Mathematics Teacher Education and Development 2004, Vol. 6, 63-78

Coaching Teachers to Implement Mathematics Reform Recommendations

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Historically, teachers implemented mathematics reform recommendations by infusing new activities into the curriculum. However, mathematics instruction continues to be teacher centred, challenging professional developers to find new ways to encourage teachers' growth. This study used activity-reflective cycles (Tzur & Simon, 1999) to examine different coaching approaches to support teachers' use of rich mathematical tasks and questions to promote students' mathematical thinking. We suggest a new coaching approach characterised as *evoking teachers' pedagogical curiosity* to advance teachers' professional growth.

Over the past century, mathematics educators and mathematicians used recommendations from early reform advocates (Brownell, 1935; Dewey, 1902) to influence the teaching of mathematics. Mathematics educators designed workshops and summer institutes to encourage teachers to adopt reform recommendations (Loucks-Horsley, Hewson, Love, & Stiles, 1998). Teachers infused their teaching practice with new activities and manipulatives that shifted student engagement from completing drill and practice worksheets to active participation in mathematical activities. Still, teachers managed the classroom discourse as though mathematics contained only right and wrong answers (Cohen, 1990). Students' opportunities to explore mathematical ideas were limited by many teachers who retained an authoritative role even though the activities within the classroom changed. Despite forty years of efforts to change classroom practices, mathematics instruction continues to be dominated by teacher demonstrations followed by student practice (Stigler & Hiebert, 1999).

Professional developers are utilizing new models for professional development to encourage teachers to modify their teaching practices (Loucks-Horsley et al., 1998). For example, McLaughlin (1990) orchestrated discussions about the impact of reform recommendations (National Council of Teachers of Mathematics [NCTM], 2000) on student learning and provided feedback on the use of new teaching strategies. She found that these coaching approaches supported teachers' professional growth. Arbaugh (2003), Franke, Carpenter, Fennema, Ansell, and Behrend (1998), Ferrini-Mundy and Graham (1997), and Schifter and Fosnot (1993) found that teachers in small cohorts modified their practices when they participated in sustained professional development that deepened their understanding of mathematics while discussing pedagogical implications. These small cohorts of teachers explored mathematical ideas by solving non-routine problems, reflecting about their teaching practices, examining student work, and using theoretical frameworks to design curriculum that developed students' mathematical thinking. Teachers in a slightly larger cohort demonstrated professional growth after exploring, discussing, and reflecting on problems that could be adapted to classrooms (Farmer et al., 2003). The inquiry stance assumed by these professional developers encouraged teachers to adapt the problems explored during the professional development sessions for their own classrooms. This process promoted the professional growth of the teachers in the cohort.

The number of teachers who participated in these initiatives was relatively small (from 7 [Arbaugh, 2003] to 80 [Farmer, Gerretson, & Lassak, 2003]). To support the professional growth of teachers in large districts, the National Science Foundation funded large systemic change initiatives with both summer institutes and school-year support. These initiatives reflected the belief that effective professional development for teachers should involve follow-up activities (i.e., classroom coaching, school-year meetings) to sustain teachers' professional growth after summer institutes. But, little research describes or examines features of follow-up activities that actually support teachers' growth (Ball, 1996).

Recently, Suber, Garrison, and Martin (2001) explored classroom coaching by partnering teachers in a low-achieving school with exemplary educators. The classroom coach assumed the role of a consultant who (a) made observations and suggestions to improve teachers' classroom practice by deepening their knowledge of best practices, (b) validated alternative teaching strategies, and (c) addressed instructional deficiencies. Thus, classroom coaching occurs when a professional developer observes a classroom teacher and offers support that enables the teacher to change aspects of her or his teaching practice. The study reported here builds on Suber, Garrison, and Martin's work by investigating approaches to classroom coaching that promote teachers' professional growth. Specifically, we sought to describe a coaching approach that could be utilised by a classroom coach working with large numbers of teachers to encourage them to pose good mathematical tasks and ask questions, creating opportunities for students to explore mathematical ideas.

Theoretical Orientation

This research study was part of a larger project investigating teacher growth in a systemic-change initiative between 2000 and 2003 that scaled-up professional development in a mid-size urban school district in the United States. The Midwestern school district had a high minority population with a history of low scores in mathematics on state tests. The systemic change initiative, Primary Mathematics Education Project (PRIME), (Thornton & Barrett, 2000), was a collaborative partnership between mathematics education faculty from Illinois State University, school district administration, and 337 elementary teachers. PRIME provided teachers with opportunities to develop new conceptions of how to teach mathematics using the reform curriculum, Investigations of Number, Space and Data (Investigations), (Akers et al., 1997).

One key aspects of PRIME was to improve teachers' pedagogy by developing better practices in three specific areas: posing worthwhile mathematical tasks, improving questioning techniques, and promoting mathematical thinking by listening and responding to students' responses (PRIME strategies). Teachers' professional growth was supported by (a) three summer seminars (b) four half-day seminars during each school year, (c) monthly classroom coaching, and (d) team meetings with project staff. The seminars were conducted by mathematics educators who had an association with Illinois State University.

The authors Olson and Barrett interpreted changes in teachers' actions within their classrooms to indicate learning (Lave & Wenger, 1991), believing that sustained changes signified a modification in teachers' conceptual knowledge about teaching. Piaget (1969) describes two processes for integrating new ideas into an existing conception: new ideas are assimilated into existing conceptions or existing conceptions are reorganised to accommodate the new knowledge. We anticipated that teachers would experiment with the PRIME strategies during their classroom instruction and consider how to incorporate them into their existing practice; incorporating these strategies would require either modifying the recommendations or altering their conception about teaching. Classroom coaching was conceptualised as both a site for learning and a mechanism to unmask the process by which teachers integrated the PRIME strategies into their teaching practice. We wanted to support learning that changed the individuals' conception about teaching and would influence subsequent experiences. Mezirow (1991) describes these changes as transformational and theorizes that they would influence the interpretation of all subsequent experiences.

Professional developers (Barrett et al., 2002; Cohen, 1990) found that transforming teachers' current knowledge into more advanced concepts was problematic. One explanation of this problem is that teachers need more specific support when they are attempting to develop entirely new cognitive structures as foundations for their teaching practices. Simon, Tzur, Heintz, Smith, and Kinzel (1999) describe a model to support the development of a new cognitive structure by engaging a teacher in a sequence of cyclical activities. In an activity-reflective cycle, the professional developer (a) assesses the teacher's current knowledge, (b) describes a conceptual advance, (c) creates a learning trajectory, (d) selects activities, and (e) supports the teacher's reflection. Key to advancing teacher's learning is their reflection on the activity or activities that cause perturbation. Steffe and Cobb (1988) found that prompting perturbation facilitated learning by providing an opportunity for the individual to reconsider a conception in light of a new experience. But, they found that sometimes individuals were unable to resolve the ensuing cognitive dissonance because he or she lacked the conceptual understanding or did not understand the conflict. Merely drawing a teachers' attention to an inadequate conception about teaching would not lead directly to resolution of the conflict in the manner intended.

In this situation, Tzur and Simon (1999) theorize that an exploration by teachers on the relationships between the activity and its outcomes promotes learning. They describe three stages of reflection on the activity-effect relationships, initial, reflective, and anticipatory. In the initial stage, a purposeful activity is designed to provide an opportunity for the teacher to attend to and explore a conceptual advance. The teacher considers and recognises the relationship between an activity and particular outcomes during the reflective stage. Teachers move to the anticipatory stage when engagement in the activity is no longer necessary to predict outcomes. Thus, the activity-effect relationships evolve into more complex conceptions of teaching.

Method

To help teachers shift their focus from students' classroom behaviour to students' mathematical thinking, the research team utilised a teacher-development experiment (Simon, 2000) in a large systemic change project. Within this context, three first-grade teachers were selected as case studies to investigate coaching approaches that support teachers' professional growth to generate a coaching approach that could be utilised by other classroom coaches in PRIME.

The three case-study teachers identified place value as a problematic concept for students. We selected seven lessons from *Investigations: Building Number Sense* (Russell & Kliman, 1998). These lessons were designed to develop children's conceptual understanding of number and place value and met the teachers' expressed need. The teaching practice for each first-grade teacher was characterised by constructing an account of practice (Simon & Tzur, 1999). Researchers create a hypothetical learning trajectory for a teacher by (a) characterizing her or his teaching practices, (b) articulating a desired change (conceptual advance), and (c) selecting a task that will promote professional growth. Olson and Barrett created a hypothetical learning trajectory for each teacher to describe the conceptual advances that would help her shift from listening for memorised answers to using students' responses to build mathematical ideas. Initially, we theorized that providing the first-grade teachers with rich mathematical tasks would create an opportunity for students to independently explore and articulate mathematical ideas. Yet we found the tasks alone did not provide an adequate support for the teacher to reflect upon a different conception of the learning process.

Cognitive coaching (Costa & Garmston, 1994) supports the professional development of teachers through a process of reflection. The teacher identifies a concern during a pre-observation conference, the coach gathers data in a classroom observation, and the pair analyses and reflects on the lesson in a post-observation conference. Olson and Barrett used the format of cognitive coaching with pre- and post-observation conferences to support the three case-study teachers' reflection during five lessons over a three-week period. After each lesson, we analysed the lesson for evidence of learning and used this to adjust our coaching approach to better advance the teacher's professional growth. This cycle of analysing, adapting theory, defining new activities, and observing was repeated for each coaching cycle (see Figure 1).

Teacher's practices		**Conceptual advance**		**Learning trajectory**
The research team creates an account of practice to characterize the teacher's practice.	⇨	The teacher utilizes good mathematical tasks and asks questions that create opportunities for students to explore mathematical ideas. In addition, the teacher listens carefully to students' explanations.	⇨	The teacher uses rich mathematical tasks to create opportunities for students to independently explore and articulate mathematical ideas.

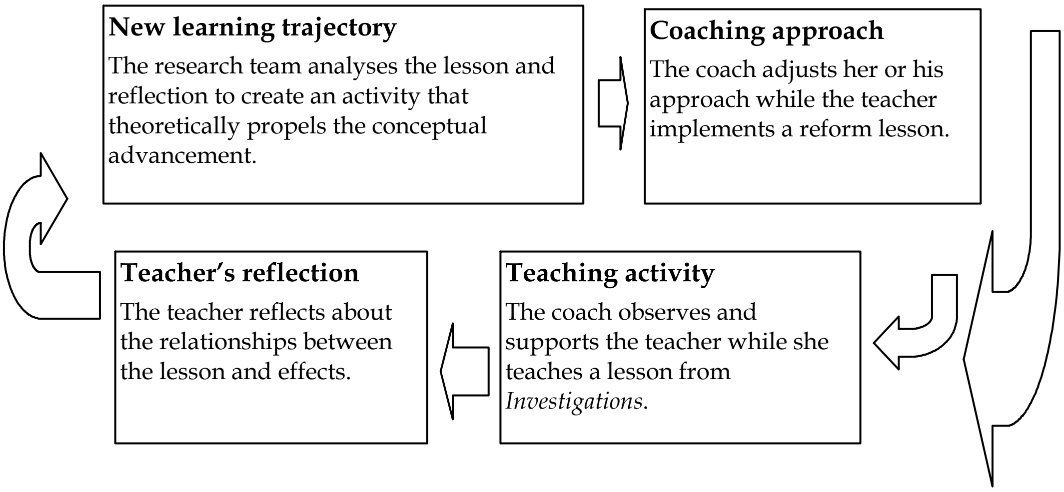

Figure 1. Learning trajectory that guided the development of coaching approaches utilised by the classroom coaches.

Participants

Three case-study teachers with different levels of experience were selected to explore the use of classroom coaching to support teachers' implementation of PRIME strategies. Anne and Rachel (pseudonyms are used for all individuals) were novice first-grade teachers establishing their own classroom routines and the intersection of their professional development with classroom coaching is discussed in a related research report (Barrrett et al., 2002). The third case-study teacher, Ellen, was an experienced first grade teacher who had her Master's degree in curriculum and instruction. She previously participated in professional development designed to support mathematics reform and considered her practice exemplary of reform recommendations. The focus of this paper is on Ellen, precisely because she characterised an experienced teacher who voiced beliefs in mathematics reform recommendations and because she claimed to have already extended her pedagogical understanding of mathematics through Master's Degree coursework.

The research team comprised the two authors. Olson was a graduate research assistant who provided classroom coaching for approximately 70 elementary teachers participating in the PRIME project and conducted a case study of the experienced first-grade teacher, Ellen. Barrett was a co-PI of the PRIME project and coached the two novice first-grade teachers, Anne and Rachel.

Data Collection and Analysis

Data collection included field notes, samples of student work, and audiotape of lessons and pre and post-observation conferences. Data for each case-study teacher were analysed using constant comparative analysis (Maykut & Morehouse, 1994) for evidence that the teacher maintained the integrity of the mathematical task, encouraged students to express their mathematical thinking, and used students' responses to develop mathematical ideas. A conceptual matrix (Miles & Huberman, 1994) was constructed to describe the coaching strategies utilised during each intervention with the teachers' responses. The data were further analysed using stages of reflection on the activity-effect relationships (Tzur & Simon, 1999) to describe the way a coaching strategy influenced the teachers' actions, which were interpreted as evidence of their learning. Cross-case analysis (Miles & Huberman) was used to generate a coaching approach that could be utilised by classroom coaches who worked with a large number of teachers in the PRIME project.

Results

The results are presented in two parts. The first part describes the activity-reflective cycles utilised by Olson as she coached Ellen to use good mathematical tasks and ask questions that created opportunities for students to explore mathematical ideas. The dilemmas of coaching an experienced teacher with traditional practices are presented with illustrative examples to describe the challenges of creating new learning trajectories that initiate a conceptual advance. The second part compares and contrasts the coaching approaches utilised with all three case-study teachers.

Coaching Ellen

Ellen participated in a one-week summer workshop that focused on ways to develop children's geometric thinking using tasks from a reform curriculum, Investigations (Akers et al., 1997). Olson coached Ellen in her classroom to implement the three PRIME strategies during the following year. Ellen, an experienced first-grade teacher, was teaching for her sixteenth year and recently completed her Master's degree. We first describe Ellen's initial practice from which the conceptual advance was created.

Ellen's initial teaching practice. Two accounts of practice were created to characterise Ellen's teaching practices. The first was made prior to the PRIME project in the spring of 2000 and the second occurred in September 2000 after Ellen participated in a one-week summer seminar. Both accounts of practice indicated that Ellen typically modelled a solution strategy whenever students were frustrated. For example, when they forgot the procedure for adding multi-digit numbers, she gave them base-ten blocks with a piece of paper which was divided into two regions, tens and ones (April, 2000). Ellen guided her students to represent and solve 45 + 24 using the procedure of representing the 4 with sticks and 5 with cubes. Then, the students were directed to "put the sticks in the tens box and the cubes in the ones box." Next she asked, "Do we need to count out the 24 little ones?" and a student responded, "No, take 2 sticks." Ellen modelled a solution strategy, "Right, 2 tens and 4 little ones. How much do we have?" During the post-observation conference she reflected, "There are lots of students in my class who will never figure out two-digit addition. By modelling a strategy, the ones who will never get it will know how to solve the problem, learn the procedure, and eventually understand it." Her reflection indicated a belief that her students could not independently solve mathematical problems and that understanding was demonstrated by correct answers.

Ellen used manipulatives to teach children a procedure that was memorised and her questions reinforced the procedure. She listened to students' responses to determine whether an answer matched her own and then filled in the missing details. Ellen's pedagogy was characterised as a sequence of four reported practices: (a) telling the children a procedure, (b) asking them to recall the procedure, (c) practicing until they were successful, and (d) reviewing the procedure by asking questions designed to solicit predictable responses (observations, April and September, 2000). Thus, we characterised Ellen's teaching practices and beliefs about learning as traditional.

Conceptual advance and initial learning trajectory. After attending the summer geometry institute, Ellen used activities from Investigations (Akers et al., 1997) to develop children's understanding of geometry. She was surprised at their ability to examine the geometric properties of shapes and appeared to modify her beliefs about learning mathematics. "If I can get my kids to understand the shapes and the relationship between them... They'll have a deeper understanding that's natural instead of it's something they have to learn for now and won't know it next time" (conference, September, 2000). Ellen also wanted her students to gain a conceptual understanding of place value. Olson provided Ellen with seven lessons that developed students' conceptual understanding of place value and discussed the unit's mathematical goals. We theorized that Ellen would implement the lessons as described by the curriculum designers. Furthermore, we expected her to reflect about the relationships between the curriculum and student learning during the post-observation conference. Five lessons are presented to describe the activity-reflective cycles utilised while coaching Ellen and then summarized in Table 1 at the conclusion of this section.

Coaching using cognitive coaching. The first lesson was called "Cats and Dogs" (Russell & Kliman, 1998) in which students explored combinations of numbers that made 12 using the context of cats and dogs to represent the two addends. Ellen and Olson discussed the lesson during a pre-lesson conference. On the agreed upon day, Ellen changed her lesson plans and used a Halloween story to introduce counting by twos (November 1, 2000). Ellen showed her students how to use cubes to represent children's feet in a trick-or-treat line which led a counting routine. After the lesson, Olson asked Ellen why she changed the lesson plan. Ellen explained, "My students enjoyed listening to stories and in my master's program I learned that good teachers make connections between content areas." Then, she shifted the conversation to misbehaving students.

This was a dilemma for Olson and she wrote in her field notes, "I wanted to build a relationship with Ellen in which I could help her examine her own practices, but Ellen deflected conversations away from an examination of mathematical tasks for their potential to provide students with opportunities to investigate significant mathematical ideas." The lessons from Investigations had the potential to launch an initial stage in which Ellen explored the relationships between her actions and students' learning. Olson and Barrett theorized that Ellen would consider this relationship the following day when she utilised the planned lesson, Cats and Dogs.

Coaching using a rich task with discussion. Olson reviewed the lesson plan for Cats and Dogs with Ellen at the conclusion of the post-observation conference held the previous day. Ellen modified the lesson plan by reading a story about a frog that wanted a pet (November 2, 2000). After a brief discussion about pets, Ellen launched the problem, "I have a friend who had 12 cats and dogs, how many cats and how many dogs could they have? We're going to figure this out at our seats. But before I send you to your seats, let's see if we can figure out a few together. Here's my cats and dogs." She placed a basket of bottle caps on the floor. Ellen modified the task by selecting a new manipulative instead of using unifix cubes "because I get tired of using the same manipulative." The change was made to provide variety without considering the curriculum's use of unifix cubes to develop place value. Charles, "a good problem solver," shared a solution.

Charles: Well, I got out 12 cats (He took out 6 blue caps and 6 purple caps and placed them in a circular pattern on the table.)

Ellen: Pick a volunteer to count and make sure he has 12 caps. (A student counted them.) Now, I have a good question. Which ones are cats and which ones are dogs?

Charles: The blue ones are dogs and the purple ones are cats. (As he made this distinction, he placed the purple caps in a column and the blue caps in another column making a 6 x 2 array.)

A good question for Ellen required students to think beyond a rote response. The question, "Which ones are cats and which ones are dogs?" was definitional. The PRIME strategies suggested that a good question probed the mathematical thinking of a student. Even though Ellen spent time working with patterns in geometry, she did not notice the opportunity to discuss the information displayed by the two patterns. The array display was a powerful tool for exploring other combinations and could provide a platform to discuss an exchange strategy that was taught later. As a coach, Olson could only sit, listen, and consider how to encourage Ellen to reflect about the relationships between PRIME strategies and her students' mathematical thinking.

During the post-observation conference, Olson asked Ellen to talk about her decision to use the story as an introduction. Ellen shared, "I was thinking about the pet idea and remembered the story about pets. I like to integrate literature into math lessons whenever I can." Olson asked how the story helped her launch the investigation to find different combinations of numbers that made 12. Ellen bypassed the question and ended our conversation stating, "stories helped first graders see connections in math." Ellen was comfortable discussing modifications but resisted Olson's attempts to help her analyse how the modification might influence student learning.

Again, this lesson had the potential to launch an initial stage of reflection but Ellen attended to superficial aspects of instruction, which prevented her from attending to substantive mathematics. Olson and Barrett theorized that Ellen was unable to utilise the rich mathematical tasks to explore mathematical ideas with children because she focused on modifications instead of the mathematical concepts and needed help to unpack the mathematics in the lesson. We created a new teaching activity in which Olson modelled a lesson and used it to discuss the relationships between the task, questions, and the development of students' mathematical thinking. Therefore, Olson asked permission to teach a lesson and discuss it with her afterwards.

Coaching by modelling. The following week, Olson taught the lesson "Double Compare" following the instructional plan provided by the curriculum (November 6, 2000). A student partner turned over two cards, a 5 and a 4. Olson turned over an 8 and 5 and asked, "Which is bigger?" The students responded that Ms. Olson's combination was bigger. Olson asked, "How could you tell that I had more?" Three students quickly shared their strategies, "Because 13 is more than 9." "Yours has 13 pictures and the other has 9 pictures." And finally, "You can see there is a 5 there and there (the student pointed at the two fives that were turned over). Then the 8 is more than the 4." After modelling the game stressing the importance of justifying which sum was greater, the students played the game for fifteen minutes. Olson gathered the cards and asked what they learned. Students shared a variety of strategies (i.e., "doubles facts" and "just looking at the numbers"). Olson asked for an example. A student explained, "If I get 5 and 6 then I think 5 plus 5 is 10 and 1 is 11. That's less than 10 plus anything except 1." Olson asked how they used combinations that made 10 to compare the sums. Another student shared, "I make two combinations that make 10 and then compare the ones." The questions that Olson asked prompted students to articulate their mathematical ideas and to demonstrate a variety of strategies for comparing numbers.

During the post-observation conference, Olson commented on the different strategies the students used to compare numbers. Ellen remarked with surprise, "My students can share strategies and explain their thinking. I didn't think they could talk about their thinking. They know how to explain what they are thinking." Ellen identified that her students could articulate their mathematical thinking when Olson implemented the lesson without modification. Thus, Ellen recognised a relationship between teaching a lesson as described in the curriculum and a desired outcome, signalling an initial stage of reflection on activity-effect relationships. Olson and Barrett theorized that Ellen needed help to implement the lessons as described by the curriculum developers and Olson jointly planned the next lessons with Ellen.

Coaching by collaboration. Ellen and Olson discussed the mathematical concepts of the lesson and crafted questions during the pre-observation conference. Olson asked Ellen questions like: “What do you think that students will learn playing the game Towers of Ten?” and “What questions could you ask that would help them think about regrouping?” to help her focus on important mathematical ideas during the lesson. Ellen followed the lesson as described in the curriculum and modelled the game with a student partner. She rolled a sum of five and constructed a partial tower with five cubes. John rolled a sum of seven and placed five cubes on the partial tower making a tower of ten.

Ellen: What are we going to do with these two extra cubes?

John: Put them back.

Ellen: Here? (Pointed to John’s pile of extra cubes.)

John: No. Over there (he pointed to the extra cubes in a basket).

Ellen: What are you going to do? Here you have a stack of ten. What are you going to do with these two extra ones?

John: Put them on top.

Ellen: But that would be 12. We can only make stacks or towers of 10. We can only make towers of 10, so what are we going to do with these two extra ones?

John: Put them back in the pile.

Ellen: No, we can’t put them back. What are we going to do?

John: You’re going to try and make another stack.

Ellen wanted John to recognise that seven cubes could be first broken apart into two smaller groups and then combined with the partial tower to form a group of ten cubes plus two cubes. She restated the question, “What are we going to do with these two extra cubes?” several times until John arrived at the anticipated answer. Ellen did not probe John when he replied, “Put it back” and missed an opportunity to develop the concept of equality. She assigned each student a partner to play the game and write equations to record the roll of the dice. Student pairs played the game and Ellen stood near Olson to listen to conversation between her students and Olson as they explained how many towers were represented by the recorded equations.

Three significant changes occurred in this lesson. First, Ellen did not modify the lesson from the suggested lesson plan. Second, the problem presentation was brief (about five minutes) and students independently explored the connections between notation and towers of ten. Third, Ellen altered her classroom discourse pattern. In the previous lessons, she told her students how to solve problems and then checked their ability to follow the prescribed steps. This was the first time that Ellen posed a problem without providing a solution strategy. These changes provided a context from which we could discuss the conceptual advancement of using mathematical tasks and questioning to promote student thinking. Ellen noticed that her students "seem[ed] to understand how to put numbers together to make ten," indicating that she recognised that her students could independently solve problems and articulate their thinking. Ellen moved to the reflective stage of the activity-effect relationships as she listened to the questions asked by the coach and reflected on her students' ability to articulate mathematical ideas. During the post-lesson conference, Ellen recognised and considered the relationship between the activity and particular outcomes.

Coaching by reflection. Ellen considered her students' learning after teaching a series of seven lessons (November 15, 2000). She articulated five observations about her students' learning after using the lessons.

1. Students understood that there were lots of combinations to make a number. The lessons helped students develop a concept. It [teacher notes in *Investigations*] helped teachers look at different ways of thinking.
2. Students *can* share strategies and explain their thinking. I didn't think they could talk about their thinking. It made their math stories easier. They knew how to explain what they were thinking.
3. Students enjoyed the activities but it will be hard for them to get back to doing worksheets.
4. Students lost ground on their facts. They weren't as quick as they were before doing these lessons… Perhaps they were slower because they were thinking. They figured out the answer instead of pulling an answer from memory. Thinking takes longer.

Ellen recognised the relationships between the PRIME strategies and student learning (see Table 1). She observed her students engaged in mathematics in new ways when they used reasoning to solve problems but felt "they [students] lost ground on [learning] their facts." This dilemma prompted a perturbation. Implementation of the PRIME strategies provided evidence that students could use reasoning to construct mathematical ideas. But, Ellen continued to evaluate students' mathematical knowledge using timed tests, indicating that she valued rapid-automatic responses. Accommodating the PRIME strategies into her teaching practices required the reorganization of her existing conceptions about teaching and learning. To resolve the dilemma, Ellen dismissed the PRIME strategies as "too time consuming" and resumed her practice of "showing and telling" because it was "more efficient and students mastered their facts quicker." Olson and Barrett interpreted Ellen's reflections to indicate that she was willing to assimilate the PRIME strategies by changing them to support her conceptions about teaching and learning mathematics. We were frustrated by Ellen's dismissal of the PRIME strategies for her traditional practices in light of evidence indicating the development of students' conceptual understanding.

Olson and Barrett theorized that Ellen needed opportunities to listen to the discussions of her colleagues as they reconsidered the relationships between the PRIME strategies and student learning before she would reconsider her own beliefs and concluded that coaching alone would not support her professional growth.

Table 1
Description of the Activity-Reflection Cycles Used to Modify Olson's Coaching Approach to Support Ellen's Conceptual Advancement

Learning Trajectory	Coaching Strategy	Teaching Activity: Lessons from *Investigations*	Ellen's Reflection
Ellen would implement lessons from *Investigations* and explore the relationships between the PRIME strategies and students' mathematical thinking	Cognitive coaching (Costa& Garmston, 1994)	Ellen did not use the lesson. She designed a lesson using a story that did not involve significant mathematics	Good teachers make connections between content areas and integrate literature with mathematics
The lessons from *Investigations* had the potential to launch an initial stage of reflection-on activity. Ellen needed help to recognise how the lessons develop mathematical concepts	Using a rich task and discussing the mathematical concepts	Ellen modified the lesson and focused her attention on the modifications instead of the mathematical concepts	Stories help students make connections in mathematics
The lesson from *Investigations* had the potential to launch an initial stage. Ellen attended to the superficial aspects of instruction and prevented her from attending to substantive mathematics. She needed help to unpack the mathematics	Modelling instruction	Olson taught a lesson as described by the curriculum designers while Ellen observed	Students articulated their mathematical thinking and independently solved problems
Observing Olson teach a lesson initiated an initial stage. Ellen identified a relationship between teaching a lesson as described in the curriculum and students' articulation of mathematical ideas. Ellen needed help to plan and question students	Collaborating during planning and instruction	Ellen taught two lessons as described by the curriculum designers. She listened to the questions asked by the coach	Asking questions suggested by the curriculum elicited students' mathematical ideas
The activity initiated a reflective stage. Ellen observed that her	Reflecting on the series of	Ellen reflected about the relationships	PRIME strategies build students'

students could independently solve problems and articulate their thinking. She needed to reflect on student learning and consider implications for teaching practice	lessons	between PRIME strategies and students' learning after teaching a series of seven lessons	conceptual understanding but they take too long and students did not gain number fact proficiency

Coaching strategies utilised with three case-study teachers

Olson and Barrett utilised cognitive coaching (Costa & Garmston, 1994) as a coaching strategy to propel a conceptual advance. Constant comparative analysis indicated that both Olson and Barrett expanded the cognitive coaching strategy with different approaches while they created new learning trajectories and teaching activities. We characterise these coaching approaches with descriptors from the literature (a) using a rich mathematical task and discussing the development of concepts (Stein & Smith, 1998), (b) modelling instruction (Becker, 2001), (c) collaborating or co-teaching (Showers & Joyce, 1996), and (d) reflecting on teaching (Schon, 1987). These coaching approaches are compared with the teaching activity and reflection (see Table 2).

Table 2
Coaching Approaches Utilised by Olson and Barrett to Advance Teachers Toward Using PRIME Strategies to Develop Students' Conceptual Understanding of Mathematics

Coaching approach	Olson	Barrett	Barrett
Cognitive coaching (Costa & Garmston, 1994)	Ellen portrayed herself as a "good" teacher by designing a lesson that integrated literature with mathematics but did not create a task that involved significant mathematics	Anne was positive about using student thinking to inform her instructional decisions but struggled in implementation	Rachel was sceptical about using student thinking to inform instruction. She also was concerned that multiple solutions would confuse students
Using a rich task and discussing the mathematical concepts (Stein & Smith, 1998)	Ellen modified a lesson from an innovative curriculum and focused her attention on the modifications instead of the mathematical concepts	Anne gave her students opportunities to independently solve problems and share their mathematical thinking	Rachel reduced or eliminated the cognitive demand of the problem by providing a procedure to follow
Modelling instruction (Becker, 2001)	Ellen noticed that her students could articulate their mathematical thinking and independently solve	Not used	Not used

	problems		
Collaborating during planning and instruction (co-teaching; Showers & Joyce, 1996)	Ellen listened to the questions asked by the coach and reflected on her students' ability to explore mathematical ideas	Anne embraced working collaboratively and depended on the coach to lead the classroom instruction	Rachel found it difficult to embrace a strategy where students shared their mathematical thinking
Reflecting on the series of lessons (Schon, 1987)	Ellen felt the PRIME strategies built students' conceptual understanding but they were time consuming and students did not gain number fact proficiency	Anne felt comfortable using PRIME strategies but had concerns about actually doing it	Rachel felt the PRIME strategies had merit but she remained committed to teacher-centred instruction

Barrett coached two-novice teachers while they implemented seven lessons reported in a related article (Barrett et al., 2002). Olson and Barrett encouraged the three first-grade teachers to reflect about their students' mathematical thinking and how their actions influenced students' opportunities to learn. Each of the three teachers noticed that their students had the capacity to think independently about mathematical ideas but they were not able to develop a practice that capitalised on their students' natural abilities. Ellen believed that building on students' mathematical ideas took too long and rapid recall of basic facts suffered. Anne had trouble constructing her own mathematical ideas and was unable to use students' responses to build their mathematical knowledge. Rachel believed that multiple strategies confused students and mathematical knowledge was best demonstrated when students reproduced a modelled solution strategy. Olson and Barrett anticipated that helping the three teachers reflect about students' mathematical thinking would create dissonance and prompt them to reconsider some of their beliefs. Instead, they assimilated their observations about student thinking into their existing beliefs about learning and used the reflections to confirm the beliefs that we hoped would be modified. Even though we were discouraged, we could not ignore these results. Other teachers in the district portrayed some of the same characteristics and the systemic change initiative required that we support the professional development of all teachers. This challenged us to create a new coaching approach that could support the professional growth of teachers like Ellen, Anne, and Rachel.

Conclusion and Discussion

This study examined the use of cognitive coaching by two classroom coaches as they worked with three first-grade teachers with different teaching experiences to support the implementation of recommendations from mathematics reformers (NCTM, 2000). The cognitive coaching strategy was modified in-action by the coaches as they created learning trajectories to advance teachers' use of PRIME strategies to develop students' conceptual understanding. We believed that if the teachers listened carefully to their students' responses then they might ask follow-up questions that developed students' mathematical thinking.

Analysis of the first-grade teachers' practices indicated that providing them with rich mathematical tasks and discussing the embedded mathematical concepts did not promote the anticipated professional growth. Like Mrs. Oublier (Cohen, 1990), Ellen and Rachel used innovative materials in traditional ways, as though mathematics contained only right and wrong answers, and managed the discourse in ways that discouraged exploration of students' understanding. Anne used the tasks to elicit multiple solutions but was unable to interpret solution strategies that were different from the ones presented in the textbook. Thus, Anne also utilised the reform materials in traditional ways and referred to the textbook for acceptable solutions. Olson and Barrett characterised these three first-grade teachers as resistant to change because they retained traditional practices in light of evidence that students could create mathematical ideas. Clearly, the evidence was insufficient to overcome prior conceptions about teaching and learning mathematics.

The three teachers noticed that their students could independently construct mathematical ideas after the coach modelled a new teaching strategy or co-taught a lesson by posing questions to students. Ellen positioned herself near Olson to listen to the questions asked by Olson and her students' responses. While Ellen characterised these practices as "inefficient," she did show curiosity and this curiosity was interpreted as an avenue to support her further professional growth. Anne showed curiosity when Barrett questioned her students. Anne was hesitant to question the mathematical ideas of her students because she lacked confidence in her own understanding of the *Investigations* materials. Anne demonstrated curiosity about Barrett's questions, interactions, and interpretations of her students' mathematical thinking.

Teachers who voluntarily participate in professional development sometimes have difficulty implementing reform recommendations (Nelson, 1997). While other research indicates that one-on-one coaching can support the professional growth of teachers (i.e., Early Numeracy Research Project (McDonough & Clarke, 2003), this study found that individually coaching three teachers with different levels of experience and conceptions about teaching mathematics did not enable them to enact the desired teaching practices. The four coaching approaches described in the professional development literature had limited success in supporting teachers' professional growth. Ellen and Rachel's beliefs about teaching and learning mathematics interfered with the adoption of PRIME strategies like other teachers with a traditional view toward teaching mathematics (Manouchehri & Goodman, 1998; Nelson, 1997). Anne's content knowledge reflected Schifter and Fostnot's (1993) finding that a limited understanding of mathematics impacted the implementation of reform recommendations.

After examining instances when the teachers noticed that students could articulate their mathematical ideas, Olson and Barrett theorize that if we could evoke their curiosity, then they would investigate students' mathematical thinking.

We found that the three teachers had difficulty building instruction based on student's responses and theorized that if they could satisfy their curiosity then they might start to wonder about why students respond as they do. Through wondering, we hoped that they might seek answers to their own questions. To coach teachers using this strategy, we suggest that coaches ask teachers to predict how their students might respond to a related question. For example, during the lesson on "Cats and Dog," Olson might prompt curiosity about her students' representations of mathematical ideas by stating, "I noticed that Charles made an array to illustrate a combination of 12 animals. What other ways might children represent combinations of 12 dogs and cats?" We predict that teachers will listen more carefully to their students' responses and investigate their mathematical thinking when they are curious about what they might do or how they might respond to a question. This coaching approach is characterised as evoking teacher's pedagogical curiosity and we theorize that coaches can utilise this approach to promote the professional growth of teachers that struggle to implement mathematics reform recommendations. More research is needed to explore this coaching approach as a strategy to support the professional growth of teachers.

Acknowledgements

The work described in this report was funded in part by the National Science Foundation, grant number ESI-9911754. Opinions expressed are those of the authors and not necessarily those of the Foundation.

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