Examining Preservice Elementary Teachers' Epistemological Beliefs, Views about Mathematics, and the Effect of their Prior Mathematics Experience

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This study examined preservice elementary teachers' (PETs') epistemological beliefs (EB) and their views about mathematics (VM) and the impact on these parameters by their prior experience in learning mathematics. A total of 541 PETs from an American university completed EB and VM questionnaires. It was found that participants' EB and the relationship to their VM are different compared to those of other college students or middle/high school students. Participants hold strong beliefs about "how to study" but not "what knowledge is." Furthermore, the more participants believe that knowledge is constructed by authority and a collection of facts, the more they make efforts to improve their knowledge of mathematics. Concerning their prior mathematics experience, the more mathematics courses they have taken during secondary education, the more developed EB and VM they have, but their prior experience in college mathematics education did not have any significant correlation with their EB and VM.

Keywords · teacher education · epistemological beliefs · views on mathematics · preservice elementary teacher

Introduction

Epistemological beliefs (EB) are often considered as beliefs about the nature of knowledge about the certainty, source, acquisition, and structure of knowledge (Hofer & Pintrich, 1997; Metallidou, 2013; Muis, 2004b; Schommer-Aikins, 2002). It has been reported that elementary teachers' EB is one of the main factors that determine their instructional approaches for engaging students in learning mathematics (Boz & Boz, 2014; Dunekacke et al., 2016; Fives & Buehl, 2014) and enhancing their instructional practice (Huling, 2014; Lunn Brownlee et al., 2017; Tezci et al., 2016). Several studies have also provided evidence that teachers' EB play a vital role in forming their students' attitudinal foundations toward mathematics even though they are unaware that they hold these beliefs (Depaepe et al., 2016; Metallidou, 2013; Schommer et al., 1992; Tsai et al., 2011). Thus, understanding preservice elementary teachers' (PETs') EB is critical in assisting their integration of mathematical content knowledge into their instructional practices. Furthermore, it has the potential to influence their implementation of appropriate and effective teaching methodologies for their future students' learning success (Boz & Boz, 2014;

Tezci et al., 2016; Yilmaz & Sahin, 2011). Although various aspects of PETs' EB have been examined in a few studies (Stohlmann et al., 2015; Yoon & Kim, 2016), little research has investigated their unique relationship to the views about mathematics (VM), especially compared to that of other groups of students.

We have noticed that the PETs in our elementary teacher education programs often encounter challenges not only in understanding certain mathematical contents but also in reconciling ways of mathematical thinking. PETs often have no more previous experience of tertiary-level mathematics than their K-12 education. Hence, their mathematics-related beliefs have mainly been influenced by their former teachers in K-12 education or outside the community of mathematicians (Gill et al., 2004; Ng et al., 2010; Stohlmann et al., 2015). Transforming them into teachers who can effectively teach mathematics with a positive attitude requires an ongoing reflective practice and an awareness of their EB. This would require them to have better beliefs in teaching and learning in general and hence their VM. Therefore, an elementary teacher education program must provide them with experiences that can improve their VM and related EB that will help them to teach mathematics effectively (Huling, 2014; Ng et al., 2010; Stohlmann et al., 2015). For this reason, EB should be taken into consideration throughout PETs' teacher education training because an understanding of EB provides insights into the challenges they may face during their teaching practices (Aslan, 2017; Cam et al., 2015).

PETs' EB and VM may be formed by various factors that affect their belief's system differently as opposed to college students of other majors of study (Schommer-Aikins et al., 2005; Schommer, 1993; Schommer & Walker, 1995). For a PET, a shift to a better teaching practice depends fundamentally on their system of beliefs such as EB or VM (Ernest, 1989). Teaching reforms may not occur unless teachers profoundly change their beliefs about mathematics and its teaching and learning (Ernest, 1989). Therefore, understanding PETs' EB and VM can support teacher educators' efforts to increase the effectiveness of their students' learning of mathematics. Moreover, PETs usually do not have specialized training in mathematics and their future mathematics teaching largely depends on their understanding derived from their K-12 education. Examining how their educational experiences in mathematics affects their EB and VM has the potential to inform on how best to utilise such experience in their future teaching.

Hence the two purposes of the present study are:

1) To examine PETs' EB and VM, and their relationships, and
2) To examine the effects of PETs' prior mathematics experiences on their EB and VM.

Theoretical framework

Epistemological Beliefs

EB are often considered as beliefs about the nature of knowledge including beliefs about the certainty, source, acquisition, and structure of knowledge (Hofer & Pintrich, 1997; Metallidou, 2013; Muis, 2004b; Schommer-Aikins, 2002). Hofer and Pintrich (1997) stated that EB are how individuals come to know, the beliefs they hold about knowing, and the cognitive processes of thinking and reasoning. Schommer-Aikins et al. (2005) further suggested that EB should be considered as a system of independent beliefs that include the following factors: stability of knowledge; structure of knowledge; source of knowledge; speed of learning; and (e) ability to learn (Schommer-Aikins et al., 2005). In Schommer-Aikins and her colleagues' previous studies, the same factor structure, consisting of four out of those five factors (i.e., the stability of knowledge, structure of knowledge, speed of learning, and ability to learn), was replicated with college and high school students but not with middle school students. In other words, although middle school students' EB appeared to be also multidimensional, their beliefs seemed to have a simpler structure than high school and college students' (Schommer-Aikins et al., 2000; Schommer, 1993, 1994).

The Schommer-Aikins model and questionnaire for assessing students' beliefs initiated a line of research linking EB to learning (Hofer & Pintrich, 1997; Schommer-Aikins & Duell, 2013; Schommer-Aikins & Easter, 2015; Schommer-Aikins & Easter, 2006). Following this research line, various studies have revealed different connections between the beliefs and learning outcomes (Buehl et al., 2002; Chen & Pajares, 2010; Muis, 2004a; Rastegar et al., 2010; Viholainen et al., 2014; Wood & Kardash, 2002; Woolley et al., 2004). For example, Kloosterman (2002) identified a connection between EB factors and *effort*. Other researchers associated the EB factors with *motivation*, *confidence*, or *personal interests* (Breiteig et al., 2005; Metallidou, 2013). Spangler (1992) claimed that students' learning experiences contributed to their beliefs about mathematics, and such beliefs, in turn, influenced their attitudes in learning mathematics. Recent studies have also shown that general EB are critical to a person's ways of learning, information processing, and problem-solving (Dunekacke et al., 2016; Lunn Brownlee et al., 2017; Metallidou, 2013; Schommer-Aikins & Duell, 2013; Schommer-Aikins & Easter, 2015; Trakulphadetkrai, 2017; Viholainen et al., 2014). Nevertheless, questions about how unique a certain group's EB are and their relations to other aspects of learning such as attitudes, motivation, and self-efficacy in learning specific domain areas such as mathematics remain unanswered.

Views About Mathematics

VM are often interpreted as an individual's understanding and attitudes that shape the ways the individual conceptualises and engages in mathematical behavior (Kloosterman & Cougan, 1994; Muis, 2004a; Schommer et al., 1992; Viholainen et al., 2014). Some studies have provided a deeper understanding of students' VM and their effects on learning mathematics (Kloosterman & Cougan, 1994; Schoenfeld, 1989; Schommer-Aikins et al., 2005). For example, VM can be a predictor of students' attitudes in learning such as completing a task or giving up on it (Kloosterman & Cougan, 1994; Schoenfeld, 1989; Schommer-Aikins et al., 2003). Sample attitudes asked in Kloosterman's VM questionnaire include "I can solve time-consuming mathematics problems" or "There are word problems that cannot be solved with simple, step-by-step procedures" (Kloosterman, 1992, p. 110). Schoenfeld (1989) also showed that many high school students believe mathematics can be learned by only memorisation without real application problems to practice. Researchers also claim that teachers' instructional methods may influence not only their students' mathematical learning but also their VM (Kloosterman & Cougan, 1994; Schoenfeld, 1989).

Relationship Between EB and VM

Researchers have also examined the relationship between EB and VM (Muis, 2004a; Schommer-Aikins & Duell, 2013; Schommer-Aikins et al., 2005) and possible connections between EB and specific learning outcomes in mathematics (e.g., Hofer & Pintrich, 1997; Kloosterman, 2002). However, there has been a call for further research on EB and their effects on different groups of learners at different levels of domain specificity (Buehl et al., 2002; Cam et al., 2015; Schommer-Aikins & Duell, 2013; Schommer-Aikins et al., 2005), and it has been pointed out that students have different EB by age and majors. Also, such variables made different relationships between the students' EB and VM (Buehl et al., 2002; Carlson et al., 1999; Chen & Pajares, 2010; Schommer-Aikins & Duell, 2013; Schommer-Aikins & Easter, 2006).

Studies on middle school, high school, and college students have been conducted to investigate the relationship between their EB and VM (e.g., Buehl et al., 2002; Schommer-Aikins & Duell, 2013). These studies suggest that as college students gain more academic experiences in domains of interest, they may begin to adjust their general EB developed from childhood (Paulsen & Wells, 1998; Schommer-Aikins et al., 2003; Schommer & Walker, 1995). On the other hand, PETs are a unique group of college students as most of them choose their major not by their academic interest but by their aspiration in a teaching career or love of children (UNESCO Institute for Statistics, 2017). Although PETs' EB have been examined in several studies, how distinctive they are and how distinctive the relationship between their EB and VM is as compared to the other groups, have not been sufficiently investigated.

Research Questions

The research questions addressed in this study were:

1. What are the participating PETs' EB and VM?
2. What are the relationships between their EB and VM?
3. Is there a correlation between the number of mathematics courses taken during their secondary or college education and their EB and VM?

Regarding research question 1, we hypothesised that participants may have different factors and structures from those revealed in previous studies about middle/high school students and college students in other disciplines (Schommer-Aikins et al., 2005; Schommer-Aikins et al., 2013). The factors that consist of their EB and VM were explored and identified by Exploratory Factor Analysis (EFA). The identified factors and the associated model were also compared with those in previous studies with different groups of students.

Regarding research question 2, based on the previous studies by Schommer-Aikins, we hypothesised that there might be high correlations between the participants' EB and VM and between the factors of EB and those of VM. Then, the hypothesised relationships were tested by Structural Equation Modeling (SEM) path analysis using AMOS 19.

Regarding research question 3, we hypothesised that students who took more mathematics courses from their secondary or college education have somewhat developed EB and VM. The correlation between their mathematics background and their EB and VM mean scores were tested using Multivariate Generalised Linear Model.

Methods

Participants

The participants were undergraduate college students (N = 541) in an elementary teacher education program at a midwestern U.S. university. There were 67, 56, 65, 78, 102, 118, and 57 participants from 2010 to 2016 academic years, respectively, of whom 14 were second-year , 239 were third-year, and 278 were fourth-year undergraduate students and 10 were graduate students. When reporting their ethnic origin, 82.3% chose White-American, 7.6% Hispanic, 4.7% African American, 3.2% Asian, and 2.2% Native American. When asked how many college mathematics courses they had completed, 2 reported none, 6 one, 315 two to three, 182 four to five, and 36 six or more. Since there were only 38 males, it was not a balanced sample in terms of gender, even though it appeared to represent the gender ratio of elementary teachers in many U.S. elementary schools (UNESCO Institute for Statistics, 2017). The majority of the participants (64.4%) also reported that they had completed more than five secondary mathematics courses. English was not the first language for 6.4%. All participants were recruited from a course of mathematics inquiry nature required of elementary education majors and taken mostly in their junior or senior year of college.

Instruments

EB Questionnaire

A 30-item questionnaire was used to assess the participants' general EB. The instrument was originally developed by Schommer (1990) and has been continually revised. This study used its latest version, , last used in Schommer et al. (2005). It consists of items to which students respond on a Likert scale from 1 (strongly disagree) to 5 (strongly agree). The questionnaire was repeatedly tested with middle school, high school, and college students and generated a four-factor structure (Schommer-Aikins et al., 2005; Schommer-Aikins et al., 2000; Schommer, 1993, 1994). Factors originally composed to assess students' EB include:

1. Structure of Knowledge (ranging from *knowledge is organised as isolated bits* to *complex, interrelated concepts*),
2. Stability of Knowledge (ranging from *knowledge is unchanging* to *evolving*),
3. Omniscient Knowledge (ranging from *knowledge is constructed by an authority* to *by reason and evidence*),
4. Speed of Learning (ranging from *learning is quick or not-at-all* to *gradual*), and
5. Ability to Learn (ranging from *the ability to learn is fixed at birth to the ability* to *learn is acquired*) (Schommer-Aikins et al., 2005).

For these constructs, according to previous studies, Cronbach's alphas ranged from .63 to .85 for college students (Duell & Schommer-Aikins, 2001) and from .36 to .77 for middle school students (Schommer-Aikins et al., 2005). The instrument was developed in the way that the higher the score, the less epistemologically developed the respondent. According to Schommer-Aikins (2004), to be more epistemologically developed indicates that the belief supports higher order thinking. For example, a student who believes that knowledge is simple and certain would be considered to have a less developed epistemology than a student who believes that knowledge is complex (Schommer-Aikins, 2004). In previous studies, the factors were named in two different ways using either a positive or negative meaning, that is, *Control of Learning* or *Study Aimlessly*. In this study, we selected the titles with negative meaning in that higher scores indicate less developed epistemological beliefs.

VM Questionnaire

The participants' VM were assessed using a combined questionnaire with all items from both the Indiana Mathematics Belief Scale (Kloosterman & Cougan, 1994) and the Usefulness of Mathematics Scale (Fennema & Sherman, 1976). Those two instruments have been used with both high school and college students. The combined questionnaire consists of 7 factors: (a) effortful mathematics, (b) useful mathematics, (c) persistence in mathematics, (d) mathematics confidence, (e) understand mathematics concepts, (f) word problems, and (g) nonprescription mathematics (Schommer-Aikins et al., 2005). Cronbach's alphas for these scales ranged from .54 to .84 in Schommer-Aikins et al.'s studies (Schommer-Aikins et al., 2005; Schommer-Aikins et al., 2000). Unlike the EB instrument, the VM instrument was developed in the way that *the higher the score, the higher motivation to learn to solve mathematical problems* in the respondent (Kloosterman & Stage, 1992). According to Kloosterman and Stage (1992), students with a high motivation to learn about solving mathematical problems will be more likely to become a good mathematical problem solver. In all questions in the instrument, students with high motivation to learn to solve mathematical problems were expected to agree or strongly agree with positively worded items (Kloosterman & Stage, 1992). Thus, a student who has lower scores on EB survey (indicating belief supports higher order thinking) may have higher scores on VM (indicating higher motivation to learn about solving mathematical problems).

Procedure

These two questionnaires were administered at the beginning of each semester during 2010-2016. Participants completed the surveys through Google forms, and their responses were collected electronically and downloaded to MS Excel for further analysis. Participants signed an informed consent form in which the purposes of the research and their rights as a participant were explained before they started the surveys. In addition to the two measures, to assess participants' mathematical background, a question "How many mathematics courses have you completed in K-12 and college education?" was asked. On average, students spent 15–25 minutes completing the online surveys.

Data Analysis Procedure

Research Question 1

To identify the participants' EB and VM factors, EFA was repeatedly conducted until it produced the best model fits such as Eigenvalues (higher than 1), Scree Plots, and percentage of the total variance the factors account for using SPSS 23. The Principal Component for Extraction and Varimax for Rotation was used to assess internal consistency which yielded the factors from the data (Loewen & Gonulal, 2015). The Scree Plots of the initial eigenvalues were also used to determine the number of possible factors in the data.

Research question 2

To find the relationships among participants' EB and VM factors, the correlations between the EB and VM factors were examined with SEMusing AMOS 19. For evaluating the SEM path model, the following indices have been used: Chi-square, Goodness-of-Fit Index, Comparative Fit Index (CFI), and Root Square Mean Residual (RSMR). For Goodness-of-Fit and Comparative Fit Indices, a value of 1 indicates a perfect fit; a value higher than .90 has been considered a reasonable fit. For RSMR, an ideal value is 0, whereas a value smaller than .05 is considered a reasonable fit.

Research question 3

Multivariate General Linear Modelling is a type of ANOVA test that was used in this study to examine the effects of the participants' mathematics education experiences on their EB and VM. The tests were conducted for each belief separately to identify not only the group differences but the effects on the beliefs by the number of mathematics courses taken in their college and K-12 education. In the GLM model, the number of courses taken by the participants was set as a fixed factor and the EB and VM as dependent variables.

Results

Research Question 1, EB

The EFA yielded four factors in 23 out of 30 items: (a) *Speed of Learning* (*Cronbach* α = .78), (b) *Ability to Learn* (*Cronbach* α = .55), (c) *Structure of Knowledge* (*Cronbach* α = .47), and (d) *Omniscient Knowledge* (*Cronbach* α = .76). Seven items, Q2, Q7, Q10, Q20, Q23, Q26, Q30, were removed because they were not associated with any of the factors. The factors were named as specified in previous studies (e.g., Schommer-Aikins et al., 2000; Schommer, 1993, 1994) with the equivalent items constructed. These four factors were determined by the Eigenvalues (higher than 1), and Scree Plots and these four factors account for 40% of the total variance. The items and their factor loadings are shown in Table 1.

The results also revealed both differences and similarities in the factor structures found in Schommer's studies (Schommer-Aikins et al., 2005; Schommer-Aikins et al., 2000; Schommer, 1993, 1994). For example, 8 out of 9 items of the first factor were the same as Speed of Learning in Schommer-Aikins et al., (2000). The item "You cannot learn anything more from a textbook by reading it twice" belonged with the Speed of Learning factor in the present study. However, it was tied with the *Ability to Learn* factor for middle school students in Schommer (2005) and with *Speed of Learning* in Schommer-Aikins et al. (2000). The items in the second factor include Q8, Q29, Q22, Q11, Q13, Q25, and Q12 which indicate beliefs related to ways of learning or control of learning. More than half of the items of this factor, Study Aimlessly, are the same as those of the factor of Study Aimlessly extracted in Schommer et al. (2005). The third factor with the items of Q1, Q3, Q4, Q17, and Q5, named *Structure of Knowledge*, was not found in Schommer-Aikins et al. (2005) with middle school students. The factor *Certain Knowledge* with Q22 and Q25, which was one of the factors extracted in the study with middle school students (Schommer-Aikins et al., 2005), however, did not emerge in this study.

Table 1.
Factor Loadings in Each Factor for the Epistemological Belief Scale

	Item	Factor Loadings* 1	2	3	4
	Speed of Learning (SL):				
Q15	An expert is someone who is really born smart in something.	.69			
Q14	Working hard on a difficult problem only pays off for the really smart students.	.68			
Q18	If I cannot understand something quickly, it usually means I will never understand it.	.65			
Q21	If I am ever going to be able to understand something, it will make sense to me the first time I hear it.	.61			
Q27	Students who are "average" in school will remain "average" for the rest of their lives.	.60			
Q16	Successful students understand things quickly.	.57			
Q24	The really smart students don't have to work hard to do well in school.	.57			
Q6	Some people are just born smart; others are born dumb.	.55			
Q9	You cannot learn anything more from a textbook by reading it twice.	.46			
	Ability to Learn (AL):				
Q8	What students learn from a textbook depends on how they study it.		.57		
Q29	Getting ahead takes a lot of work.		.56		
Q22	Today's facts may be tomorrow's fiction.		.54		
Q11	A class in study skills would probably help slow learners.		.53		
Q13	Thinking about what a textbook says is more important than memorizing what a textbook says.		.48		

Q25	The only thing you can be sure of is that nothing is sure.	.44		
Q12	Learning something really well takes a long time.	.43		
	Structure of Knowledge (SK):			
Q1	It is hard to learn from a textbook unless you start at the beginning and learn one chapter at a time.		.65	
Q3	The best thing about a science course is that most problems have only one right answer.		.56	
Q4	You will get mixed up if you try to combine new ideas in a textbook with what you already know.		.52	
Q17	I really do not like listening to teachers who cannot seem to make up their minds as to what they really believe.		.46	
Q5	I like it when experts disagree.		.45	
	Omniscient Knowledge (OA):			
Q19	Scientists can get to the truth if they just keep searching for it.			.87
Q28	If scientists try hard enough, they can find the truth to almost everything.			.82

* Extraction Method: Principal Component Analysis. Rotation Method: Varimax with Kaiser Normalisation.

Table 2 shows the participants' means and standard deviations for the factors of EB. Such results suggest that the participants were likely to believe in *Speed of Learning* (*M* = 1.75 out of 5) and *Ability to Learn* (*M* = 2.03). The higher mean score of *Omniscient Knowledge* (*M* = 3.15), however, indicates that they were more likely to believe that knowledge is rather constructed by authority than evidence or reasoning. Also, the mean score for *Structure of Knowledge* (*M* = 2.87) indicates that they were more likely to believe knowledge is organised as isolated pieces than interrelated concepts.

Table 2
Means and Standard Deviations for the EB Factors Extracted

Factors	*M*	*SD*
Speed of Learning (SL)	*1.75*	*.52*
Ability to Learn (AL)	*2.03*	*.53*
Structure of Knowledge (SK)	*2.87*	*.67*
Omniscient Knowledge (OA)	*3.15*	*.88*

Research Question 1, VM

In the present study, an 8-factor model was hypothesised to determine the participants' VM structure. All 36 items were included in EFA using the principal extraction method and the varimax rotation to maximise the independence of the factor scores. The first extraction produced eight factors, and all the factor loadings were over .4, accounting for 58.6% of the total variance. All the factors have eigenvalue greater than 1, and the scree plot indicated that eight factors were plausible. The name of the eight factors and Cronbach's alpha scores between the items are as follows: *Effortful* (6 items, .57), *Confidence* (8 items, .85), *Useful* (5 items, .80), *Understand Concepts* (5 items, .79), *Word Problems* (4 items, .70), *Useful Word Problem* (3 items, .98), *Critical Word Problem* (3 items, .48), and *Memorizing Concept* (2 items, .28). The same 8-factor model was also tested using EFA with middle and high school students' data in previous studies by Schommer-Aikins and colleagues (2000; 2005). Factor loadings are shown in Table 3.

Table 3
Factor Loadings of Items in Each Factor for the Views about Mathematics Scale

	Questionnaire Item	Factor Loadings							
		1	2	3	4	5	6	7	8
	Effortful maths (EM)								
M23	I can get smarter in math if I try hard.	.85							
M17	I can get smarter in math by trying hard.	.82							
M30	Ability in math increases when one studies hard.	.77							
M4	By trying hard, one can become smarter in math.	.76							
M12	Hard work can increase one's ability to do math.	.72							
M36	Working can improve one's ability in mathematics.	.66							
	Maths confidence (MC)								
M35	I'm not very good at solving math problems that take a while to figure out		.78						
M24	I feel I can do math problems that take a long time to complete.		.77						
M5	Math problems that take a long time don't bother me.		.75						
M10	If I can't solve a math problem quickly, I quit trying.		.63						
M14	If I can't do a math problem in a few minutes, I probably can't do it at all.		.49						
M28	I find I can do hard math problems if I just hang in there.		.48						
M6	I study mathematics because I know how useful it is.		.46						
M8	Time used to investigate why a solution to a math problem works is time well spent.		.39						
	Useful maths (UM)								
M9	Mathematics is of no relevance to my life.			.70					
M13	Mathematics is a worthwhile and necessary subject			.67					
M34	Studying mathematics is a waste of time.			.66					
M27	Knowing mathematics will help me earn a living.			.62					
M19	Mathematics will not be important to me in my life's work.			.54					

	Understanding maths concepts (UMC)					
M2	It doesn't really matter if you understand a math problem if you can get the right answer.	.75				
M16	Getting a right answer in math is more important than understanding why the answer works.	.74				
M32	It's not important to understand why a mathematical procedure works as long as it gives a correct answer.	.71				
M33	A person who doesn't understand why an answer to a math problem is correct hasn't really solved the problem	.69				
M22	In addition to getting a right answer in mathematics, it is important to understand why the answer is correct.	.57				
	Word problems (WP)					
M21	Most word problems can be solved by using the correct step-by-step procedure.		.83			
M31	Any word problem can be solved if you know the right steps to follow.		.71			
M7	There are word problems that just can't be solved by following a predetermined sequence of steps.		.63			
M1	Learning to do word problems is mostly a matter of memorizing the right steps to follow.		.63			
	Useful word problem (UWP)					
M25	Learning computational skills is more important than learning to solve word problems.			.76		
M20	Math classes should not emphasize word problems.			.72		
M3	Word problems are not a very important part of mathematics.			.66		
	Critical word problem (CWP)					
M29	Computational skills are of little value if you can't use them to solve word problems.				.78	
M18	Computational skills are useless if you can't apply them to real life situations.				.68	
M11	A person who can't solve word problems really can't do math.				.52	
	Memorizing math (MM)					
M15	Memorizing steps is not that useful for learning to solve word problems.					.60
M26	Word problems can be solved without remembering formulas.					.49

* Extraction Method: Principal Component Analysis. Rotation Method: Varimax with Kaiser Normalisation. a. Rotation converged in 7 iterations.

Participants' mean scores and standard deviations for the VM factors are as follows: Word Problem (2.90, .78), Effortful Mathematics (3.37, .36), Mathematics Confidence (3.71, .75), Useful Mathematics (4.43, .63) and Understand Concepts (4.43, .62). The high mean scores of the factors indicate they had well developed VM beliefs. Such results show that students have better developed beliefs of 3) Useful Maths and 4) Understand Concepts than the other factors. It suggests that they believe that maths is useful and understanding maths is important, but they are not confident in their mathematical ability or whether they can improve even though they study hard.

Research Question 2

To examine the relationship between the participants' overall EB and VM, Pearson's correlational analysis was used with all the items in the instruments first. Results indicated a significant but reversed relationship between the mean scores of the EB and VM, $r(541) = -.48$, $p < .001$. Such a result suggests that a student who has higher scores on EB (the more naïve beliefs) will have lower scores on VM (the less epistemologically developed on VM). After that, we tested all the extracted factors' correlations by combining the two EB and VM models into one SEM model.

The results showed statistically significant negative correlations between most of the factors of the participants' EB and VM. For example, the more likely the participants were to believe that mathematics is *Effortful* and *Useful,* the less they believed in *Speed of Learning* (Table 4). The less they believed in *Ability to Learn*, the more likely they were to believe that mathematics is effortful and useful and understanding mathematics is more important than getting the correct answer. The results also revealed relationships between the participants' beliefs. For example, the belief about the *Structure of Knowledge* is not correlated with their beliefs about *Effortful Math*. Interestingly, a significant positive correlation between the factor *Omniscient Knowledge* in EB and the factor *Effortful Math* in VM was also revealed. It indicates that the more students believed in *Authority of Knowledge* (less developed EB), the more likely they believed in *Effortful Math* (more developed VM).

Table 4

Correlations and Covariance Between the Factors in EB and VM

Factors in EB	Factors in VM	Covariance	Correlation	S.E.	C.R.	*p*
Speed of Learning	Useful Maths	-.10	-.57	.02	-6.27	***
Speed of Learning	Word Problem	-.06	-.21	.02	-3.35	***
Speed of Learning	Effortful Maths	-.05	-.27	.01	-4.21	***
Speed of Learning	Maths Confidence	-.08	-.37	.02	-5.41	***
Speed of Learning	Understanding Maths	-.09	-.63	.01	-6.60	***
Ability to Learn	Understanding Maths	-.06	-.35	.01	-4.35	***
Ability to Learn	Useful Maths	-.09	-.46	.02	-5.06	***
Ability to Learn	Effortful Maths	-.11	-.53	.02	-5.78	***
Ability to Learn	Word Problem	.03	.12	.02	1.65	.100
Ability to Learn	Maths Confidence	-.10	-.43	.02	-5.20	***
Structure of Knowledge	Understanding Maths	-.05	-.27	.02	-3.45	***
Structure of Knowledge	Maths Confidence	-.08	-.26	.02	-3.37	***
*Structure of Knowledge	Effortful Maths	.01	.05	.02	.72	.469
Structure of Knowledge	Word Problem	-.11	-.33	.03	-3.83	***
Structure of Knowledge	Useful Maths	-.09	-.38	.02	-4.25	***
Omniscient Knowledge	Word Problem	-.19	-.34	.03	-5.54	***
Omniscient Knowledge	Useful Maths	-.03	-.08	.02	-1.44	.149
Omniscient Knowledge	Understanding Maths	-.06	-.17	.02	-3.08	.002
Omniscient Knowledge	Maths Confidence	.03	.03	.03	.99	.321
*Omniscient Knowledge	Effortful Maths	.08	.18	.02	3.26	.001

* Unique relations revealed only in this study, which were not revealed in Schommer-Aikins' studies.
*** $p < .001$

Research Question 3

Multivariate General Linear Modelling revealed the effects of the participants' mathematics education experiences on their EB and VM. The test was conducted separately for each belief.

Secondary Maths Background on EB and VM

Table 5 shows descriptive statistics about participants' EB and VM by the number of mathematics courses taken during secondary education, defined as the last seven years of formal education from grades 6 through 12. To assess whether there were significant mean differences of EB and VM between groups by secondary mathematics education, Multivariate Analysis of Variance (MANOVA) was conducted, $F(4, 541) = 0.640$, $p = 0.640$, n.s. for EB, and $F(4, 541) = 1.230$, $p = 0.296$, n.s. for VM.

Table 5
Descriptive Statistics of EB and VM by the Number of Secondary Mathematics Courses Taken

	Number of Secondary Maths Courses	M	SD	N
Epistemological Beliefs	1 - 2	2.32	.11	22
	3 - 4	2.33	.08	148
	5 - 6	2.32	.04	182
	7 -	2.26	.08	189
	Total	2.28	.30	541
View about Mathematics	1 - 2	3.54	.12	22
	3 - 4	3.53	.09	148
	5 - 6	3.54	.05	182
	7 -	3.57	.09	173
	Total	3.58	.339	541

We hypothesised that the more mathematics courses are taken during secondary education, the more developed EB and VM the participants have. The data showed that the difference between the means is very small and statistically insignificant in both their EM and VM by the number of mathematics courses taken. It is rather surprising that secondary mathematics education does not have a significant influence on the students' beliefs. This ran against our expectations as we believed that students who have higher motivation to learn mathematics or have more developed EB might have taken a larger number of courses, which, in turn, would have influenced their beliefs or vice versa.

College Mathematics Background on EB and VM

Table 6 shows descriptive statistics of the participants' EB and VM by the number of courses taken in their college education. A MANOVA test also produced a nonsignificant result: $F(3, 541) = 1.25$, $p = 0.29$ for EB and $F(3, 541) = 0.94$, $p = 0.419$ for VM. This result also indicates that the participants' EB and VM did not show any significant differences associated with their college mathematics education experiences.

Table 6
Descriptive Statistics of EB and VM by the Numbers of College Mathematics Courses Taken

	Number of College Maths Courses	Mean	Std. Deviation	N
Epistemological Beliefs	0 - 1	2.39	.13	8
	2 - 3	2.26	.02	315
	4 - 5	2.26	.04	182
	6 -	2.29	.09	36
	Total	2.28	.30	541
View about Mathematics	0 - 1	3.44	.15	8
	2 - 3	3.57	.03	315
	4 - 5	3.58	.04	182
	6 -	3.60	.09	36
	Total	3.58	.33	541

Discussion

The results from this study revealed several aspects of the participants' EB and VM and the relationships among the factors such as the factor of *Ability to Learn* or the correlation between *Omniscient Knowledge* and *Effortful Maths*. The findings also lead to several practical implications for educators in elementary teacher education programs as described below in detail. Such implications are consistent with the literature on teachers' EB and VM, particularly with regard to their powerful impact on their practice of teaching (Ernest, 1989). As Ernest (1989) argued, the transformation of teaching practice can start with teachers' awareness of the gap between beliefs and practice such as self-evaluation with regard to putting beliefs into practice.

Regarding research question 1, the results showed how participants' EB and VM are structured with the factors extracted by EFA. As stated earlier, our participants' beliefs are similar to that of middle school students in that they also had this factor "Study Aimlessly." Then, the participants' mean score for *Study Aimlessly* was lower than that of the middle school students in Schommer-Aikins et al.'s (2005) study. The middle school students' higher score on this belief indicates they showed relatively less-developed belief about *Control of Learning*. The presence of the factor of *Ability to Learn* in this study is also an interesting finding because it did not emerge in previous studies with high school or other college students but only emerged with middle school students. It suggests that PETs have a similar belief system as middle school students, that is, they may see learning as strategic or as a chance event. However, the mean score of the factor is low in which more of the participants view learning as strategic and effortful rather than as out of their control. It may be interpreted as meaning that the participants were aware of this belief that they have formed through their experiences in the early years of secondary schools and have developed more sophisticated directions along with their aspiration for their teaching career.

The participants in this study were less likely to believe in *Speed of Learning* (the lower score indicating more developed EB) as well and were more likely to believe in *Structure of Knowledge* and *Omniscient of knowledge* (the higher score indicating less developed EB). Such results suggest that the participants have more developed beliefs in "how to learn" consistent with contemporary educators' ideas but not about the nature of knowledge. Similar aspects were revealed in other studies in which there were positive relationships between students' EB and preferences for constructivist learning environments (Gunstone, 1991; Taylor & Fraser, 1991). According to Gunstone (1991), a student who asserted that knowledge was a collection of proven facts and corresponding formulas did not see any advantage in the constructivist learning strategy. Such results are reinforced when PETs' EB are compared to other college students' EB examined in Schommer-Aikins et al.'s (2003) study. It suggests that PETs' aspiration in the teaching career may have influenced their EB and their relation to their VM.

The findings in this study have important practical implications for teacher educators. Although PETs' EB in an elementary education program can vary, teacher educators may assume that they have some common aspects in the ideas about how to learn. However, there may be a greater discrepancy between the PETs' belief and their teacher educator's beliefs about *what knowledge is*. For example, in the authors' experiences, if asked to write a lesson plan, some students may focus on listing concepts and facts (without connecting them through the class activities), making it rather teacher-centred. Another example is, when given a project, they may try harder than other college students but they still want rather detailed instructions for it by their instructors. Thus, teacher educators may need to allow students more freedom regarding their project ideas or their lesson plan templates or structures. If a teacher education program has a specific lesson plan template or format for their students, it may be giving an impression that knowledge is unchanging and constructed by authority. Thus, giving students more freedom on tasks and assignments would allow them to develop EB more congruent with constructivists' views.

Regarding research question 2, the correlations that emerged between the extracted EB and VM factors provide interesting insights into the participants' beliefs in learning and mathematics. The belief about *Speed of Learning* is significantly and negatively correlated with all five VM beliefs. It is consistent with the findings from previous studies (Schommer, 1993; Schommer et al., 2005; Schommer-Aikins et al., 2013). However, some unique relationships were also revealed in this study. For example, the participants' belief in *Omniscient Knowledge* is significantly but positively correlated with their mathematical views about *Effortful Maths*. This positive relationship indicates that the more students believe knowledge is constructed by authority (i.e., higher score indicating less developed EB), the more likely they are to make efforts to improve (i.e., higher score indicating more developed VM). The participants' beliefs in *Structure of Knowledge* is significantly and negatively correlated with all the VM beliefs but "*Effortful Maths*." It indicates that the more the participants believe that knowledge is organised and interrelated collection of concepts, the more they value the importance, usefulness, and understanding of mathematics. However, it is less likely related to how much they make efforts to improve themselves in mathematics. Such relations are unique and suggest that PETs' aspiration in their teaching career may have influenced their EB and its relations to VM.

These findings have important, practical implications for educators in elementary education programs. PETs' aspiration in their teaching career may affect their EB and their relations to VM, which, in turn, may affect how they will teach mathematics in the future. Thus, learning how to teach mathematics in ways of contemporary educators' views may also change their EB in that way as well. For example, instead of practicing on mathematical problems repeatedly, giving physical or virtual manipulatives to come up with a creative solution to the problem may help

PETs to see how conceptual understanding can be formed from experience. Or, instead of memorizing formulas or equations, having them see how the concepts are related to real-world situations or scientific phenomena may improve their beliefs about how knowledge is structured.

Regarding research question 3, we hypothesised that the more mathematics courses the participants take, the more likely they have well-developed beliefs, and vice versa. However, the correlation between the numbers of secondary or college mathematics courses and the participants' EB or VM were not significant. This may indicate that how mathematics is being taught did not have any effects on their EB or VM. Such a result suggests that well-developed EB such as constructivists' views about learning, higher thinking skills, or learning to solve mathematical problems and associated knowledge should be explicated in mathematics courses.

This study has limitations and yields several suggestions for future studies. First, although we assumed that the participants' EB and their relations to VM may be influenced by their career aspiration, we did not examine how their career interest affects the beliefs directly. Thus, using an instrument to gauge the level of PETs' career aspiration and to test its relations to their EB would provide better insight into their beliefs. Second, the PET sample used in this study is limited locally to an urban area in a Midwest state in the U.S. Their characteristics may be different from that of PETs in other regional areas or other countries. Thus, if the instruments could be administrated more widely, the assumptions from the results can be generalisable to a larger extent. Moreover, how the environments of the college or characteristics of the student populations affect the PETs' EB and VM should also be investigated. Lastly, the number of secondary or college-level mathematics courses taken by PETs may not be an indicator for their mathematical understanding or proficiency. Thus, the use of an instrument to measure students' level of mathematical understanding may be more accurate to determine its relations to their EB or VM.

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