Self-Study of a Mathematics Learning Consultant: Supporting Teachers to Plan Lessons for Implementing Differentiation in the Classroom

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There is widespread belief that addressing differentiation within classroom practice is critical for school improvement in mathematics. School mathematics leaders and consultants are faced with the challenge of interpreting the various models of differentiation that are presented throughout professional learning programs and aligning these with existing beliefs and cultures within schools. This paper reports on a self-study of a learning consultant who supported primary mathematics teachers to plan lessons for implementing differentiation in their classrooms and aims to highlight the Mathematical Knowledge for Teaching (MKT) required to lead such planning sessions. Over three weeks, a mathematics consultant assisted teaching teams across three different schools by facilitating planning meetings. The consultant recorded self-reflections after each session and the qualitative data were coded to identify the categories of MKT the consultant relied on during her interactions with teachers. Overall, the results highlight the dominance of three categories, Knowledge of content and students (KCS), Knowledge of content and teaching (KCT) and Specialised content knowledge (SCK) emphasised by the consultant, as well as the influence different contexts had in developing a shared understanding of differentiated learning in mathematics. The findings have implications for ways in which schools and facilitators develop strategies that comprehensively support teachers to plan suitable differentiated lessons for their students. Furthermore, this study highlights the need to develop greater understanding around models of planning that adequately support teachers to plan effectively for differentiating instruction in mathematics.

Keywords · self study · differentiation · school mathematics leader · teacher knowledge · primary

Introduction

The need to differentiate instruction to cater for students' diverse learning needs is considered a critical component of teacher practice. Policy documents reflect the expectation that all students have maximised opportunities to access a comprehensive curriculum (Sullivan, 2015). As a consequence, school systems and leadership teams are required to adhere to differentiated practices. Most teachers are familiar with the concept of differentiated instruction (Lynch et al., 2018) but attaining proficiency in differentiated instruction is a dynamic process that continually develops as teachers strengthen their knowledge of how students respond to different tasks (Hackenburg et al., 2020). Others would argue that differentiated instruction is one of the most difficult aspects of teaching behaviour for teachers to develop (Maulana et al., 2020; van der Lans et al., 2018).

Framing differentiated instruction as a philosophy of practice is complex and requires teachers to obtain considerable knowledge about both the content and the students they are teaching (Tomlinson & Imbeau, 2010). In our study we agree with Tomlinson and Imbeau (2010) that differentiation is best viewed as a set of principles to guide teacher practice rather than a set of methods or instructional models to implement. Tomlinson (1999) described differentiated instruction as when content, process and/or product is modified by the teacher in response to student learning needs. Teacher modifications should adhere to the general principles that underpin differentiated instruction: respecting the readiness of learners; expecting growth and provision of support for all learners; offering all students opportunity to explore essential skills and understanding at an appropriate level of challenge; and a provision of tasks that are engaging and interesting to students of all abilities.

Mathematics pedagogies that aim for student inclusion also align with the principles described by Tomlinson (1999). Specifically, the challenging task approach described by Sullivan et al. (2015) encapsulates differentiated instruction principles as the pedagogical approach begins with communal classroom experiences where students work to solve open ended and open middled tasks through an inquiry pedagogy. Variations in student readiness, while often addressed through the open nature of the tasks, are further supported by enabling prompts that simplify the steps of the main task while students maintain the same mathematical focus as the rest of the class (Sullivan et al., 2009; Sullivan et al., 2020). Likewise, extending prompts can be utilised for students who are ready to take their learning further as these prompts encourage students to explore the connectedness and depth within the original task (Sullivan et al., 2009; Sullivan et al., 2020). Rather than view the diversity in student responses and readiness as problematic, a challenging task approach capitalises on the different thinking processes of students, building communities of practice where rich and robust mathematical thinking opportunities are fostered and encouraged for all students (Sullivan, 2015).

One barrier facing mathematics educators in supporting schools to implement differentiated instruction is the gap between research findings and current school practices. Despite significant progress within the research that aligns differentiated instruction with reform approaches to mathematics, such as the challenging tasks model proposed by Sullivan et al. (2015), current classroom teaching is still likely to reflect models of traditional instruction pertaining to fixed ability mindsets (Cobb & Jackson, 2015). Fixed ability mindsets can devalue student diversity leading to instructional models that rely on ability grouping, streaming or individualised work plans (Marks, 2013). Long term evidence indicates ability grouping leads to an overall decline in student performance (Cheeseman & Klooger, 2018) and influences how students' value and perceive the worth of mathematics in adulthood (Boaler, 2012).

Traditional mindsets of mathematics education coupled with a lack of clarity about differentiated instruction are proving problematic for teachers in creating inclusive classrooms for students when learning mathematics (Cobb & Jackson, 2015; Dack et al., 2019). In mathematics, a teacher's discipline knowledge is heavily shaped by their own experiences as students (Beswick et al., 2016) and it is common that there are gaps in their subject matter knowledge (Livy & Vale, 2011). Surface level content knowledge can prevent teachers from appreciating how a recommended differentiated instruction pedagogy such as an inquiry approach or challenging tasks model can support students' conceptual understanding (Lynch et al., 2018).

Jane, the first author, was transitioning from classroom teacher to Mathematics Learning Consultant (MLC) at the time of the study. School leaders such as MLCs support teachers to develop deeper knowledge for teaching when planning for effective differentiated instruction. In supporting excellence in mathematics teaching practices Jane had become aware of and concerned about the lack of consistency and clarity with differentiated instruction practices amongst the teams of teachers she led. Jane had started to realise her beliefs and practices, including the interplay of her own subject matter and pedagogical knowledge (e.g., Ball et al., 2008) were being challenged as she planned with groups of teachers and was interested in how she might rely on her knowledge for teaching as a leader.

Jane faced a situation, similar to that researched by Chick and Beswick (2018), who advocated that it is time to question the assumption that being a successful and effective school mathematics teacher is enough to qualify one as a mathematics teacher educator. In addition, Beswick et al. (2016) agree there is limited research on the knowledge required to lead school based professional learning of mathematics. There is literature that identifies the meta-knowledge mathematics teacher educators develop when placed in university settings (Beswick & Chapman, 2015), but limited research exists on the Mathematical Knowledge for Teaching (MKT) for school-based leaders. Therefore, a self-study into the practice of Jane becoming an MLC and her role to support teachers when planning and implementing differentiated instruction will contribute to the literature. The Domains of MKT framework (Ball et al., 2008) will guide the inquiry, in order to respond to the following research question:

> What categories of MKT does an MLC emphasise when supporting different groups of primary mathematics teachers to plan for differentiated instruction?

Literature Review

The degree to which teachers understand and successfully implement differentiated instruction is influenced by two types of knowledge: subject matter knowledge and student knowledge (Tomlinson & Imbeau, 2010; van Geel et al., 2019). In mathematics education is it recognised that there is a difference between the knowledge one needs to be proficient with mathematics compared to the knowledge required to teach mathematics (Ball et al., 2005). The following review of literature first presents a theoretical framework and the different types of knowledge teachers require to teach mathematics effectively. We then explore how this knowledge for teaching might influence the various approaches teachers use when implementing differentiated instruction in mathematics. The final part of the review considers how teacher educators utilise their knowledge for teaching when supporting practising teachers to plan for differentiated instruction in mathematics.

Theoretical Framework and Knowledge for Teaching

Within the literature, different studies have categorised the knowledge teachers rely on when teaching mathematics (Ball et al., 2008; Shulman, 1987). Expanding upon subject matter knowledge and pedagogical content knowledge defined by Shulman (1987), Ball et al. (2008) created a theoretical framework: Domains of MKT. Figure 1 shows the MKT framework including three domains that categorise subject matter knowledge and three domains that categorise pedagogical content knowledge (Ball et al., 2008).

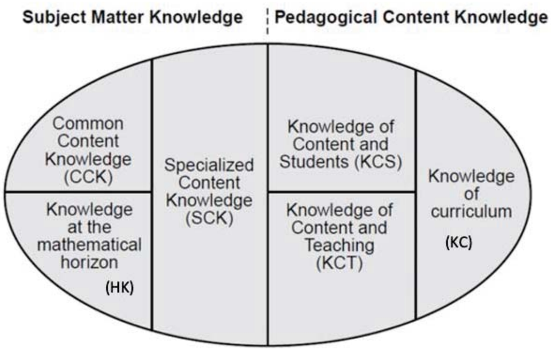

Figure 1. Domains of Mathematical Knowledge for Teaching (Ball, et al., 2008, p. 403).

The six categories (Figure 1) can be used by teachers and their leaders to critically focus on how to utilise MKT for teaching including when planning for differentiated instruction. Subject matter knowledge includes: Common content knowledge (CCK); Specialized content knowledge (SCK); and Knowledge at the mathematical horizon (Figure 1). CCK consists of mathematics knowledge used in everyday settings. SCK is the knowledge that distinguishes mathematics teachers from mathematically competent adults. SCK is important for mathematics teaching and requires an ability to decompress conceptual understandings into developmental stages to scaffold student learning and is unique within the field of mathematics teaching (Ball et al., 2008). The third provisional category is Horizon knowledge (HK) and is evident when a teacher knows the larger mathematical landscape beyond the level they are teaching.

The second domain of the Ball et al. (2008) MKT framework is pedagogical content knowledge and includes: Knowledge of content and students (KCS); Knowledge of content and teaching (KCT); and Knowledge of curriculum. KCS represents the ways teachers interact with their students and the mathematics simultaneously. KCS can be enacted in the analysis of student work and the degree to which teachers can evaluate student responses when determining a future course of action (Sullivan & Mornane, 2014). KCT refers to the particular decisions teachers make when selecting tasks for a purpose and the way these tasks will be sequenced, based on the mathematical concepts they teach and assess (Ball et al., 2008). The final category Knowledge of curriculum (KC) has been researched less extensively (Hill et al., 2008).

Many agree that professional development programs support teachers' growth in their subject matter knowledge and pedagogical content knowledge (Cobb & Jackson, 2015; Sullivan, 2011). Whereas some teachers [and leaders] seem unaware of the need to change their practice, others understand the intent of practice change, yet lack the clarity or skill to do so effectively (Dack et al., 2019) without support from others. Cobb and Jackson (2015) recommended that focus should be on supporting teachers longer term within their school contexts. Therefore, there is a need for effective MLCs who can provide ongoing support to teachers in their school so as to sustain such change and approaches to differentiated instruction. Such support may be improved when mathematics leaders [and teachers] focus on the MKT categories (Ball et al., 2008) when planning for differentiated instruction and implementing professional development initiatives such as teaching with challenging tasks (Sullivan et al., 2009; Sullivan et al., 2020).

MKT for Differentiated Instruction

For leaders and MLCs who support practising teachers with planning and implementation of their mathematics curriculum, the MKT framework can shed light on the extensive knowledge required for differentiated instruction. Deep knowledge of content and students is a salient theme throughout the literature when describing effective conditions for differentiated instruction (Mills et al., 2017; Tomlinson & Imbeau, 2010; van Geel et al., 2019). Yet as current classroom practices continue to reflect surface level teaching in mathematics (Cobb & Jackson, 2015), it can be argued that current levels of MKT are insufficient in supporting teachers to implement differentiated instruction models.

Traditional practices focus heavily on fixed ability notions where speed and neatness correlate with competence (Anthony & Hunter, 2017). Often, such practice reflects a teacher centred pedagogy where content is disseminated by the teacher for students to remember and retain. It is likely that traditional practices influence a widespread use of ability grouping as a default model to differentiate instruction (Mills et al., 2017) as such grouping can continue to operate in a teacher centred model (Marks, 2013). Even when physical grouping of students has ceased and classes are heterogeneous in nature, the undercurrent of traditional approaches hold strong and observed teacher-student interactions reveal some teachers continue to hold fixed ideals about mathematical ability (Marks, 2013). Differentiating by way of individualised instructional models can be another by-product of traditional mindsets. Not only do such models of differentiation create an unsustainable workload for teachers, segregating learning opportunities by different groupings and the provision of individualised programs exacerbates the gap in student achievement (Hopkins, 2011).

Student centred differentiated instructional models based on inquiry pedagogies align with reform approaches in that they position mathematics as a social endeavour (Mills et al., 2017) and encourage teachers to create cultures that foster inclusive practices (Lambert, 2015). However, a particular challenge teachers have in executing inquiry-based lessons is to reframe their role in guiding the learning and avoid telling students the processes and procedures for solving tasks. Warshauer (2015) describes this as allowing students to experience productive struggle. To ensure students continue to take ownership of their learning, teachers are encouraged to use probing questions to elicit student thinking. Ensuring that students' thinking directs lessons provides greater opportunity for the development of conceptual understanding, yet teachers need to be prepared for the many pathways that students take in achieving the mathematical intent (Askew, 2015). As a mathematics teacher, it is one thing to notice a deficient approach in student learning, but quite another level of knowledge to determine an appropriate response during the course of a busy lesson (Mason & Davis, 2013). Therefore, it is critical that teachers have substantial knowledge across all of the six categories within the MKT framework in order to make suitable adaptations to differentiated instruction.

Teachers are more likely to adopt differentiated instruction practices if proposed models and approaches are straightforward and compatible within their current context for teaching mathematics (Cobb & Jackson, 2015). Expecting teachers with traditional notions of mathematics to adopt an inquiry approach to teaching without adequate planning support creates environments which become chaotic and inadequate for deep learning (Reeve & Halusic, 2009). When preparing for teaching with open ended tasks, Sullivan (2020) recommends that teachers begin their planning by trying to do the task first themselves. By doing the task, teachers gain a greater insight into the range of possible solutions and begin to appreciate the depth and connectedness of the mathematical knowledge required within the task (Brown & Coles, 2010). Solving the tasks in collaborative planning time can support teachers to anticipate the range of student responses that may arise within the lesson and prepare adaptations accordingly (Smith & Stein, 2018). Making time to plan in this way places teachers in a stronger position to be responsive to their student needs in the moment of teaching and more likely to implement differentiated instruction modifications more effectively.

Knowledge Development when Transitioning from Teacher to System Leader

When making the professional transition from classroom teacher to system leader (in this case MLC) a challenge is the change in role of working with theory on a deeper level and translating theory into relevant teaching practices (Prescott, 2011). Knowing how to develop the skills and knowledge teachers need is complex and relies upon pedagogical adaptations being made as the learning unfolds (Chick & Beswick, 2018). Without specific training or support, many MLCs initially rely upon their experiences as classroom teachers and find it is inadequate (Loughran, 2014). To support teachers dynamically, MLCs require extensive subject matter knowledge, familiarity with a range of pedagogies and knowledge to interpret many different structures and system requirements schools adhere to (Cobb & Jackson, 2015). This study addresses these issues by reporting on a self-study of an MLC to determine the MKT she used during planning for differentiated instruction with teachers in three schools.

Methodology

Context of Study

With the intent to better understand and analyse her role as an MLC, Jane commenced a self-study investigation to identify the knowledge required to undertake the role effectively. Self-study methodology begins from a personal situated inquiry context and provides a means through which practitioners can closely learn from their practice (Samaras, 2011) and was suitable for answering the research question. Jane had 15 years of experience as a primary school teacher, and in the last seven of those years held the dual role of school mathematics leader and classroom teacher. She had been working as an MLC for three years when conducting this study. As both a school mathematics leader and MLC, Jane had the opportunity to participate in a range of extensive professional learning programs that developed her knowledge for mathematics teaching and leadership in primary schools. The contexts for this study were onsite mathematics planning sessions with teaching teams across three primary schools. Prior to the study, Jane met with each school's principal to gain insights into some of the broader contextual issues that then in turn influenced the way she worked with three groups of teachers.

The focus of this study was to report on how Jane emphasised different components of MKT when supporting teachers to plan for differentiated instruction through a facilitated planning design. Facilitated planning occurs when teachers collaboratively plan with the guidance of a mentor [school mathematics leader and/or MLC]. Jane based her approach on a combination of prior experiences she found effective as well as knowledge gained from the literature about mathematics professional development including teaching with challenging tasks. For example, some of the strategies she considered effective were to encourage teachers to do the mathematics tasks themselves and to prompt collaborative discussion (Smith & Stein, 2018); create opportunities for analysis of student work samples; as well as working from the planning processes that currently existed within the school (Roche et al., 2016). Jane wanted to ensure her actions were responsive to teachers' needs, by helping them to consider how different students might respond to the same problem, as well as planning and discussing ways to implement differentiated instruction.

Data Collection and Analysis

In this self-study, data were collected by observing, analysing and interpreting one's own practice within a situated context (Samaras, 2011); therefore, the nature of data collection for this project was qualitative. The MLC chose to collect data at three schools, during six planning sessions over a three-week period providing sufficient data to code when responding to the research question. All schools and teachers remained anonymous. In addition, to ensure the self–study met the rigorous design recommendations that include transparency and credible data sources (Loughran, 2007), Jane invited a colleague (also an MLC) to participate in the investigation in the role of critical friend. Throughout the three weeks of data collection (Table 1) they both met once a week for approximately 30 minutes to discuss progress and observations gathered for each week of data collection.

Table 1. School planning sessions and data collection schedule (n = number of teachers).

Week	Tuesday	Wednesday	Thursday	Friday
Term 4 Week 2		School A Years 5/6 Multiplication (Session 1) n=4	School B Foundation Structure of number (Session 2) n=3	School B Years 1/2 Area (Session 3) n=4 Critical friend meeting
Term 4 Week 3	School C Years 1/2 Fractions (Session 4) n=2		School C Years 5/6 Measurement (Session 5) n=2	Critical friend meeting
Term 4 Week 4		School A Years 5/6 Multiplication (Session 6) n=4		Critical friend meeting

Table 1 shows when qualitative data were collected; the year levels taught and the mathematics topics that were the focal point of the planning sessions. After each planning session one journal entry was completed. The journal entry numbers correlate with each session number as reported in the results section.

First data were collected by reflecting on planning and implementation of differentiated teaching with Foundation to Year 6 teachers (students aged 5-12) including when planning number, geometry and measurement lessons (Table 1). Each planning session lasted for about 90 minutes; then after each session Jane recorded reflections in a journal. She developed questions to guide her reflections, for example:

> How did I support teachers to plan their mathematics in order to cater for the different learning needs within their class?
>
> Were my pedagogical choices appropriate for the learning intention of the session?
>
> In what ways can I reconceptualise my knowledge for teaching mathematics so that the teachers engage more readily with the mathematics themselves?

Three additional journal entries were completed to reflect upon each meeting Jane had with her critical friend at the end of each week during the study. After all data were entered into a word document both authors independently colour coded data using the six categories from the

Domains of Mathematical Knowledge for Teaching (Ball et al., 2008). They then checked each other's coding and agreed on any discrepancies to ensure consistency. Table 2 shows the coding system including samples from the data.

Table 2. Sample of self-study log entries and 6 categories of coding.

Colour	Code	Category from MKT	Example indicator from self-study log entries
Yellow	CCK	Common content knowledge	Solving a 4-digit multiplication equation
Green	SCK	Specialized content knowledge	Fractional representations that cover more than partitioning and include other constructs such as fraction as a measure and fraction as a divisor.
Purple	HK	Knowledge at the mathematical horizon	Development of extending prompts to allow students to demonstrate breadth of mathematical understanding.
Pink	KCS	Knowledge of content and students	Conversations about learning dispositions of students and unpacking student responses in order to make inferences about student understanding.
Orange	KCT	Knowledge of content and teaching	Anticipating some of the possible student responses and preparing suitable questions for students to elicit their mathematics understanding.
Blue	KC	Knowledge of Curriculum	Referencing achievement standards from Victorian Curriculum when interpreting different student work samples.

All six categories of the MKT framework were used to code the journal entries (Table 2). The multidimensional nature of mathematics knowledge for teaching (Ball et al., 2008) resulted in several indicators being coded twice demonstrating the interconnectedness within several domains. For example, adapting a task to enable students to engage with mathematics required knowledge of KC and KCS.

Results

The results are presented in two sections. First the findings unique to each planning session as documented by the first author in her journal. Each planning session (see, Table 1) presents a sequence of MKT categories emphasised by the MLC along with a short description of key observations that highlight the context the MLC was working within. The next section reports overall trends emerging from the MLC's practice as a facilitator of planning for differentiated mathematics teaching.

Mathematical Knowledge for Teaching Identified in School-Based Contexts: School A

School A conducted online assessments as part of a pre- and post- unit testing schedule. This assessment data was an initial focus of the first meeting to highlight the range of student achievement levels. The two planning sessions for School A occurred two weeks apart, Session 1 and Session 6 (Table 1) are reported first.

Session 1: Year 5/6, multiplication (*n* = 4)
To engage the teachers in the mathematics and promote collaborative dialogue, I made a conscious choice to start the first planning session by asking the teachers to solve the following task, "What two numbers multiply to give 4800?"

I anticipated that this question would be considered accessible for adults to do, essentially drawing upon the category of CCK. As expected, all teachers were able to solve this task and did so using various strategies and methods. With encouragement, I asked all teachers to share their solution and strategies. Everyone had approached the task differently and appeared surprised and interested in the various solutions. Having had this shared experience, I was able to ask the teachers,

> What are the big mathematical ideas this problem explores … How could you use a question like this to create a lesson for your class? (Journal entry 1)

The first question led to a conversation about knowledge (SCK) required to complete the task while the second question anticipated how Years 5/6 students may solve the task (KCS). I was able to use the teachers' experience of solving a task at the beginning of the session, to exemplify how students may come up with various strategies. After discussing the possible strategies that students may use (KCS), the group was able to consider some of the supportive teaching models such as exploring number patterns in multiplication, along with questions teachers might pose during the lesson to support students with different strategies (KCT).

Session 6: Year 5/6, multiplication (*n* = 4)
Prior to this planning session, I had requested that teachers bring student work samples to analyse as a way to elicit the range of student knowledge before determining where to go next for planning. Discussion of the work samples created a forum for teachers to explore different strategies and thinking that students were able to demonstrate (KCS). During this analysis I was able to lead the teachers into a more precise evaluation of what students did and did not understand by focusing on the different student approaches rather than the completed answer (KCS). While this technique was not new, for this group of teachers it was a shift away from using spreadsheet test data to determine where to take student learning next. The student work samples helped me to prompt questions such as:

> Is this what you expected of this student? What surprises you about this student's response? Where will you take their learning next and why? What question might you ask to develop this thinking? (Journal entry 6)

Interpreting the work samples, discussing teacher reflections and student responses drew upon the categories of KCS and KCT respectively. Viewing these samples as formative assessment also led to KC, SCK and HK being emphasised albeit to a limited capacity.

Mathematical Knowledge for Teaching Identified in School-Based Contexts: School B

School B was part of a research project and professional learning program investigating the implementation of sequences of challenging tasks (Sullivan et al., 2020). Their participation in this project meant that tasks were provided for teachers to adapt into lessons. Additionally, differentiated practices were emphasised within the philosophy of the project encouraging schools to create a classroom culture of inclusivity and student agency (Sullivan et al., 2020). While foundational elements of differentiated instruction were shared through the school's participation within the project, implementing these within each teacher's classroom continued to be a challenge. Tasks provided were open in nature, yet classroom observations indicated that teachers were still gaining confidence in guiding classroom discussion to support differentiated instruction.

Session 2: Foundation teachers, structure of number (*n* = 3)
For School B I chose to start their first planning session by analysing student work samples with teachers (KCS) from a recent lesson. Unpacking the different student responses and strategies prompted a natural discussion about how the samples addressed curriculum content (KC). Connecting student responses from the task to the curriculum content descriptors in this mode instigated an important conversation through which the group were able to interpret the curriculum content to a greater depth. With my prompting, the group were able to discuss different pathways students had taken to solve the task (KCS). Additionally, interpreting ways in which students had recorded their thinking revealed the extent to which students could link the mathematics in the task to other concepts and use prior knowledge to build new knowledge (KCS). My role within this process was to help teachers identify interesting differences in student thinking (KCS) and raise questions to prompt teachers to think about "what next?" for their teaching. With a common understanding of lesson structure (Sullivan, 2020) and the use of enabling and extending prompts to support the differences in student learning, I was able to lead the conversation to discuss the important mathematical ideas that were to be drawn out in additional tasks for the remainder of the sequence (SCK). Knowing the classroom setting through prior lesson observations, I was able to link back to particular examples to prompt teachers' thinking, when making considerations for how different students learn. We discussed the use of enabling and extending prompts that can support different students when attempting the mathematics (KCS & KCT). This team of teachers had established the practice of solving the task themselves prior to planning for teaching and using this opportunity to then anticipate how their students may respond to the task once implemented. I recalled the difficulties teachers had with eliciting student understanding during class discussions (KCT). Throughout this meeting I was able to prompt teachers in thinking about the probing questions they may use to elicit student thinking and anticipate different solution pathways students may take in order to prepare them for teaching this part of the lesson (KCT).

Session 3: Year 1/2, area (*n* = 4)
Session 3 consisted of a different cohort of teachers. Using the same practices for planning (Sullivan, 2020), teachers were encouraged to do the tasks themselves and anticipate student responses before implementing lessons (KCS). I began the session by asking the teachers to share how they had found the tasks from the project (they were trialling). This debrief proved to be important in order to identify misconceptions about the implementation of the project materials (KCT) but also allowed for insights into how teachers were feeling about teaching mathematics with a more open pedagogy as they considered how to adapt their teaching to support different student responses (KCT). The teachers shared many insights about how their students were responding and we discussed examples of student thinking (KCS). The teachers demonstrated a clear interpretation of student work samples and were able to articulate the different approaches or strategies. I was able to build upon the comments shared by the teachers as we planned future lesson foci to meet student learning needs (KCS). In looking at the upcoming tasks, the group considered how the task goal encompassed the curriculum indicator for that level (KC). Prompting teachers to anticipate different student responses that may arise during the lesson, I encouraged teachers to consider how the different responses may demonstrate various stages of student mathematical understanding (SCK). When possible and appropriate, I referred to

examples of student strategies offered by the teachers themselves (KCS). Finally, the planning session moved towards the ways in which teachers may use different work samples to guide classroom discussion as well as how the use of enabling and extending prompts could support the mathematical development of all students (KCT).

Mathematical Knowledge for Teaching Identified in School-Based Contexts: School C

School C used summative data sets to group and stream students into different ability mathematics classes from Years 3-6 (students aged 8 to 12 years old). The teachers realised that despite students being grouped according to end of year summative scores, they were still needing to modify their lessons considerably to cater to the wide range of abilities within each streamed cohort. As part of my role the Principal was keen to consider different approaches for differentiating the learning and to share these ideas with the teachers.

Session 4: Year 1/2, fractions (*n* = 2)
First, I asked both teachers to solve a fraction task that would be considered non routine task but did not draw upon the partition model of fractions that teachers were accustomed to using (SCK). I purposely selected this task to highlight how traditional models of fractions can limit student understanding. However, I misjudged the teachers' prior knowledge (and beliefs) as they did not engage or connect with the task. In an attempt to re-engage the teachers, I asked them to anticipate how students would respond to a similar type of task. Their responses prompted me to infer that perhaps the task I had used was misaligned with the teachers' level of SCK about fractions. Acutely aware of my responsibility as the facilitator of the planning and the intention of professional learning as intended by the principal, I made an *in-the-moment* decision to proceed with a brief instructional approach to unpack the SCK about the big ideas that students should explore when learning about fractions. It was in this moment that I realised that I had incorrectly made assumptions about what would work best for these teachers based on my own experiences rather than taking more time to critically interpret the teachers' knowledge and the context within which this group were situated. I was able to quickly access alternative resources and task types that were used to prompt teachers to anticipate student responses and open up discussion about their students (KCS). Once teachers were able to focus on students' learning with more familiar task types and engage with the mathematics themselves, I was able to provide suitable support for planning lessons with adjustments, catering for the different abilities described within each class (KCT).

Session 5: Year 5/6, measurement (*n* = 2)
The situated context of both teachers and School C influenced the way in which I approached this planning session as well. My prior observations had revealed the senior classes were grouped according to their end of (previous) year summative test. Furthermore, I was conscious of not making a similar mistake to Session 4 by assuming teacher beliefs were aligned with mine when it came to classroom pedagogies and differentiation strategies. I started by asking teachers to share what lessons they were planning next and why (KCT and KCS). Letting teachers talk about their class, the students and their prior experience when teaching this topic allowed me to develop greater insight into their situation and better informed the interpretations I had made from an earlier classroom visit. Encouraging teachers to talk about their immediate circumstances as a starting point seemed to adequately engage them without the need for an additional task to solve. I was able to support the teachers by making suggestions on how to adapt selected mathematical tasks (KCT) that should allow for students to respond in various ways, with a goal of supporting differentiated instruction. Interest and engagement from both teachers prompted

me to point out some of the critical SCK that they may need to navigate in response to the different strategies and thinking students could use throughout the unit of work.

Summary of MKT Category Activation

Each school planning session provided opportunities for the MLC to rely on her MKT when supporting the different groups of teachers. Table 3 reports each of the MKT categories that were emphasised by the MLC during the six planning sessions.

Table 3. *Summary of MKT category activation.*

	Sequence of school visits	Facilitator initiated category	Sequences of categories addressed throughout planning time
School A	1	CCK →	SCK→ KCS→ KCT
School B	2	KCS →	KC →SCK/KCS → KCT
	3	KCS →	KC→SCK/KCS → KCT
School C	4	SCK→	KCS → KCT
	5	KCS→	KCT→SCK
School A	6	KCS →	KCT→ SCK/KC → HK

Coding of the data in Table 3 shows that all categories of the MKT framework were emphasised by Jane. The results demonstrate an obvious emphasis and interplay of SCK, KCT and KCS that dominated throughout the planning process. The other three categories in this study CCK, HK and KC tended only to occur sporadically.

Overall Insights when Facilitating Planning

Differences with school mathematics leader versus consultant

A recurring theme that arose throughout the journal entries was the difference in leading mathematics as an external consultant compared to how Jane had previously led her teams when working as a school mathematics leader. Deciding how to approach each challenge with a consultant approach instead of a school mathematics leader approach came up each week in the critical friend conversations. The following journal excerpt portrays this challenge:

> When I worked as a school mathematics leader and classroom teacher, I had worked for several months to align teachers in their vision of teaching mathematics across the school. Having the opportunity to create this collaboratively and build joint expectations provided a strong foundation of expectation and common beliefs amongst all staff. I feel that this element is missing when coming in as an external consultant and making the planning session all the more challenging. At times to me it feels obvious that I am an outsider and I question how much the teachers are valuing the planning session. (Critical Friend Reflection Journal 2)

Having had prior conversations with the principals of each school, Jane knew that differentiated instruction was a key pedagogical focus. Throughout each session it was obvious that teachers were aware that they had to cater for the different ability levels of students within their classes, which was reflected in the constant occurrence of KCS and KCT within each session. However, what became obvious to Jane throughout her reflections and discussion with her critical friend was the lack of shared understanding about differentiated instruction. This was particularly evident from her experiences in School C where Jane found it difficult to engage teachers in conversations about SCK unless these discussions were in conjunction with KCS. The misalignment in common understanding about differentiated instruction between herself and each group of teachers was a significant challenge to overcome and required Jane to draw upon knowledge that extended beyond her own first-hand experiences.

Professional learning through a planning model is a co constructed process

At the beginning of the study, Jane approached planning for differentiated instruction by focusing on SCK, thus activating her own SCK in the process. Early journal entries reflect her initial approach, "I started the session by asking the teachers to solve a task" (Journal entry 1). However, as the study progressed, and she experienced different school contexts and teacher interactions she realised that starting with the SCK that she deemed important for planning did not necessarily lead to productive teacher discussions about how such SCK would support teachers in implementing their future lessons.

A significant point in the study occurred when Jane came to realise how she might adapt as a leader, so as to effectively support teachers with their differentiated planning:

> There was an underlying tension throughout the start of today's session and I quickly realised that the example task I had provided for the teachers to try was too far removed from their current practice and it became difficult to 'get on the same page' when it came to the important points for teaching. Once we refocused on the students and particular classroom practices, I was able to support the teachers much more effectively by sharing a selection of suitable tasks and worked with them to decide the ways in which these could be best implemented. (Journal entry 4)

This journal entry shows how Jane's understanding of her own MKT was developing. While differentiated instruction for mathematics remained the focus for each planning session, the way Jane was able to support each group varied according to the circumstances of each teacher.

The coding identified that Sessions 5 and Session 6 (Table 3) started with KCT and KCS whereas during earlier sessions Jane had chosen to use SCK to start the sessions. The change reflects how Jane was able to emphasise different approaches after reflecting on her practice. Jane was considering the impact on teachers and planning including the importance of placing students and teachers at the centre of planning for differentiated instruction. She also realised that when asking teachers to engage in the mathematics, her choice of selected tasks required careful consideration so as to provide a purposeful lead into the planning.

Putting Theory into Practice

The iterative nature of self-study, involving cycles of practice, reflection and further practice resulted in Jane reconceptualising her MKT. Additionally, the conversations with her critical friend helped Jane to evaluate and clarify her understanding of the categories of the framework and reframe her MKT as an MLC.

At times Jane felt and recorded in her journal entries that her MKT and approaches for adapting teaching to meet the learning needs of all students was increasing at a rate faster than she had a chance to implement when leading teachers. In addition to supporting teachers Jane was reading research articles and teacher reference books to extend her knowledge and skills as a leader. Throughout the study the conflict between theory and teacher practice was continuous as described in this excerpt:

> I have come to realise that in this role I am constantly working with an underlying tension – on one side are my strong beliefs and experience of effective differentiated teaching practices in mathematics and on the other is supporting teachers (who at times appear to start from a very different place from me) to reach the goal of effective differentiated practice. Working out how to

> get them there is a challenge – the path I took worked for me, but it won't necessarily work for everyone else. Having patience as a consultant to encourage them and not want to change everything all at once is something I have to be mindful of. (Critical Friend Reflective Journal entry 3)

Having talked about her experiences with a critical friend who was more experienced in the role, Jane came to appreciate many of the challenges of her leadership role, particularly developing knowledge and skills so she could support teachers when planning for differentiated instruction. Moreover, the experience prompted Jane to realise the importance of considering the context of each group of teachers, anticipating and preparing for potential constraints and challenges that may arise in professional mathematical discussion when leading planning.

Discussion

Undertaking this investigation has enabled Jane, the MLC, to better understand the categories of MKT she emphasised when supporting groups of teachers from three schools when planning for differentiated instruction. By analysing her practice across several groups of teachers the self-study emphasised how she grappled with the complexity of supporting groups of teachers. The different planning experiences identified the various mindsets and beliefs teachers held about differentiated instruction and thus became a challenge for the MLC to navigate when such knowledge became misaligned with her own.

As a result, a challenge for the MLC was to establish shared differentiated instruction principles (Tomlinson, 1999) with teachers prior to planning the content or tasks for the student experiences. As part of her role, the MLC needed to consider how to differentiate when extending teachers' understanding of approaches to differentiated instruction. Like a teacher differentiating for students, the facilitator has to be responsive to the teachers' needs and adjust their interactions accordingly (Askew, 2015). This mirrors the challenge of "differentiated instruction" for students as the MLC considered the different needs of her teachers when planning. The lack of clarity for teachers regarding the various approaches to differentiated instruction is concerning (Dack et al., 2019). Centralising teacher planning processes around student learning needs (KCS) aligns with the principles of differentiated instruction described by Tomlinson (1999).

Differentiated teaching approaches that came naturally to the MLC from her own professional learning were the use of open-ended, challenging tasks. Aware of the recommendations in the literature which promote learning opportunities that stem from within the parameters of current teacher practice (Cobb & Jackson, 2015; Sullivan, 2011), the MLC underestimated how difficult it would be to determine the point of learning for each cohort of teachers within the limited allocated planning. At times she had to critique her own MKT in order to support teachers from a different starting point to the one she intended prior to a planning meeting. When comparing the groups of teachers, School B could be considered somewhat of an anomaly within this study. They had adopted a consistent school approach that was a result of their participation in a professional development program. The teachers were more familiar with using challenging tasks through inquiry pedagogies, which allowed the MLC to easily infer the school's context and therefore draw upon research approaches supportive of differentiated instruction as well as effectively anticipate ways to approach planning.

The importance of context continually emerged when coding the data from the MKT. Teacher decisions are heavily context dependent (Dack et al., 2019) and context was continually reflected throughout this study in the many ways that teachers' discussions reverted back to the students in their class (KCS). The experience and study enabled the MLC to reflect on how knowledge of students was critical in the planning process for teaching differentiated instruction (Tomlinson &

Imbeau, 2010), even to an outsider such as herself. The MLC was able to recognise that her initial advice to teachers was based on her own experiences as a classroom practitioner and drawn from a place where she remembered feeling comfortable with her own MKT. As the study progressed, the MLC came to appreciate the importance of placing teachers and their context at the centre of planning. Furthermore, centring the discussion on students allowed teachers to view differentiated instruction more holistically. Starting with the student, not the content knowledge, naturally led to the discussion of more inclusive modifications rather than focus on the process of modifying tasks (Marks, 2013). Teachers know their learners much more deeply than an external facilitator (Ross & Chan, 2016) and the MLC came to value the information teachers were able to share about their students in order to plan for differentiated instructions with tasks.

Another point highlighted in the results was determining effective strategies the MLC might draw upon to readily engage teachers with necessary SCK needed for upcoming units of work. Influenced by the literature on differentiated instruction, the MLC initially placed a large emphasis on SCK in order to adapt tasks for differentiated instruction (Lynch et al., 2018). Strategies such as asking teachers to trial tasks themselves and do the mathematics (Brown & Coles, 2010) were employed within the early sessions of this study but the MLC found that relying on this strategy did not always engage teachers or lead to engaging discussions on SCK. Supporting teachers to anticipate the range of student solutions and responses, as recommended by Smith and Stein (2018), was an additional layer to planning. Encouraging teachers to anticipate student responses allowed the MLC to gain insight into teachers' KCS and KCT. Anticipating student responses in planning enables the preparation of suitable adaptations that can be made for different students to access the learning focus (van Geel et al., 2019). There was an added challenge of noticing and interpreting the nuances of group dynamics that influenced the way suggestions from an external facilitator were received.

The emergence of context as a focal point throughout this study has encouraged a reflection of leadership practice that will change the way in which the MLC considers how to most effectively support schools in planning for differentiated mathematics learning. While Ball et al. (2008) has addressed the multidimensional nature of the MKT categories and acknowledges that they coexist in practice, how the different categories are emphasised throughout different types of teacher professional development is less obvious. The relationship between KCS, KCT and SCK reported within this study appears to be prominent when planning for differentiated instruction. The other three MKT categories (HK, CCK & KC) were emphasised in a smaller capacity by the MLC, suggesting that these areas were less of a focus when supporting planning. It is likely that HK, CCK & KC may be addressed in other types of teacher meetings such as planning yearly overviews of curriculum and therefore less critical for the MLC to address when planning lessons that improve differentiated instruction.

Conclusion

This self-study provided an opportunity to highlight the different categories of MKT an MLC emphasised when supporting different groups of primary school teachers to plan for differentiated instruction by utilising an approach using challenging tasks. Reporting on the six categories of MKT (Ball et al., 2008) provided insights into how such knowledge was emphasised by an MLC when supporting planning for teaching. The iterative reflection cycles provided an opportunity to better understand how different categories of MKT were emphasised and ordered, enabling flexibility in the way learning was adapted when fostering the varying support teachers required. Overall, the results highlight the dominance of three categories, KCS (Knowledge of content and students), KCT (Knowledge of content) and teaching and SCK (Specialised content knowledge) emphasised by the MLC when leading planning. Such insight is imperative into the preparation and knowledge required by facilitators working with and leading teachers when planning for differentiated mathematics instruction.

The findings reported difficulties leaders may have when supporting teachers to plan for differentiated instruction in primary mathematics classrooms. Given difficulties teachers have in developing understanding differentiated instruction as a desirable approach to teaching (Maulana et al., 2020; van der Lans et al., 2018), schools and facilitators must consider ways to develop close working relationships that foster a mutual understanding of what underpins effective differentiated instruction as part of planning. Focusing on the categories of MKT when leading teachers to differentiate instruction may help other mathematics leaders to provide greater depth when leading planning for teaching. In other words, comprehensive support for teachers, provided by facilitators who understand the nuances of their MKT and make connections between the six categories of MKT when supporting planning for differentiated instruction, is pivotal in improving mathematics learning and teaching in our schools.

While we acknowledge the short timeframe of the study as a limitation, the intensity of self-study methodology allowed Jane, an MLC to examine her practice to a depth that could not be achieved by external data. However, we also recognise the inclusion of external data sources, such as teacher interviews and session transcripts would have contributed to more informed conclusions. Further studies would be beneficial to these leaders and contribute to the literature.

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