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Professional Development: Mathematical Content versus Pedagogy

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Recent teacher professional development in California tends to focus on increasing teachers' mathematical knowledge. In Australia, pedagogically focused programs are preferred. The two approaches are contrasted by comparing a Californian professional development model based on improving mathematical knowledge in number and algebra with a New South Wales school-based model involving the teaching of angle concepts. In both approaches, strengths and weaknesses are identified in terms of appropriate mathematical content, research-based pedagogy, and accreditation. Examples of courses incorporating all these components are given, and difficulties associated with implementing such courses are identified. Two important criteria for success, perceived relevance and teacher enthusiasm, are seen to be dependent on cultural factors in the education systems involved.

Improving numeracy among students, particularly in the primary years, has received considerable attention in Australia over recent years. A number of teacher professional development programs have built upon extensive research into children's mathematical understanding. For example, *Count Me In Too* (CMIT), an initiative of the Department of Education and Training (DET) in New South Wales (NSW), has three major components: (a) a theory of numeracy development, based on research by Steffe and Cobb in the USA and Wright in Australia (Steffe, von Glasersfeld, Richards, & Cobbs, 1983; Wright, 1994), developed over several years and formalised into a Learning Framework in Number; (b) an individualised Schedule for Early Number Learning, used to place each child at a point within this framework; and (c) a professional development program designed to assist teachers better understand how children learn arithmetic (Stewart, Wright, & Gould, 1998). CMIT has now also extended its brief to Space and Measurement. The *Early Numeracy Research Project* (Clarke, 2001) in Victoria and the *Numeracy Development Project* in New Zealand have similar programs. Recognition of the importance of improving teacher's pedagogical understanding is not, of course, restricted to Australasia (Carpenter, Blanton, Cobb, Franke, Kaput, & McClain, 2004; Loucks-Horsley, Love, Stiles, Mundry & Hewson, 2003).

Pedagogy in its own right has also been a major focus for professional development in Australia (NSW DET, 2003a; Queensland School Reform Longitudinal Study, 2001) and elsewhere (Farmer, Gerretson, & Marshall, 2003). Many programs have been promoting pedagogy models derived from Authentic Pedagogy (Newman & Associates, 1996), with a special emphasis on the development of authentic mathematics learning experiences for the classroom. In general, consideration of pedagogical practices is regarded as crucial in teacher professional development. For example, Renyi (1998) reported that, when the National Foundation for the Improvement of Education asked teachers what motivates them to seek professional development, 55% responded that they wanted to improve their teaching skills.

Professional development programs frequently deal with other topics besides pedagogy. Loucks-Horsley et al. (2003) argue for programs that help teachers develop in-depth knowledge of their disciplines as well as pedagogical content knowledge. Many programs do, in fact, aim to increase teachers' mathematical knowledge. This approach has been common in California and is gaining impetus with the US federal initiative "No Child Left Behind" (NCLB, California Department of Education, 2003). The relevant Act of 2001 requires that, by the academic year 2005-2006, all students will be taught by highly qualified teachers in all core academic subject areas (including mathematics). Two of the main objectives of this legislation are that:

1. all students gain proficiency in mathematics (as measured by standardised tests);
2. teachers demonstrate subject area knowledge for the level they teach.

This article examines the two types of professional development programs (pedagogy and content) by looking at specific examples of each. The pedagogically focused program, which involved teaching angles in Grades 3 and 4, was conducted by the first two authors as part of CMIT in Sydney. We contrast that with two content-focused programs which were conducted as summer institutes for elementary and middle school teachers at San Diego State University, California, and coordinated by the third author with the first and fourth authors as teaching assistants. Finally, desirable attributes of professional development programs are identified and examples of some initiatives which incorporate these attributes presented.

The Pedagogy-Based Program

In 2001, the NSW DET identified angles as a key area of student learning to be included in CMIT for Grades 3 and 4. The angle concept is significant because it arises in so many different contexts. For example, angles are not only used to describe the shape of the corner of a geometrical figure but they are also used to specify a direction, an amount of turning or opening, and an inclination or slope. The DET were able to call on recent research conducted by Mitchelmore and White (2000), who had investigated young children's learning of angles in a large scale research project and proposed a method of teaching called Teaching for Abstraction.

The Angles Project

In 2001, the first two authors were invited to do three things: (a) develop a draft angles teaching and assessment package for Year 3 which drew on their previous research, (b) use the package as the basis for a professional development program in the teaching of angles, and (c) assess the effectiveness of the package and the professional development program.

The original angles unit consisted of outlines of 10 lessons (Mitchelmore & White, 2001). A total of 12 volunteer teachers from five nominated DET schools in Sydney trialed the unit. The teachers first attended a one-day workshop at which the researchers outlined their recent research on student understanding of angle concepts and described how the activities in the unit were designed to build the appropriate understanding. The teachers then worked through many of the student activities and the student assessment interview. On returning to school, they administered the assessment interview to a target group of eight students, taught the unit at the rate of about one lesson per week for 10 weeks, and re-administered the assessment interview to the target students. Finally, the teachers met with the researchers for a one-day de-briefing workshop that evaluated the unit and the professional development program.

Assessments showed substantial learning benefits to students. Teachers felt that the variety of material, the sequential nature of the lessons, and the hands-on nature of the activities were the best features of the unit but indicated several areas for improvement. Following this feedback, a second trial was conducted (White & Mitchelmore 2002). Seven DET schools participated in the second trial. In each school, there was at least one class containing Year 3 students and at least one class containing Year 4 students. A total of 25 teachers (20 female and 5 male) were involved. The teachers were assisted by the schools' district mathematics consultants.

The format for the trial was the same as in 2001. All the teachers who participated in the project again attended two one-day workshops, one before and one after teaching the unit. This second workshop was again used to evaluate the professional development aspects of the trial.

Outcomes

As a professional development exercise, the Angles Project appears to have been successful. Teachers' written evaluations and comments in focus groups during the de-briefing workshop indicated that their involvement had resulted in broadening their teaching skills. The best reported learning features for teachers were their increased understanding of how to use hands-on tasks which related to the environment. As one teacher said, she would never have thought of using pattern blocks, scissors, or clocks for teaching angles. Teachers also increased their knowledge of curriculum development through exposure to a soundly sequenced series of lessons that were easily resourced. The assessments also helped them identify key features of student learning about angles. As another teacher said:

> The unit had activities that the children were interested in. They enjoyed the hands on work. The build up of the lessons was good. Students had time to explore before moving on to the next lesson.

From the comments teachers made, we infer that they deepened their understanding of several important pedagogical principles—in particular, the value of hands-on materials, links to students' environment, interactive lessons, use of correct terminology, careful sequencing of topics, and the need to build on students' previous knowledge.

The teachers also agreed that being involved in the project had been professionally rewarding because it had increased their own knowledge about the concept of angle. The fact that the teaching sequence had a research basis was credited with this increase in knowledge. However, various misconceptions by teachers suggest that there was room for more emphasis on teachers' knowledge. For example, in a lesson devoted to angles of slope, students easily recognised the angle of slope when a ruler was placed on a table, but they had difficulty when this prop was removed. Many drew arbitrary second lines, which were accepted by some teachers as correct (see Figure 1).

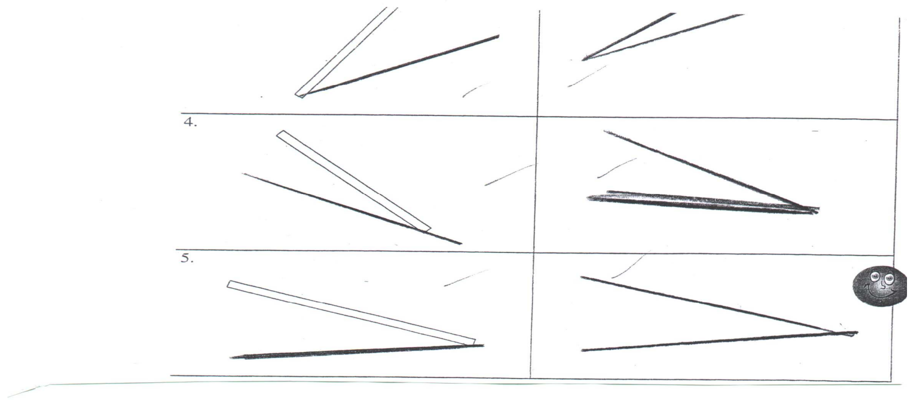

Figure 1. Student representations of angle of slope (marked correct by teacher).

Slope was also a point of contention in the second workshop. Most students were happy to take the acute angle between the ruler and the horizontal in Figure 2(a) as representing the slope. However, in the case shown in Figure 2(b), the obtuse angle was often seen by students and accepted by teachers as the angle of slope. Many teachers had not been aware of the common convention of using the acute angle to define slope and felt slope needed a more detailed discussion in the unit notes.

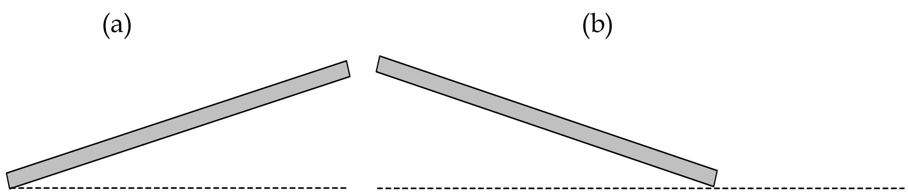

Figure 2. Rulers sloping in different directions.

The teachers all commented that being able to contribute to curriculum development in NSW in a very real way was professionally rewarding and assisted them in becoming more autonomous in their approach to teaching. For example, instead of following the design of the units in a rigorous way, they wanted to use them as a guide to move through as appropriate to their own teaching situation. Specifically, the group suggested that the units be set up as support lessons for teaching angles in Grades 3 and 4 and that more emphasis be put in the lesson notes on suggestions and alternatives rather than "the way" to teach angles.

In summary, the professional development program contributed positively to the teachers' confidence and pedagogical practices, but showed some deficiencies in addressing mathematics content knowledge. A positive professional development result for all teachers is the curriculum document (NSW DET, 2003b) which has now been produced and disseminated.

The Knowledge-Based Programs

At least six cultural factors impact directly on the nature, timing and content of professional development offerings in San Diego (and the United States in general):

1. High school begins in Grade 9 and Grades 6–8 are middle school. Many of the middle school teachers do not have special credentials in mathematics.
2. In California, all teachers (elementary and secondary) complete a four-year bachelor's degree with a supplementary credentialing year for teaching. To teach High School mathematics, they must major in that subject. Elementary teachers have no subject-matter specifications about their degree.
3. July and August is a long summer holiday which provides opportunities for teachers to attend courses.
4. Funding has been available to run such courses. The maximum number who can attend a course is determined by the amount of funding.
5. By attending courses in mathematics, teachers can move up the salary scale or gain a 'specialist' status which may result in teaching more mathematics or teaching higher grades
6. Attending approved professional development courses attracts a 'stipend' payment and thus provides extra income for teachers.

One major reason for a content focus is the firm belief in the United States that "teachers must know and understand deeply the mathematics they are teaching" (NCTM, 2000, p.17) and the fact that research has shown that many teachers do not possess "this rich and deep knowledge of mathematics" (Mewborn, 2003, p.47). Added to the already strong conviction for content-based professional development is the impetus of NCLB and an associated increase in the emphasis on teachers improving their mathematical knowledge. By the end of the 2005-2006 school year, new elementary teachers must pass a California Commission on Teacher Credentialing (CCTC) approved mathematics examination in order to demonstrate subject matter competence. New middle and high school teachers must either have a mathematics major in their degree or pass the same CCTC examination. The trend is towards the use of approved subject matter exams to satisfy credentialing requirements.

The Summer Institutes

The Centre for Research in Mathematics and Science Education (CRMSE) is an interdisciplinary research centre associated with San Diego State University (SDSU). A major function of the Centre is to implement externally funded research projects — funded from either government or industry. As well as research projects, the Centre also conducts government-funded institutes, which are summer courses of two weeks duration with 40 hours of follow-up classes spread throughout the academic school year. Although these institutes have no official assessment, gain in teacher knowledge is evaluated using an externally designed survey administered before and after the institute.

In 2003, two such institutes were offered. The first, attracting 105 elementary and middle school teachers (Grades K-8), focused on number concepts. The second was an algebra institute and attracted 40 middle school and high school teachers (Grades 6-10). Numbers for algebra courses are likely to grow as a result of NCLB requiring all school students to show competency in an examination on algebraic manipulation and necessitating some study of algebra for most middle school teachers.

Both the institutes predominantly used an interactive, problem-solving model in which those involved were given substantial amounts of time to solve both closed and open ended tasks. The teachers mostly worked in groups and prepared visual and/or verbal presentations of their findings. The most important part of the participant presentations was the justification for their results. Unsupported mathematical claims made by one participant would be questioned and investigated until other participants could explain if and why the claim was true. Occasionally during these discussions, instructors and participants would consider how the mathematical ideas being discussed could be implemented in the classroom. Through these tasks and discussions, the teachers accessed some quite sophisticated mathematics. For example:

> *The Locker Problem.* In a certain school there are 100 lockers lining a long hallway. All are closed. Suppose 100 students walk down the hall, in file, and the first one opens every locker. The second student comes behind the first and closes every second locker, beginning with locker #2. The third changes the position of every third locker; if it is open this student closes it; if it is closed, this third student opens it. The fourth changes the position of every fourth locker, and so on, until the 100th student changes only the position of the 100th locker. After this procession, which lockers are open? The discussion following this problem led to the result that all the lockers with numbers that had an odd number of factors would remain open, and further investigation showed that these consist of the perfect squares.

Investigating Irrationals. Given a square of side one unit, the area of the square on the diagonal is shown to be two square units. The argument used was the triangle making half the unit square occurs four times in the square on the diagonal. What is the length of the diagonal? Teachers were then asked to imagine that they only had the tools of the ancient Greeks and use these to work out at the length of the diagonal (the side of the square with area 2 square units). This exercise produced a variety of imaginative results. For example, some drew their square on grid paper and used a compass to mark off the length of the diagonal onto an extended side of the square. The activity led on to a discussion of irrational numbers and an informal proof by contradiction that $\sqrt{2}$ is irrational. The argument used the perfect squares again: If $\sqrt{2}$ were a fraction (say $\frac{p}{q}$), then $p^2 = 2q^2$; but $2q^2$, and therefore p^2, has an even number of factors - which is impossible, since p^2 is a perfect square.

Rates of Change. Using 'The Red Box' (Shell Centre for Mathematical Education, 1985), teachers first investigated graphs 'without numbers'. Examples were heights and ages of people in a bus queue, height and volume in emptying containers etc. Next speed and time were investigated numerically leading to the principles of differential calculus - average rate of change and secants and then instantaneous rate of change and tangents.

Outcomes

Both groups of teachers involved were very committed to the courses. Many indicated that the stipend they received was a factor in their choice to attend, but were attracted by the way the classes were conducted and felt they learnt a great deal of good mathematics. For example, a number of teachers who had studied calculus in their bachelor's degree claimed their basic understanding of what calculus is had been expanded through these investigations into rates of change. Also, the pretest survey data indicated major conceptual weaknesses in a number of areas which the posttest data showed had been rectified. However, the group nature of classes made individual assessment difficult. (It may be noted that after one or two days about 10% of students withdrew from the institute. Some of these explained that the level of mathematics required was too difficult for them.)

Even though the focus was on mathematics in these classes, discussion naturally turned from time to time to classroom implications. For example:

> The course enabled me to meet several teachers from other schools dealing with the same issues and struggles while teaching and learning mathematics.

Individual teachers also indicated that they had experienced good pedagogy and intended to use activities from the course in their future practice. As one student said:

> They (instructors) modeled their instruction around what a learning environment should look like.

During the follow-up sessions throughout the academic year, when the teachers were back in their classrooms, discussions around practice increased. At the end of the academic year, the teachers described how they had used ideas learned in the institute to change their practice. However, there were no formal assessments or visits to classrooms to evaluate the effectiveness of any implementations.

Discussion

The data suggest that both models were successful and that they both had strengths and weaknesses that were, in effect, mirror images of each other. On the one hand, the Angles Project was strong on pedagogy with content knowledge coincidental, while the summer institutes were strong on content with pedagogy coincidental. One other difference was that the institutes had some accreditation, with possible associated benefits for the participants, whereas involvement in the Angles Project resulted in no form of accreditation. We shall discuss these differences below.

One similarity between all the programs was that they were all supported by government funding. Indeed, funding was a necessary component for the existence of the programs.

Content Knowledge

The knowledge-based model in California resulted in many teachers of primary age children gaining deeper mathematical knowledge than would be common in New South Wales. In fact, their knowledge would certainly exceed what they required to teach K–6 mathematics. Teachers of Grade 7 and 8 would gain knowledge comparable to their NSW colleagues, who would generally have some specialist background in mathematics.

Teachers' difficulties over content in the Angles Project suggest that mathematical knowledge is an important issue which should be considered in designing primary professional development programs in New South Wales. We cannot ignore consistent feedback from teachers that they felt their own mathematical understanding had been enriched, but any increased knowledge was incidental and not a result of specifically focusing on the mathematical content. Teachers had been briefly exposed to the underlying mathematical ideas, but the emphasis was on the pedagogical implications of the lessons. More specific emphasis on teachers' mathematical understanding in such professional development programs would appear to be desirable.

Pedagogy

Data from the NSW Angles Project showed conclusively that the pedagogical ideas presented had been successfully implemented in the classroom. Teachers and children both had new, enjoyable learning experiences. The increased skills and knowledge attained by teachers can be attributed to the fact that the teaching sequence was based on extensive research into children's understanding of angles and involved an extensive in-school component where teachers experimented with what they had learnt. As Mewborn (2003) states:

> Professional development opportunities for teachers need to occur in a context in which teachers can try what they have learned in the classroom (p. 49).

Apart from incidental learning in teacher's informal presentations, no implementation information about pedagogical learning was available in the knowledge-based model. Yet, the teachers in San Diego were exposed to an interactive teaching model and discussions with other teachers which must have had some positive impact on their own practice. However, it may have been useful to include more specific attention to pedagogical issues and classroom applications.

Accreditation

Teachers in California attended courses during summer holidays which attracted some accreditation. We can not imagine many teachers in New South Wales attending professional development programs, as currently conceived, in school holiday time. On the other hand, many do attend vacation courses as part of an accredited university degree. Hence, it appears that some accreditation is a desirable outcome of professional development programs.

Conclusions

The three components (pedagogy, content and accreditation) were crucial factors in the professional development programs examined in this paper. A shortfall in any of the three components appeared to reduce the effectiveness of the learning situation for teachers. This finding supports the claim of Loucks-Horsley et al. (2003) that professional development programs need to promote teaching practices based on understanding of how children learn as well as building teachers' content knowledge. A program combining mathematics and pedagogy could be more effective than the present professional development models in both systems.

Such integrated programs do exist. For example, in California, the Professional Development Collaborative (PDC), aligned with CRMSE and San Diego State University, provides a variety of professional education programs to mathematics and science teachers in San Diego County. One such program is the Mathematics Specialist Certificate Program (MSCP) for teachers in Grades 4-6. This is a two-year program with all-day classes during a two-week summer session and 3 hours (one evening) per week during the academic year. The course is an even balance of mathematical content and pedagogy. As well, MSCP instructors visit schools and assist participants' in the teaching of mathematics. The total cost (thanks to industry funding) is a modest $750 (US) per year. Successful participants earn a supplementary certificate as a specialist mathematics teacher, and the pedagogy units may be used toward a master's degree in education. Even though this program has not been evaluated in detail, it has been well received and is in demand. It annually enrols the maximum number of 25 students and comments from participants are often as positive as this one:

> The Math Specialist Program was a superb blend of pedagogy and application. I learned practical teaching strategies which could be implemented the following day in class. I also learned the theories involved in those teaching strategies which made implementation intuitive.

In Australia, one approach to blending content with pedagogy is through partnerships between universities and education systems to provide postgraduate courses in numeracy which can contribute to master's degrees. Examples are those based on the ENRP and run by ACU National in partnership with the Catholic Education Offices in Sydney and Melbourne. Students complete four units, each requiring attendance of 3 hours per week for 12 weeks, and receive a post graduate certificate (half of a master's degree). Fees are paid by the student's sponsoring system. These courses are well received but have a pedagogical rather than mathematical focus; there is a strong in-school focus, with students expected to complete a major project as part of the course. However, given the mounting evidence to support the value of more content, those responsible for the Sydney course are now examining the possibility of including specific mathematical ideas.

The implication is that in courses like the summer institutes in California, the inclusion of more classroom applications would be beneficial. In New South Wales, more content is called for. However, in both cases there are likely to be practical obstacles. For example, in California, the in-term follow-up to the two-week summer blocks was initially designed to provide for some pedagogical discussion and reflection on using skills learnt in the summer program - but no funding was made available for its implementation. Again, the NSW Angles Project was a DET initiative whose main objective was pedagogical and curriculum development, not improving teachers' mathematical knowledge. Also, our experience suggests that most Australian primary teachers would react negatively to a content-based inservice course. Moves to include content would have to be well promoted, well supported, and subtly introduced. Pure mathematics lectures would certainly not be welcomed (neither by teachers nor the DET), but problem-solving courses of the San Diego style might well be successful if they were clearly related to curriculum goals.

The success of any professional development activity depends very much on teacher enthusiasm and commitment, so it is imperative to provide programs that teachers feel positive about and see as relevant. However, what constitutes relevance varies according to cultural factors in the different education systems. In the United States, teachers are required to undertake some postgraduate study periodically in order to maintain their credentials and are paid to attend summer institutes. Teacher salaries are based on a combination of academic qualifications and years of experience. In Australia, on the other hand, there is no obligation for teachers to undertake inservice programs and systematic postgraduate study is not normally funded. But teachers are generally interested in learning about new ways to implement curriculum in the classroom, and professional associations and school systems respond by offering a wide variety of corresponding professional development opportunities. Teachers currently reach a maximum salary relatively quickly and can only advance further through promotion into executive positions - promotion which is facilitated by higher degree study in education.

This discussion shows that teachers generally welcome professional development in some form, but that what they see as relevant and thus are enthusiastic about is influenced by their working conditions, the way funding is allocated, and the rewards that are associated with participation. Therefore, if an appropriate balance of content, pedagogy and accreditation is to be achieved, then career path policies, credentialing requirements, and funding priorities may all need to be re-examined.

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