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# Learning and Unlearning Through Questioning Practices: Middle Grades Mathematics Teachers' Transformations to Support English Learners

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Questioning is recognised as a well-established aspect of good mathematics teaching, and this study draws on this instructional practice to provide multilingual learners high-level mathematics, discourse, and cognitively demanding mathematics. Three seventh grade mathematics teachers engaged in professional learning to develop their questioning practice over the course of a semester, with this research seeking to answer the following questions: (1) How did three seventh grade mathematics teachers develop their questioning practices over the course of a semester?; and (2) What opportunities did these mathematics teachers' questioning provide for discourse, particularly for English learners? The research was framed using the concepts teacher learning and unlearning, discourse, and questioning. Qualitative data analysis of seven observations and four interviews from each teacher examined which types of questions teachers asked, how teachers' questioning developed over time, and how students responded to questioning. The findings illustrate that teachers' questioning strategies were fairly limited initially and that teachers rarely planned to support multilingual students. As teachers' questioning practices developed, they asked a wider variety of questions. In addition, there were interesting developments in the teachers' conceptions of their students, particularly related to how teachers identified multilingual students' mathematical capabilities. A focus on questioning allowed teachers to develop their instructional practices while also reconsidering their expectations for multilingual students, both linguistically and mathematically.

**Keywords** · questioning · professional development · English learners · middle grades

## Introduction

Questioning is a generally well-established aspect of good mathematics teaching (Martin, 2015). This classroom discourse technique allows mathematics teachers to model mathematics academic discourse for their students and pushes students for greater intellectual work in the form of justifications and explanations (Shein, 2012). Many teachers avoid using high-level discourse and cognitively demanding mathematics with English learners (ELs; Iddings, 2005; Planas & Gorgorió, 2004), often stripping away language in mathematics classrooms out of concern that ELs can handle neither the language nor the mathematics (Roberts, 2013). Teacher questioning supports students to be more detailed and explicit in their explanations (Franke et al., 2009), suggesting that questioning could provide scaffolding for ELs as they develop mathematics discourse, such as explanations.

This study describes the development of three relatively novice seventh grade, junior high mathematics teachers' questioning practices over the course of a semester as they participated in professional learning organised around questioning and ELs in the United States. As teachers partook in this process of inquiry

around questioning, they not only saw their questioning practices develop, but they also engaged in a process of self-examination of their philosophy of teaching mathematics to English learners. This study sought to answer the following research questions:

- (1) How did three seventh grade mathematics teachers develop their questioning practices over the course of a semester?; and
- (2) What opportunities did these mathematics teachers' questioning provide for discourse, particularly for English learners?

## Literature Review

This study is framed around the ideas of teacher learning and unlearning, discourse, and questioning.

### *Teacher Learning and Unlearning*

This study takes an approach to teacher learning that is grounded in Cochran-Smith's (2003) inquiry as stance. Teacher inquiry, undertaken within a community of learners, allows for the development of knowledge and new teaching practices (Cochran-Smith, 2003). In such a learning situation, educators have the possibility for both learning and unlearning. They are able to learn practices and knowledge while also unlearning practices, ideas, and beliefs that are often difficult to relinquish, and this unlearning can be a sign of growth (Cochran-Smith, 2003). A facilitator helps with this work, everybody is an equal member of the inquiry community, and an inquiry question drives the work of this learning and unlearning. Cochran-Smith noted that individuals in this community of learners work to generate knowledge and also to interrogate their practice. In the case of the teachers in this study, they examined their learning and unlearning around questioning to provide opportunities for discourse, particularly for ELs, such as through their self-reflection in interviews and in their work in professional development.

### *Discourse*

Internationally, as reform-based mathematics has taken root, there has been a shift in mathematics instruction that includes a greater focus on communication and reasoning (Brodie, 2008; Khisty & Chval, 2002; Lee & Kim, 2016; National Council of Teachers of Mathematics [NCTM], 2000). It is vital to support students to be part of the larger mathematics discourse community, or the ways of talking, being, interacting, reading, writing, and believing that are associated with doing mathematics competently (Moschkovich, 2002). It is critical for teachers to go beyond simply increasing students' vocabulary (Moschkovich, 2002; Musanti & Celedón-Pattichis, 2013), but to instead inculcate ELs in this mathematics discourse community (Willey, 2010).

Teachers play a crucial role in helping ELs to learn and become part of a mathematics community. They provide explicit guidance for students about how to engage in the discourse (Esquinca, 2013), orchestrating ELs' participation in the discourse (Lee & Buxton, 2013), so that ELs learn what is privileged in a classroom (Moje, 2008). Part of this explicit guidance is creating environments purposefully in which linguistically diverse students develop language in context, through active and meaningful use of new language. Teachers model academic language and use supports as they help students develop meaning (Khisty & Chval, 2002). This is not a passive process; teachers must offer students the chance to use and hear language, so that students can develop the language they need for the mathematics they are completing (Musanti & Celedón-Pattichis, 2013). Such classrooms and teachers afford students the opportunity to participate in rich mathematical learning while drawing on students' resources (Moschkovich, 2013).

The ELs, their classmates, and their teachers in this study, as they engaged with the mathematics discourse, also learned the mathematics classroom's sociomathematical norms (Yackel & Cobb, 1996) and its meta-discursive tools (Sfard, 2001). The teachers and students prepared to use questioning; in turn, they considered what counted mathematically as an explanation, as a question, and as an answer to a question

while simultaneously drawing on prior experiences about what it meant to do mathematical work (Yackel & Cobb, 1996). More specifically, students and teachers engaged with meta-discursive tools, those understood classroom communicative tools that are part of regular activities (Sfard, 2001). Students and teachers learn these meta-discursive tools in practice in the classroom, usually as part of the “hidden curriculum” of the classroom (Sfard, 2001, p. 31), although some teachers may make them more explicit and model them, such as through the use of questioning and providing model responses (e.g., Khisty & Chval, 2002).

### *Questioning in the Mathematics Classroom*

Questioning serves a number of roles in the classroom. For instance, it provides teachers with the opportunity to assess student learning (McCarthy, Sithole, McCarthy, Cho, & Gyan, 2016), to help students explain their thinking in more detail (Franke et al., 2009), and to promote a higher level of mathematical thinking (Hong & Choi, 2018). Questioning also supports the development of mathematical discourse, the focus of this study; teachers can model mathematical discourse, outputs, and expectations. Some researchers have found that specific types of questioning are particularly useful with ELs for developing discourse practices. For example, Celedón-Pattichis and Turner (2012) found that with consistent questioning, a kindergarten teacher’s students developed new discursive habits, such as the use of more extensive explanations of solution strategies. Shein (2012), in an additional application of classroom questioning, found that a fifth-grade teacher’s use of gestures in tandem with questioning aided ELs in understanding questioning and content.

### *Types of Questions*

There are a number of categories of questions that a teacher can ask to elicit student thinking and to assess student thinking to develop students’ mathematical discourse practices. Table 1 provides an overview of the categories of questions that guided this study, including a definition, description, and some examples for each type of question. This study used these categories of questions to synthesise the types of questions found in the literature to understand the study teachers’ learning and unlearning of questioning to develop discourse. Moving down the table, each question type requires a higher cognitive demand than the one before.

The first question type is Gathering Procedural Explanations and Known Facts. Such questions might be fall into the classic Initiate-Respond-Evaluate/Feedback (IRE/F) category (Cazden, 2001), where teachers question to gather information to get to a desired procedure or conclusion. These questions have a limited desired path, in that there are few opportunities for student sense-making and there are few possible ways for students to respond (NCTM, 2014), and these questions often involve student rehearsal of known facts and procedures (Boaler & Brodie, 2004).

The second category of questions is Probing Thinking, in which a teacher supports students to elaborate, expand, or articulate their thinking (Boaler & Brodie, 2004; NCTM, 2014). These questions develop depth, in that students explain and clarify their thinking (Boaler & Brodie, 2004; NCTM, 2014).

The third category of questions is Generating Mathematical Connections, Discussions, and Applications. Here, students should make connections and link relationships among mathematical ideas and structures. These questions ask students to make connections to prior mathematical work, such as whether they had worked on a similar problem previously (Martino & Maher, 1999), or ask students to make applications to relationships among mathematical ideas (Boaler & Brodie, 2004).

The fourth category of questions guiding this study is Encouraging Reflection, Justification, and Extension of Student Thinking. These questions push student thinking through the use of justification and reflection, such as considering whether there are similar types of problems where students would use a particular solution strategy (Boaler & Brodie, 2004).

Table 1  
*Types of Mathematical Questioning*

Question Type	Description	Examples
Gathering Procedural Explanations and Known Facts  (Boaler & Brodie, 2004; Brodie, 2010; Cazden, 2001; NCTM, 2014; Paoletti et al., 2018)	Questioning where teachers ask a question that requires a limited response with limited dialogue. The teacher asks only asks short questions to which they know the answer.	“Do you just estimate?” (Cazden, 2001, p. 42)  “What is the formula for finding the area of a rectangle?” (NCTM, 2014, p. 36)
Probing Thinking  (Boaler & Brodie, 2004; Childs & Glenn-White, 2018; Chin, 2007; NCTM, 2014; Oliveira, 2010; Paoletti et al., 2018; Sahin & Kulm, 2008)	Supporting students to articulate, elaborate, clarify, expand, and explain their thinking and ideas.	“Can you show and explain more about how you used a table to find the answer to...?” (NCTM, 2014, p. 36)  Can you explain your idea for how you got 10? (Boaler & Brodie, 2004)
Generating Mathematical Connections, Discussions, and Applications  (Boaler & Brodie, 2004; Childs & Glenn-White; Chin, 2007; Martino & Maher, 1999; NCTM, 2014)	Soliciting contributions to make connections to and link relationships among mathematical ideas, relationships, and structures.	What does your equation have to do with the ____ situation in the problem? (NCTM, 2014)  To what other situation could you apply this? (Boaler & Brodie, 2004)
Encouraging Reflection, Justification, and Extension of Student Thinking  (Boaler & Brodie, 2004; Martino & Maher, 1999; NCTM, 2014)	Supporting student to “reveal deeper understanding of their reasoning and actions, including making an argument for the validity of their work” (NCTM, 2014, p. 37).	“How did you reach that conclusion?” (Martino & Maher, 1999, p. 57)  “How do you know the sum of two odd numbers will always be even?” (NCTM, 2014, p. 37)
Non-Content Question	Questioning is not related to content.	“Did you get out your notebook?”

Using a lens of teacher learning and unlearning, this study considers how three seventh grade mathematics teachers developed their questioning practices over the course of a semester and what opportunities these questioning practices provided for discourse, particularly for English learners.

## Design, Methods, and Analysis

### *Professional Learning Design and Participants*

Three seventh grade teachers participated in this study. Ms Heller was a White, female, monolingual English speaker with four years of mathematics teaching experience. Ms Kim was an Asian, female, bilingual Vietnamese-English speaker, who was in her third year of teaching mathematics. Mr Weston

was a White, male, bilingual Spanish-English speaker with two years of mathematics teaching experience. Each teacher had a single focal class period from the six periods of classes that they taught (e.g. Ms Kim's last period class was her focal period) where the members agreed to participate in the study.

The project team collected qualitative data for five months, while holding professional learning, or professional development, meetings approximately monthly, for a total of three meetings over the course of the study. Two meetings were after school for two hours, and one meeting was a full-day meeting. Two teachers attended all meetings, and Ms Kim missed the last after school meeting.

Each of the three professional development meetings focused on a single mathematics task. The teachers worked to develop associated questioning in their implementation of each task. These tasks aligned with the teachers' district curriculum and associated standards. These tasks were: (1) "Orange Fizz Experiment" (Georgia Department of Education, 2016), (2) "Sports Bag" (Mathematics Assessment Resource Service, 2015), and (3) "Who's Watching What?"<sup>1</sup> (Illustrative Mathematics, 2017). These were rich tasks in that they had multiple entry and exit points, included multiple representations, and engaged students in reasoning, all qualities of good mathematical tasks (Silver & Stein, 1996). Additionally, they provided multiple opportunities for students to engage with content, to employ language, and to communicate their thinking in meaningful ways (Roberts & Bianchini, 2019).

In each professional development meeting, the project team provided teachers with information about questioning, such as background information on questioning, different types of questions, and the importance of using questioning with their English learners. The research team then provided the teachers with time during the professional development to solve a task, to create a lesson plan, and to consider the questions they would utilise while implementing the task. Teachers finalised their plans during their regular planning time and within their professional learning community meetings.

## Context

The teachers worked at Central Junior High School in California in the US, which had an enrolment of approximately 450 students. Demographic data for the school is shown in Table 2. The school had approximately 44% of their students designated as ELs, 34% as Fluent English Proficient, and 17.9% as Redesignated Fluent English Proficient. This project acknowledges that ELs are not a homogeneous group. While the majority of the ELs in the district were Latinx and spoke Spanish, there were ELs in the participating teachers' classes who spoke at varying levels of English proficiency.

Table 2  
*Junior High vs. District Demographics*

School	Black or African American	Native American or Alaska Native	Asian	Hispanic	White (not Hispanic)	Multiple or No Response	Socio-econ. Disadv.	Students with Disabilities
Central Jr. High	0.6 %	0.4 %	0.6 %	91.3 %	6.2 %	0.6 %	80.2 %	18.5 %
District (Total)	1.16%	0.54%	3.21%	58.63%	33.59%	2.77%	Unknown	Unknown

<sup>1</sup> Abbreviated as WWW in this text

### *Data Collection Procedures*

The project team video recorded teachers' professional development lessons, which included seven lessons for each teacher, for a total of 21 lessons: a pre-study initial lesson; three professional development lessons, two of which spanned two days ("Sports Bag" and "Who's Watching What?") and one that lasted one day ("Orange Fizz"); and a post-study final lesson. The research team placed a video camera on a tripod at the back of the classroom to follow the teacher, and the teacher wore a lapel microphone.

The second source of data for this paper was four semi-structured interviews (Glesne, 2011), with each lasting approximately one-hour. These interviews attended to how teachers considered questioning, how they prepared for their questioning, how they attended to ELs in their questioning, how they felt that students responded to their questioning, what number and types of questions teachers they asked during lessons, and what questions teachers asked in their classrooms. The interviews and observations served to triangulate each other (Mathison, 1988).

### *Data Analysis*

The first step of data analysis involved creating transcripts of all the lessons and interviews. The author then proceeded through two stages of coding. The first stage of analysis involved the classroom lessons. The author coded the lessons by the types of questions listed in Table 1.

The author and a research team member trained together to achieve reliability, using a single question as the unit of analysis. After two rounds of coding across multiple teachers, reconciling our codes, and updating our codebook, we achieved reliability with a Cohen's Kappa of 88.4%, which is considered substantial agreement (McHugh, 2012).

The author coded the teacher interviews to triangulate the teachers' classroom observations. Using a focused coding approach (Maxwell, 2005), the author first read through all the interviews and identified all instances related to questioning and/or ELs. The author developed codes around the following categories: questioning ELs, number of questions, types of questions, planning for questions, questioning practice, questions asked, and students' responses to questions. The author looked across these interview codes for consistencies and inconsistencies. The author then looked for alignment with the classroom analyses, while looking for consistencies and inconsistencies across the sets of data and the participants.

## Findings

This section shares the findings, highlighting the teacher learning around the evolution of the teachers' questioning practices and the opportunities for discourse that those practices created for students, particularly ELs. To answer the first research question, I examine teachers' initial questioning strategies and how teachers' questioning practices transformed. I then consider my second research question and review how there were challenges for students to engage in the mathematics discourse as teachers' questioning changed. While some changes in teachers' questioning were nominal, there were interesting changes in the teachers' conceptions of what their students, particularly their ELs, were capable of mathematically.

### *Teachers' Initial Questioning Strategies*

The teachers developed their questioning practices across the project, starting with their initial questioning, as shown in Table 4 and indicated in the "initial" lesson in the top row for each teacher. Teachers' initial questions were composed of known information types of questions in their first lesson, illustrating the teachers' starting point for their questioning and the starting point for their inquiry and learning and unlearning as this project began (Cochran-Smith, 2003). Ms. Heller and Mr. Weston asked only Gathering Procedural Explanations and Known Facts questions and Non-Content questions in their

Initial lessons. For example, in that first lesson, they asked questions such as, “Exactly, how do we undo multiplication?” (Mr. Weston, Initial Lesson, 101) and, “How many does it say she has at the beginning?” (Ms. Heller, 80), while Ms. Kim similarly asked mostly Gathering Procedural Explanations and Known Facts questions and also had fewer than a handful of Probing Thinking questions, as shown in Table 5. With such questioning from these teachers, students had limited opportunities to engage in prolonged discourse, as Gathering Procedural Explanations and Known Facts questions generally provided students with opportunities for only single answer responses, with students providing answers such as, “Some,” to Ms. Keller’s question, and, “Division!” to Mr. Weston’s question, as noted in Table 6.

Planning for questioning was new for two of these teachers. Mr. Weston and Ms. Kim noted that prior to this study, they had not been particularly intentional about their questioning. Ms. Kim explained that, previously, her planning of questioning was generally extemporaneous: “I’m more on the fly” (Ms. Kim, Orange Fizz, 205-206). Mr. Weston similarly remarked:

It was different [to plan for questioning]. I normally don’t. I normally sort of ‘fly by the seat of my pants.’ I look at what students are able to do and try to ask them a question that takes it one step further. (Mr. Weston, Orange Fizz, 202-204)

All three teachers noted that they did not plan separately for their questioning for ELs, because, as Ms. Heller noted, the majority of their students were ELs or had perceived language issues.

To be honest...I think it’s the majority of our classes are English learners...And so, and even the ones that aren’t the English Language Learners, a lot, the majority of our students, are reading at or below grade level. And so, knowing that there’s, even if it’s not an English Language issue, it’s just an academic language issue. (Ms. Heller, Orange Fizz, 302-309)

Throughout the study, this sentiment rang true with the teachers – there was no special attention paid to their ELs with regard to developing their questioning, perhaps because teachers had already adapted their instruction for ELs with regards to the language and language supports they were using (e.g., with the regular use of sentence frames, etc.).

### *Teachers’ Transforming Questioning Practices*

The teachers’ professional development lessons included a wider variety of questions as compared to their initial lessons. In Table 4, each row for a teacher provides the number and types of questions within a lesson for each lesson the teacher taught throughout the study. The questions teachers asked varied more widely in the teachers’ professional development lessons as compared to their initial and final lessons. While a majority of the questions the teachers asked in the professional development lessons were still Gathering Procedural Explanations and Known Facts, they asked a wider variety of questions and began asking their students more and different kinds of questions. For example, in the WWW, Day 2 lesson, Ms. Heller asked all the types of questions, and 35% of her questions were Probing Thinking questions, a higher-level questioning type. Similarly, in the WWW, Day 1 Lesson, Mr. Weston also asked all the types of questions, and 18% of his questions were Encouraging Reflection, also a higher-level question type. Ms. Kim also used all types of questions on her WWW, Day 1 lesson, but she had fewer high-level questions.

The teachers took notice of their learning and the changes in their questioning practices over the course of the project. For instance, Ms. Heller shared that she asked fewer procedural questions in her final professional development lesson: “I feel like there was definitely different levels, different depths of questions that we asked, from ones that were definitely more procedural” (Ms. Heller, WWW, 50-52). However, the teachers wanted to continue to move beyond their “procedural” questioning. For example, Ms. Heller mentioned following her first professional development lesson, “I definitely wished I had asked some deeper questions” (Ms. Heller, Orange Fizz, 750-754). Similarly, Mr. Weston explained after his final professional development lesson, “I think I asked a good number of questions, but too many of them were still procedural” (Mr. Weston, WWW, 352). Table 4 illustrates that Mr. Weston was indeed

asking a good number of questions but was still asking many procedural questions even if he was asking a wider variety of questions.

Table 4

*Types of Each Question Type Asked across Study by Each Teacher in Each Lesson*

Lesson	Teacher	Types of Questions Asked				
		Gathering Procedural Known Facts #Qs (%) Qs/lesson)	Probing Thinking #Qs (%) Qs/lesson)	Generating Mathematical Connections #Qs (%) Qs/lesson)	Encouraging Reflection #Qs (%) Qs/lesson)	Non-Content #Qs (%) Qs/lesson)
Initial	Heller	92 (79.31%)	0	0	0	24 (20.69%)
Orange Fizz		157 (76.96%)	18 (8.82%)	0	0	29 (14.22%)
Bag, Day 1		135 (83.33%)	5 (3.1%)	10 (6.17%)	0	12 (7.41%)
Bag, Day 2		84 (70%)	15 (12.50%)	8 (6.67%)	4 (3.33%)	9 (7.5%)
WWW, Day 1		76 (73.79%)	4 (3.88%)	0	7 (6.8%)	16 (14.56%)
WWW, Day 2		90 (56.60%)	54 (34.96%)	2 (1.26%)	1 (.63%)	12 (7.55%)
Final		70 (81.4%)	1 (1.16%)	0	0	15 (17.44%)
Initial	Kim	106 (65.03%)	3 (1.84%)	0	0	50 (30.67%)
Orange Fizz		101 (69.66%)	6 (4.14%)	0	0	38 (26.21%)
Bag, Day 1		98 (42.79%)	29 (12.66%)	6 (2.62%)	2 (.87%)	94 (41.05%)
Bag, Day 2		193 (61.27%)	24 (7.62%)	8 (2.54%)	10 (3.17%)	80 (25.40%)
WWW, Day 1						
WWW, Day 2		81 (55.86%)	11 (7.59%)	1 (.69%)	2 (1.38%)	50 (34.48%)
Final		95 (66.43%)	7 (4.9%)	0	0	41 (28.67%)
Initial	Weston	145 (79.24%)	0	0	0	38 (20.77%)
Orange Fizz		71 (66.98%)	9 (8.5%)	0	0	26 (24.53%)
Bag, Day 1		121 (82.88%)	3 (2.05%)	4 (2.74%)	0	18 (12.33%)
Bag, Day 2		134 (77.46%)	27 (15.03%)	2 (1.16%)	0	10 (5.78%)
WWW, Day 1		72 (66.67%)	8 (7.40%)	3 (2.78%)	19 (17.59%)	6 (5.56%)
WWW, Day 2		90 (77.59%)	10 (8.62%)	0	1 (.86%)	15 (12.93%)
Final		135 (76.70%)	9 (5.11%)	0	5 (2.84%)	27 (15.34%)

As they progressed through their professional learning experiences, the teachers considered how to make their questioning less procedural. For instance, part of their learning was about being more aware of their questioning and part of the work was about being exposed to other's questioning. Ms. Heller explained that she felt she was thinking more broadly about her questioning practice, such as trying not to "funnel" her students (a type of questioning considered in the second professional development meeting and

explored in NCTM, 2014). Funneling is a type of questioning pattern that uses questioning to lead students to an expected conclusion or procedure (NCTM, 2014). Ms. Heller shared:

I mean, just even that word, of just funneling...I don't know if I had really heard the term before, and I think for me it is just such a strong image that it, like, stuck with me, where a lot of the times I was, like, that's exactly what you are doing. Stop! So I was kind of able to check myself with that...And having that collaboration time for me was really helpful...It was really awesome being able to bounce things off of [Mr. Weston] and realising that he was seeing it in a slightly different way...Just having that collaboration time made my questioning better as well. (Ms. Heller, WWW, 586-599)

Ms. Heller developed her questioning through working with other teachers and being observant about her own questioning.

### *Developing Classroom Discourse Practices*

The teachers' questioning practices began to diversify, as seen in Table 4 and in the teachers' own reflections; however, as these questioning practices began to transform, the vital supporting discourse practices to develop students' discourse were not necessarily present. For example, teachers did not accordingly provide enough wait time and or model responses to questions (i.e. What does a good explanation or justification entail?).

Table 5 provides a sampling of questions that teachers asked in the professional learning lessons they taught individually but planned with one another. The questions in Table 5 show higher-order questioning further to the right, shown chronologically in descending order. Table 5 is useful for illustrating some of the questions that teachers exhibited as they began to ask different types of questions within lessons containing different content in each of the professional development lessons.

Teachers' questioning did not happen in isolation; this questioning occurred within the classroom discourse community, hopefully with students responding with answers to such questions. Table 6 provides students' answers to these questions (when there were answers to teachers' questions). What is interesting looking across Table 6 is how many of the higher-level questions had a teacher response to a teacher question. As noted in Table 4, there were fewer of these higher press teacher questions, or those questions that required: explanations consisting of mathematical arguments, mathematical thinking involving understanding among multiple strategies, opportunities to make sense of errors, or collaborative work involving accountability through mathematical argumentation (Kazemi & Stipek, 2009). Therefore, the teacher questions in Table 5, with aligned responses in Table 6, are representative of responses teachers received from students when their questioning developed. As teachers began to ask higher-order questions, which were unfamiliar to both teachers and students, teachers often answered their own questions. For example, during the third professional development lesson, "Who's Watching What?" Mr. Weston, asked, "Who can tell me your justification? Why do you think it is Reading Rainbow?" There were no student responses, and, instead, Mr. Weston responded with the following: "Ok, it sounds childish. What do you think? Why do you think that kids watch it? Let's focus on what show it is. What is the connection there that it seems old, and child show?" (Mr. Weston, WWW, 86-91). Similarly, also in her "Who's Watching What?" lesson, Ms. Keller asked a series of questions of her students:

Why do you think "Science Experiments You Can Do" went with this age group? You put it there for a reason. Why? Why do we think it went with that one? What did a lot of you guys say to me when I first started playing that Bill Nye video?...You watched it yesterday! Is this one closest in age to you guys? Hmm! [Student], were you thinking something different! You guys just told me right! We watched this yesterday! Are you guys closer to 6, 11.7, or almost 55 years old? (Ms. Keller, WWW, 304-310)

Teachers and students appeared to struggle with how to respond at times to these richer questioning practices, with teachers providing answers to their own questions as a result.

## *Countering Low Expectations – Changing Discourse Expectations*

Teachers' learning about questioning provided a context to counter some of the low expectations that the teachers had about their work with their students, and with ELs in particular, in ways that had the potential to change the mathematics discourse expectations in their classrooms. Very early on in the project, Ms. Heller raised the idea that, through using questioning, she was rethinking some of her approaches to using less rigor with ELs. Ms. Heller explained that questioning could counter some of the historic and institutional approaches within her department to working with ELs:

It's something we've kind of talked about as a department... We have a tendency to modify on the assumption that most of our students have language concerns...It's kind of made me think about...modifying doesn't always need to be, you know, stepping it down. How can we modify with still keeping the same rigor? And that's just been, I mean, a department concern for years. But, you know, it's definitely thinking about this and thinking about the different types of questioning, [and it] is letting me see different ways I can keep that rigor with still providing some of the scaffolds that my students might need to be able to, you know, just even access the problem. (Ms. Heller, Orange Fizz, 608-622)

What is powerful is that this insight came after the first professional development lesson for Ms. Heller. By the third professional development lesson, Ms. Heller noticed differences in how her ELs and non-ELs provided reasoning in response to her questioning, showing that they could participate in a richer mathematics discourse. Ms. Heller exhibited an unlearning about her previous thinking related to expectations for rigor for ELs.

Mr. Weston commented at the end of the study about his learning on the role of focusing on content versus providing opportunities for students to justify their thinking. He noted:

Sometimes when somebody struggles with the language, the easy way out is to say, "Well, then let's just focus on the content. Can you add these things together? Can you solve this?" And so, when I am engaging with you, I am going to see if you can do that as opposed to making those justifications, as opposed to asking you more open-ended questions. I've sort of limited folks before, or I should say that limits students to being asked to justify. (Mr. Weston, WWW, 425-430)

Mr. Weston highlighted that, often, there is an idea that mathematics should be devoid of language (Roberts, 2013), when, in fact, content and language are deeply entwined, as the mathematics discourse as a whole. Mr. Weston learned that he needed to provide ELs with discourse opportunities embedded with language, reasoning, and justification.

Finally, Ms. Kim shared that this study accentuated for her the need to keep high expectations for students, not lower ones.

I think it was a reminder of why we're here...[W]e need to set the bar high for our students not lower it – and, be okay with not knowing everything and not always feeling comfortable and trying something new. (Ms. Kim, WWW, 766-772)

Ms. Kim brought together the learning process, and perhaps the learning and unlearning process, that many of the teachers engaged in, as they felt uncomfortable developing their burgeoning questioning practices. These three teachers all provided examples of how they unlearned lowered expectations for ELs and learned to have higher expectations around reasoning and justification through the use of questioning.

Table 5  
*Examples of Teachers' Questions in Lessons*

Lesson	Teacher	Type of question			
		Gathering Procedural Known Facts	Probing Thinking	Generating Mathematical Connections	Encouraging Reflection
Initial	Keller	How many does it say she has at the beginning?	N/A	N/A	N/A
	Kim	What is step zero?	Why?	N/A	N/A
	Weston	Exactly, how do we undo multiplication?	N/A	N/A	N/A
Orange Fizz	Keller	And then were going to need to do four times five and what are we going to get?	Could you give me a more specific, an exact amount? Because you found a rate to show me? Why do you think?	N/A	N/A
	Kim	One of these, two, right, but we would think which one is the best?	What do you notice about every time you plot?	N/A	N/A
	Weston	If we have three of orange concentrate, how much do we need?	How would you do it?	N/A	N/A
Sports Bag	Keller	And what does the C stand for, do you remember?	What is the other thing that we have to think about if we're going to be sewing this together?	<b>What would they look like now if we're adding that on to it, onto your flat ones? What is that gonna make it look like?</b>	N/A
	Kim	How many shapes are we going to see?	Because it's being multiplied by two, multiplied by 3.14 and there's an r; but why are we getting that?	<b>If I were to take apart this sports bag, what shapes will you see?</b>	Someone told me it's now 11.5. Why do you agree or why do you disagree? What's going on?
	Weston	How big are they?	Okay, what was her mistake? What should it be?	<b>So, if I put that there, how long is it going to need to be so that I can fit in both my rectangular</b>	N/A

				<b>piece of fabric and my two circles?</b>	
Who's Watching What?	Keller	What are we talking about when we are looking for the mean?	Which show goes with sample one, and be prepared to tell me why you thought that show went with sample one. [Student]?	What are we seeing in our data?	<b>You put it there for a reason, why? Why do we think it went with that one?</b>
Kim		Jeopardy because the average age is?	What happened? Why would there be a higher amount?	So what can we conclude looking at this if we're talking about the sample means for learning to read?	Why are you choosing Jeopardy?
Weston		What is our number going to be?	What makes you say yes?	What could we do with these two numbers to get a more accurate mean?	<b>Who can tell me your justification? Why do you think it is Reading Rainbow?</b>
Final	Keller	And notice our outer scale is counting by what?	<b>What do we know about it if it's obtuse?</b>	N/A	N/A
Kim		How many sixes are there?	<b>Why do you think that would be the number that would come up most?</b>	N/A	N/A
Weston		How far did I just spin?	Can we think about how it relates to 90 degrees?	N/A	<b>Why? Can we look at anything to help us figure it out?</b>

*Note.* Bold font indicates a teacher question in which the teacher answers their own question.

Table 6  
*Examples of Students' Responses to Questions in Lessons*

Lesson	Teacher	Type of Responses			
		Gathering Procedural Known Facts	Probing Thinking	Generating Mathematical Connections	Encouraging Reflection
Initial	Keller	Some.	N/A	N/A	N/A
	Kim	Distributive Law.	Because the negative is bigger?	N/A	N/A
	Weston	Division!	N/A	N/A	N/A
Orange Fizz	Keller	20.	Because it has the lowest orange syrup thingy.	N/A	N/A
	Kim	Adrian's.	The dots show up.	N/A	N/A
	Weston	Six.	I divided by five, by two.	N/A	N/A
Sports Bag	Keller	Circumference.	The fabric.	Teacher response: <i>I'm not going to tell you that. I'm going to let you struggle through it, but is that what you're getting for both of them now?</i>	N/A
	Kim	Three.	Because we are trying to figure out the radius.	Teacher response: <i>There's three pieces, what shapes will you see?</i> Student: <i>A triangle.</i>	I disagree because you don't actually use the half an inch because that's what you are going to use.
	Weston	20 inches.	It should have been pi times 12.	Teacher response: <i>What I would like you to do is start laying it out and start drawing it out. And see if you can figure out the shortest amount of fabric you're going to need.</i>	N/A
Who's Watching What?	Keller	It is an average of numbers.	Learning to read, because most people at that age start to go to school and learn things.	They're at the age, that's 15.	Teacher response: <i>What did a lot of you guys say to me when I first started playing that Bill Nye video?</i>

	Kim	45.	There's like a 15-year-old and a 25-year-old.	That a lot of people are learning to read.	Jeopardy, oh, because they have an older average age.
	Weston	6.1.	Because they're older.	The middle, the median.	Teacher response: <i>Ok, it sounds childish. What do you think? Why do you think that kids watch it?</i>
Final	Keller	10.	Teacher response: <i>That it's greater than 90 degrees.</i>	N/A	N/A
	Kim	One.	Teacher response: <i>I'll let you think about it.</i>	N/A	N/A
	Weston	180.	It's closer.	N/A	Teacher Response: <i>Can we think about how it relates to 90 degrees?</i>

*Note.* "Teacher Response" identifies instances when there were no student responses to a teacher question. We have in these cases, shared part of the teacher response.

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## Discussion

Teachers went through a process of learning and unlearning (Cochran-Smith, 2003) as they participated in this professional development. At the forefront of this process was learning about questioning and becoming more nuanced in their questioning practice. Questioning is a key component in mathematics instruction (NCTM, 2014), and it was a practice that the three teachers in this study illustrated that they enhanced over the course of a semester.

As there is very little research currently on professional development related to teacher questioning, with some work at the preservice level (Weiland, Hudson, & Amador, 2014), this study provides a contribution to the field through its examination of inservice teachers. In particular, this study examined how these teachers' questioning practice developed across the study as they engaged in a professional learning experience, demonstrating how teachers' participation affected their practice.

These three teachers did not begin with a very diverse questioning practice. Their baseline initial lesson included questioning very much situated in the Gathering Procedural Explanation and Known Fact questions, which required limited responses from students and allowed for limited discourse in the classroom (Boaler & Broadie, 2004; Cazden, 2001; NCTM, 2014). The teachers did not plan for their questioning, whether that was for ELs or other students.

Participating teachers also shared that they came from a place of low expectations for their students, especially their ELs. Over the course of a semester, they asked more questions and more diverse questions, such as Probing Thinking questions (Boaler & Broadie, 2004; Chin, 2007, NCTM, 2014), during their professional development lessons. Teachers' evolution of questioning practices occurred in a relatively short period time as the study progressed. With a more extended and sustained professional learning experience, as the professional learning literature highlights (e.g., Garet, Porter, Desimone, Birman, & Yoon, 2001), teachers may have continued to develop these practices and supports for student discourse.

Even when the teachers asked higher press questions (Stipek & Kazemi, 2009), or used higher order questioning strategies, these questions did not necessarily provide greater opportunities for discourse or even perhaps learning opportunities for ELs or other students, as teachers often answered their own questions. Attention to such interactions involving high press questions is important and provides insight into a burgeoning mathematics discourse community. As teachers learned new practices, in their communities, the teachers expanded their questioning in their mathematics classrooms. Perhaps it was not surprising that this was how students reacted to the teachers' questioning. Students and teachers were not used to such questioning; the class was not familiar with this discourse structure and the associated sociomathematical norms (Yackel & Cobb, 1996) or meta-discursive rules (Sfard, 2001). Students needed someone to model and support the new mathematics discourse structures (Khisty & Chval, 2002) to expand their own learning. The support structures to accompany the teachers' mathematics questioning were lacking, even if the teachers were working to broaden ELs' participation in mathematical reasoning (Moschkovich, 2012). Students likely did not know what was expected of them when teachers asked them these richer questions. However, it should be noted that Ms. Heller shared that her students did respond with justifications and reasoning to her questioning—data collection, however, was not capturing it. Perhaps there were some of these structures being developed over time in her classroom. This study demonstrated that teachers and their students require larger mathematics discourse structure supports throughout such professional learning.

Ultimately, the biggest story of this study was not perhaps the development of questioning and discourse but the learning and unlearning related to teachers' expectations for what students,

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especially ELs, were capable of mathematically. Teachers unlearned previous low expectations, and they learned new expectations for their ELs. There was a story that the study teachers had constructed around how to teach ELs and their students, at large, and the type of mathematics they could do—because of reading levels, prior mathematics achievement, etc.— something that is not uncommon for teachers of ELs (Iddings, 2005; Planas & Gorgorió, 2004). However, there was an unlearning, a reconsideration as teachers worked on their development of questioning. Yes, there were small changes in questioning, but it was important that all three teachers, independent of each other, made comments about raising the bar on their teaching. Questioning created an interactional space that allowed for the unfolding of teachers' expectations of students and created capacity for students to do a different type of mathematics than they had done before, both linguistically and mathematically. Teachers moved from a deficit view of their EL students to a more positive view of what ELs were capable of doing in the mathematics classroom (Moschkovich, 2002) in terms of mathematics discourse and content. Imagine the possibilities if all ELs had opportunities to learn with high cognitive demand tasks and discourse and if all mathematics teachers had EL-specific professional learning opportunities, something sorely missing currently (de Araujo et al., 2018). This research provides implications for future research with its attention to professional learning on questioning and its focus on ELs, in particular. Developing teachers' practices around ELs' discourse practices and helping highlight teachers' beliefs about ELs to combat deficit perspectives of students is important.

## Conclusions

We currently have very few studies about how inservice teachers develop their questioning practices (Weiland et al., 2014), particularly through an inquiry process of professional learning. This study adds to this research literature, illustrating that in a short period of time, this study's teachers, in their professional learning community, were able to expand their questioning practices from limited, known-answer questions to include a variety of higher-level types of questions. Participating teachers also learned that they needed more in their mathematics classroom discourse community for students to be able to respond to such questions, besides simply posing such questions, particularly related to the expectations and quality of mathematics that teachers provided all students, especially ELs.

First, teachers learned how to provide richer questioning to their students, changing the discourse in their classrooms. Second, and possibly more importantly, the teachers learned about their students and what they were capable of in the mathematics classroom. The teachers engaged in a learning and unlearning process (Cochran-Smith, 2003) – unlearning their previous low expectations, and learning to have higher expectations. Unfortunately, these expectations were not individual expectations from one teacher; these were pervasive expectations that the mathematics department, the group of all seventh and eighth grade mathematics teachers at the school, as a whole, held for ELs. Using teacher questioning as a means to develop ELs' mathematics discourse helped the teachers think about what their students could do. This shift in disposition was more than just asking different questions; it was seeing their students in a different light. Attending to discourse was more than asking questions, it was teachers starting to notice their own dispositions toward their students, particularly their ELs, and understanding how changing those approaches could change the opportunities teachers provided their students.

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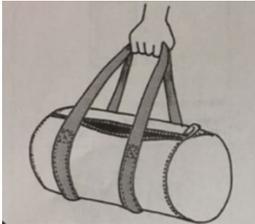
## Appendix

### Orange Fizz Experiment (Modified from Georgia Department of Education, 2016)

Students were given an opener about a cola company changing their soda formula. As a scientist, students had to find the right formula based on concentration and best flavor. They were then provided with three tables of three different formulas, each labeled with different part-to-part ratios (1 part orange concentrate to 2 parts carbonated water; 2 part orange concentrate to 5 parts carbonated water; and 2 part orange concentrate to 3 parts carbonated water). After filling in missing parts of a data table, students needed to find the unit rates in terms of water and in terms of orange concentrate. Students then used the ratios to for each recipe based on different amounts of water or orange concentrate (e.g., 8 cups of orange concentrate and how much carbonated water make 28 servings?)

### Designing a Sports Bag (Modified from Mathematics Assessment Resource Service, 2015)

You have been asked to design a sports bag:



- The length of the bag will be 20 inches.
- The bag will have circular ends of diameter 11 inches.
- The main body of the bag will be made from three pieces of fabric: a piece for the curved body and the two circular end pieces.
- When cutting out pieces of fabric for the bag, each piece will need an extra  $\frac{1}{2}$  inch all the way around it. This is the seam allowance and allows the pieces to be stitched together.
- Don't worry about making the straps.

Make a sketch of all the pieces you will need to cut out for the body of the bag. On your sketch, show all the measurements you will need.

### Who's Watching What? (Modified from Illustrative Mathematics, 2017)

This task began with students first finding the mean of three different samples (of 10 ages) and guessing which show they thought individuals might watch based on the mean. During the launch of the lesson, students had watched short clips of Jeopardy, Bill Nye, and Reading Rainbow and associated their choices based on these shows. Using their data, they matched their analysis with a given show. "Which show do you think your sample represents? Write your answer in the table above and justify." Students repeated this work once more with another set of three samples of data (again, with 10 pieces of data), finding the mean. The third page of the handout provided students with synthesis questions on their work:

1. If you take a random sample of 10 viewer of Trivia the Game Show, do you think the sample mean will be a good estimate for the population mean based on the dot plot?

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Explain or show your reasoning. (For example, Jeopardy has about 10,000,000 viewers each week.)

2. Marketing research shows that advertising investment in retirement planning appeals to people 40 to 55, but people younger than that are not interested and people older than 55 do not invest enough to make it worth advertising to them. Based on the information you calculated above, is it worth advertising retirement planning during any of these three shows? Explain your reasoning.
3. An advertising agency is trying to sell three products: Play-Doh, prom dresses, and river cruises. What program should they advertise on and why? (Include data in your reasoning.)

The next page had students find the Mean Absolute Deviation of three samples.

Then students answered:

4. An advertising director for a video game company has a commercial that appeals to 15 to 16-year-olds. Based on these samples, are any of these shows a good fit for this commercial? Explain or show your reasoning.