

Prospective Teachers' Pedagogical Considerations of Mathematical Connections: A Framework to Motivate Attention to and Awareness of Connections

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Research findings and reform-oriented standards emphasise the importance of mathematical connections in support of students' conceptual development. Previous research on teachers attending to mathematical connections has tended to focus on expert teachers' practice. Complementing previous research, this study describes how a cohort of twelve prospective mathematics teachers attended to and made sense of mathematical connections that arose when working with secondary students in small-group instruction. Results indicated prospective teachers were able to attend to mathematical connections during instruction and made several pedagogical considerations around such connections. We present a framework, the *Pedagogical Considerations of Mathematical Connections* (PCMC) framework, which offers mathematics teacher educators a new model to expand prospective teachers' attention to and awareness of mathematical connections. The study contributes to the existing literature on teacher noticing by providing a new kind of theme-specific noticing (i.e., mathematical connections) and informing mathematics teacher educators of how prospective secondary teachers attend to mathematical connections.

Keywords · attention · awareness · mathematical connections · noticing · teacher education

Introduction

Hiebert and Grouws (2007) found, from their review of literature, that when teachers and students explicitly attended to connections among mathematical facts, procedures, and concepts during instruction, students more likely acquired a conceptual understanding. Reform-based standards emphasise mathematical connections as a goal of school mathematics. For example, in the Australian Mathematics Curriculum, a key idea of understanding mathematics is that students will "make connections between related concepts and progressively apply the familiar to develop new ideas" (Australian Curriculum, Assessment and Reporting Authority, n.d.). In the United States (US), the National Council of Teachers of Mathematics (NCTM, 2000) emphasise instructional programs should enable students to "recognise and use connections among mathematical ideas; understand how mathematical ideas interconnect and build on one another to produce a coherent whole, and recognise and apply mathematics in contexts outside of mathematics" (p. 64).

In response to calls for students and teachers to explicitly attend to mathematical connections, mathematics teacher educators (MTEs) will have to contemplate how to assist teachers in supporting these goals. One approach is to examine how expert teachers explicitly attend to mathematical connections (e.g., Ball, 1993; Boaler & Humphreys, 2005; Even, et al., 1993; Lampert, 2001). Examining experts' practice has led to the development of models for instruction. For

instance, the five practices of anticipating, monitoring, selecting, sequencing, and connecting is a model to support teachers in productively using students' thinking in mathematical discussions (Stein, et al., 2008). In particular, the practice of *connecting* supports teachers in explicitly attending to mathematical connections between students' strategies and how ideas within those strategies are related to important mathematical ideas. However, models such as these do not describe *how* teachers *begin* attending to mathematical connections.

Russ et al. argued, "If we are truly interested in teacher learning as a process, then we must pay more attention to the starting state and how learning progresses from this starting state to the end state" (2016, p. 410). Teachers, even novices, bring a range of experiences. Just as teachers should pay attention to and use students' ideas to further their instruction, MTEs should attend to the ideas of prospective teachers (PSTs) and use them to develop PSTs' understandings of significant instructional practices.

While some have described how novice teachers attend to mathematical connections (e.g. Borko & Livingston, 1989; Even et al., 1993; Livingston & Borko, 1990), the focus has been on describing the novice teachers' actions, or lack thereof, from the perspective of the expert or researcher. This study takes a complementary approach by seeking to understand, from PSTs' perspective, what mathematical connections they explicitly attend to in their instruction and how they make sense of those connections. We illustrate how PSTs begin to attend to mathematical connections and suggest activities MTEs could implement to support PSTs to attend to mathematical connections.

Drawing on Mason's construct of the discipline of noticing (1998, 2002), we investigated how twelve secondary PSTs enrolled in a mathematics methods course attended to mathematical connections during their field experience. We illustrate (a) the kinds of mathematical connections the PSTs anticipated before and recalled encountering after their instruction and (b) the pedagogical considerations they made surrounding these instances while working with students.

Literature Review on Teachers Attending to Mathematical Connections

In this review, we highlight some results from teacher noticing studies and expert-novice studies regarding teachers attending to mathematical connections during instruction. We situate our study within these two areas of research and explain how the findings from our study informs MTEs to prepare PSTs to notice mathematical connections.

Teacher Noticing

In a complex sensory world, several phenomena compete for attention. Scholars have argued the way professionals select what to attend to and interpret among phenomena competing for attention is unique and distinguishable from the layperson (Goodwin, 1994; Mason, 2002; Stevens & Hall, 1998). Mathematics education researchers have identified this expertise in mathematics teachers as *teacher noticing* or simply *noticing*.

Dreher and Kuntze (2015) noted that studies on teacher noticing focus on a particular instructional feature, which they termed as *theme-specific noticing*. Two significant, but not necessarily distinct, lines of theme-specific noticing in mathematics teacher education research are noticing *students' mathematical thinking* and noticing *indicators of equitable mathematics instruction* (Jacobs & Spangler, 2017). Both bodies of literature indicate teachers *can* develop their

noticing skills (e.g., Jacobs, et al., 2010; Schack et al., 2013; Sherin & van Es, 2008). In line with this perspective, our study contributes to the existing literature on teacher noticing by offering a new theme-specific noticing, namely, mathematical connections, and providing MTEs with an understanding of the scope of what secondary PSTs *can* attend to in their field experience.

Teachers Attending to Connections

Comparison studies of expert-novice teachers imply novices experience difficulty attending to mathematical connections in their lesson planning and instructional explanations. Experts' plans often connected previous lesson discussions or used the same representations or contexts across several lessons (Even et al., 1993; Leinhardt, 1989). Furthermore, experts used unplanned opportunities to make mathematical connections during lessons (Even et al., 1993). Novices' plans did not include relationships between concepts across lessons, and they did not typically leverage unplanned opportunities to make connections with students (Even et al., 1993). In addition, experts' explanations during lessons focused on critical concepts and made explicit connections across problems; whereas novices' explanations focused on procedures not linked to concepts (Borko & Livingston, 1989; Livingston & Borko, 1990). Novices either expressed there was no need or time to make connections or that connection-making was essential but challenging to plan and facilitate during instruction (Even et al., 1993). Additionally, while not a traditional expert-novice study, Star and Strickland (2008) studied what first-semester secondary PSTs attended to when watching Year 8 mathematics lessons from the TIMSS 1999 Video Study (see Hiebert et al., 2003) at the beginning and end of a methods course. The research team (experts) designed a survey from what they noticed in the video and administered the survey to the PSTs. They found many PSTs did not recall a moment in the lesson when a student made a mathematical connection between two algebraic expressions.

In comparison to the expert-novice studies, different results emerged from teacher noticing studies that sought to describe what PSTs attended to rather than what they did not in contrast to experts. For example, Walkoe (2015) asked PSTs to identify moments in a video of "interesting student algebraic thinking" and found that some PSTs attended to connections. Walkoe described a PST's (Heidi's) early participation in a video-club as follows: "Heidi was able to infer aspects of the student's understanding. She explored connections the student was making" (p. 539). Another example comes from a study by Dreher and Kuntze (2015). They asked PSTs to read a vignette of a classroom situation in which a teacher makes a change in mathematical representations in response to a student's comment or question. In a response to one of the vignettes, a PST wrote the following about the teacher's change in representation, "It confuses the student more instead of helping him. As a result, the understanding of a connection between multiplication and addition gets worse, since different representations are used" (Dreher & Kuntze, 2015, p. 104). These studies and others (e.g., Krupa et al., 2017; Monson et al., 2020) suggest that PSTs can attend to mathematical connections with some coincidental evidence. Findings from our study contribute to this line of study not only in terms of adding more evidence that PSTs can attend to mathematical connections, but also extends the literature base by presenting a framework that captures the different kind of pedagogical considerations PSTs made around mathematical connections.

Even though reform efforts have focused on students' conceptual development in Australia and the US since the TIMSS 1999 Video Study, we argue Star and Strickland's (2008) study

suggests attending to mathematical connections during instruction may still be difficult for novices. However, other studies (e.g., Walkoe, 2015) tangentially suggest PSTs can attend to connections with support. Operating on the premise that teacher noticing is a learnable practice from previous research (Jacobs & Spangler, 2017), we pursued understanding how PSTs attended to mathematical connections from their perspective to identify potential productive opportunities for developing PSTs' noticing of mathematical connections that MTEs may leverage, which others have yet to explicitly study.

Theoretical Orientation and Conceptual Framework

To reiterate, our focus of this study was to investigate (a) the kinds of mathematical connections the PSTs anticipated before and recalled encountering after their instruction and (b) the pedagogical considerations they made surrounding these instances while working with students. In this section, we describe our theoretical orientation toward mathematical connections and explain our conceptual lens used to address (a) and (b) above, which include Mason's (1998, 2002) discipline of noticing and Singletary's (2012) Mathematical Connections Framework (MCF).

Theoretical Orientation Towards Connections

We take the constructivist orientation in viewing mathematical connections as constructions individuals make and not as ideas that live within mathematics in-and-of-itself. That is, we consider mathematical connections arise from reflectively abstracting from and reorganising activity; they are not perceived or intuited from the world (Cobb, 1988; von Glasersfeld, 1995). Teaching actions consistent with this orientation towards connection include, but are not limited to, constructing models of students' understanding of mathematical connections (Confrey, 1990), selecting and sequencing conceptually-rich mathematical tasks that afford students opportunities to construct mathematical connections (Stein et al., 1996), and careful initiating and eliciting ideas to promote students' engagement with mathematical connections (Lobato et al., 2005).

Modifying Singletary's (2012) definition, we define a mathematical connection as a relationship one constructs between a mathematical entity and another mathematical (or non-mathematical) entity. In other words, we view the relationship between entities as becoming one only when an individual making the connection establishes one. In the context of this study, an individual refers to either PSTs, secondary students, or MTEs.

Mathematics Connections Framework

As our analytical framework for distinguishing different kinds of connections PSTs anticipated before and recalled encountering after their instruction, we used Singletary's (2012) Mathematical Connections Framework (MCF) (see Table 1). *Connecting through comparison* is when an individual makes a comparison between concepts A and B; A is similar to B. *Connecting specifics to generalities* is when an individual relates a specific case A to a more generalised concept B; A is an example of B. *Connecting methods* is when an individual relates two or more methods to a solution; A or B can be used to find C. *Connecting through logical implications* is when an individual provides a connection through implication or proposition; if A, then B. Finally, *connecting to the real world* is when an individual presents an application of a mathematical

concept outside the domain of mathematics; A is an example of B in the real world. Singletary developed the MCF from connections expert teachers made during instruction. We used the framework to analyse the kind of mathematical connections secondary students made (as noticed by the PSTs) or PSTs made in the field and described in online discussion board (ODB) posts.

Table 1
Examples of Kinds of Connections Using the MCF

Kind	Description	Examples
Connecting through comparison	A is similar to B. A is the same as B. A is not the same as B. A or B similarly defines or describes B.	Solving the equation $3^{5x+4} = 3^{x+8}$ for x is similar to solving $5x + 4 = x + 8$ for x.
Connecting specifics to generalities	A is an example of B.	The distance formula is an example of the Pythagorean theorem.
Connecting methods	A or B can be used to find C.	Completing the square or the quadratic formula can be used to find the roots of a quadratic polynomial.
Connecting through logical implication.	If A, then B. If A, then B but not C.	If two linear equations have the same slope, then the lines are parallel.
Connecting to the real world	A is an example of B in the real world.	The Hohenzollern Bridge in Cologne, Germany is an example of a catenary in the real world.

Discipline of Noticing and Our Notion of Considerations

Following our aforementioned theoretical orientation, we draw upon the notions of markings and considerations to *infer* PSTs' attention to mathematical connections. According to Mason (2002), the discipline of noticing is a purposeful set of actions one takes to improve their professional practice. It involves iteratively refining one's attention and reflection in-the-moment to act in a more disciplined way. Mason distinguished this *intentional noticing*, from which professional practice develops, from that of *ordinary-noticing*, which is easily forgotten or only available when explicitly reminded. Intentional noticing is bringing specific data to mind through *marking* or *recording*. Marking is recalling and describing a salient incident to oneself or others. Recording, a higher form of noticing than marking, is to make a record of an incident within some medium to externalise thoughts and to create data to be able to re-enter the incident for analysis and reflection for future action. In our study, we draw on the notion of marking as recalling incidents salient to PSTs during their field experience.

Mason (2002) argued, "To develop your professional practice means to increase the range and to decrease the grain size of relevant things you notice, all to make informed choices as to how to act in-the-moment, how to respond to situations as they emerge" (p. xi). Relatedly, Mason

(1998) conceptualised *awareness* different from *attention*. Attention is what an individual is attending to in-the-moment. Awareness is how an individual makes sense of or interprets what she attended to, either consciously or unconsciously. Teacher noticing entails what teachers attend to and how they make sense of what they attend to (Sherin, Russ, & Colestock, 2011). Building on these notions, we use the term consideration to refer to a phenomenon related to teacher noticing.

What we refer to as *pedagogical considerations* are the ways teachers, in our case PSTs, interpret the pedagogical implications surrounding an instance they marked. This sense-making may include their interpretations or responses to the objects of their attention. We use the term consideration to note that we inferred, for some PSTs, the object of their attention as mathematical connections. In some instances, it was unclear whether PSTs were making sense of or responding to a mathematical connection, although from our perspective, we saw the instance they marked as being one. A consideration only infers the object of teachers' attention from the teacher's sense-making actions, while noticing is a direct relation to what teachers attend to (i.e., attention) and how they make sense of it in-the-moment (i.e., awareness).

Research Design and Procedures

To study the nature of PSTs' markings of mathematical connections, we examined PSTs' marking of a salient moment in the ODB or their reflections of a mathematical connection they made or observed students making. We extracted and coded each marking from PSTs' written text. First, we coded each PSTs' markings using two coding schemes: (a) whether the instance was an explicit or non-explicit connection marking and (b) kind of mathematical connection using Singletary's (2012) MCF. Whereas the MCF aided us to identify the kind of connections PSTs marked, it did not capture PSTs' introspection of the pedagogical potentiality of the connection. Therefore, we used thematic analysis (Braun & Clarke, 2006) to describe how PSTs made pedagogical considerations of the mathematical connections they marked.

Participants and Context

The study occurred at a large public university in the south-eastern US. The participants, a cohort of twelve second- or third-year undergraduate PSTs were enrolled in their first semester (approximately 16 weeks) methods course of the secondary mathematics education program. Data were collected from this methods course, in which the second author was the instructor. The main objective of the course was for PSTs to make sense of and facilitate students' mathematical thinking. The course included a field experience for PSTs to engage in small-group instruction at a local secondary school (Years 9–12) once a week for approximately 90 minutes over 8 weeks. The PSTs worked with three Year 11 classes: Advanced Algebra¹, Advanced Algebra Support², and Accelerated Pre-Calculus. Table 2 lists the weeks allotted to and topics covered in each class. Each PST worked with the same students for each class.

¹Advanced Algebra is equivalent to a traditional Algebra II course in the US with the addition of some statistical topics.

² Advanced Algebra Support was an elective class for secondary school students who needed additional assistance completing Advanced Algebra. The support class either reviewed or previewed Advanced Algebra course material.

The secondary teachers shared their lesson plan and materials for each lesson before PSTs visited the school each week. To enhance PSTs' learning from the field experience and attention to secondary students' mathematical thinking, the instructional team (the university instructor, a teaching assistant, and classroom teachers) assigned PSTs to engage in an ODB before working with students and write reflections after working with students (Fernández et al., 2012; Krupa et al., 2017; Mason, 2002; van Es & Sherin, 2002). These two assignments were data sources and are further described in the next section. PSTs also attended lab sessions with the university instructor to review the lesson plan and the mathematical content in the lesson with PSTs. These lab sessions were hour-long class meetings on the university campus a day before the field visit, designed to allow PSTs to discuss and prepare their instruction with secondary students. Lab sessions also provided PSTs the opportunity to further discuss topics posted on the ODB.

Full ethical approval was gained for this study from the authors' institutional review board. As the instructor was also part of the research team, a neutral-party individual affiliated with the university was responsible for recruiting and obtaining PSTs' informed consent. PSTs' participation was voluntary, and they could withdraw their course assignments from later analysis at the end of the course by the research team by submitting a written request. Participants were not provided any compensation for participating in the study. All names that appear in this paper are pseudonyms.

Table 2
Course and Topics Summary by Week

Field Visit Week	Class	Topics
1-3	Advanced Algebra	Operations with polynomials: Polynomial multiplication, long division and synthetic division, factoring polynomials
4-5	Advanced Algebra Support	Interval notation, characteristics of polynomial functions
6-8	Accelerated Pre-calculus	Solving trigonometric equations, law of sines, law of cosines

Data Collection

Online Discussion Board

Before each lab session, the PSTs were required to solve all the problems in the lesson and engage in an ODB to discuss the upcoming lesson (See Figure 1 for an example of an ODB post). The PSTs were required to post at least one comment or question and then reply to at least one post. The ODB posts were counted for completion as part of the PSTs' participation grade. There were no specific prompts for the PSTs to follow when posting. It was left open for any type of comment or question about the field visits.

During the lab session, the university instructor organised questions or comments from the ODB posts into four of the five practices of orchestrating productive mathematical discussions (Smith & Stein, 2011)—anticipating, selecting, sequencing, and connecting—to be discussed. For

instance, the instructor used Sherry's post in Figure 1 to facilitate a discussion on the relation between the methods identified by Sherry for polynomial division as an instance to support PSTs' practice of connecting. The text produced by PSTs in the ODB for all eight weeks served as one data source.

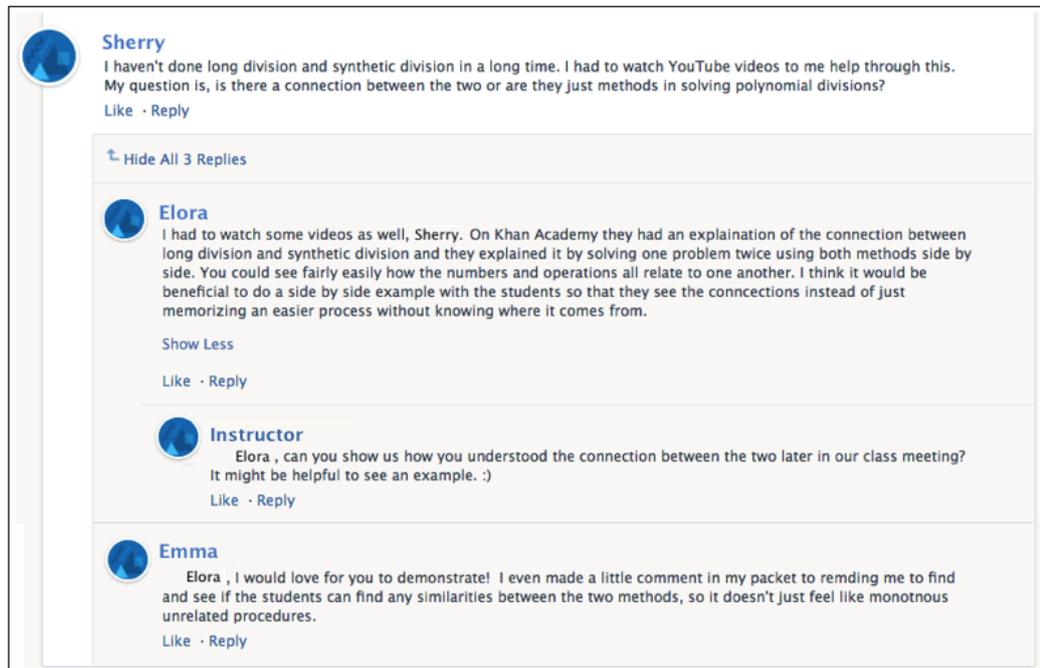


Figure 1. A post made by a PST, Sherry, on the ODB with comments by PSTs, Elora and Emma, and the methods instructor in the reply thread to the post. This post is a recreation of the original post to preserve anonymity, but the content and structure of the post and replies appear exactly as on the ODB.

Reflections

PSTs wrote two reflections for each group of students they worked with within a class, one after their first visit and one after their last, for a total of six reflections. The instructor asked PSTs to respond to a set of focused prompts common across the weeks and a weekly prompt that changed for each reflection. See Appendix A for the focused prompts and weekly prompts. The reflections were graded as a course assignment. The text written in these six reflections (Weeks 1, 3, 4, 5, 6, & 8) served as the other data source.

Analysis

Phase 1: Identifying Markings of Mathematical Connections

From the ODB posts and written reflections, we identified markings of mathematical connections first by identifying explicit connection keywords, which either started with connect (e.g., connected, connecting, connection, disconnect etc.), referred to a relation (e.g., relate, related to, related, relating, relationship), or involved some comparison (e.g., alike, similar to, different from). As we investigated the data, we noticed there were other instances when the PSTs alluded to, from our perspective, connections but did not use such explicit keywords. Therefore, we identified markings of mathematical connections to include (a) when PSTs explicitly used connection keywords and (b) when we inferred some mathematical connection from their writing. We considered the use of explicit connection keywords to be evidence of PSTs' attention and awareness of a connection while non-explicit connections may or may not entail conscious awareness by the PST, but we inferred they were attending to a mathematical connection. (See Table 3). The unit of analysis was a collection of text written around one mathematical connection. We compiled extracted markings of mathematical connections into a spreadsheet. In either explicit or non-explicit connections, a marking had to involve a discussion of some relationship between a mathematical entity (e.g., objects, topics, concepts, procedures, etc.) and some other mathematical or non-mathematical entity. For example, in Amanda's reflection, she wrote:

At one point, she [the student] looked up and complained, "I don't understand something, and I don't know what it is. Something is missing, and I don't feel like I'm getting the right answer." I assumed this was because she was forming relationships on the worksheet and not producing numerical answers. (Week 8 Reflection).

While Amanda discussed a student "forming relationships," it was unclear to us what mathematical entities were being connected and so did not count as a marking of a mathematical connection.

Phase 2: Identifying Types of Mathematical Connections Using MCF

To study what kind of mathematical connections PSTs marked, we coded each marking according to Singletary's (2012) MCF. When a PST explicitly marked a connection, but we could not infer the kind of connection due to ambiguity, we coded the kind of connection as 'undefined' to capture when a PST was attempting to relate two entities (i.e., students' previous knowledge to the ideas, procedures, concepts, or topics in a future lesson) but the nature of the relationship between the

entities was left unstated. Table 3 lists examples of mathematical connections in conjunction with explicit and non-explicit markings.

Table 3
Examples of Kinds of Mathematical Connections in the Data

Kind of Connection	Explicit	Not explicit
Comparison	I asked my students if they noticed something about the area model that could be <i>related</i> to something other than drawing tedious boxes out. [A student] was the first to say how you could distribute. I asked if he could explain.	How could I have used a visual representation to show why distributing an exponent to a polynomial is incorrect?
Logical implication	[A student] said [the solution interval] was between 0 and 2π but did not make the <i>connection</i> that you cannot divide by 0. Thus, I asked, "Since we know $\sin \theta$ is between 0 and 2π , if $\sin \theta$ is equal to 0 what is $\sin \theta / \sin \theta$?"	[The students] were able to realise that a problem had no solution when the solution was greater than one, or greater than the radius of the unit circle.
Methods	My mathematical learning goal for the lesson was to create <i>connections</i> between solving trig[onometric] equations using the unit circle and using a graph...	I had the students use both distribution and the area model for each problem. Here they could try both strategies on the same problem.
Real-world	Any thoughts on how to <i>relate</i> negatives to the real world other than owing money (or something else)?	I am curious if there would be a way to provide a high-school level real-world situation using trigonometric functions.
Specifics to generality	We worked through different examples to see the <i>connections</i> between these different depictions [inequality statement, number-line, and interval notation] of inequality statements.	[Understanding sine and cosine as inverse functions] will help [students]...generalise their understanding to new situations.
Undefined	...the [curriculum] material often seemed <i>disconnected</i> .	

Note. We added the *italicised* emphasis to highlight the explicit connection keywords.

Phase 3: Development of Pedagogical Consideration Themes

Reviewing the data, we observed PSTs were drawing some pedagogical considerations around their markings. We drew upon thematic analysis (Braun & Clarke, 2006) to understand what pedagogical considerations PSTs made when marking a mathematical connection. Thematic analysis is a systematic approach to identifying and interpreting *themes*, i.e., a patterned response

or meaning, in qualitative data. After developing an initial set of themes, we coded two weeks of written reflections (third and fifth week randomly chosen). We elaborate on the final themes—students making connections, suggested practice, knowledge to draw connections, curriculum, and affect—in the Results section.

As we continued re-examining the data according to these themes and compared our codes, we refined our descriptions of each theme, consistent with the thematic analysis methodology as well as Mason's (2002) discipline of noticing. Mason stated, "Where several themes emerge, it is useful to look at how the various themes interact, and whether there really are differences between them or whether they are all manifestations of a yet more general theme" (p. 120). For instance, there were markings originally coded as students making connections as the focus was on students. However, as the affective considerations recurred in our data, we noticed a pattern in which our PSTs were attending to affective elements, related to but distinct from, the actual activity of students making connections. Therefore, we generated a new theme and coded for instances of *affect*.

Findings

We identified a total of 282 connection markings, 246 from PSTs' written reflections and 36 from the ODB. First, we present the kinds of mathematical connections the PSTs explicitly or implicitly (from our perspective) marked. Second, we present the *Pedagogical Considerations of Mathematical Connections* (PCMC) framework outlining the five themes of pedagogical considerations: students making connections, knowledge to draw connections, suggested practice, curriculum, and affect.

Mathematical Connections

PSTs considered a variety of connections: connecting through comparison ($n=167$), connecting specifics to generalities ($n=31$), connecting through logical implication ($n=28$), connecting methods ($n=23$), and connecting to the real world ($n=19$). Among the 282 connection markings, 183 were markings in which PSTs used explicit language, which was evident across each kind of connection. The percentage of explicit connections was highest in connecting through logical implication (75%) and lowest in connecting methods (39%).

Looking across the weeks, certain kinds of connections appeared at different times during PSTs' field experience. For example, connecting through comparison showed prominently in weeks 1 through 7, whereas connecting specifics to generalities occurred predominantly in week 8. Table 4 provides the frequencies of each kind of connection in the written reflections and ODB.

Table 4
Mathematical Connection Types Marked Across Weeks

Kind of Connection	Week								Total
	1	2	3	4	5	6	7	8	
Comparison	32	4	30	39	23	5	31	3	167
Explicit	24	4	25	26	18	5	16	2	117
Not explicit	8		5	13	5		15	1	50
Logical implication	3		2	1	9		6	7	28
Explicit	3		2	1	9		3	3	21
Not explicit							3	4	7
Methods	7	2	1				11	2	23
Explicit	2	2	1				4		9
Not explicit	5						7	2	14
Real-world	4		1	4	3	1	2	4	19
Explicit	3			2	3		1		9
Not explicit	1		1	2		1	1	4	10
Specifics to generality	1		2	3	1		5	19	31
Explicit	1		2	3			2	5	13
Not explicit					1		3	14	18
Undefined	3		2	1	4		3	1	14
Explicit	3		2	1	4		3	1	14
Not explicit									
Total	50	6	38	48	40	6	58	36	282

Note. Week 2 and 6 only contain markings from ODB posts.

To summarise, the PSTs considered all five mathematical connection types identified in the MCF. The kind of connections differed across the weeks in the field and the explicitness of mathematical connection markings varied by kind of connection and the mathematical content involved.

Pedagogical Considerations of Mathematical Connections Framework

The PCMC framework organises the pedagogical considerations of mathematical connections made by PSTs. See Table 5 for an overview of the framework with select examples. We elaborate on each component of this framework.

Table 5
Pedagogical Considerations of Mathematical Connections

Pedagogical Considerations	Description	Example PST quotes
Students making connections	Consideration of students' connection-making process	"[Student] ... was the one who helped connect distribution, factoring, and the Area Model to the other two students before I even had the chance."
Suggested Practice	Consideration of practices that assisted or could assist students' connection-making	"I anticipate students having a hard time connecting the ambiguous case reasoning to these wonky SSA triangles... Thus, I think some helpful things to ask when we're going over the ambiguous case with our students are: [lists potential questions]"
Knowledge to draw connections	Considerations of knowledge (or lack of) to facilitate students' connection-making	"One question I have about trigonometric functions is how I could, as a teacher, provide a contextual problem at a high-school level."
Curriculum	Consideration of how curriculum impacts connection-making for students	"...when I left [the school] I wondered if I could have introduced the ideas and topics in a different order that would have better set the students up to engage and make connections."
Affect	Consideration of students' affective behaviours (motivations, feelings, beliefs, etc.)	"I asked [my student] why he disliked it the area model and he responded that it's too complicated for him to make the connections..."

Students Making Connections

Students making connections refers to PSTs' considerations of their students' connection-making process. These instances included when the PSTs discussed (a) students' roles in making connections, (b) connections students developed or were yet to develop, or (c) their instructional goals regarding students making mathematical connections. We present illustrative examples of each.

First, in their reflections, some PSTs marked students' roles in making connections. For example, Natalie noted a student contributed a connection to the group:

"[A student], a lover of the Distributive Property, was the one who helped connect distribution, factoring, and the Area Model to the other two students before I even had the chance." (Week 3 Reflection).

This instance, and others like it, contrasted with moments in which PSTs described the roles *they* took to support students' connection-making (See Suggested practices). Instead, PSTs emphasised the role that a *student* had in assisting in a group in establishing a connection.

Second, PSTs discussed connections their students developed or were yet to develop. For example, Amanda reflected on a discussion with her student on how to factor trigonometric expressions in a manner similar to polynomial expressions:

[My student] did have a significant 'ah-ha' moment concerning factoring and expanding polynomials and seemed happy with herself once she realised how she could manipulate trigonometric equations the same way. (Week 7 Reflection)

Amanda considered her student as developing a connection between factoring and solving trigonometric equations similar to her previous understanding of factoring and solving polynomial equations. Conversely, Amanda considered when students had not yet developed certain connections as well:

I realise that the students did not understand what the axis stood for. This showed me that the students had not understood the meaning of their answers in context of the graphs, but rather the unit circle. They had not connected the two yet. (Week 7 Reflection)

There were also instances in which PSTs considered connections students developed even though these connections might not be valid from a mathematician's perspective. For example, in one of his reflections, Cory wrote:

[A student] asked, "Well if law of sines is $\frac{\sin(A)}{a} = \frac{\sin(B)}{b}$, is the law of cosines $a \cdot \cos A = b \cdot \cos B$?" I believe that she thought since sine and cosine were 'opposites' of each other that you would do the opposite operation (Week 8 Reflection).

We interpreted Cory's understanding of the student's question as a connection because of a suggested relation between sine and cosine as 'opposites.' While this connection is not mathematically valid, from our perspective, it is a connection taken from the student's perspective, as interpreted and described by Cory.

Third, PSTs considered their instructional goals regarding *students* making mathematical connections. One of Ali's reflections provides a descriptive example: "I decided that my goal for my students would be to recognise the similarities between regular equations and trigonometric equations and be able to solve trigonometric equations and have a conceptual understanding of the answer" (Week 7 Reflection).

Suggested Practices

Suggested practices were markings where PSTs considered ways in which they might assist or did assist students in making mathematical connections (e.g., a sequence of questions, a demonstration, selecting tasks, etc.). For instance, before teaching the law of sines, in the ODB, Joann posted,

I anticipate students having a hard time connecting the ambiguous case reasoning to these wonky SSA triangles... Thus, I think some helpful things to ask when we're going over the ambiguous case with our students are: (a) Which side "swings" in SSA triangles? (b) How does this swinging side relate to what is given? (c) From which angle do we drop h ? (d) Why do we drop h from this angle as opposed to the other 2 angles? If your students happen to finish early, see if they can retry #2

on the Warm-Up, taking a second triangle into consideration, assuming that they only found one solution the first time they went through this problem (Week 7 ODB).

Joann suggested a thoughtful sequence of questions and tasks to her peers to guide students to make a connection through generalisation. Furthermore, she suggested as a closure having students revisit the warm-up activity to potentially elicit another solution strategy.

In contrast to Joanna, Emma reflected on what she *could have* done to make a more explicit connection with students. She stated,

I could have used this as an opportunity to make more explicit connections about the even and odd characteristic being directly related to the parity of the leading variables exponent as well as the end behaviour, and so on (Week 5 Reflection).

Knowledge to Draw Connections

Knowledge to draw connections were markings when PSTs considered their knowledge (or lack thereof) of mathematical connections or (b) their knowledge (or lack thereof) in aiding students in drawing mathematical connections or demonstrating connections. Although these instances involved, to some extent, considerations of students making mathematical connections or a suggested practice, the main focus was on *their knowledge* to support students' making mathematical connections.

The ODB and reflections provided differing opportunities for PSTs' considerations of the knowledge to draw connections. The ODB provided an opportunity for PSTs to reach out to peers and the instructional team to address gaps in their knowledge whereas the written reflections provided an opportunity for PSTs to reflect on their knowledge to draw connections. For example, in the ODB, Sherry posted about a question about the connection between long division and synthetic division for polynomials (see Figure 1). In her reflection, Elora revealed her misunderstandings of a connection that emerged when working with students. She noted,

My questions this time have to do with ... the connection between the graph and the unit circle. I got confused at one point while trying to explain a connection between the unit circle and the graph of trig functions (Week 7 Reflection).

Curriculum

Curriculum was markings when PSTs focused on how the curriculum materials aided or hindered opportunities for their students to make connections. These included PSTs' consideration of the (a) coherence in instructional material such as a task or worksheet, (b) sequencing of mathematical topics across or within tasks, and (c) appropriateness of the context in a task (or sequence of tasks).

A few PSTs considered the coherence of the instructional material. Joann posted how the coherence of the task impacted her thinking when solving the task:

While I was going through this worksheet, I had a really hard time making connections and keeping my thinking consistent from the first page to the last page of the packet (Week 7 ODB).

Similarly, Cory wrote in his reflection,

Another problem I came across was as the students did more practice problems, they lost the connection between multiplying polynomials and area. With the task being long, they were focused more on completion rather than conceptual understanding" (Week 1 Reflection).

These examples illustrate how PSTs considered the task's features (e.g., length of the task) limited the coherence and thus distracted from the goal of the task for making connections between concepts and procedures.

PSTs also considered the sequencing of mathematical topics across or within tasks. In a reflection, Emma stated,

When I left [the school] I wondered if I could have introduced the ideas and topics in a different order that would have better set the students up to engage and make connections (Week 1 Reflection).

Later in the course, Emma posted,

I'm trying to determine the best way to introduce this to my students, and what sequence of problems will set them up to make the most connections. Do you guys have any suggestions? (Week 3 ODB).

Emma considered the sequencing of the problems or tasks within a lesson appeared to be important. Amanda, on the other hand, considered the sequence of topics across lessons by questioning the sequencing in which students learned concepts and how it impacted their connection-making. In a reflection, Amanda wrote,

Regarding the specific topic of coordinate planes, I wonder how it was first taught to them, how they have used graphs in the past, and what kind of connections were missing that caused this lack of foundation (Week 5 Reflection).

The lack of foundation that Amanda was referring to is the coordination of the independent and dependent variables on a graph.

Furthermore, some PSTs considered the appropriateness of a context in a task (or sequence of tasks) for fostering students' connection-making. For example, Cory wrote,

The only thing I would change about this lesson would have been... coming up with a similar question to the baseball one that would have been more relevant to my students (Week 8 Reflection).

Cory recognised that the context of the task was not meaningful to his students and so he articulated a need for attending to culturally relevant contexts (c.f., Aguirre et al., 2013).

Affect

We coded markings as *affect* when PSTs considered their students' affective behaviour, such as motivation, feelings, or beliefs, related to making mathematical connections. For example, Sherry reflected on her student's feelings about using the area model:

I asked [my student] why he disliked it, the area model, and he responded that it's too complicated for him to make the connections and felt it was much more complicated than the FOIL method (Week 1 Reflection).

In contrast, Emma considered a positive affective experience with her student:

[He] seemed truly interested when I was explaining to him why the Law of Sines doesn't work to find obtuse angles. I think this is because students really do want to make sense of the material they are learning. They like filling in the blank spaces with information that connects ideas (Week 8 Reflection).

Table 6 summarises the frequencies across the five pedagogical considerations in the data. Across both data sources, PSTs most frequently considered suggested practice. In their reflections, PSTs marked several instances of students making connections. We see this focus as a natural one in that it is more likely PSTs will discuss students making connections after having worked with them, and also a celebratory shift in that such transition is not guaranteed.

Table 6
Frequency of Each Consideration Across Data Sources

Pedagogical Considerations	ODB	Reflections	Total
Students making connections	0	112	112
Suggested practice	24	118	142
Knowledge to draw connections	10	18	28
Curriculum	3	7	10
Affect	0	10	10
Total	37	265	302

Note. There are more pedagogical considerations than the mathematical connection instances identified, as a PST may have considered more than one pedagogical consideration per connection marking.

Discussion

The findings address two interrelated ideas: (a) kinds of mathematical connections PSTs marked and (b) the pedagogical considerations PSTs made about these mathematical connections. The PCMC framework addresses the second of these ideas. We discuss and interpret these main findings and then offer potential activities that MTEs could use to support PSTs in attending to and becoming aware of mathematical connections during instruction.

PSTs' Attention and Awareness of Mathematical Connections

PSTs in our study attended to various kinds of mathematical connections, whether those connections were constructed by them or students. The majority of the connections that we identified in the data were explicitly identified by PSTs as connections. Each kind of mathematical connections in the MCF was evident in PSTs' markings. The PSTs often attended to connecting through comparisons. The high frequency of attending to connection through comparison found in PSTs' markings is consistent with expert teachers' practice (Singletary, 2012). Overall, these findings highlight that secondary PSTs are able to attend to and explicitly identify the kinds of mathematical connections found in experienced mathematics teachers' practice. Demonstrating such ability is important because explicit attention to mathematical connections during instruction is generative for students' learning, promotes recall, and impact students' beliefs about mathematics (Hiebert & Carpenter, 1992).

The MCF (Singletary, 2012) afforded us a framework to interpret the kinds of connections PSTs attended to across particular strands of mathematics and different contexts (ODB posts and reflections). Our finding that some PSTs were aware of all five kinds of mathematical connections suggests that PSTs can develop an explicit awareness-in-discipline (Mason, 1998) of these different kinds of connections described in the MCF. Therefore, we find it an appropriate framework to introduce to PSTs to support them in reflecting on the explicit mathematical connections evident in their work with students and call for future studies to investigate the effects of such an approach.

We attribute the kinds of connections PSTs marked to (a) the tasks implemented in the field, (b) PSTs' mathematical knowledge in relation to the topics covered, and (c) the nature of the small-group discussions with secondary students during the field experience. First, mathematical tasks have considerable influence on students' opportunities to make connections (Stein et al., 1996). The tasks PSTs worked on with their students varied in potential opportunities for connection-making. For instance, some tasks focused on the execution of procedures (e.g., factoring quadratics) while others were supportive of connecting procedures with concepts (e.g., deriving the law of cosines). However, we also emphasise the importance of the PSTs' agency in building on such opportunities when implementing tasks. Recall, Joann suggested a thoughtful sequence of questions to support students in making connections as they worked through tasks. Joann's vision for implementing the task was not an explicit feature of the tasks nor an explicit goal stated in the lesson plan. She designed an attentive implementation plan using the tasks given to her by the classroom teacher that could provide opportunities for students to make connections.

Second, teachers' mathematical knowledge for teaching is considered influential in leading to opportunities for students to build mathematical connections (Hill & Charalambous, 2012). Teachers with robust mathematical knowledge for teaching are able to traverse the mathematical terrain, flexibly respond to students' mathematical thinking, and support students in making mathematical connections. Relatedly, some PSTs became explicitly aware of the gaps in their mathematical knowledge for teaching in preparation for or during their field experience. This awareness was evident in one of Elora's reflections when she recalled getting confused in trying to explain a connection between the unit circle and the graph of trigonometric functions. Therefore, the PSTs' mathematical knowledge for teaching likely contributed to the opportunities PSTs and secondary students had to make mathematical connections and whether PSTs marked the mathematical connections.

Finally, the nature of the small-group discussions likely had an influence on the mathematical connections PSTs had an opportunity to observe. While facilitating small-group discussions, PSTs were learning how to attend to student thinking and build a math-talk community (Hufferd-Ackles et al., 2004). As PSTs and their students moved to higher levels of math-talk learning community, there were more openings to a diverse set of students' ideas and hence more opportunities to build on and connect students' mathematical ideas. For instance, Natalie marked an instance in her reflection when a student took on the role of explaining the connection between the distributive property and the area model for multiplying polynomial expressions. Giving students opportunities to contribute ideas during small-group discussions allowed PSTs and their students to make mathematical connections among ideas.

Our finding that PSTs attended to and explicitly identified mathematical connections in working with students somewhat contrasts with Star and Strickland's (2008) study, which found that few PSTs attended to a connection when watching a video of a mathematics lesson. They

conjectured that PSTs may have not attended to the connection due to limited content knowledge, lack of recent experience with the particular content, or just failing to notice. We believe such contrast between the two findings may be, in some ways, attributed to different approaches took in the studies. From a lesson recording, Star and Strickland identified several significant features of a teacher's instruction, including making mathematical connections, and sought to determine if PSTs would identify such features. Our approach was to highlight the perspective of PSTs in identifying and marking mathematical connections that arose in their own instruction with secondary students. In other words, the data generated was from PSTs' markings. While it is important for PSTs to attend to significant mathematical connections in the discipline as identified by experts, it is also important to consider the implications of what PSTs attend to. For example, Cory recognised a student's connection-making that was not mathematically valid (i.e., If law of sines is $\frac{\sin(A)}{a} = \frac{\sin(B)}{b}$, then the law of cosines $a \cdot \cos A = b \cdot \cos B$ because sine and cosine are "opposites"). Even though the connection might not be valid or one that a mathematician would make, it is evident that Cory recognised an attempt by a student to make a connection between the law of sines and law of cosines. We believe both approaches are important for informing MTEs and mutually supportive for PSTs' conceptual development. PSTs should attend to mathematical connections that experts have identified as important for students' learning and should receive support from MTEs in developing the mathematical content knowledge to do so because teachers' mathematical knowledge for teaching enables teachers to successfully support students in explicitly making mathematical connections (Hill & Charalambous, 2012). MTEs also need to examine the mathematical connections PSTs attend to and how those connections may provide insight into understanding students' mathematical thinking.

Contribution of the PCMC Framework

While there are exemplars in the literature detailing expert teachers' instruction of mathematical connections and their pedagogical decisions (e.g., Ball, 1993; Boaler & Humphreys, 2005; Lampert, 2001), the field can benefit from insights on how to prepare novice teachers for such intricate work. There is a need to support MTEs' awareness to the sensitivities that PSTs need to design and facilitate discussions that explicitly attend to mathematical connections (i.e., awareness-in-counsel; Mason, 1998).

The PCMC framework contributes to the field of mathematics teacher education as a way to structure MTEs' awareness-in-counsel so they may guide and develop opportunities for PSTs to design and facilitate discussions of mathematical connections. The framework was developed from PSTs' attempts to make sense of mathematical connections within the context of teaching and learning in the field by considering their pedagogical implications. The framework outlines five pedagogical considerations PSTs marked: (a) students' connection-making, (b) practices that assisted or may potentially assist students' connection-making, (c) their knowledge (or lack thereof) to facilitate students' connection-making, (d) curricular influences on connection-making, and (e) students' affective behaviours (e.g., motivation, feelings, beliefs, etc.) towards connection-making. These pedagogical considerations are not exhaustive; other considerations may arise in-the-moment. Although not exhaustive, the strength of the PCMC framework lies in the fact that it emerged from PSTs' markings. The findings revealed that PSTs *can* engage in these pedagogical considerations; and therefore, the PCMC framework can serve as a starting point for MTEs to draw

upon in supporting novice teachers to design instruction that explicitly attends to mathematical connections.

Table 7

Potential activities for developing PSTs' attention to and awareness of PCMC.

Pedagogical Considerations	Potential Activities for Developing PSTs' Attention and Awareness
Students making connections	<ol style="list-style-type: none"> 1. When PSTs observe or watch videos of instruction, design reflection questions that prompt PSTs to attend to students' roles in connection-making and the different ways students appear to understand the mathematical connections. 2. Have PSTs write instructional goals for future lessons with a focus on students' mathematical connections.
Suggested practices	<ol style="list-style-type: none"> 3. Have PSTs solve tasks and anticipate potential connections students might make and generate ways to support students in making connections they identified. 4. Have them reflect on how a teacher's or their support afforded or constrained students' connection-making after an observation or watching a video of instruction.
Knowledge to draw connections	<ol style="list-style-type: none"> 5. Ask PSTs to reflect on their knowledge of mathematical connections in various activities such as while working on a mathematical task with their peers or watching different ways students solve the same problem.
Curriculum	<ol style="list-style-type: none"> 6. Ask PSTs to examine how tasks are sequenced in curriculum materials and consider how the sequence afford or constrain students' connection-making. 7. Ask PSTs to examine when and how connections could potentially occur in curriculum materials and learning progressions. 8. Ask PSTs to examine if the context of the task will allow students to make meaningful connections and if so, how.
Affect	<ol style="list-style-type: none"> 9. Have PSTs observe and describe students' cognitive, behavioural, and emotional engagement surrounding mathematical connections. 10. Have PSTs share successful ways they found to engage students in making connections with peers.

In Table 7, we offer some activities MTEs could use in their instruction for each of the pedagogical considerations. For instance, MTEs can explicitly direct PSTs' attention and awareness (Mason, 1998) to the five pedagogical considerations for a lesson or sequence of lessons (e.g., in the planning phase or analysing a video of a lesson).

Conclusion

Our study revealed that PSTs could consider various kinds of mathematical connections in their field experience. When marking connections, PSTs also recognised an assortment of pedagogical considerations: students' mathematical connection-making, suggested practices, knowledge to draw connections, curriculum, and affect (Table 5). These pedagogical considerations are productive beginnings for novice teachers, and we described potential activities MTEs might use

to foster such considerations in their instruction (Table 7). Future research studies may evaluate these activities or design others to understand how an activity developed or refined teachers' pedagogical considerations for mathematical connections. Furthermore, future studies may use the PCMC framework to understand the qualitative differences between the considerations of novice and expert teachers, thus providing further direction in understanding how to develop and refine teachers' pedagogical considerations for supporting explicit attention to mathematical connections during instruction.

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Appendix A

Focused Prompts	<p>What did you learn about your student(s) at the beginning?</p> <ul style="list-style-type: none"> • What did he/she know at the beginning? • What was he/she struggling with at the beginning? • How do you know? <p>What did you do/say to promote learning?</p> <ul style="list-style-type: none"> • How effective was it? • How do you know? <p>What did the student(s) know at the end of the session?</p> <ul style="list-style-type: none"> • How do you know? <p>If you had it to do over, what would you do differently (in planning/preparing or in working with your student)?</p> <p>Describe an interesting interaction you had with the student around a mathematical idea.</p> <ul style="list-style-type: none"> • What made this interaction interesting? • What did this interaction tell you about the student's mathematical thinking, motivation, or learning preferences? <p>What questions do you have now?</p> <ul style="list-style-type: none"> • About learning? • About the mathematical topic? • About teaching?
Weekly Prompts	<hr/> <p>Week 1 From the Jacobs & Ambrose (2008), Jacobs et al. (2014), Chapin (2003) readings, which tip did you try out with your student(s)? Describe how you used it and reflect on the experience implementing it.</p> <p>Week 3 Some students prefer using area models and some students don't. This week reflect on the effectiveness of the area model. Describe how your student used (or did not use) the area model and how it was helpful or hindered your students' mathematical thinking.</p> <p>Week 4 How did you sequence your lesson? Which examples/problems did you start with? Which examples/problems did you use towards the end? Why did you make such instructional decisions?</p> <p>Week 5 Reflect on your anticipation activity of the 5 practices. What did you anticipate your student to do/think? Was anticipating helpful in your teaching? Why or why not?</p> <p>Week 6 What was the mathematical learning goal you set for this lesson? Assess whether you think your student achieved this goal or not. What makes you think so?</p> <p>Week 8 Based on your observations with this student, what problem or problems might you pose next? Why? Do you think your student would be motivated in solving this problem? Why or why not?</p> <hr/>