriverdi, 2012). Indeed, some evidence exists that attending a program known for evidence-based practices could move everyone closer to reform-based mathematics instruction and dynamic mathematical epistemic beliefs over time (Leavy & Hourigan, 2018; Marbach-Ad & McGinnis, 2009), and that teacher preparation can facilitate belief change (Gill et al., 2004; Morine-Dershimer, 1993). Yet, whether most mathematics teacher preparation programs spend time ascertaining PTs' beliefs initially and throughout the delivery of relevant instruction is not known, despite teaching methods courses—where PTs' beliefs are most likely to surface—dominating the time devoted to teacher preparation (Mortimer, 2018; Otten et al., 2015). Moreover, PTs' beliefs would likely become less accessible after leaving teacher preparation due to the demands of teaching and other duties (Jansen et al., 2017; O'Meara et al., 2017). This is why evaluating PTs' mathematics instruction and mathematical epistemic beliefs before leaving university would make the best sense (Farfan, 2021).

Furthermore, as noted in the introduction, experts have argued that for reform-based mathematics standards to be implemented as intended, an alignment between teachers' content knowledge, beliefs, and practices and the principles informing the standards' development should be sought (Olson et al., 2014; Opfer et al., 2017). This is because misalignment might compromise the effectiveness of mathematics instruction, as has been observed, for example, with other reform-based mathematics initiatives such as Multi-tiered System of Supports (Donnell & Gettinger, 2015). Thus, a stronger correlation between PTs' beliefs regarding mathematics and mathematics instruction and the principles behind reform-based mathematics standards could indicate PTs' readiness to teach these standards as intended.

Reform-based Mathematics Standards and Teacher Preparation in the United States

The CCSSM were created by the Common Core initiative, a joint effort by the National Governors Association and the Council of Chief State School Officers, with feedback from teachers and content experts (McCallum, 2015). Released in 2010, and adopted by 46 states, the CCSSM follow research-based learning progressions on how students build mathematical knowledge, skill, and understanding over time (Schoenfeld, 2014; Wu, 2011), and were designed to provide explicit and clear descriptions of what students should understand and be able to do in mathematics at grade level across the United States (National Governors Association, 2010).

Alongside the CCSSM, which describe *what* to teach children, the Common Core initiative produced eight MPs, which describe *how* children should be practicing their mathematical learning (Common Core State Standards Initiative, n.d.):

- Make sense of problems and persevere (MP1)
- Reason abstractly/quantitatively (MP2)
- Construct arguments and critique others (MP3)
- Model with mathematics (MP4)
- Use tools strategically (MP5)
- Attend to precision (MP6)
- Make use of structure (MP7)
- Express regularity in repeated reasoning (MP8)

The MPs were based on processes and principles derived from two guidance documents: the process standards found in National Council of Teachers of Mathematics' *Principles and Standards for School Mathematics* (2000), and the strands of mathematical proficiency found in the National Research Council's *Adding It Up* (2001). Together with the CCSSM, the MPs were meant to help students not only learn mathematical content but "do" mathematics meaningfully, setting expectations for the kind of mathematics instruction United States primary and secondary teachers were asked to provide (Coburn et al., 2016).

Despite their high profile, little is known about how the CCSSM or MPs are integrated into teacher preparation. The few studies conducted have shown a limited understanding of reform-based mathematics standards among PTs (Courtney & Caniglia, 2021; Mortimer, 2018). Besides, many states that originally adopted the CCSSM and MPs are either modifying the standards or replacing them with new ones, adding to the confusion. For instance, Florida—home to approximately 3 million school students—adopted the CCSSM in 2010 but delayed their implementation until 2014 to produce aligned curricula, training, and assessments (Florida Department of Education, 2019). In the interim, to avoid controversy, the state legislature renamed the K–12 CCSSM as the Mathematics Florida Standards (MAFS) but kept the MPs the same. Although the MAFS remained very close to the CCSSM, they did include some minor revisions (Farfan & Schoen, 2021; Postal, 2020).

These and other considerations serve as a reminder that learning to teach does not take place in a political and socio-economical vacuum but occurs within ever-changing contexts (Peressini et al., 2004). Hence, it is important not only to acknowledge national and state-led initiatives like the introduction of reform-based mathematics standards but also to consider the impact these might have on the overall professional education of PTs, including how PTs' beliefs may or may not align with the principles behind the standards' development.

Motivation and Purpose of the Study

Although several factors beyond beliefs influence teaching (e.g., accountability systems, coaching, etc.; Sun et al., 2014), growing evidence exists that teachers' beliefs play a part in the quality of instruction, , and mathematics achievement of students (Hofer & Bendixen, 2012; Muis & Foy, 2010; Stahnke et al., 2016). Thus, this mixed-methods study reported in this article aimed to examine PTs' knowledge of reform-based mathematics standards as well as their mathematics instruction and mathematical epistemic beliefs. It was done to better anticipate how people entering the teaching profession might respond to changes, such as the implementation of reform-based mathematics standards, affecting how content is presented to students (Farfan et al., 2020; Yadav et al., 2011). Specifically, we examined the survey and interview responses of PTs majoring in primary education or secondary mathematics education at a Florida public research university to answer the following research questions (RQ):

1. *How familiar are PTs with reform-based mathematics standards (i.e., MAFS and MPs)?*
2. *What type of mathematics instruction and mathematical epistemic beliefs do PTs have, and how strongly are these beliefs related across the different majors?*

3. *What are PTs' understandings of reform-based mathematics standards considering their reported beliefs, teacher preparation, and future teaching practice?*

Method

Research Context: Teacher Preparation Programs

The primary education program is a five-year Bachelor/Master's degree that is designed to prepare students to teach K–6 grades. The program is consistently ranked as one of the best in the state and nationally (Farfan, 2021). PTs majoring in primary education take three education methods courses focused on early mathematical learning, teaching mathematics to children, and mathematics learning progressions beginning in their fifth term or third year (academic years in the United States. traditionally have two terms), which is when the MAFS and MPs are officially introduced to primary majors.

Highly successful in the state, the secondary double-major program allows students majoring in mathematics, physical sciences, or computer science to earn a second major in secondary mathematics teaching as part of their four-year degree (Farfan, 2021). PTs double-majoring in secondary mathematics take five education methods courses covering mathematical learning, inquiry-based learning, and teaching secondary mathematics beginning in their third term or second year, which is when the MAFS and MPs are officially introduced to secondary double-majors.

Instructors in both programs consisted of faculty and graduate teaching assistants who were well-informed regarding reform-based mathematics standards, who actively implemented evidence-based practices in their teaching, and who confirmed the MAFS and MPs were discussed in the mathematics education courses.

Research Design

Given the limitations of relying solely on survey data to infer beliefs, we chose a cross-sectional, sequential mixed-methods design combining survey research with follow-up interviews (Creswell, 2009). The online survey consisted of demographic questions, questions regarding participants' knowledge of reform-based mathematics standards, and two scales to collect information about beliefs. Follow-up interviews were done in person or virtually, using participants' previous survey responses to facilitate discussion. Florida's MAFS and MPs were introduced in 2014, and data were collected for this study between 2017 and 2021.

Participants

Participants were 244 undergraduate students majoring in primary education or majoring in mathematics and secondary education at a Florida public research university who indicated their willingness to participate in the study and their intention to teach after graduation. Most primary majors (n = 154) identified as female (96%) and White (84%); among secondary double-majors (n = 90), 71% identified as female and 70% as White. Those from minoritised racial and ethnic backgrounds were particularly underrepresented among primary majors, making up less than a sixth of the total (16%). Thus, the sample was representative of the teacher population in the state (Florida Department of Education, 2021). The median age for both primary and secondary double-majors was 20 years, with a greater spread of ages in the latter group (M = 20.8, SD = 3.2). All PTs included in the analytic sample were taking or had taken at least one education methods course. More than 80% of PTs indicated having no prior teaching experience, with only a small minority of either primary or secondary double-majors declaring having experience as a substitute teacher or similar. Unlike primary majors, almost half of the secondary double-majors (44%) entertained considering other careers besides teaching.

We extended interview invitations to PTs who completed the survey in its entirety, who were at least in their second year of studies, and who reaffirmed their intention to teach immediately after graduation. We invited 41 participants, out of which 14 were interviewed, seven for each major (11 females; *Mdn* =

21 years old). The subsample included two people who identified as Black and two people who identified as Hispanic of mixed race.

Data Collection Instruments

Demographic questions and familiarity with state standards

In addition to basic demographic information, participants were asked: (1) if they had any teaching experience before enrolling in their academic program, and (2) whether they planned to teach mathematics after graduation. Participants reported on their familiarity with the MAFS and the MPs by answering four questions (e.g., "How familiar are you with the Mathematics Florida Standards (MAFS)?"). Participants could choose one out of five different responses to these questions (e.g., "I know and understand the MAFS very well"), including an open-ended response ("Other: Please specify").

Mathematics instruction beliefs survey

Because no comparable scales existed for determining teachers' mathematics instruction beliefs (Colen, 2019) it was necessary to develop an appropriate scale that aligned participants' agreement with mathematics instruction beliefs with the eight MPs. The Mathematics Instruction Beliefs Survey (MIBS) scale developed was comprised of 24 items, three for each of the MPs (e.g., "I believe students should know their answers and solutions are open to revision"). Participants could choose one out five Likert-type responses (0 = "I do not have an opinion," 1 = "Strongly disagree," 2 = "Disagree," 3 = "Agree," and 4 = "Strongly agree"). The scale was developed from a conceptual base (Colen, 2019; Koestler et al., 2013), had at least three items per subscale, was vetted by two content experts and one mathematics teacher, and was pilot-tested on undergraduate students to ensure intelligibility, and meeting criteria for face validity (Hardesty & Bearden, 2004). The overall reliability score (McDonald's ω) for the MIBS was .84, 95% *CI* [.80, .87], indicating good reliability. Correlational analyses using Spearman's ρ (r_s) showed none of the MIBS subscales were strongly correlated (r_s > .70), nor were there any substantial negative correlations, suggesting multidimensionality and basic discriminant validity of the scale (Shiu et al., 2011)

Discipline-focused epistemological beliefs questionnaire

To assess participants' mathematical epistemic beliefs, we adapted Hofer's (2000) Discipline-focused Epistemological Beliefs Questionnaire (DFEBQ), which assesses beliefs about knowledge in any discipline by simply adding the name of the discipline to the items. The DFEBQ used here consisted of 15 items making up four factors regarding beliefs about knowledge as described in Hofer (2000): Certainty, which is the degree one understands knowledge to be fixed or more fluid (e.g., "Truth is unchanging in mathematics"); Simplicity, which is the degree by which one sees knowledge as an accumulation of discrete facts or as highly interrelated concepts (e.g., "All mathematicians would probably come up with the same answers to mathematics questions"); Justification, which is how much individuals justify knowledge by referring to external authority or their own (e.g., "First-hand experience is the best way of knowing something in mathematics"); and Source, which is the degree one sees knowledge as originating outside oneself or from within (e.g., "The most important part of work in mathematics is coming up with original ideas"). Participants chose one out five Likert-type responses (0 = "I do not have an opinion," 1 = "Strongly disagree," 2 = "Disagree," 3 = "Agree," and 4 = "Strongly agree"). A higher score on these items indicates stronger agreement with dynamic mathematical epistemic beliefs (some items were reverse-scored). The overall McDonald's ω for the DFEBQ was .84, 95% *CI* [.76, .89], indicating good reliability. Correlational analyses using Spearman's ρ (r_s) showed none of the DFEBQ subscales were strongly correlated (r_s > .70), nor were there any substantial negative correlations, suggesting multidimensionality and basic discriminant validity of the scale (Shiu et al., 2011).

Interview protocol

The interview protocol was designed to be semi-structured, allowing participants the opportunity to unpack, explain, or change their prior responses to the survey (Creswell, 2009). An effort was made to

avoid leading questions by using participants' survey responses and employing neutral language as much as possible.

The protocol included questions about their prior school experiences (e.g., "What was the quality of mathematics instruction you received in school?"), questions about responses to the survey (e.g., "When asked about the new mathematics standards, you said *x*. Could you elaborate?"), as well as more specific questions regarding the MIBS and DEQB items (e.g., For this item *x*, you answered *y*. Could you elaborate?"). The interviews ended with questions about the participants' future teaching practice (e.g., "How would you know if a student understood a mathematical idea or concept?"), including using classroom instruction scenarios aligned with one or more MPs to unpack their instructional readiness (Koestler et al., 2013).

Procedures

PTs were recruited through the university's Research Subject Pool and by email from Fall 2017 to Summer 2021. PTs taking courses listed in the Subject Pool could access the survey and receive course credits for their participation, which added to their overall grade. PTs not enrolled in courses included in the Subject Pool were emailed the survey link via their instructors. The survey asked at the start if PTs planned to teach after graduation to continue.

The survey had four sections: (1) half of the demographic questions and questions regarding familiarity with the MAFS and MPs, (2) MIBS scale, (3) the second half of the demographic questions, and (4) DFEBQ scale. The survey had a median completion time of 9 minutes. Participants were also asked about participating in a follow-up interview for a $20 Amazon Gift Card incentive. If interested, participants entered their names and email in a text box before the DFEBQ part of the survey. Information entered in the text box was checked to determine if those interested in the interview met the criteria delineated earlier. Invitations were sent to schedule an interview over the phone, online, or in a public place (pre-COVID-19 pandemic). Most interviews were conducted in the 2019–20 and 2020–21 academic years and averaged 28 minutes. The median time from survey to interview was approximately 30 days for primary majors and 60 days for secondary double-majors. Transcribed documents of the student interviews were made available afterward to interviewees to allow for an opportunity to change or add to their responses or provide other feedback.

Data Analyses

We ran correlational and multiple regression analyses on the survey data to investigate associations of beliefs among PTs and performed thematic analyses of the interview data to gain more nuanced insights about the quantitative findings. Unless otherwise indicated, belief survey data were assumed to be rank ordered. All correlational and reliability analyses were done in JASP 16.0 (JASP Team, 2021), and regression analyses were done in SPSS 27.

To test for associations, PTs' median (*Mdn*) responses for the MIBS and DFEBQ subscales were used. A higher score indicated stronger agreement (e.g., "3 = Agree") with either reform-based mathematics instruction beliefs or dynamic mathematical epistemic beliefs, respectively, while the response "I do not have an opinion" was coded as 0 and treated as missing. Missing responses (n = 100 for primary majors, n = 47 for secondary double-majors) did not show any patterns of concern and thus were excluded by pairwise deletion when applicable.

Transcribed interview data went through at least two cycles of coding: an initial open identification and documentation of codes emerging from the data, and a subsequent re-organisation and reduction of codes to a priori categories relevant to the research questions (Deterding & Waters, 2021). The coding of interviews was done by the first author using MS Word and QDA Miner.

Results

Knowledge of Reform-based Mathematics Standards

Overall, 18% of primary majors and 17% of secondary double-majors indicated they knew the MAFS very well. Only 15% of primary majors said they knew the MPs very well, whereas 23% of secondary double-majors stated the same. About a fifth of each group, however, did not distinguish between the MAFS and the MPs (Table 1).

Table 1
Percentage of Responses to Knowledge about the Standards (N = 244)

Question	Primary Majors	Secondary Double-majors
How familiar are you with the Mathematics Florida Standards (MAFS)?		
Not interested in knowing about the MAFS.	5%	6%
I don't know the MAFS but would like to know more about them.	31%	31%
I know about the MAFS but don't understand them very well.	47%	47%
I know and understand the MAFS very well.	18%	17%
What would you say is the main purpose of the MAFS?*		
To ensure students are college-ready	6%	5%
To make sure no one fails standardised testing.	8%	12%
To teach students new skills they'll need in the higher grades.	14%	30%
To help students develop conceptual understanding.	72%	53%
Do you believe the MAFS currently serve their purpose?*		
I am not aware of what the MAFS are intended to do.	8%	5%
No, I think the MAFS in their current form are inadequate.	5%	12%
Yes, I think the MAFS are great at what they intend to do.	20%	16%
Yes, but I've some reservations.	66%	65%
Are you familiar with the Mathematical Practice standards?		
Not interested in knowing about the Mathematical Practice standards.	4%	4%
I don't know the Mathematical Practice standards but would like to know more.	56%	51%
I believe the Mathematical Practice standards and the MAFS are the same thing.	21%	19%
I know and understand the Mathematical Practice standards very well.	15%	23%

Note.: Questions marked with (*) were only shown to preservice primary (n = 100) and secondary (n = 57) teachers who knew the MAFS partially or very well.

The survey also included two follow-up questions for those who answered they knew the MAFS partially or very well, accounting for 65% of all primary majors (n = 100) and 63% of all secondary double-majors (n = 57). In these subsamples, 72% of primary majors and 53% of secondary double-majors correctly identified the main purpose of the MAFS as developing students' conceptual understanding of mathematics. Almost a third of secondary double-majors thought the main purpose of the MAFS were to teach students new skills for higher grades, whereas less than 15% of primary majors said the same.

Most PTs in these subsamples expressed their reservations as to whether the MAFS served their intended purpose, with 66% of primary majors and 65% of secondary double-majors declaring it so. Only 20% of primary majors and 16% of secondary double-majors thought the MAFS were great at what they intended to do, with 12% of secondary double-majors also indicating that the MAFS were inadequate at achieving their intended goal (Table 1).

Preservice Teachers' Beliefs

Correlations

Among primary majors, only three MIBS subscales were significantly correlated with a DFEBQ subscale: Regularity in repeated reasoning (MP8) and Source of knowledge (SRC) were positively correlated, r_s = .19, p = .029, whereas the next two were negatively correlated: Make sense of problems and persevere (MP1) and Simplicity of knowledge (SIMP), r_s = -.18, p = .029; and Make use of structure (MP7) and Justification of knowledge (JUST), r_s = -.20, p = .026. More importantly, however, all correlations were small ($r_s \leq .20$). Conversely, among secondary double-majors, half of the MIBS subscales were significantly and positively correlated with the DFEBQ subscale standing for Simplicity of knowledge (SIMP): Construct arguments and critique others (MP3), r_s = .24, p = .031; Model with mathematics (MP4), r_s = .28, p = .008; Use tools strategically (MP5), r_s = .26, p = .019; and Regularity in repeated reasoning (MP8), r_s = .28, p = .010. Notably, the correlations of SIMP with MP4 and MP8 had values close to r_s = .30 (Table 2), which indicated moderate strength.

Table 2

Correlations for MIBS and DFEBQ Median Scores in Primary Majors (n = 154) and Secondary Double-majors (n = 90)

Subscale		CERT	SRC	SIMP	JUST	CERT	SRC	SIMP	JUST
	Mdn (IRQ)	3.0 (2-3)	2.0 (2-3)	3.0 (2-3)	3.0 (2-3)	3.0 (2-3)	2.0 (2-3)	3.0 (2-3)	3.0 (2-3)
		Primary Majors				Secondary Double-Majors			
MP1	Spearman's ρ	-.05	.01	-.18*	-.05	.07	.03	.15	.19
	Sig. (2-tailed)	.562	.954	.029	.594	.517	.776	.182	.098
MP2	Spearman's ρ	.07	.01	-.04	-.09	.16	-.01	.18	.08
	Sig. (2-tailed)	.428	.909	.633	.306	.133	.965	.103	.507
MP3	Spearman's ρ	.15	.04	-.04	.07	.05	-.10	.24*	-.05
	Sig. (2-tailed)	.079	.628	.680	.436	.641	.382	.031	.657
MP4	Spearman's ρ	.02	.13	-.11	-.13	.14	.17	.28**	-.19
	Sig. (2-tailed)	.845	.122	.177	.158	.182	.132	.008	.095
MP5	Spearman's ρ	.06	.14	-.08	.05	.13	-.05	.26*	.12
	Sig. (2-tailed)	.467	.107	.329	.576	.232	.654	.019	.293
MP6	Spearman's ρ	.11	.07	.07	.01	.09	-.09	.10	.19
	Sig. (2-tailed)	.186	.452	.389	.924	.413	.420	.356	.113
MP7	Spearman's ρ	.08	.08	.08	-.20*	.16	.13	.01	.04
	Sig. (2-tailed)	.362	.334	.338	.026	.145	.255	.909	.708
MP8	Spearman's ρ	.03	.19*	-.01	-.13	.17	.19	.28**	-.07
	Sig. (2-tailed)	.695	.029	.232	.166	.111	.089	.010	.540

Notes. CERT = Certainty, SRC = Source, SIMP = Simplicity, JUST = Justification. For descriptions of the MPs, see p. 3. * p < .05, ** p < .01.

Regressions

We ran several categorical regressions with optimal scaling categorical regression (CATREG) to further evaluate differences in PTs' beliefs as reported in the MIBS and DFEBQ. CATREG simultaneously scales ordinal, categorical, and numerical variables to find the best-fitting model (van der Kooij, 2007). We used the overall median value on each scale as a proxy "global" score for each individual (Abell et al., 2009). For both scales, the median overall score (MIBS_Overall and DFEBQ_Overall) was set as an ordinal variable by ranking, age was set as a numerical variable, and major (primary or secondary), race, and gender were set as nominal variables by grouping. However, gender and race were highly correlated with participants' major in our data because primary majors in our sample were overwhelmingly White and female. Accordingly, to mitigate the risk of being unable to differentiate major as a predictor from predictors gender and race, we report here on a CATREG we ran separately for each major, with gender, age, and race as the only predictors.

As shown in Table 3, among primary majors, the MIBS and DFEBQ regression models explained only 2% (R^2 = .02) and 3% (R^2 = .03) of the variance in overall scores, respectively, and the variance explained with all three predictors was not significant, MIBS: F (3, 149) = .882, p = .452; DEFBQ: F (4, 145) = 1.005, p = .407. Conversely, among secondary double-majors, the MIBS and DEFBQ regression models explained 16% (R^2 = .16) and 36% (R^2 = .36) of the variance in overall scores, respectively, and the variance explained with all three predictors was significant, MIBS: F (6, 81) = 2.598, p = .024; DFEBQ: F (6, 80) = 7.624, p < .001. Gender was a significant predictor in both models. In the MIBS, gender had a partial correlation = .35 or 12% of the variance explained when removing the other variables, and Pratt's measure of relative importance = 76% (Pratt, 1987). In the DFEBQ, gender had a partial correlation = .59 or 35% of the variance explained when removing the other variables, and Pratt's relative importance = 93%. This indicated that female secondary double-majors in our sample tended to agree more with reform-based mathematics instruction beliefs and dynamic mathematical epistemic beliefs, respectively, than their male counterparts.

Table 3

Standardised Coefficients, Partial Correlations, Pratt's Relative Importance, and Tolerance Values for Categorical Regression of MIBS and DFEBQ Scores

		Bootstrap (1000) Estimate of Std.					Tolerance	
Variable	*β*	Error	*df*	*F*	Partial	Importance	After	Before
				MIBS				
gender	.35	.14	2	5.90[b]	.35	.76	.99	.97
age	-.03	.10	1	.10	-.04	.00	.98	.99
race	.19	.14	3	1.83	.20	.24	.98	.96
				DFEBQ				
gender	.59	.31	2	3.58[a]	.59	.93	.99	.97
age	.04	.14	1	.08	.04	.01	.99	.99
race	.17	.13	3	1.70	.17	.06	.99	.96

Notes. Secondary-double majors' MIBS (n = 90) and DFEBQ (n = 89) data only. Dependent Variables: MIBS_Overall, DFEBQ_Overall.
[a] p < .05, [b] p < .01.

Preservice Teachers' Understandings of Reform-based Mathematics Standards

We identified four categories and nine subcategories from our interview data after two cycles of coding; however, we only report here on categories relevant to RQ3: PTs' references to reform-based mathematics standards and their mathematics instruction and mathematical epistemic beliefs.

Mathematics standards

Location. All primary majors interviewed (n = 7) remarked having encountered the MAFS at some point in their teacher preparation. However, most of these references were brief and thin in detail. Only one primary major mentioned the MPs by name during the interview. Among secondary double-majors interviewed (n = 7), all but two recalled in detail when the MAFS were used in their education courses. Additionally, three secondary double-majors indicated they knew the MPs very well in the survey but did not refer to them by name in their interviews.

Knowledge level. Most references to reform-based mathematics standards among primary majors were related to classroom assignments that required lesson planning; participants recalled going over a list of standards but not in-depth or otherwise did not remember them too well. Only one primary major acknowledged knowing the connection between the MAFS and the MPs, adding she originally did not see the connection, but now it made sense to her. Likewise, knowledge of reform-based mathematics standards among secondary double-majors was limited to classroom assignments that required lesson planning. For example, participants acknowledged becoming familiar with the MAFS and the MPs in their two introductory mathematics education courses but indicated their acquaintance was largely confined to planning lessons for their first year. A senior in the program further remarked

PTs were supposed to always incorporate the MAFS and the MPs in their lesson plans, and another added the MPs were brought up only when working on these assignments.

Challenges and reservations. For the most part, PTs across both majors were sympathetic to teaching with reform-based mathematics standards, though many of them also expressed some reservations. Most of the criticisms coming from primary majors were directed toward issues of implementation rather than the standards' actual content. They expressed their concern that some teachers may have trouble accepting these standards because it would challenge the way mathematics is traditionally taught, a sentiment they extended to parents too. Other primary majors were unsure about how much classroom time they should dedicate to a particular standard; at least one primary major thought the introduction of reform-based mathematics standards was confusing and significantly affected standardised testing across the state.

Issues of implementation were also present among secondary double-majors but to a lesser extent. One secondary double-major acknowledged that not being deeply familiar with the MAFS could compromise her ability to see the bigger picture, and another speculated experienced teachers and parents would have the most difficulty due to how they were taught mathematics in the past. However, at least two PTs went beyond implementation and spoke of the presentation of content in the standards, though they drew different inferences from it. One of them referred to the MAFS as being too vague and indicated having difficulty finding standards that would match the lessons he was planning. Conversely, the other secondary double-major acknowledged some of the standards were vague but thought that was intentional as the standards developers hoped the new standards could be applied to other subjects besides mathematics.

Beliefs

Mathematics instruction beliefs. Primary majors interviewed agreed students should take an active role in their learning of mathematics, though they also showed reservations regarding how much control a teacher should relinquish during a lesson. PTs highlighted the importance of productive struggle and conceptual understanding over just getting the correct answers quickly; however, some acknowledged unpacking students' solutions takes time and raised concerns they would not be able to afford to spend multiple days on a single lesson. Others were more reserved in their answers regarding the amount of information or involvement a teacher should have when students engaged in problem-solving. At least one primary major acknowledged it is difficult to strike a balance between letting students explore freely and making sure they are meeting curricular goals.

For their part, secondary double-majors spoke of students being responsible for their mathematical learning and distanced themselves from the idea that mathematics teachers are there merely to tell students how to solve problems or to judge answers as right or wrong. They expressed their belief that mathematics teachers should take time to explain how and why certain answers make sense and let students draw connections and not just follow a procedure, though at least two highlighted the advantages of direct instruction and exposing students to efficient and standardised strategies for solving problems. Another idea shared by many secondary double-majors and closely connected to instruction was students' ability to understand or reason mathematically. PTs characterised students' reasoning as a form of formative assessment. For instance, one of them believed students providing their reasoning helps not only the teacher identify potential errors but also assess whether students understand a mathematical idea or not. Others stressed students being able to explain their reasoning is tantamount to determining their level of understanding and measuring a lesson's success. Interestingly, secondary double-majors who responded this way often remarked that students should be given opportunities to think or solve problems on their own to foment their reasoning. Incidentally, one PT commented that it was important to tell students that teachers are not the sole providers of all information and that if students encounter problems they do not understand, they should not rely entirely on the teacher for help.

Mathematical epistemic beliefs. Overall, primary majors spoke of their beliefs about mathematical knowledge within the context of mathematics instruction, frequently confirming their survey responses.

PTs defined mathematics primarily as problem-solving and subject to change, like any other discipline. Almost all expressed their belief that there is always a right answer in mathematics. Additionally, primary majors tied originality to something students do, such as coming up with surprising answers or using their creativity to solve problems. Two PTs, however, stated they never had an original mathematical idea and, even if students have original ideas to share, some things cannot be done in mathematics (e.g., 2 x 2 = 4 is a true statement students cannot change). Finally, primary majors reflected on whether it was the judgment of others or personal experience that better justified mathematical knowledge. Some PTs trusted other authority figures, such as mathematicians, while others relied on first-hand experience, or saw it as a mixture of their logic, reasoning, and the consensus of fellow teachers.

Secondary double-majors had more to say about the meaning of mathematics and the nature of mathematical knowledge. For instance, two believed mathematics to be the science of numbers or quantity, with direct applications to other scientific disciplines and our understanding of the world. One PT believed mathematics is primarily a creative activity, whereas another thought mathematics has connections to everything. Other PTs expressed the belief that mathematical knowledge is evolving, never fixed, with new mathematical applications discovered every day. Almost all secondary double-majors stated personal experience is the best way to know something in mathematics; however, most also believed the opinion of mathematicians and other experts had value and must be considered. For example, one PT stated using one's knowledge to come up with ways to answer questions is always preferable to having someone else dictate how it is done. Others acknowledged in certain circumstances one should reach out to more knowledgeable individuals or sources to find a solution to a mathematics problem, and at least one PT raised her concern that some mathematics textbooks were deficient or downright wrong in their content, adding she would not want her students to think they could not challenge what the book says or what anyone says.

Discussion

The findings reported here provide evidence for subtle differences in PTs' knowledge of reform-based mathematics standards and their mathematics instruction and mathematical epistemic beliefs. These differences extend from the strength of certain relationships between those beliefs by major, to whether coming up with original ideas is possible during mathematical problem-solving. Although not all these differences may be equally relevant to the future of mathematics education reform, nonetheless, their existence merits further discussion.

Despite much controversy and being in the spotlight for years (Resmovits, 2012; Schneider, 2015; Tampio, 2018), PTs' knowledge of reform-based mathematics standards in the state was not extensive. Close to half of the primary majors and secondary double-majors in the sample indicated having only partial knowledge of the MAFS, and about a third did not know them at all. Likewise, thorough knowledge of the MPs was below 50% across the two majors. Follow-up interviews confirmed PTs' knowledge of reform-based mathematics standards, regardless of major, was largely limited to lesson planning assignments.

Among primary majors who stated knowing the MAFS partially or very well, most correctly identified their primary purpose as fomenting students' conceptual understanding of mathematics (McCallum, 2015; Wu, 2011; Zimba, 2014). This was less the case among secondary double-majors, a third of whom indicated the MAFS' purpose was teaching students new skills for higher grades. Similarly, although almost a quarter of all secondary double-majors stated they knew the Mathematical Practice standards very well, only 15% of primary majors said the same.

These discrepancies may reflect differences in professional expectations across the two majors, as primary teachers focus more on foundations, whereas secondary teachers focus more on preparing students for college or life beyond high school (Book & Freeman, 1986; Hamman et al., 2013). Additionally, primary PTs are expected to teach more than one subject and thus have less of an incentive to learn one area in-depth, whereas secondary PTs are often expected to teach only one subject and draw more on their knowledge for instruction (Hamman et al., 2013; Opfer et al., 2017). The way the standards were written could also have influenced some of these responses, as the emphasis on

conceptual understanding in the content standards becomes less pronounced in the higher grades (Koestler et al., 2013; Schoenfeld, 2014). Nonetheless, the fact that between one-quarter and one-half of participants who claimed to know MAFS partially or very well seemed to misunderstand their purpose suggests surface-level knowledge of the standards. We also note that regardless of what answer they chose, most PTs in our survey and interviews had reservations about the standards achieving their intended primary purpose.

What could explain PTs not being conversant with reform-based mathematics standards? One factor may have been timing: when these standards went into effect in 2014, many PTs in our sample were transitioning from middle school to high school and would have had less time to become familiar with either the MAFS or the MPs (McCallum, 2015; Zimba, 2014). Another consideration is the number of opportunities PTs had to become acquainted with reform-based mathematics standards during their teacher preparation program. As mentioned previously, primary majors' first serious introduction to the MAFS and MPs usually occurred in their fifth term, whereas secondary double-majors were often introduced to these standards in their third term. Nonetheless, there was some indication PTs' exposure to the MAFS and MPs in either program may have been insufficient to overcome the "washout" effect, which states PTs would often resort to traditional forms of instruction once they moved to in-service teaching (Zeichner & Tabachnick, 1981).

Unlike what some of the literature suggested, the mathematics instruction and mathematical epistemic beliefs of PTs were not strongly correlated overall. The case was particularly marked among primary majors, as statistical tests failed to find any substantial correlations, a finding somewhat contrary to prior research implying certain associations between these beliefs (Hofer, 1999; Muis & Foy, 2010). However, there is some evidence ambivalent beliefs are more common among preservice primary teachers (Roscoe & Sriraman, 2011; Seaman et al., 2005), which could explain part of the discrepancy. Conversely, some moderate relationships between these beliefs were found among secondary double-majors. Specifically, those secondary double-majors who agreed with reform-based mathematics instruction beliefs standing for mathematical modelling (MP4) and reasoning about procedures (MP8) were also more likely to agree with dynamic epistemic beliefs standing for mathematical knowledge as highly interrelated (SIMP).

What can explain these correlations? Both mathematical modelling and reasoning about procedures take a more predominant role in the secondary grades as students begin to handle increasingly more abstract material (Kaminski & Sloutsky, 2011; Tall et al., 2001). It is possible, though far from certain, that some secondary double-majors were thinking of this when answering the items making up these MIBS subscales, as the literature suggests preservice secondary mathematics teachers generally have more experience with symbolic reasoning and mathematical modelling than do preservice primary teachers (Birkeland, 2019; Castro, 2004; Stillman & Brown, 2011), further amplified in our sample as secondary double-majors were taking advanced mathematics courses as part of their program.

We also found when looking at secondary double-majors' survey data (which had more male participants) that gender was the only significant predictor of overall belief scores. That is, females tended to hold more reform-based mathematics instruction beliefs and dynamic mathematical epistemic beliefs than their male counterparts. Yet, we cannot rule out the possibility this reflects certain response biases, such as social desirability bias (i.e., the tendency to be seen in a favourable light) or extreme response bias (i.e., the tendency to select items at the endpoints of a scale; Bernardi & Guptill, 2008; Dalton & Ortegren, 2011). Both types of biases may restrict the range of responses and reduce the correlations among items, which could make the strength of the relationships between beliefs look lower than it was (DeMaio, 1984; Dolnicar & Grün, 2007).

Most PTs interviewed also expressed beliefs closer to reform-based mathematics instruction during their interviews, perhaps as a reflection of the messages conveyed in their teacher preparation programs (Leavy & Hourigan, 2018). For instance, most PTs believed children should take control of their learning, but not everyone understood this in the same way. Primary majors took that to mean students leading the class, although some struggled with the idea of when to intervene during a lesson. On the other hand, some secondary double-majors interpreted that to mean the students were responsible for seeking assistance if having difficulties or finding materials beyond the ones a teacher provided.

Incidentally or not, those advocating for this form of self-learning also happened to disagree with most dynamic mathematical epistemic beliefs in the survey, suggesting a connection between their conceptualisation of mathematical knowledge (e.g., top-down, fixed) and their understanding of mathematics instruction (e.g., direct, teacher-led; Buehl & Fives, 2016; Weinstock & Roth, 2011).

Limitations

This study used a non-random sample to make inferences about PTs' beliefs and knowledge of reform-based mathematics standards at a prominent Southeastern research university in the United States, and thus our findings have limited generalisability due to the characteristics and politics of the state. Sampling PTs in different states that have also implemented reform-based mathematics standards would help strengthen our results. Furthermore, other researchers have pointed out issues about inferring instructional practices from belief measures (Bullough, 2015; Wilke & Losh, 2008). To overcome some of these limitations, survey and interview responses were compared with the effects theorised in the literature for pattern-matching (Rothland, 2012; Yin, 2003). Finally, given political partisanship in the state, both the MAFS and the MPs were replaced in 2023.

Implications for Practice

Experts agree teachers play a decisive role in implementing standards as originally intended and correct implementation requires a careful combination of mathematical and pedagogical content knowledge, beliefs, and practices (Olson et al., 2014; Opfer et al., 2017; Schmidt & Houang, 2012). PTs' lack of knowledge regarding reform-based mathematics standards may result in these standards being incorporated into lessons in ways that are perfunctory or ineffective and, thus, do not promote students' mathematical learning (Drost & Levine, 2017; Silver et al., 2005). Furthermore, the kind of mathematics instruction these standards are meant to support cannot be achieved unless there is a close integration between the content standards and the MPs. Yet, on-the-job training and professional development for teachers often miss this close relationship entirely, leaving teachers to draw the connection on their own (Bostic & Matney, 2013; Koestler et al., 2013; Olson et al., 2014). Additionally, it is conceivable that being more familiar with reform-based standards may, by extension, make one more willing to accept the reform-based practices that inspired them (Kruse et al., 2017). If so, in-depth knowledge of reform-based mathematics standards could represent a kind of "competitive advantage" PTs could possess before entering the profession and could ease whatever difficulties in teaching with the standards may surface in their future instructional practice (Farfan et al., 2020; Schoen et al., 2003; Wang, 2002), such as being comfortable with students' thinking deviating from standard formulas and procedures during problem-solving (Stein et al., 2017). For countries looking to improve mathematics instruction through reform-based mathematics standards, however, the biggest lesson might be to ensure that their teacher preparation programs include several opportunities for PTs to become well acquainted with said standards beyond lesson planning assignments, as relaying solely on those here resulted in insufficient alignment with the principles behind the MPs.

Our findings also suggest primary and secondary preservice teachers held similar mathematics instruction and mathematical epistemic beliefs on the surface but that the strength of the relationship between these beliefs varied depending on PTs' program of study and gender. Given the need for more work done in this area (Beswick & Callingham, 2014; Rott & Leuders, 2017), our findings contribute to the evidence necessary to make better predictions. It is also important to reiterate other factors may exert positive or negative influences on individuals' mathematics instruction and mathematical epistemic beliefs. For example, senior teaching colleagues or instructional coaches can help nurture new teachers' beliefs and practices, or they can bring those beliefs and practices into conflict with the school culture (Sun et al., 2014; Zeichner et al., 1987). Thus, PTs should not only be made aware of their beliefs regarding mathematics and mathematics instruction during their teacher preparation but also be provided with opportunities to explore how the beliefs could be challenged or sustained in their future practice, including when instructing with reform-based mathematics standards.

Conclusion

The findings reported here show subtle differences in PTs' knowledge of reform-based mathematics standards as well as of their mathematics instruction and mathematical epistemic beliefs. Specifically, the fact that many PTs held reform-based mathematics instruction beliefs while at the same time not being thoroughly familiar with reform-based mathematics standards is something that needs to be evaluated carefully. It does not bode well for the future of mathematics education reform in the United States if, after years of significant state-wide investments, PTs are only partially or superficially acquainted with reform-based mathematics standards—or any standards for that matter. Only time will tell if this was a particularity of these mathematics standards, or if this is a phenomenon that repeats itself whenever new standards are introduced. Amid doubts regarding mathematics education reform worldwide (Friedberg et al., 2018; Shimizu & Vithal, 2023), more can be done to assist PTs to examine their beliefs and their learning of reform-based mathematics standards so that they can teach them in ways consistent with the principles behind mathematics education reform.

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Competing interests

The authors declare there are no competing interests.

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