

# Exploring Instruments for Assessing Initial Mathematical Knowledge in Primary Teacher Education

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Knowing the mathematical knowledge of students entering primary teacher education is key to guiding admission or diagnoses to inform teacher education programs. To understand how students' initial mathematical knowledge is organised, the study draws on the idea of Fundamental Mathematical Knowledge, which had been defined as the mathematical knowledge necessary for students to be able to successfully follow subjects about mathematics and teaching during their teacher education. This study examined two questionnaires that were designed in Catalonia and Chile to gauge the disparity between students' mathematical knowledge at the beginning of their teacher education, and a specification of Fundamental Mathematical Knowledge defined around curricular content blocks. The scores obtained in both questionnaires by two groups of students—one in Barcelona and another in Santiago de Chile—constituted the data. The quantitative analysis of the data enabled the psychometric validation of the instruments for the measurement of students' initial mathematical knowledge and revealed that the structure of this knowledge was unidimensional and behaved as a single latent construct. From the study it was concluded that teacher education programs should be aimed at progressing towards the development of a deeper understanding of elementary mathematics and its conceptual connections.

**Keywords:** fundamental mathematical knowledge • mathematics teacher education • student teachers • assessment instrument validation

## Introduction

A major concern in mathematics education is whether teachers have sufficient content knowledge to teach mathematics effectively (Hiebert et al., 2019). This concern is shared by teacher education programs as content knowledge is considered a crucial part of the curriculum (Feuer et al., 2013). Different theoretical frameworks have been developed to characterise mathematics teachers' knowledge, addressing various aspects of their education and professional practice (Carrillo et al., 2018; Rowland et al., 2009). In particular, there has been a focus on the mathematical knowledge required to teach primary school mathematics (Ball et al., 2001; Ball et al., 2012; Hiebert et al., 2002; Hill et al., 2007). Additionally, there are studies that explored the development of mathematical knowledge during teacher education (Ball et al., 2001; Ball et al., 2008; Cooney & Wiegel, 2003; Hiebert et al., 2002). Also, there are studies that established the characteristics of mathematics knowledge for teaching and identified subject areas to inform the curriculum of teacher education programmes (Association of



Mathematics Teacher Educators, 2017; Ball et al., 2008). There is, however, scant research exploring the mathematical knowledge of the students at the beginning their education as teachers.

The lack of a thorough theoretical debate on desirable prior knowledge for students entering teacher education programmes, combined with varied admission methods, poses significant challenges for teacher education institutions. Miller-Levy et al. (2014) pointed out that teacher education programmes admission requirements should assess, among other aspects, candidates' prior knowledge of the content of the different subjects they will later have to teach. Teacher education students have successfully passed the previous educational stages established by each system and the applicable university entrance exams. Several studies, however, show that candidates admitted to the programs continued to display gaps in their mathematical knowledge (Ball, 1990; Beswick & Goos, 2012; Ingram & Linsell, 2013; Ryan & McCrae, 2005/06; Senk et al., 2012; Tatto et al., 2008). Even so, a review of the published literature revealed that little attention has been given to the mathematical knowledge possessed by students beginning teacher education (Gorgorió et al., 2021; Linsell & Anakin, 2012, 2013).

At a theoretical level, there is no consensus on the specific mathematical content or competences desired of incoming students. Nor has it been defined what the instruments should be and how they could be used to characterise students' knowledge when they join teacher education programs (Gorgorió, Albarracín, & Villarreal, 2017). At the social level, too often, training programs do not have validated instruments to assess the mathematical knowledge of incoming students, either for selective or diagnostic purposes. It is, therefore, necessary to create instruments to assess applicants or obtain diagnostic data to inform and guide training programs. It is also important to have a theoretical framework to understand this knowledge and valid tools to assess it.

This study draws on the idea of Fundamental Mathematical Knowledge as that mathematical knowledge that students entering teacher education need to successfully pursue the mathematics and mathematics teaching syllabi included in their courses (Castro et al., 2014; Gorgorió & Albarracín, 2020; Gorgorió et al., 2021). Following on from the work presented in Albarracín et al. (2021), this study examined two questionnaires that had been designed in two different national contexts to study the disparity between students' mathematical knowledge at the beginning of their teacher education and a specification of Fundamental Mathematical Knowledge (Gorgorió & Albarracín, 2020). The scores obtained by administering the questionnaires to two groups of students, one in Barcelona and the other in Santiago de Chile, constituted the data for the study. A quantitative analysis of the data made it possible to achieve the study goals, namely: to psychometrically validate both questionnaires as instruments to measure the mathematical knowledge of students starting a teacher education program and to analyse whether the structure of the mathematical knowledge exhibited by these students matched the thematic grouping established theoretically when defining fundamental mathematical knowledge. To ascertain the mathematical knowledge structure of students entering a primary school teacher training programme, it is necessary to employ psychometrically validated instruments. The result of this validation has the potential, in addition to obtaining a valid instrument, to give an account of how this knowledge is organised, either by blocks of mathematical content or as an interrelated whole.

### *Context of the Study*

This study focused on two different educational contexts: Catalonia in Spain and Chile and is the result of a collaboration between two teams, one from each country. In Spain, most studies assessing students' mathematical knowledge at the start of teacher training used instruments aimed at schoolchildren (Arce et al., 2017; Nortes & Nortes, 2013; 2018). For this reason, in Catalonia, the group led by Gorgorió developed a specific instrument to assess student teachers' mastery of the basic mathematical knowledge they would need to successfully complete their teacher education (Gorgorió & Albarracín, 2020). The results of Gorgorió's group suggested the need to consolidate tools to ensure that evaluation of students' mastery of basic mathematical knowledge is sufficiently solid. The group's efforts led to the requirement that in Catalonia, from the 2017–2018 academic year, candidates for primary teacher



training programmes must pass a personal aptitude test that assesses their communicative and mathematical competence<sup>1</sup>.

In Chile, the *Sistema de Desarrollo Profesional Docente* or Teacher Professional Development System (Law 20.903, April 1, 2016) mandates the use of two diagnostic assessments, one at the beginning of the programme and another in the penultimate year, to monitor student knowledge. However, training institutions, however, are responsible for developing their own diagnostic instruments, which has resulted in the use of a wide variety of instruments that are often not reliable enough to be able to make decisions about how to support students in their initial studies (Giaconi et al., 2022). To advance towards common instruments among universities, Martínez et al. (2019) studied whether students starting teacher education in Chile had a primary education level of mathematical knowledge and explored their beliefs on mathematics, its teaching and learning. This study reported that regardless of the university of origin, students perform poorly in algebra, statistics and probability, together with an arithmetic perception of mathematics taken as a whole. In this way, unified studies between educational institutions provide a comprehensive overview of the mathematical knowledge that entrants possess; new unified studies applied to various universities confirm the results. For example, Rojas et al. (2021) showed that those who enter teacher education mostly have a medium or low mastery of school mathematics.

## Fundamental Mathematical Knowledge

The mathematical knowledge that students already possess when they enter teacher education programmes is a topic scarcely researched even though their background mathematical knowledge may be predictive of their success in developing mathematical knowledge for teaching (Baumert et al. 2010). The variety of theories developed so far that describe the professional knowledge of mathematics teachers reflects the complexity of the knowledge and competences demanded by mathematics teachers' professional practice. Concepts such as *subject knowledge*—as a component of Pedagogical Content Knowledge, PCK (Shulman, 1987), *common content knowledge*—as part of Mathematical Knowledge for Teaching, MKT (Ball et al., 2008), or *knowledge of the topics* within Mathematical Teacher Specialised Knowledge, MTSK (Carrillo et al., 2018), guide much of the research being carried out today related to mathematics teachers' education.

None of the frameworks described above indicate how subject knowledge, common content knowledge, or knowledge of the topics, are articulated or characterised when students enter initial teacher education. The Knowledge Quartet (Rowland et al., 2005), which is a much more knowledge-oriented framework for mathematics classroom management, introduces the *foundations dimension*, which includes knowledge and understanding of mathematics that informs the decisions of the future classroom teacher. Again, the mathematical knowledge with which prospective teachers arrive at initial teacher education is not covered, since the focus is on mathematical knowledge that is instrumental to classroom activity.

It appears that the relevance of student teachers' initial mathematical knowledge goes unmentioned in the theories that discuss the knowledge needed to teach mathematics at school. Consequently, the assessment of student teachers' mathematical knowledge, either as a precondition for admission or for diagnostic and training purposes at the beginning of their studies, has rarely been the subject of research. Moreover, the kind of prior mathematical knowledge that should be considered desirable and the methods that could be used to assess it has only been described tentatively up to now (Gorgorió, Albarracín, & Villarreal, 2017). The research reported in this article is intended to contribute to the debate through an exploration of two instruments used for assessing students' mathematical knowledge at the beginning of their primary teaching degrees.

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<sup>1</sup><https://www.educaweb.cat/continguts/educatiu/estudis-universitaris/prova-aptitud-personal-pap/pap-educacio-infantil-educacio-primaria/>



This study draws upon the idea of *Fundamental Mathematical Knowledge* (from now on FMK). In a definition in Castro et al. (2014), FMK was presented as the disciplinary knowledge of mathematics needed to successfully progress through courses of mathematics and its teaching, considering the requirements of professional practice and the mathematical competences of primary education. FMK is the desirable initial disciplinary knowledge that the student can draw on to develop the specialised and pedagogical content knowledge needed to work as a teacher.

FMK incorporates elements of the deep knowledge discussed by Ma (1999), in that it can serve as the cornerstone that supports the didactics of mathematics and student teachers' learning during mathematics courses. The idea of FMK also ties in with the interpretation that Hiebert et al. (2019) make of common content knowledge as "[the] knowledge that people learn in school" (p. 6), when they specified the definition of Ball et al. (2008). FMK is also like the foundation content knowledge described by Linsell and Anakin (2012), which includes indivisibly linked conceptual and procedural knowledge. However, none of the research noted here (Hiebert et al., 2019; Linsell & Anakin, 2012; Ma, 1999), explicitly addressed specific mathematical content.

### *Specifying Fundamental Mathematical Knowledge*

In the FMK specification process, the study initially worked with two groups of experts. One was an international team of researchers from Finland, England, and Sweden (see Gorgorió, Albarracín, Ärlebäck et al., 2017), and the other group was made up of representatives of all the Catalan universities (see Gorgorió & Albarracín, 2020). Subsequently, the Chilean research team was enlisted for the development of this study, given their interest in the definition of mathematics diagnostic questionnaires for their institutions.

From a pragmatic standpoint, an approach used by Ryan and McCrae (2005/06) and Linsell and Anakin (2012) was adopted for the study. Their approach used the curricula of their respective countries' school systems as a framework for examining future teachers' initial knowledge. Although school mathematics may be bounded by its context and use different resources and procedures, the core concepts that lie at the root of elementary mathematics are universal. Therefore, our starting point was the mathematics syllabi set by the national curricula of schools in Catalonia (Departament d'Ensenyament, 2015), Finland (National Board of Education, 2016), Sweden (Skolverket, 2011) and England (Department of Education, 2013), limiting our attention to primary education in each case. The present study also incorporates the analysis of the Chilean primary education curriculum (Martínez et al. 2019; MINEDUC, 2012).

Considering that the aim in Gorgorió, Albarracín, Ärlebäck et al. (2017) was to characterise or assess explicable knowledge in a written questionnaire designed for easy implementation in institutions, a content analysis of the different syllabi was conducted. The authors' first step was to identify the common areas, analysing the scope and importance given to each area, and then to decide which concepts and procedures should be predetermined when specifying FMK and which were important but not essential (see Gorgorió & Albarracín, 2020), thereby obtaining a concrete proposal for FMK.

The FMK content blocks chosen for the development of this study are those specified by Gorgorió, Albarracín and Villarreal (2017; p. 61):

- **Numeration and Calculation.** Students must demonstrate an understanding of and ability to represent and use natural numbers, integers, and rational numbers in different situations; an understanding of the meaning and properties of operations and the relationships between them; a knowledge of the meaning of divisor; and a mastery of the skills needed to solve problems of factorisation and division of natural numbers.
- **Relationships and Change.** Students must demonstrate an ability to identify and generalise patterns that are not necessarily numerical; to identify and interpret relationships of dependence between variables; to interpret and construct graphs that express relationships of change. It is also necessary to demonstrate an integrated understanding of the meanings of numerical proportionality and ratio and the ability to use these concepts to solve different problems.



- **Space and Shape.** Students must demonstrate a knowledge of the characteristics and properties of two- and three-dimensional geometric figures and the ability to apply them in different situations; an understanding and ability to represent and use reflections, rotations, and translations; an integrated understanding of the meanings of geometric proportionality, similarity, and scale; and the ability to use visualisation strategies to solve geometric and non-geometric problems.
- **Measurement.** Students must demonstrate a knowledge of the meaning of measurable magnitude (angle, length, area, volume, capacity, mass and time) and of measurement processes; a knowledge of the corresponding decimal and sexagesimal units of measurement and of the mechanisms required to solve problems with a change of units; and a mastery of the knowledge and skills needed to solve diverse problems related to the ideas of perimeter, area and volume.
- **Statistics and Chance.** Students must demonstrate the ability to interpret, analyse, draw conclusions, and make predictions from statistical data; to interpret and construct statistical graphs; to interpret and calculate measures of centralisation; and to understand the meaning of chance.

This specification of mathematics content guided the development of the data collection instruments used in this study.

## Initial Questionnaires

The knowledge of a discipline includes knowledge of a formal and explicit nature, together with tacit knowledge. Explicit knowledge can be assessed in various ways: through observation, interviews and written tests, among others. The instruments in this study were two written questionnaires developed in response to an institutional need, namely, to assess the explicit mathematical knowledge of students beginning their education as teachers. The questionnaires were designed taking the FMK specification as a reference. Since the goals of this research refer specifically to the questionnaires that were used, the questionnaires are presented first, followed by a description of the goals of the study.

The *Fundamental Mathematical Knowledge Questionnaire* was created in Catalonia. It was comprised of a set of 25 items, from O1 to O25, 23 of which were short-answer questions and the other two were multiple-choice questions. (for further detail, see Gorgorió & Albarracín, 2020). In Catalonia, students are familiar with this type of test, as it is often used for their assessment in schools. The *School Mathematical Knowledge Questionnaire*, based on the work of Martínez et al. (2019) in Chile, was comprised of a set of 25 items, from C1 to C25, all were multiple-choice. Each item has four possible answers of which only one is correct. Chilean students are accustomed to doing multiple-choice tests both during their schooling and in university entrance exams. Throughout this article, the Catalan questionnaire will be referred to as the *open questionnaire* and the Chilean one as the *multiple-choice questionnaire*.

The *open questionnaire* was part of the system of assessment of the Mathematics for Teachers course, studied in the first year of the primary teacher education programme at the Universitat Autònoma de Barcelona (UAB). It assessed knowledge of mathematical content related to the curriculum of the Catalan compulsory education system: Numbers and Arithmetic; Space, Shape, Magnitude and Measurement; Relationships and Change; and Statistics and Randomness. In the questionnaire, the organisation of the content areas and the weight given to them (see Table 1) was deliberately linked to their weight and organisation in classroom practice. Answering the questions required an understanding of the concepts and procedures involved, and the results had to be interpreted within the context of each problem.



Table 1  
*Distribution of Items by Content Block in the Open Questionnaire*

Content Block	Number of Items
Numbers and arithmetic	8
Space, shape, magnitude and measurement	9
Relationships and change	4
Statistics and randomness	4

The questionnaire content and structure were defined by a panel of experts from four Catalan universities within the framework of the *Estudi per a l'avaluació diagnòstica de les competències matemàtiques dels estudiants del grau en Educació Primària*<sup>2</sup> [Study for the Diagnostic Evaluation of the Mathematical Competences of Students of the Degree in Primary Education]. The questionnaire was pilot tested on three occasions with students at the UAB, who provided feedback that resulted, among other aspects, in the correction of problematic wording of statements. Furthermore, the pilot-tests at the UAB served to contrast the reliability of the instrument (see Gorgorió et al., 2021) and confirm its criterion validity through a study of the correlation between the students' results in the questionnaire and the grades obtained in the Mathematics for Teachers course (for a complete description of the process, see Gorgorió & Albarraçin, 2020). By way of an example, Figure 1 shows the O8 and O21 items.

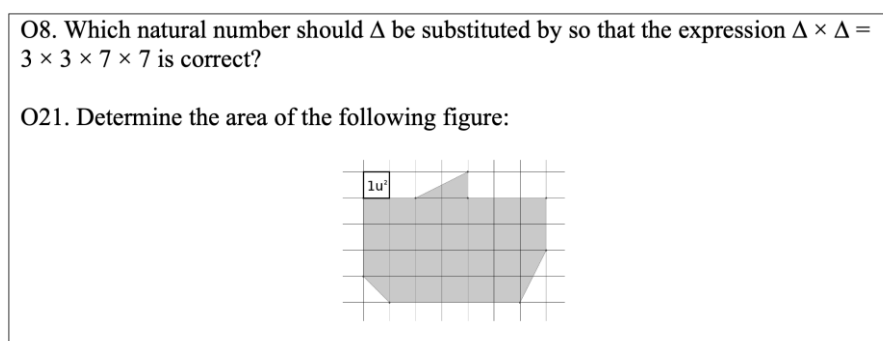


Figure 1. Example of two short-answer items.

Item O8 focused on the multiplicative structure of natural numbers. To solve it, students had to have a conceptual knowledge of prime factorisation, as they could not use any specific algorithm to solve the problem. In Item O21, the content to be assessed was related to the measurement of surfaces; students had to understand the concept of unit of measurement and know that the diagonal of a parallelogram divides it into two triangles of equal surface area.

The *multiple-choice questionnaire* was based on a test developed by Martínez et al. (2019) that sought to measure the school mathematical knowledge acquired during primary education by those entering primary teacher education in Chile. The study drew on the official primary education curriculum (Grades 1 to 6) of Chile (MINEDUC, 2012) to construct a set of items that covered the four thematic blocks, namely: Numbers and operations, Patterns and algebra, Geometry and measurement, and Data and probability. Following its validation by experts and its implementation with over 500 student teachers in their first year and the respective psychometric analyses, a validated scale with 40 items was obtained (for further details, see Martínez et al., 2019). Based on this validated instrument, 25 items were selected for this study, from C1 to C25, to equate it with the *open questionnaire*. Bearing in mind that the *multiple-choice questionnaire* was to be applied in Catalonia as well as in Chile, various aspects were

<sup>2</sup> 2014 ARMIF-00041, AGAUR, Agència de Gestió d'Ajuts Universitaris i de Recerca.

given consideration when selecting the 25 questions, including the curricular relevance of the items in the two contexts. The set of items selected for the *multiple-choice questionnaire* maintained the proportion per block of content of the Martínez et al. (2019) instrument, which was already constructed according to the curricular weight of each block, as shown in Table 2.

Table 2

*Distribution of Items by Blocks of Content in the Multiple-choice Questionnaire*

Content Block	Number of Items
Numbers and operations	12
Geometry and measurement	6
Patterns and algebra	3
Data and probability	4

In each of the multiple-choice items, the incorrect options were intended to be plausible answers based on students' misconceptions when trying to solve the problems. For example, the aim of Item C11 (see Figure 2) was to find out if students knew how to compose and decompose numbers.

C11. Which of the following equalities is correct?

A.  $30,000 + 2,000 + 50 + 700 + 2 = 32,572$

B.  $11,000 + 200 + 105 + 30 = 11,335$

C.  $60,000 + 5,000 + 30 + 1 = 6,531$

D.  $400 + 132 = 400,132$

Figure 2. Example of a closed item.

Option A confused 50 and 500, with students being distracted by the place occupied by the 50 in the order of the sum instead of considering the positional value of the 5 in 32,572. Option C involved recognising the digits with the highest positional value while adding up and interpreting them as the digits that should be placed in the corresponding positions. Here, there was no grouping of hundreds and the positional value of 6 was not interpreted correctly. Finally, in option D the addends were joined in the order they appeared. This illustrated a common mistake when reading numbers, since it associates four hundred thousand one hundred and thirty-two with four hundred and one hundred and thirty-two, using the thousands separator comma as an operator to additively break up the number.

## Study Goals

The concept of FMK fills a gap at a theoretical level and provides reflection around a specific need of teacher education institutions: the assessment of mathematical knowledge possessed by students on entering teacher education. Its specification allowed us to design instruments for its assessment. From now on, Initial Mathematical Knowledge (IMK) will refer to the knowledge that student teachers made explicit in the written questionnaires designed according to the specification of FMK.

Within this framework, the aim of this study was to validate the above-mentioned questionnaires and to find out what information they provided for the discussion of the IMK of prospective teachers. Therefore, our specific goals were as follows:

- Goal 1: To psychometrically validate the *open questionnaire* and the *multiple-choice questionnaire* as instruments to measure the IMK of students beginning their teacher education.

- Goal 2: To analyse whether the structure of the IMK exhibited by students beginning their teacher education matched the thematic grouping established theoretically when defining FMK.

## Data Collection

This research considered two comparable populations in terms of curriculum, at two institutions where the authors taught on teacher education programmes. Therefore, the two questionnaires were administered to first-year students enrolled in primary teacher education programmes at the Universitat Autònoma de Barcelona (UAB), Spain, and the Pontificia Universidad Católica de Chile (PUC), Chile. A total of 283 students answered the questionnaires: 158 in Chile in August 2019 (2019 cohort) and 125 in Catalonia April 2019 (2018-2019 cohort). At the UAB, the data collected were part of the assessment system for the first-year subject Mathematics for Teachers. At the PUC, the study had the authorisation of the Ethics Committee for Social Sciences and Humanities. In both contexts, students gave their consent authorising the use of their anonymised responses to both questionnaires as data for the analysis. Each questionnaire was graded by assigning a score of 1 or 0 to each question depending on whether the answer was correct or incorrect. The resulting sets of scores constitute the data of this study since the responses constituted the students' IMK as it had been defined.

## Analysis and Results

To achieve the research goals, a quantitative analysis of the data was carried out with a double intention. First, to psychometrically validate the two questionnaires and, second, to obtain a measure of the students' IMK to contrast whether the thematic grouping established when specifying the FMK was empirically confirmed. The analysis was carried out in three steps, which are schematised in Figure 3.

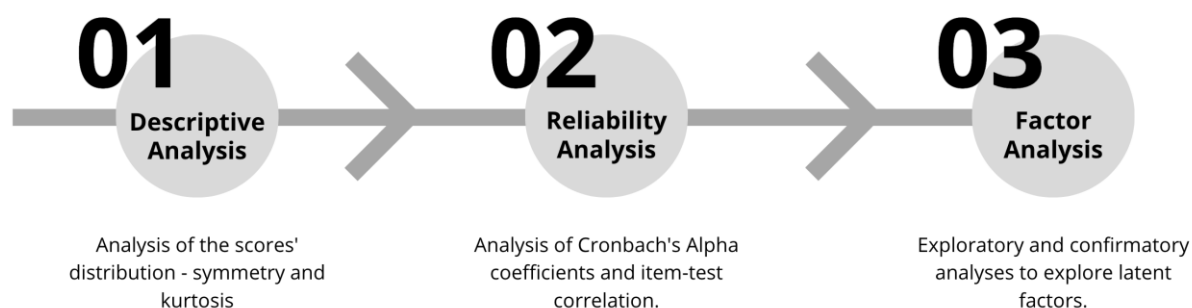


Figure 3. Steps of analysis.

The first step entailed a descriptive analysis of the number of correct answers to each item of the questionnaires to understand their distribution. Of interest was the symmetry of the distribution—whether the students' scores were skewed to one side or the other of the mean, and kurtosis—whether the students' scores of the questions were concentrated around the mean or not.

In the second step of the analysis, item reliability was studied using Cronbach's alpha coefficients, and test-item correlation was calculated using Pearson's  $r$  coefficients. Cronbach's alpha coefficients allowed for the determination of each question's suitability in measuring the students' IMK. The item-test correlation made it possible to assess the extent to which each item contributed to the assessment of students' IMK. Both coefficients were calculated for the entire questionnaires—taking FMK as a single theoretical construct—and for the questionnaires according to the content blocks that had been used to create them. Dealing with the entire questionnaires and eliminating from them those questions with low reliability and low item-test correlations, was the first step towards achieving Goal 1, validation of



the questionnaires. The analyses of the questionnaires organised by mathematical content blocks suggested an answer to Goal 2, already indicating a single structure for the IMK.

The analyses carried out in the first and second steps indicated that the initial questionnaires behaved as single scales after some of items had been eliminated. In the last stage of the process, factor analyses (both exploratory and confirmatory) of the two reduced questionnaires were carried out. The exploratory factor analysis served to find out to what extent the reduced questionnaires—and the grouping of the items by content blocks included—were latent factors, that is, whether the items adequately represented FMK when taken as a whole, as well as by blocks. The content blocks could be understood as latent factors since their significance was proposed non-empirically. Confirmatory factor analysis was used to confirm the only latent structure identified in the previous steps, while trying to conserve as much variance as possible and determining the final set of items on the validated scale for each questionnaire. The third step of the analysis offered a definitive response both to Goal 1 by defining the set of items on the validated scale, and to Goal 2 because it confirmed that once psychometrically adjusted, each of the questionnaires behaved as a single scale.

### *Descriptive Analysis: Distribution of Scores*

From now on, the term "item score" will be used to refer to the number of correct answers collected for a given item. Table 3 shows the distribution of scores, symmetry, and kurtosis, with the two questionnaires and the two institutions considered separately.

Table 3  
*Score Distribution, Symmetry, and Kurtosis*

Questionnaire	Score		Symmetry			Kurtosis		
	mean	<i>sd</i>	mean	<i>sd</i>	interval	mean	<i>sd</i>	interval
UAB Open	0.62	0.17	-1.33	1.61	[-6.15;0.51]	2.28	8.51	[-1.96;36.06]
UAB Multiple choice	0.83	0.14	-2.69	1.89	[-7.62;0.05]	8.79	13.58	[-2.02;56.55]
PUC Open	0.58	0.21	-0.48	1.07	[-2.51;1.42]	-0.65	1.73	[-2.01;04.32]
PUC Multiple choice	0.71	0.25	-1.20	1.66	[-3.79;3.79]	2.13	4.48	[-1.77;12.42]

Table 3 shows that the group of UAB students performed better on average than the PUC students in both questionnaires. The symmetry analysis of the scores showed that it was negative for both instruments and both institutions, indicating that the distribution is skewed to the right. That the distribution was concentrated in the higher scores meant that the answers tended to be correct rather than incorrect. The kurtosis analysis showed that the number of correct answers to the *open questionnaire* items was far from the mean in both institutions. This deviation indicated that students tended to have consolidated the knowledge that was part of this questionnaire. Conversely, the correct answers to the *multiple-choice questionnaire* items were mainly concentrated around the mean.

### *Reliability Analysis: The Questionnaires as a Whole*

Taking FMK as a single theoretical construct, Table 4 presents the Cronbach's alpha and test-item correlation coefficients for the *open questionnaire*. In parallel, Table 5 shows the coefficients obtained for the *multiple-choice questionnaire*. The UAB and PUC results are considered separately since alpha is a property of the scores obtained in a test by a specific sample of participants.



Table 4  
Cronbach's Alpha Reliability Coefficients and Pearson's  $r$  Item-test Correlations for the Open Questionnaire

Item	UAB		PUC	
	Alpha	r.cor	Alpha	r.cor
O1	0.71	0.43*	0.74	0.42*
O2	0.70	0.47*	0.74	0.39*
O3	0.71	0.37*	0.73	0.43*
O4	0.70	0.46*	0.73	0.45*
O5	0.71	0.40*	0.74	0.37*
O6	0.71	0.38*	0.74	0.37*
O7	0.71	0.46*	0.75	0.11
O8	0.69	0.58**	0.73	0.52**
O9	0.72	0.13	0.75	0.24*
O10	0.71	0.33*	0.74	0.39*
O11	0.71	0.36*	0.74	0.30*
O12	0.72	0.25*	0.74	0.35*
O13	0.72	0.18	0.73	0.46*
O14	0.71	0.44*	0.73	0.44*
O15	0.72	0.19	0.73	0.44*
O16	0.73	0.15	0.74	0.29*
O17	0.72	0.19	0.73	0.50**
O18	0.71	0.34*	0.73	0.46*
O19	0.73	0.09	0.74	0.32*
O20	0.72	0.17	0.73	0.45*
O21	0.71	0.44**	0.73	0.55**
O22	0.73	0.01	0.74	0.30*
O23	0.72	0.23	0.75	0.29*
O24	0.70	0.52**	0.74	0.32*
O25	0.72	0.14	0.76	0.13
GLOBAL	0.72 ( $SD = 0.01$ )	0.31 ( $SD = 0.15$ )	0.75 ( $SD = 0.02$ )	0.37 ( $SD = 0.11$ )

Note: The Alpha of each item corresponds to the reliability of the scale after eliminating that item. It should lie between 0 and 1. In turn, r.cor corresponds to the item-test correlation, with \* $p < 0.05$ ; \*\* $p < 0.01$ .

Table 4 shows that in the case of the *open questionnaire* the Cronbach's Alpha coefficient was 0.72 for the UAB sample and 0.75 for the PUC, which is considered adequate reliability (Novick & Lewis, 1967). The same table also reports that in the case of the *open questionnaire* the scores in the UAB and PUC samples behaved differently. Despite these differences, the average item-test correlations were relatively high (Novick & Lewis, 1967), with a mean equal to 0.31 ( $SD = 0.15$ ) for the UAB and 0.37 ( $SD = 0.11$ ) for the PUC.

Looking at the correlations of the different items, Items O9, O16, O19 and O22 had the lowest correlations with the total scale ( $r.cor$ ) in the UAB sample and that the reliability (Cronbach's alpha) of the scale was maintained or increased when these items were excluded. Regarding the PUC sample, Item O7 had the lowest item-test correlation ( $r.cor$ ), with reliability (Cronbach's alpha) of the scale maintained or improved by excluding it. Also, Item O25 showed very low correlations in both samples, with the respective reliabilities maintained or increased when the item was excluded.



Table 5  
Cronbach's Alpha Reliability Coefficients and Item-test Correlation for the Multiple-choice Questionnaire

Item	UAB		PUC	
	Alpha	r.cor	Alpha	r.cor
C1	0.56	0.16	0.75	0.16
C2	0.56	0.12	0.74	0.30*
C3	0.56	0.10	0.74	0.26*
C4	0.56	0.14	0.73	0.45**
C5	0.54	0.38*	0.73	0.52**
C6	0.55	0.27*	0.73	0.39*
C7	0.57	0.08	0.75	0.07
C8	0.54	0.33*	0.76	-0.07
C9	0.53	0.38*	0.75	0.14
C10	0.54	0.32*	0.73	0.45**
C11	0.56	0.25*	0.74	0.33*
C12	0.56	0.22	0.73	0.44*
C13	0.57	0.17	0.72	0.58**
C14	0.54	0.41**	0.73	0.59**
C15	0.57	0.06	0.74	0.35*
C16	0.57	0.05	0.73	0.48**
C17	0.56	0.16	0.74	0.33*
C18	0.55	0.32*	0.73	0.38
C19	0.55	0.31*	0.73	0.41**
C20	0.55	0.25*	0.73	0.43**
C21	0.55	0.30	0.73	0.39**
C22	0.56	0.14	0.74	0.35*
C23	0.55	0.26*	0.73	0.55**
C24	0.53	0.36*	0.73	0.40*
C25	0.53	0.35*	0.76	-0.19
GLOBAL	0.56 ( <i>SD</i> = 0.01)	0.05 ( <i>SD</i> = 0.1)	0.75 ( <i>SD</i> = 0.08)	0.11 ( <i>SD</i> = 0.14)

Note: The Alpha of each item corresponds to the reliability of the scale after eliminating that item. In turn, r.cor corresponds to the item-test correlation, with \* $p < 0.05$ ; \*\* $p < 0.01$ .

Table 5 shows that the items in the *multiple-choice questionnaire* exhibited totally different behaviours in the UAB and PUC samples. In the UAB, 15 items displayed a low item-test correlation that affected the Cronbach's Alpha coefficient, which did not exceed 0.57 while that of the complete scale was 0.56—considered very low for a scale (Novick & Lewis, 1967). In the PUC, the scale had very good item-test correlation indices (Novick & Lewis, 1967) in 19 of its items, as well as in the reliability coefficients, obtaining 0.72 as the lowest index and 0.75 for the complete scale. These results suggest that the items with this structure were better suited to the PUC sample than the UAB sample.

### *Reliability Analysis: The Questionnaires According to Content Blocks*

The low reliability of Items O7, O9, O16, O19, O22 and O25 suggested that they should be removed from the *open questionnaire* to ensure its homogeneity. Before doing so, however, Cronbach's alpha and item-test correlation coefficients were determined by grouping the questions according to content blocks into which the FMK had been packaged (Tables 1 and 2), both for the *open questionnaire* (Table 6) and for the *multiple-choice questionnaire* (Table 7). It is important to note that, for this purpose, analogous content blocks from the curricula of Catalonia and Chile were assimilated and designated with a generic name. Therefore, for example, the GEO (geometry) block brought together those called Space, Shape, Magnitude and Measurement in Catalonia and Geometry and Measurement in Chile, and



the ALG (algebra) block brought together those called Relations and Change in Catalonia and Patterns and Algebra in Chile.

Table 6

*Cronbach's Alpha Reliability Coefficients and Item-test Correlations Grouped by Content for the Open Questionnaire*

Block	Item	UAB			PUC		
		Alpha_G	Alpha	r.cor	Alpha_G	Alpha	r.cor
NUM	O1	0.65	0.61	0.49*	0.55	0.51	0.38*
	O2		0.62	0.43*		0.48	0.48*
	O3		0.63	0.41*		0.51	0.38*
	O4		0.62	0.47*		0.49	0.46*
	O5		0.62	0.42*		0.53	0.31*
	O6		0.64	0.34*		0.54	0.26
	O7		0.61	0.49*		0.57	0.08
	O8		0.61	0.46*		0.48	0.48*
ALG	O9	0.32	0.23	0.35*	0.44	0.42	0.26
	O10		0.14	0.49*		0.32	0.46*
	O11		0.25	0.38*		0.38	0.37*
	O12		0.33	0.12		0.41	0.29
	O13		0.38	0.07		0.37	0.38*
GEO	O14	0.36	0.27	0.40*	0.61	0.57	0.44*
	O15		0.33	0.27		0.59	0.35*
	O16		0.31	0.34*		0.60	0.29*
	O17		0.34	0.22		0.58	0.38*
	O18		0.33	0.25		0.60	0.34*
	O19		0.39	0.06		0.60	0.32*
	O20		0.35	0.24		0.55	0.53**
	O21		0.33	0.24		0.54	0.57**
DATA	O22	0.23	0.31	-0.09	0.07	-0.02	0.31*
	O23		-0.01	0.42*		0.08	0.16
	O24		0.05	0.38*		0.03	0.11
	O25		0.22	0.14		0.14	0.05

Note: The Alpha\_G corresponds to reliability by grouping, and the Alpha of each item corresponds to the reliability of the grouped block after eliminating that item. In turn, r.cor corresponds to the item-test correlation in the subscale, with \* $p < 0.05$ ; \*\* $p < 0.01$ .

As shown in Table 6, the theoretical grouping of questions in the *open questionnaire* generated very low Cronbach's Alpha coefficients, so it was decided not to consider the theoretical grouping, accepting that all the questions measure a single construct—the students' IMK. This was supported by the assumption of tau-equivalence of the questions as regards the reliability coefficient and the high correlation of the items in the complete scale (Table 4).

Subsequently, to improve the reliability and correlation indices, Items O7, O9, O16, O19, O22, O23 and O25 were eliminated from the questionnaire in both samples in view of their low correlations with the total scale (Table 4). The elimination of those items increased reliability from 0.72 to 0.75 and from 0.75 to 0.78 for the items of the *open questionnaire* in the UAB and PUC samples respectively, attaining a figure of 0.76 for the joint samples.



Table 7  
Cronbach's Alpha Reliability Coefficients and item-test Correlations Grouped by Content for the Multiple-choice Questionnaire

Block	Item	UAB			PUC		
		Alpha_G	Alpha	r.cor	Alpha_G	Alpha	r.cor
NUM	C1	0.25	-0.10	0.40**	0.47	0.47	0.28
	C2		0.01	0.12		0.40	0.41**
	C3		0.12	-0.16		0.39	0.41**
	C4		0.03	0.04		0.40	0.41**
	C5		0.02	0.18		0.41	0.36**
ALG	C6	0.22	0.09	0.36*	0.44	0.01	-
	C7		0.14	0.22		-0.08	-
	C8		0.18	0.22		0.01	-
	C9		0.24	0.16		-0.18	-
GEO	C10	0.40	0.35	0.33*	0.61	0.69	0.46**
	C11		0.37	0.34*		0.71	0.33*
	C12		0.35	0.40**		0.70	0.39**
	C13		0.43	0.12		0.68	0.62**
	C14		0.37	0.30*		0.70	0.50**
	C15		0.39	0.22		0.71	0.38**
	C16		0.39	0.16		0.69	0.51**
	C17		0.43	0.14		0.72	0.31*
	C18		0.34	0.38*		0.70	0.39**
	C19		0.37	0.34*		0.69	0.47**
	C20		0.38	0.20		0.70	0.41**
	C21		0.40	0.22		0.71	0.34*
C22		0.39	0.22		0.71	0.32*	
DATA	C23	0.21	0.04	0.43**	0.07	0.00	0.36*
	C24		0.30	0.13		-0.21	0.50**
	C25		0.15	0.34*		0.43	-0.13

Note: The Alpha\_G corresponds to the reliability per subscale, and the Alpha of each item corresponds to the reliability of the subscale after eliminating that item. In turn, *r.cor* corresponds to the item-test correlation in the subscale, with \* $p < 0.05$ ; \*\* $p < 0.01$ .

Table 7 shows that considering grouping the items by content blocks in the *multiple-choice questionnaire* produced low indices of both item-test correlation and Cronbach's Alpha coefficient, the same as in the *open questionnaire*. Again, this prompts the consideration of the *multiple-choice questionnaire* as a single scale, given the accuracy of the assumption of tau-equivalence of the items for the reliability coefficient.

To conclude the reliability analysis, considering both samples individually and the multiple-choice questionnaire as a single scale, the items that presented low item-test correlations were eliminated (Table 5), namely items C3, C7, C8, C15, C16 and C25. This action increased the reliability of the *multiple-choice questionnaire* to 0.57 for the UAB, 0.77 for the PUC, and 0.73 for the two samples together.

### Exploratory Factor Analysis

In the final phase of the validation process, factor analyses (exploratory and confirmatory) were carried out on the questionnaires reduced to the remaining items after the reliability analysis. The purpose of the exploratory factor analysis was to uncover the underlying structure of the questionnaires. During this analysis, each content block was treated as a latent factor. This assisted in understanding how an item's responses are influenced by the construct measured.



Before conducting the exploratory factor analysis, it was necessary to confirm certain statistical assumptions for both questionnaires by calculating the Kaiser-Meyer-Olkin index (KMO) and the Bartlett  $\chi^2$  test of sphericity. KMO measured the adequacy of the data for this type of analysis and had to be greater than 0.6 to be acceptable. The Bartlett test of sphericity indicated that the variables were not correlated in the sample when a  $p$ -value (significance) of less than 0.05 was obtained (Tabachnick et al., 2007). For both exploratory and confirmatory factor analysis, reference was made to Marsh et al. (1988) to determine which values could be considered acceptable. The values obtained are displayed in Table 8 for the *open questionnaire* and Table 9 for *the multiple-choice questionnaire*. The values displayed in Table 8 and in Table 9 indicated that it was appropriate to perform an exploratory factor analysis for both questionnaires.

Table 8  
*Assumptions for the Factor Analysis of the Open Questionnaire*

	KMO	Bartlett $\chi^2$
UAB	0.64	(153, $n = 125$ ) = 311.48, $p < .001$
PUC	0.75	(153, $n = 158$ ) = 380.73, $p < .001$
Joint	0.82	(153, $n = 283$ ) = 585.35, $p < .001$

Table 9  
*Assumptions for the Factor Analysis of the Multiple-choice Questionnaire*

	KMO	Bartlett $\chi^2$
UAB	0.51	(171, $n = 125$ ) = 302.06, $p < .001$
PUC	0.76	(171, $n = 158$ ) = 483.98, $p < .001$
Joint	0.79	(171, $n = 283$ ) = 602.84, $p < .001$

For the exploratory factor analysis, the statistical indices—fit indices—used were the absolute  $\chi^2$  and the relative Tucker-Lewis index (TLI), considered acceptable if greater than 0.9 and good if greater than 0.95. The root mean square error of approximation (RMSEA) indicated the error in the measuring the fit. In the case of the *open questionnaire*, the exploratory factor analysis was performed with a PA (Principal Axis) extraction method and Varimax rotation. The number of factors to be retained was determined by the Kaiser-Gutman rule, the Cattell scree test and parallel analysis (Table 10).

Table 10  
*Fit Indices of the Single-factor Model for the Open Questionnaire*

	$\chi^2$	TLI	RMSEA
UAB	(153, $n = 125$ ) = 151.14, $p > 0.10$	0.90	0.04
PUC	(153, $n = 158$ ) = 151.99, $p > 0.10$	0.92	0.03
Joint	(153, $n = 283$ ) = 145.68, $p > 0.20$	0.97	0.02

For the *open questionnaire*, the Kaiser-Guttman rule (eigenvalues greater than 1.00) suggested the retention of a single factor, as did the scree test. As shown in Table 10, the single-factor model displayed good absolute and relative fit indices for the samples, both separately and jointly, independent of the university where the data were collected. In this process only one item—Item O23—was eliminated. The same procedure was then repeated for the *multiple-choice questionnaire*. Table 11 illustrates the fit indices obtained for the single-factor model.



Table 11  
Fit Indices of the Single-factor Model for the Multiple-choice Questionnaire

	$\chi^2$	TLI	RMSEA
UAB	(65, $n = 125$ ) = 154.48, $p < 0.01$	0.47	0.07
PUC	(65, $n = 158$ ) = 116.99, $p < 0.001$	0.87	0.05
Joint	(65, $n = 283$ ) = 139.36, $p < 0.001$	0.90	0.04

As shown in Table 11, for the *multiple-choice questionnaire*, the Kaiser Gutman's rule and Cattell's scree test suggested the retention of a single factor. However, items with loadings below 0.3—C1, C2, C9, C11, C17 and C21—were eliminated to improve the relative and absolute fit indices (Lloret-Segura et al., 2014). Factor loading refers to the correlation coefficient that indicates the strength and direction of the relationship between a given item and a factor in factor analysis. It helps to determine how much the item contributes to a factor, providing insight into the underlying structure of data.

### Confirmatory Factor Analysis

The analysis was concluded by performing a confirmatory factor analysis of the single-factor structure for each of the questionnaires. This analysis enabled the correlation—or shared covariance—of the items to be determined using the comparative fit index (CFI), the root mean square error of approximation (RMSEA) and the standardised root mean square residual (SRMR) as a measure of absolute fit. CFI is considered acceptable if greater than 0.9 and good if greater than 0.95. RMSEA with values less than 0.05 is considered a good fit, and between [0.05; 0.08] a regular fit. When SRMR has a value of 0 it indicates a perfect fit and a value of less than 0.08 is good (Hu & Bentler, 1999).

To confirm the existence of a single factor in the *open questionnaire*, a confirmatory factor analysis was performed considering both samples together (UAB and PUC students), since the samples taken separately were rather small and the fit indices of the exploratory factor analysis were better for the joint sample (see Tables 10 and 11). To that end, the maximum likelihood extraction model and the *probit* link function were used—given that the variables were the scores 0/1 and therefore ordinal in nature. Table 12 shows that very good absolute and relative fit indices were obtained. Nevertheless, two of the items (O10 and O11) still exhibited absolute loadings lower than 0.3 so they were eliminated. After this, the model improved its fit indices as shown in Table 13.

Table 12  
Confirmatory Factor Analysis of the Open Questionnaire

	$\chi^2$	CFI	TLI	RMSEA	SRMR
Joint	(135, $n = 283$ ) = 150.17, $p = 0.17$	0.97	0.96	0.02	0.05

Table 13  
Confirmatory Factor Analysis of the Open Questionnaire After the Elimination of Two Items

	$\chi^2$	CFI	TLI	RMSEA	SRMR
Joint	(104, $n = 283$ ) = 116.57, $p = 0.19$	0.97	0.97	0.02	0.04

The final loadings of the items in the *open questionnaire* in the one-factor model are shown in Table 14. It should be noted that in the *open questionnaire* what remained of the initial distribution by topic consisted of seven items on Numbers and Arithmetic, two on Relations and Change, six on Space, Shape and Measurement, and only one on Statistics and Randomness.



Table 14  
Factor Loadings of the Open Questionnaire Items in the One-factor Model

Item	Factor Loading	Item	Factor Loading
O1	0.44**	O13	0.33**
O2	0.42**	O14	0.46**
O3	0.49**	O15	0.36**
O4	0.38**	O17	0.40**
O5	0.34**	O18	0.40**
O6	0.34**	O20	0.37**
O8	0.51**	O21	0.48**
O12	0.36**	O24	0.44**

Note: \* $p < 0.05$ ; \*\* $p < 0.01$ .

Finally, a confirmatory factor analysis of the *multiple-choice questionnaire* was carried out using the maximum likelihood extraction model—once again taking the UAB and PUC samples together for the same reason as cited above. As shown in Table 15, acceptable absolute and relative fit indices were obtained.

Table 15  
Confirmatory Factor Analysis of the Multiple-choice Questionnaire

	$\chi^2$	CFI	TLI	RMSEA	SRMR
Joint	(78, $n = 283$ ) 449.88, $p = 0.00$	0.91	0.90	0.04	0.05

As shown in Table 16, the final loadings of the *multiple-choice questionnaire* items in the single factor model were all higher than 0.3, so no items were eliminated at this stage. Note that what remained of the initial distribution by topic in the *multiple-choice questionnaire* consisted of eight items on Numbers and Operations, two on Patterns and Algebra, two on Geometry and Measurement, and only one on Data and Probability.

Table 16  
Factor Loadings of the Multiple-choice Questionnaire Items in the One-factor Model

Item	Factor Loadings	Item	Factor Loadings
C4	0.37**	C18	0.46**
C5	0.53**	C19	0.42**
C6	0.39**	C20	0.35**
C10	0.32**	C22	0.38**
C12	0.45**	C23	0.56**
C13	0.42**	C24	0.43**
C14	0.38**		

Note: \* $p < 0.05$ ; \*\* $p < 0.01$ .

This last stage of the analysis provided a definitive answer to the two goals of the study. Two different psychometrically validated instruments were obtained by eliminating the items with poor reliability and low factor loadings from each of the initial questionnaires. Therefore, Goal 1 was met by two instruments containing the items in Tables 14 and 16, which are published here: <https://ddd.uab.cat/record/257026?ln=ca>. Regarding Goal 2, the reliability analysis suggested an initial structure of the students' IMK, indicating that the scale would be unidimensional. This finding was subsequently corroborated by the factor analysis. Therefore, it was observed that the way the students constructed their IMK was not separated by mathematical content blocks and presented a unidimensional structure.





## Discussion and Conclusions

This study set out to validate two instruments designed to assess the mathematical knowledge of students accessing primary teacher education programmes in two different contexts. The development of these instruments had been based on the conceptualisation of Fundamental Mathematical Knowledge (FMK) (Castro et al., 2014; Gorgorió & Albarracín, 2020; Gorgorió, et al., 2021), understood as the mathematical knowledge necessary to successfully complete their training and acquire the mathematical knowledge needed for teaching. The results of the study allowed conclusions to be drawn at the empirical and theoretical level, discussing the implications for teacher education despite the limitations of the study, including the idiosyncrasies of the contexts in which it was defined.

At the empirical level, a validated set of questions was determined to enable the measurement of students' Initial Mathematical Knowledge, understood as the knowledge that student teachers made explicit in the written questionnaires designed according to the specification of FMK. Although the psychometric validation process of the *open questionnaire* and the *multiple-choice questionnaire* yielded fewer items than the original versions (16 and 13 questions respectively), this differed from the original criterion-referenced instruments with 25 items each to being psychometrically validated instruments. As in any knowledge measurement process, further development of questions is needed to discriminate between what the groups of students know, which raises new areas of research for the development of this type of instrument. Regarding this set of questions, it is important to point out that items in the Numbers block were predominant in both questionnaires, in accordance with the curricular weight of this content block in both the Chilean and Catalan curricula. This does not, however, imply that the students' knowledge at the beginning of their education as teachers was based exclusively on number content, since other content blocks were also present in the validated instruments.

This study opens new opportunities for the development of tools for the study of students' mathematical knowledge when they enter teacher education programmes, complementing previous studies (Arce et al., 2017; Ingram & Linsell, 2014; Nortes & Nortes, 2013; Ryan & McCrae, 2005/06; Senk et al.; 2012; Tatto et al., 2008). It should be noted, however, that the transferability of the results of this study requires differentiation. Regarding the validated instruments, necessary adaptations must be considered according to the curricular contexts, evaluative cultures, and training goals of the teacher education programmes in each country and context. Directly applying these instruments in other education systems appears to be a challenge, as assessment differences to and demands in other contexts could introduce unwanted bias into the measurement of initial mathematical knowledge.

On a theoretical level, the research contributes to filling a gap in the field of studies that examine the knowledge needed to teach mathematics. These studies considered the knowledge that is developed in initial or continuing education or in classroom practice (Ball et al., 2001; Ball et al., 2008; Ball et al., 2012; Carrillo et al., 2018; Cooney & Wiegel, 2003; Hiebert et al., 2002; Hill et al., 2007; Ma, 1999; Rowland et al., 2005, 2009; Shulman, 1987). To date, however, there have been few studies that have looked at the knowledge needed to begin teacher education. The definition of Fundamental Mathematical Knowledge as the knowledge necessary for students to successfully follow mathematics-related and mathematics teaching subjects throughout their teacher education could be considered vague or noncommittal. However, the concretisation of such knowledge around curricular content allowed for the development and validation of instruments not only to measure FMK but also, and more importantly, to study the structure of mathematical knowledge made explicit by students through these instruments.

In doing so, the reliability analysis carried out in the study suggested that the questionnaires for the realisation of fundamental mathematical knowledge behaved as single scales, despite being constructed from the different blocks of curricular content. In doing so, the content groupings were found to display low reliability with respect to the full scale, both in the *open questionnaire* and in the *multiple-choice questionnaire*. This finding suggested that the students' initial mathematical knowledge could not be explained in terms of the mathematical content blocks, implying that it was a single common underlying factor shared by the different items in each questionnaire (Segars, 1997). To test for latent uniqueness, confirmatory factor analyses of both questionnaires were carried out. The analyses confirmed the



internal and external consistency and dependent structure of the items in each questionnaire, regardless of the initial theoretical construct (Schreiber et al., 2006; Churchill, 1979; Jöreskog & Sörbom, 1989).

Therefore, it may be affirmed that the initial mathematical knowledge exhibited by UAB and PUC students is structurally unidimensional, and that fundamental mathematical knowledge is a single factor underlying the items in each questionnaire. It is, therefore, reasonable to assume that the structure of students' initial mathematical knowledge in other educational contexts may also be unidimensional. However, this hypothesis needs to be tested and deserves to be tested given its importance in relation to teacher education.

The results of the study may also have implications for teacher education, since instruments for assessing the mathematical knowledge of students entering teacher education may have different uses. On the one hand, given that these instruments measure initial mathematical knowledge, they could be used to establish admission requirements for teacher education programmes based on the attainment of a certain level of mathematical knowledge. Considering the use of the instruments to regulate admission, the negative skew of the distribution of scores showed that the percentage of correct answers was high in most cases at both universities. This would suggest that, although students enrolling in teaching degrees at these institutions show some deficiencies, their demonstrated initial mathematical knowledge could be considered sufficient in relation to FMK. However, it would be up to each education system or institution to determine what is the minimum performance required when students begin their training as teachers. In addition, the *multiple-choice questionnaire* displayed different behaviour between the two universities, with lower reliability indices for UAB students than for PUC students. This could be related to the assessment culture of each context, since multiple choice questionnaires are used widely in the Chilean context. This is not the case in Catalonia, where open-ended questions are the norm in university entrance examinations. Therefore, before transferring the assessments instruments to other contexts or institutions, it would be necessary to adapt the content according to the local mathematics curriculum.

On the other hand, since the validated instruments help to determine the structure of the initial mathematical knowledge of student teachers, they could also be used as diagnostic tools to reorganise mathematics courses and mathematics teaching courses. The unidimensional structure of their initial mathematical knowledge suggested that during their pre-university education they had developed a body of interwoven mathematical knowledge whose content could not be dissociated. This suggests that teacher training courses should integrate different types of mathematical knowledge rather than compartmentalising them by topics, which would make teacher education more challenging given the limited curricular time available and the traditions of university teaching.

To finish, the methodological limitations of the study, which are related to both the type and number of participants and the format of the instruments, are considered. These aspects are key issues that merit consideration in future studies that seek to gain an in-depth understanding of the mathematical knowledge of those entering teacher education. On the one hand, the participants in this study came from two highly selective universities in their respective contexts. This circumstance may have had an impact on the fact that the adjustment indices of the factor analyses increased when analysing the samples jointly as compared to analysing each of the groups of students separately. To make sure this does not skew the validity of the results (homogeneity of the type of participant), researchers who aim to develop an assessment instrument similar to those studied in this research but adapted to the educational system of their region should guarantee variability within the samples, and it may be preferable to use data from students registered at different universities or on different types of programmes (Begué et al., 2023). This would allow for a larger sample size, thereby strengthening the reliability of the validated instruments (Bellini et al., 2019; Bonett & Wright, 2015). On the other hand, this study only utilised "pencil and paper" type data collection instruments, which means that it was only possible to explore certain aspects of their mathematical knowledge, those that can be expressed in this format. It would be interesting to extend this study to other types of instruments and strategies, such as digital tests, performance simulations, and interviews, to obtain a deeper understanding of students' initial mathematical knowledge and its structure (Kinneer et al., 2025).



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### *Ethical approval*

Ethical approval for the research was granted by Pontificia Universidad Católica de Chile Ethics Research Committee, and informed consent was given by all participants for their data to be published.

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### *Competing interests*

The authors declare there are no competing interests.

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