Using Prompts to Empower Learners: Exploring Primary Students' Attitudes Towards Enabling Prompts When Learning Mathematics Through Problem Solving

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Teaching mathematics through problem solving is central to contemporary approaches to mathematics instruction, whilst augmenting problem-solving tasks through enabling and extending prompts ensures that a diverse community of learners are provided with opportunities to be optimally challenged, supporting an inclusive classroom environment. However, it has been frequently assumed that teachers should determine when a student should access an enabling prompt, perhaps in part due to concerns that students might be reluctant to seek prompts themselves because of social stigma associated with help seeking. In this paper, we argue that getting students to access prompts of their own volition should be central to teaching mathematics in this manner. One hundred and thirty-two Year 3-6 students completed a questionnaire disclosing their attitudes towards enabling prompts in classroom environments where they were expected to access prompts themselves. Most students consistently reported that enabling prompts empowered them as learners, allowing them to both take responsibility for, and have success with, their mathematics learning. In particular, students valued being able to access prompts when they were stuck on a task, felt that prompts had the power to increase their understanding, and to approach mathematical tasks with more confidence. Students generally did not associate accessing enabling prompts with being 'bad' at mathematics and acknowledged that even strong mathematicians might use a prompt sometimes. There was almost no evidence of any stigma or embarrassment associated with accessing enabling prompts. The implication is that classroom teachers can rapidly establish a culture where students access enabling prompts themselves to support learning mathematics through problem solving.

Keywords: Differentiation · Problem Solving · Enabling Prompts · Primary School · Student Perspectives · Self-Determination Theory

Introduction

At the turn of the century, research suggested that the primary sources of stress experienced by teachers in their roles that could be directly linked to students were teaching pupils who lacked motivation, and maintaining discipline (Kyriacou, 2001; Punch & Tuetteman, 1996). More recent research suggests that this may have changed. It appears that the most significant sources of student-related stress experienced by 21st century teachers, alongside behaviour management, has evolved into catering to a diverse range of learners (Shernoff, Mehta, Atkins, Torf, & Spencer, 2011; Skaalvik & Skaalvik, 2015). For example, Shernoff et al. (2011) undertook semi-structured interviews with 14 urban secondary teachers in the United States to examine teacher perceptions about the sources and impact of job stress. They found that "teaching large heterogeneous groups of learners" and "managing disruptive behaviour" were the two most frequently mentioned student-related stressors. The teachers in Shernoff et al.'s study noted that teaching to a broad variety of academic levels in the classroom was a significant contributor to the stress they experienced in their role, as was the need to constantly revise and adapt the curriculum to meet all learners' needs. These research findings reinforce the idea that the expectations around the role of a teacher are shifting, and that there is a growing need to incorporate pedagogies that effectively cater to students with diverse academic strengths whilst supporting an inclusive classroom environment. This is reiterated in recent attempts to support the development of more inclusive classrooms that ask teachers to focus more explicitly on the "value and contribution of pedagogical practices and tools in creating rich, collaborative learning environments" (McGhie-Richmond & de Bruin, 2015, p. 214).

Few would argue that one of the most challenging aspects of teaching mathematics to children of any age is the diversity of learners in any given classroom. As noted by Gervasoni and Peter-Koop (2020), "teachers at all levels struggle to meet the challenge of providing a high-quality inclusive mathematics education that enables all students to thrive" (p. 1). Consequently, the need to identify pedagogies that can directly support rich mathematical learning experiences for all students, whilst being mindful of the workload and emotional toil experienced by teachers, should be prioritised. One well-established approach for supporting this is to allow students to work on mathematical problem solving tasks in pairs or in small groups (Abdu & Schwarz, 2020; Cobb, Stephan, McClain, & Gravemeijer, 2001; Lester, Garofalo, & Kroll, 1989), or supporting them to engage in project-based group work that integrates mathematics authentically into inter-disciplinary problem solving scenarios (Capraro & Slough, 2008; Han, Rosli, Capraro, & Capraro, 2016; Smith, George & Mansfield, 2020). Such collaborative work has been linked to more positive attitudes towards learning mathematics and is viewed as being facilitative of students' socio-cognitive and affective development (Cesar & Santos, 2006; Lee, Capraro, & Bicer, 2019). A second approach to addressing the challenge articulated by Gervasoni and Peter-Kloop of providing all students with high-quality mathematics instruction is to present the whole class with the same low-floor, high-ceiling, open-ended, problem-solving task (Gadanidis, Borba, Hughes, & Lacerda, 2016). Such tasks enable students to access the mathematics inherent in the task at their current level of learning, with the possibility of the students being meaningfully extended (Sullivan, Mousley, & Zevenbergen, 2006a). A third approach is to teach mathematics through challenging tasks, and use enabling and extending prompts to differentiate instruction (Sullivan, Mousley, & Jorgensen, 2009).

Enabling prompts as a learning tool to support differentiated and inclusive instruction are the focus of the current study. In particular, we are interested in examining student attitudes towards enabling prompts, including whether they view them as a tool to empower themselves as learners, or whether they are reluctant to utilise prompts due to embarrassment or fears that they might be stigmatised in some manner. Although previous research has examined teacher perceptions of enabling and extending prompts, including student reactions (e.g., Clarke, Cheeseman, Roche & van der Schans, 2014; Sullivan, Borcek, Walker & Rennie, 2016; Sullivan et al., 2015), our study is the first comprehensive attempt to examine student attitudes towards prompts through asking students themselves. In addition, our study is the first significant empirical investigation into student attitudes towards prompts in classroom contexts where the responsibility for accessing (disseminating) prompts rests with students, rather than teachers.

Literature Review

Research into enabling prompts

Enabling prompts are designed to provide students with learning experiences closely connected to the initial core problem-solving task, rather than expect students having difficulties to listen to additional teacher explanations, or to pursue learning objectives substantially different from their classmates (Sullivan, 2007; Sullivan, Mouseley, & Zevenbergen, 2006b). It has been suggested that there is value in teachers anticipating those students likely to need an enabling prompt as part of the planning process, as this will guide the types of variations and modifications that would best support students in accessing the main task (Cheeseman, Downton & Livy, 2017). Through anticipating what aspects of the task are responsible for specific difficulties students are likely to encounter, the educator is in a position to develop enabling prompts to augment the student experience of the task without significantly modifying the learning objective (Sullivan et al., 2006b). As Sullivan et al. note:

> [through preparing prompts that] reduce the required number of steps, simplify the modes of representing results, make the task more concrete, or reduce the size of the numbers involved, the teacher can explore ways to give the student access to the task without the students being directed towards a particular solution strategy for the original task (p. 124).

By contrast, extending prompts can be described as being prepared for those students who finish the main task, in part to communicate to students that finishing their work early does not mean a student stops "thinking and learning" (Sullivan et al., 2015, p. 126). They are intended to expose students to an additional task that is more cognitively demanding, however requires the use of similar mathematical reasoning, conceptualisations and representations as the original task (Sullivan et al., 2006b).

To illustrate what is meant by a challenging problem-solving task, and enabling and extending prompts, an example has been included in Figure 1. The task is called Delicious Donuts and has been described previously (Russo et al., 2019). Delicious Donuts meets the definition of a challenging task in that it is likely that most students at whom the problem is targeted (foundation to year 2) would not know initially how to proceed, and it is clear from the framing of the task that students are expected to make their own decisions in relation to both solution strategies and possible solutions (Sullivan et al., 2020). The associated enabling prompt seeks to support students having difficulty through both removing a step in the problem (i.e., revealing that seven objects organised in a line are difficult to subitise) and reducing the size and complexity of the numbers (i.e., inviting students to create a recognisable pattern from any number of dots, rather than 19 dots). The associated extending prompt encourages students to build on some of the important patterns identified in the main task (Sullivan et al., 2006b).

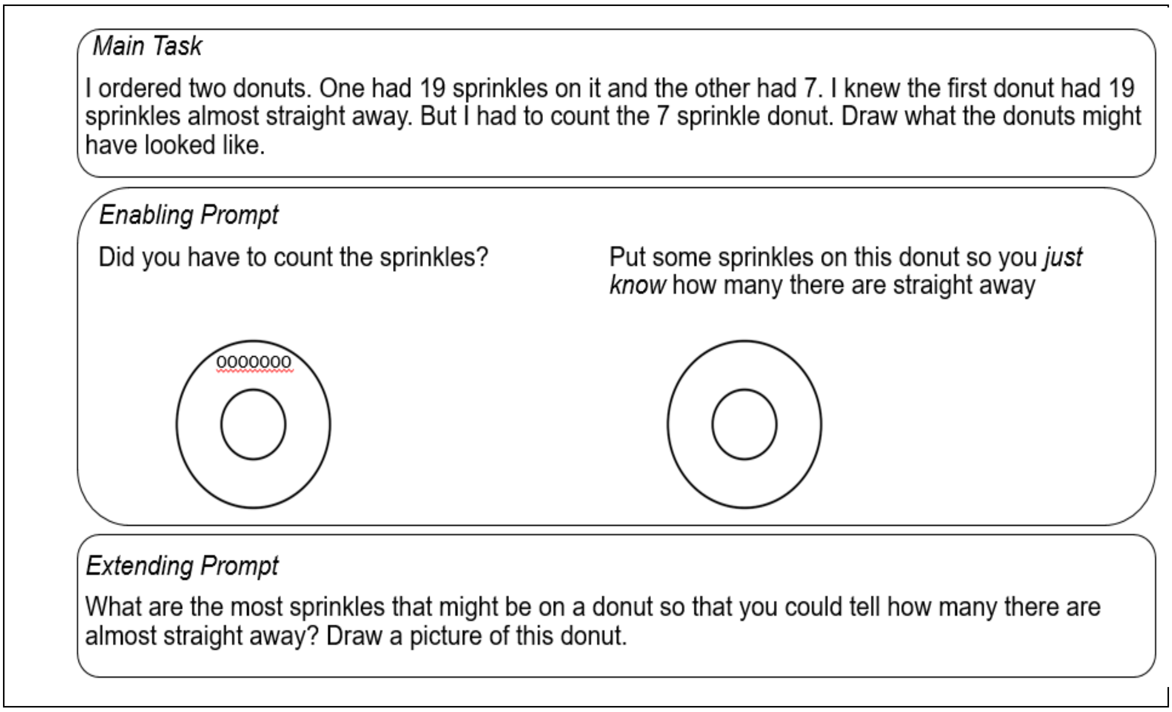

Figure 1. Delicious Donuts task and associated prompts.

It is important to note that within the challenging task literature, it is considered vital that the enabling prompt does not at any point involve explicitly telling students how to solve the problem (Sullivan et al., 2006b). The importance of teachers holding back from telling is indeed an important premise of teaching with challenging tasks more generally (Roche & Clarke, 2014). In this manner, enabling prompts can be distinguished from some other types of problem-solving prompts described in the literature. For example, Lee (2017) investigated the use of providing students with targeted technology-facilitated problem-solving prompts to support their performance on problem-solving tasks. The prompts were structured around the four step problem-solving process, originally articulated by Polya (1957), and organised into three tiers of prompts according to students' prior mathematical performance. Although the prompts that high achieving students had access to (termed *keyword prompts* and *tip prompts*) overlapped with the notion of enabling prompts, the problem-solving prompt for low achieving students (and, at times, for moderately achieving students) involved directly teaching them how to solve the problem. Consequently, Lee's (2017) conceptualisation of prompts to support problem-solving learning differs from Sullivan et al.'s (2006b) conceptualisation of enabling prompts. Lee's conceptualisation is focussed more on supporting all students to solve the mathematical problem as the central learning objective (even if it means telling students exactly what they need to do), rather than aiming to get students to engage with important mathematical ideas, and persevere, as they work towards *trying* to solve the problem (even if it means they never actually solve the problem).

Research suggests that preparing enabling and extending prompts during the planning process is viewed by teachers as central to supporting differentiation, and perceived as fundamental for encouraging persistence on challenging tasks. Clarke et al. (2014), in reporting on responses of the 164 primary teachers involved in a professional learning initiative focussing on teaching mathematics through problem solving, found that attending to differentiation, including developing and identifying prompts, was the most frequently described strategy during the planning stage for encouraging student persistence. By contrast, differentiation was the second most frequently described strategy utilised during the lesson itself for encouraging student persistence, after questioning students and supporting students to reason mathematically. Although attending to differentiation and prompts is viewed as important by teachers both during the planning process and during the actual enactment of the lesson, it is worth emphasising that twice as many teachers in the Clarke et al. study nominated differentiation during the planning phase (27%) compared to during the actual lesson (13%). One potential implication of this finding is that preparing enabling and extending prompts to support differentiation and encourage student persistence has value even if the utilisation of prompts is not central to the manner in which the actual lesson unfolds.

There appears to be a consensus that preparing prompts is an important part of planning and teaching with challenging tasks (e.g., Clarke et al., 2014; Russo & Hopkins, 2019; Sullivan et al., 2015), however opinion differs on the extent to which accessing enabling prompts should be a public act initiated and owned by the student, or a more private, tacit exchange initiated by the teacher and targeted towards particular students needing support. Sullivan et al. (2006b), pioneers in proposing enabling prompts as a means of making learning through problem solving more inclusive, appear to view the act of administering the enabling prompts as part of the teacher's role in orchestrating the lesson. For example, when describing a lesson involving a geometric reasoning task, the authors note that four enabling prompts were prepared, each that addressed a specific anticipated challenge some students might face with the task. These prompts essentially became resources at the teacher's disposal to support the student learning experience. As Sullivan et al. (2006b) note, one aspect of supporting students in this "subtle" manner "is that it is not obvious to other students when a particular student is experiencing difficulty" (p. 135).

Indeed, from Sullivan et al.'s (2006a) discussion of the issue elsewhere, it appears that it is in part the development of multiple enabling prompts that leads these authors to the view that the process of administering prompts needs to be facilitated by the teacher. For instance, when describing a task exploring multiples of 6 and the notion of remainders, Sullivan et al. note: "Even though alternate enabling prompts have been prepared, the teacher still needs to make a judgement on what enabling task to pose first to a particular student experiencing difficulty" (p. 499). The authors also note that, in addition to specifically designed prompts that require some sort of action on behalf of the student, prompts may also include asking particular questions or providing highly focussed explanations. As well as being teacher orchestrated, their conception of prompts therefore remains somewhat fluid, and seems to potentially incorporate a wide range of pedagogical actions a teacher may adopt during the lesson. In summary, Sullivan and colleagues (2006a, 2006b) appear to conceptualise prompts as a tool teachers have at their disposal to support differentiation, rather than an integral aspect of the student learning experience when engaging with challenging tasks.

Placing the onus on the teacher to make an active decision to provide a student with a prompt has been discussed by others. For example, Roche and Clarke (2015) analysed student behaviour during seatwork on a challenging mathematical task ("Work out how to add 298+35 in your head"), and incorporated the teacher action of providing a student with a prompt under the more generic description of the teacher *giving instruction*. The lesson that they elaborated in significant detail offers some insight in terms of the utility of the prompts provided by the teacher to the students. One student, Zita, who had originally solved the main task using the addition algorithm and was struggling to think of alternative strategies, appeared to benefit tremendously from the teacher providing a series of enabling prompts (28+7; 98+7; 198+7). Zita was able to adapt her thinking on these simpler problems to develop a successful mental strategy for solving the main task. In juxtaposition, Roche and Clarke described how another student Sue was instead provided with the extending prompt by the teacher, despite having not appeared to fully grasp the main task. Sue struggled to make any progress for the remainder of the session. The authors speculate that providing the extending prompt to this student was problematic and that this student may have actually benefited from engaging with the enabling prompt. These vignettes suggest that trying to accurately monitor and respond to the thinking of a heterogeneous class of 25 students in real-time as they engage in a challenging mathematical task is a highly demanding undertaking, and perhaps a responsibility that is best shared with students themselves.

In this spirit, and contrasting somewhat with the approach advocated by Sullivan et al. (2006a, 2006b), we have argued that prompts should be made highly visible and explicit to students, and be consciously identified by students as supporting their learning when they are engaged with challenging tasks (Minas, 2019; Russo, 2018). To this end, students "should be encouraged to access the enabling prompt of their own volition, after spending at least some time grappling with the problem" (Russo, 2018, p. 92). We acknowledge that initially this might result in enabling prompts being underutilised, with students being reluctant to access prompts even when unproductively engaged in a task for significant lengths of time, due to students not being used to taking it on themselves when deciding when to seek support, or because there is in fact some social stigma around help-seeking.

It is worth acknowledging there is some evidence that teachers believe that students will be unwilling to access enabling prompts on their own initiative, due to fears that they might be reluctant to seek help in front of their peers (Minas, 2019). This may be one of the reasons that Sullivan et al. (2006a, 2006b) advocated for a more teacher-directed orchestration of prompts in the first instance. For example, Minas (2019) documented his experience of working in a Year 5 classroom with a teacher named Charlotte. Charlotte was initially reluctant to embrace teaching through problem solving, due to concerns that some students might become confused, not experience success, and therefore perpetuate their negative attitudes towards learning mathematics. She anticipated that these students would be more inclined to subtly copy off other students, rather than access the enabling prompt as needed, to avoid being stigmatised as a student who required "additional help" (p. 13). However, Minas goes on to discuss how, over time, these concerns proved unfounded, as students appeared willing to independently initiate getting the enabling prompt when required. Moreover, Minas notes that "this differentiation strategy proved very capable of generating its own momentum, as the more people who used enabling prompts led to a greater proportion of students willing to walk up to the teacher's chair and grab one for themselves" (p. 13).

Indeed, there is data to suggest that teachers who have taught with challenging tasks across several lessons tend to view prompts as a useful tool for differentiating learning, and that students do not generally react negatively to receiving prompts. Sullivan et al. (2016) worked with 30 Year 3/4 teachers to support them in implementing a series of ten lessons built around challenging tasks. As part of the project, these teachers completed a proforma documenting their experience delivering the lesson, including how many students accessed enabling and extending prompts. The mean number of students receiving enabling prompts from teachers across the five lessons described by Sullivan et al. (2016) varied from 6.3 to 10.9, and almost all teachers (with one or two exceptions) provided enabling prompts to at least some students in each lesson. Importantly, teachers typically waited between 5 and 10 minutes before providing students with enabling prompts, with the mean time until prompts were given varying slightly depending on the lesson (6.3 to 7.0 minutes). Qualitative data collected during follow-up interviews with some teachers did not report any negative feedback around the prompts (e.g., students being reluctant to use them due to being stigmatised), however, it is again important to note that teachers in this study, in a similar manner to Sullivan et al. (2006a, 2006b), tended to view it as their responsibility to coordinate the administration of the prompts. In the words of one of the study teachers:

> We emphasised the need [for students] to wrestle with the questions in the zone of confusion for at least 5 minutes before asking questions. We were even more specific to say 'do not ask questions'. We will be walking around and if we think you need a prompt, we will give it to you (Sullivan et al., 2016, p. 168).

Reporting on a related project, Cheeseman et al. (2017) analysed the views of 37 Year 3 to 6 primary teachers in relation to what they were "looking for or thinking about" when they "chose to use an enabling prompt" (p. 143). Again, the expectation, at least implicit in the wording of the question put to teachers, was that teachers would orchestrate the dissemination of prompts. The authors found that teachers used prompts for several interrelated reasons, including: to assist the thinking of students who were struggling with the main task, to make the main task more accessible and to support student understanding more generally. In addition, some teachers noted that they might use prompts to encourage students who lacked confidence, and to facilitate students experiencing success.

Our central argument

Although we are indebted to Sullivan and his various colleagues for developing enabling prompts as a learning tool to support all students learn mathematics through problem solving, we disagree with the assumption that the teacher should determine if and when a student is in need of support. Moreover, we do not think that this is a trivial point. We would argue that placing the onus on students to access enabling prompts should be a foundational aspect of teaching mathematics through problem solving, just as the expectation that students will spend at least some of this lesson in the "zone of confusion" is fundamental to this pedagogical approach (Sullivan et al., 2015).

As noted by Sullivan and colleagues elsewhere, facilitating a learning environment that supports learning mathematics through challenging tasks is likely to require a shift in classroom culture, in particular, from a performance orientation to a mastery orientation (Sullivan et al., 2013). As other authors have noted (Dweck, 2000; Rollard, 2012), students possessing a mastery orientation, where they focus on mastering the content to be learnt, rather than, for example, competing with peers, tend to harbour more positive attitudes towards learning. We suggest that part of developing this mastery orientation is to create a culture where students take responsibility for their own learning, and believe that that they can become better at mathematics through their "effort, calculated risk-taking and resourcefulness" (Russo, 2018, p. 92). If there is any social stigma around accessing prompts leading to a reluctance to seek help, we would contend that a teacher should address this issue directly, because it is in itself a signal that the classroom culture is not yet where it needs to be.

This position is also supported by considering the educational implications of self-determination theory (Deci & Ryan, 2012). According to Deci and Ryan (2012), self-determination theory is premised on the idea that there are three basic psychological needs that motivate individual behaviour: autonomy, competence and relatedness. To the extent possible, the teacher should orchestrate a classroom culture that helps meet these psychological needs of their students through adopting an autonomy-supporting style, rather than a highly controlling style (Reeve, Deci & Ryan, 2004; Reeve & Jang, 2006). Providing students with choice in the classroom has been linked to them experiencing feelings of autonomy and confidence, which in turn supports intrinsic motivation (Deci, Koestner, & Ryan, 1999). Through the lens of self-determination theory, it can be speculated that preparing enabling prompts supports students to feel more *competent* learning mathematics, regardless of whether prompts are administered by the teacher or independently accessed by students. However, it is likely that meeting students' need for *autonomy*, specifically feelings of "flexibility, volition and a sense of choice" (Deci & Ryan, 2012, p. 421), is contingent on empowering students to access prompts of their own initiative.

Given the benefits of students accessing prompts for developing autonomous learners, in the event that students appear reluctant to access prompts when they are needed, we suggest that this should be interpreted by the teacher as an opportunity to positively shape the classroom culture, rather than a signal that the teacher should take back control over the student learning experience through administering prompts themselves. For example, a teacher may choose to praise a student who accesses a prompt in a particular lesson, to reframe this action as the student demonstrating independence and maturity as a learner through taking decisive action to help themselves, rather than waiting to be asked if they need support (or being provided with support without asking). Anecdotally, we would suggest that it is particularly powerful if the teacher is able to praise a student with high social status who accesses an enabling prompt, and/or a student widely perceived by their peers as being strong in mathematics. In order to better facilitate students accessing enabling prompts independently of the teacher, we also believe it is helpful if the prompts are placed in the same place in every session (e.g., on the teachers chair at the front of the classroom). More generally, strong classroom routines when teaching mathematics through problem solving seem to be important for reducing the level of extraneous cognitive load experienced by students and for building autonomy (Russo & Hopkins, 2019).

Finally, an additional benefit of getting students to access prompts themselves is that it inserts a structural barrier to prevent teachers from telling students how to do the task. It has frequently been argued that holding back from telling is fundamental to teaching mathematics through problem solving (Cheeseman, Clarke, Roche, & Walker, 2016; Livy, Muir, & Sullivan, 2018; Roche & Clarke, 2014). However, allowing students time to struggle, which can be framed as the antithesis to 'teacher telling', is something many teachers find difficult when they begin teaching mathematics in this manner (Bobis et al., 2019). By making the first point of help-seeking behaviour the student accessing the prompt independently, the teacher may feel less conflicted about how much information to disclose to support students who are struggling. More generally, it can be speculated that getting students to access prompts themselves is likely to reduce the amount of interaction teachers have with so-called struggling students in the early phases of a problem solving lesson, reducing the temptation to direct these students how to do the task.

Research in support of students accessing prompts themselves

Although we could identify no other studies that specifically looked at the importance of students accessing enabling prompts of their own volition, there is some indirect support for the notion that this may be an important aspect of students learning mathematics through problem solving. For example, the idea that students might benefit from keeping track of their own progress on problem-solving tasks, and use this gained self-knowledge to direct their own learning, has some support in the literature (DiNapoli, 2019; Wilburne & Dause, 2017). Wilburne and Dause (2017) present some evidence that opportunities for students to self-regulate their own learning will generate improvements in problem-solving skills, through developing metacognition and perseverance. The authors devised a program of work focussed around developing students' capacity to self-regulate their learning targeting fourth-grade students identified as low-achieving. They found that guided instruction in how to self-monitor learning in relation to self-nominated learning goals on a problem-solving task appeared to result in students setting more ambitious learning goals. Moreover, students were able to sustain concentration and effort whilst working on the problem-solving task despite these learning goals becoming more ambitious.

The current study

It is noteworthy that the few studies that have considered student attitudes towards, and utilisation of, enabling prompts have focussed on the perceptions of teachers, rather than directly inquired into the attitudes of students themselves (e.g., Cheeseman et al., 2017; Clarke et al., 2014; Sullivan et al., 2016). The current study attempts to address this gap in the literature. Specifically, it involves collecting data from students with varying levels of experience in learning mathematics through problem solving to inquire into their views about enabling prompts. Consistent with the arguments outlined thus far, students in the current study were generally expected to access prompts of their volition rather than being given a prompt by a teacher. The data were collected from the second, third and fourth authors' school contexts.

Method

Participants

Eight classes of students (n = 132) from two schools in the state of Victoria, Australia completed a questionnaire about their experiences learning mathematics through problem solving. Students were able to return the questionnaire anonymously. Participating students from School A included one Year 5/6 composite class (27 students), and one Year 3/4 composite class (25 students), whilst School B had three Year 5/6 composite classes participate (47 students) and three Year 3/4 classes (33 students). School A was a medium size primary school (approximately 300 students) situated in outer North-Western Melbourne. Its demographic profile was comparable to Australia as a whole, with most students being classified into the middle quartiles on the measure of community socio-educational advantage (67%). School B was also a medium size primary school (approximately 250 students) situated in a regional centre in Western Victoria. Its demographic profile was relatively disadvantaged, with almost half (45%) of students being classified into the bottom quartile on the measure of community socio-educational advantage.

Procedure

In School A, the second author was employed as a mathematics specialist. As part of this role, he was tasked with working once a week in multiple classrooms across a school term, with the objective being to model (for the classroom teachers) how to teach with challenging tasks. After working with the second author, classroom teachers were encouraged to teach mathematics through problem solving with their class at least once a week. In School B, the third and fourth authors were employed as classroom teachers and had responsibility for coordinating mathematics instruction across the school. Across the course of the school year, they had been experimenting with shifting the majority of mathematics instruction across the Year 3 to 6 levels towards learning mathematics through problem solving, including using challenging tasks and inquiry-based pedagogical approaches. This shift had been inspired by a professional development initiative in which they had been involved, which provided both authors with intensive professional learning, as well as time out of their classrooms to reform mathematics instruction across their school.

In both schools, the lesson structure being modelled resembled the launch-explore-discuss/summarise structure that is often adopted when teaching mathematics through problem solving (Lampert, 2001; Stein, Engle, Smith & Hughes, 2008). Across all classes, enabling prompts were introduced to students in line with the position put forward in the current paper. In particular, it was expected that students would access enabling prompts of their own accord, and enabling prompts were placed in the same place for every lesson. Two examples of challenging tasks, and their associated prompts, are included Table 1.

Table 1
Examples of challenging tasks and associated prompts

Main Task	Enabling Prompt	Extending Prompt
Chess tournament (Year 3/4) Nash, Isaiah, Genevieve, Rhia, Megan and Ava decided to play a round-robin chess tournament. How many matches will they have to play for each person to play each other player once?	Can you draw a *diagram* to show *how many matches Nash plays*, so that he played each of the other kids once? Can you do the same for Isaiah?	What if ten kids were playing a round-robin tournament, how many matches will they have to play for each person to play each other player once? What if there were 20 kids? How about 100?
Lucky Dice Task (Year 5/6) My dad offered me a deal. I choose any number on a hundreds chart. He'd then roll a 10-sided dice, and we'd count by whatever number he rolled (from zero). If we land on my number, he'd give me 10 dollars. If we skip my number, I'd give him 10 dollars. What are some good numbers I could choose? Should I take the deal?	What if I chose the number 13? Roll the dice 10 times, and see how many times I win the bet. Can you choose a better number than 13?	My dad changed the rules and let me choose any number up to 1000. Is there a number I could choose with the new rules that guarantees me of winning the deal? What is the smallest number that does guarantee me winning the deal?

In general, the explore phase of the lesson was structured such that students were expected to consider the initial problem for around five minutes individually, and then invited to access the enabling prompt if needed. After several more minutes of independent work, students were given the opportunity to work collaboratively, in pairs or small groups (see Russo, 2020, for a more detailed illustration). Consequently, although the current paper presents a highly individualistic framing of learning mathematics through problem solving, the reality is that these classrooms also encouraged peer-to-peer learning and collaboration to build an inclusive community of learners. Indeed, there is evidence from focus group discussions with these same learners that opportunities to work together and learn from other students was one of the most highly valued aspects of learning mathematics in this manner (Russo & Minas, 2020).

Table 2 provides a summary estimating the amount of exposure that students in School A and School B had to these types of problem-solving based lessons at the time of completing the questionnaire. Note that students in School A completed the questionnaire mid-way through Term 3, whereas students from School B completed the questionnaire mid-way through Term 4.

Table 2
Estimates of the number of problem-solving lesson hours students had been exposed to across the course of the school year.

School	Grade	Problem-solving lessons (measured in hours)
A	3/4A	25
A	5/6A	6
B	3/4A	90
B	3/4B	65
B	3/4C	65
B	5/6A	105
B	5/6B	105
B	5/6C	105

Measures

Enabling prompts as empowering mathematical learners

Students responded to six Likert-scale items designed to capture the extent to which they viewed enabling prompts as empowering mathematical learners. The scale required students to take a definite position and included four levels: Strongly Disagree (0), Disagree (1), Agree (2) and Strongly Agree (3). Agreeing with items 1, 2, 3 and 6 (and disagreeing with items 4 and 5) indicated that students viewed enabling prompts as empowering mathematical learners. The six items included:

1. Using the enabling prompt means that a student is taking responsibility for their own learning
2. If a student is stuck on a task for a while, they should go and get the enabling prompt without the teacher telling them to
3. Students who are good at maths might use the enabling prompt sometimes
4. If a student gets the enabling prompt, it means they are bad at maths
5. If a student gets the enabling prompt, they should feel embarrassed
6. The enabling prompt can help students be successful with their learning

Students were also provided with a general open-ended question after they had completed the six Likert scale items: How do you feel about enabling prompts? Qualitative data from these open-ended questions was analysed thematically using the protocol described by Braun and Clarke (2006).

Results and discussion

This section begins by presenting the quantitative Likert-scale data to convey a snapshot of students' overall impressions of enabling prompts, and specifically the extent to which prompts empowered them as mathematical learners. We then proceed to analyse and explore students' qualitative responses, to gain further insight into the reasons why students have positive or negative feelings about enabling prompts.

Results of the questionnaire assessing student attitudes towards enabling prompts are presented in Table 3. In general, students viewed enabling prompts as empowering mathematical learners. Specifically, over 70% of students in each of the schools at the two year levels agreed or strongly agreed with the four positively worded items, and over 90% disagreed or strongly disagreed with the two negatively worded items.

Table 3

Student attitudes towards prompts by year level by school (percentage)

	Using the enabling prompt means that a student is taking responsibility for their own learning	If a student is stuck on a task for a while, they should go and get the enabling prompt without the teacher telling them to	Students who are good at maths might use the enabling prompt sometimes	If a student gets the enabling prompt, it means they are bad at maths	If a student gets the enabling prompt, they should feel embarrassed	The enabling prompt can help students be successful with their learning
School A: Year 3/4						
Strongly agree	8 (32%)	11 (44%)	8 (32%)	0 (0%)	0 (0%)	15 (60%)
Agree	16 (64%)	10 (40%)	13 (52%)	0 (0%)	0 (0%)	9 (36%)
Disagree	0 (0%)	3 (12%)	2 (8%)	3 (12%)	8 (32%)	0 (0%)
Strongly disagree	1 (4%)	1 (4%)	2 (8%)	22 (88%)	17 (68%)	1 (4%)
School A: Year 5/6						
Strongly agree	11 (59%)	13 (48%)	7 (26%)	0 (0%)	0 (0%)	12 (44%)
Agree	16 (41%)	14 (52%)	17 (63%)	0 (0%)	0 (0%)	14 (52%)
Disagree	0 (0%)	0 (0%)	3 (11%)	5 (19%)	5 (19%)	1 (4%)
Strongly disagree	0 (0%)	0 (0%)	0 (0%)	22 (81%)	22 (81%)	0 (0%)
School B: Year 3/4						
Strongly agree	9 (27%)	11 (33%)	9 (27%)	1 (3%)	0 (0%)	24 (73%)
Agree	22 (68%)	12 (36%)	15 (45%)	1 (3%)	1 (3%)	6 (18%)
Disagree	2 (6%)	4 (12%)	7 (21%)	8 (24%)	4 (12%)	3 (9%)
Strongly disagree	0 (0%)	6 (18%)	2 (6%)	23 (70%)	28 (85%)	0 (0%)
School B: Year 5/6						
Strongly agree	18 (38%)	26 (55%)	19 (40%)	0 (0%)	0 (0%)	31 (66%)
Agree	27 (57%)	18 (38%)	23 (49%)	2 (4%)	1 (2%)	15 (32%)
Disagree	2 (4%)	3 (6%)	3 (6%)	10 (21%)	6 (13%)	1 (2%)
Strongly disagree	0 (0%)	0 (0%)	2 (4%)	35 (74%)	40 (85%)	0 (0%)

Notably, all 27 students in the Year 5/6 class from School A agreed or strongly agreed with the statements "using the enabling prompt means that a student is taking responsibility for their own learning" and "if a student is stuck on a task for a while, they should go and get the enabling prompt without the teacher telling them to". Despite only being exposed to enabling prompts across half a dozen mathematics lessons, it is clear that these students had formed the view that determining when to access a prompt is part of their role as a learner of mathematics. The 47 students in the Year 5/6 classes from School B, who were vastly more experienced in learning mathematics through problem solving, also strongly endorsed both these statements. In fact, these first two statements were the only items in which there were notable differences between age groups. Specifically, when pooling data from the two schools together, Year 5/6 students were more likely to positively endorse both the statement referring to enabling prompts as taking responsibility for learning and more likely to suggest that students get a prompt without the teaching telling them to, compared with Year 3/4 students.

Another important revelation from viewing Table 3 is that none of the 52 students from School A who completed the questionnaire across the two classes agreed with the statements "if a student gets the enabling prompt, it means they are based at maths", nor with the statement "if a student gets the enabling prompt, they should feel embarrassed". The rejection of these statements was almost as emphatic by students from School B, who had substantially more exposure to learning mathematics through problem solving. For example, more than 5 out of 6 students from School B strongly disagreed with the statement "if a student gets the enabling prompt, they should feel embarrassed". This suggests that there was little apparent stigma associated with accessing enabling prompts in either school, and that students were comfortable with taking public action (e.g., approaching the teacher's chair at the front) to access an enabling prompt. This finding builds on the Sullivan et al. (2016) study that found that teachers perceived that students were comfortable using enabling prompts, suggesting that this perception is shared by student themselves even in a context where they are accessing prompts in view of their peers.

Students were also given an opportunity to respond qualitatively to how they felt about enabling prompts. A thematic analysis of this data is displayed in Table 4. Over 85% of the students held positive attitudes towards enabling prompts. The two most common explanations for these positive attitudes included that enabling prompts help students when they are stuck on a problem, and the related idea that prompts support students' mathematical learning, understanding and confidence. It is notable that these explanations for the positive attitudes towards prompts reported by students in the current study resonate powerfully with teacher explanations as to why they use prompts reported in the Cheeseman et al. (2017) study.

Table 4.
Student attitudes towards prompts by year level (percentage)

	Year 3/4 students	Year 5/6 students	All students
Helps you when you're stuck, helps you make progress with the problem	29 (50%)	45 (61%)	74 (56%)
Supports mathematical learning, understanding, confidence more generally	14 (24%)	15 (20%)	29 (22%)
Provides an opportunity to collaborate, work with others	1 (2%)	3 (4%)	4 (3%)
Positive feelings, not further described	4 (7%)	2 (3%)	6 (5%)
Ambivalent feelings	6 (10%)	3 (4%)	9 (7%)
Negative feelings	2 (3%)	5 (7%)	7 (5%)

Note: three student responses were purely descriptive and could not be allocated to any theme.

With regards to the notion that enabling prompts help you when you are 'stuck' on a problem, it is apparent that students viewed the prompt as a 'just in time' support through providing students with an easier (but related) problem. Several illustrative quotations from students are provided below:

> I feel that enabling prompts are fantastic because if the person is stuck on the first problem, they can get the enabling prompts and it is just an easier problem.
>
> I feel happy about enabling prompts because I'm usually stuck and they help me.
>
> I like the enabling prompts because it helps kids who get stuck on the main one and lets them know how to work it out another way.
>
> I think they are really helpful because sometimes the problem that we have can be very difficult and hard but with the enabling prompt you can do the same problem but a bit simpler.
>
> I think enabling prompts can be very useful to students that are struggling, need a boost or need something to give them a good idea about what they need to do.

Moreover, several students noted specifically that working on the enabling prompt allowed them to switch back to working on the original problem, and have more success. This is consistent with literature that has emphasised that the intent of enabling prompts is not to replace the main task, but rather serve as a gateway into it (Ingram et al., 2019; Sullivan et al., 2009):

> I think they are great because it gives a student a way of succeeding at the main task.
>
> Good because they are just one level down to help you understand the concept of the situation.
>
> I feel like they help me because usually I get stuck with maths. Then if I figure out the enabling I can usually figure out the original question because both are related.
>
> I think they are good to have because they give you a push and help you out. And they are good because they relate really well to the tasks.

The emergence of this theme is particularly notable because providing support to students after they have spent some time in the zone of confusion is central to how enabling prompts have been described in the literature, whether this support is teacher initiated (Sullivan et al., 2016) or student initiated (Russo, 2018). It is reassuring that this maps on to how students have interpreted the value of prompts, and suggests classroom cultures consistent with a mastery, rather than performance, orientation (Dweck, 2000; Sullivan et al., 2013).

Related to our first theme, another major theme to emerge from the data was the idea that enabling prompts support mathematical learning, understanding and confidence. Although this second theme overlapped with the first theme, the major difference was that this theme involved students taking a more macroscopic view of how prompts support students. That is, rather than just helping students with a particular problem, these students responded that the value of prompts lay in the fact that they support student learning more generally. For example:

> I feel it helps me with my learning… it is easy to learn when I have it there.

> I feel like it is a good way to catch up to higher students, like it is a good way to learn new things.

> I think enabling prompts are really good and useful and they help you in your learning and help you progress on in your learning.

Other students linked the benefits of enabling prompts to dispositional, as well as cognitive aspects of learning. In particular, students emphasised that enabling prompts improved their efficacy as learners. For example:

> They help us feel more successful.

> I feel they're really good. It helps me with my learning… It makes me a lot more confident.

> I think the enabling prompt really helps me to feel more confident in my learning.

Some students experienced increased confidence from having access to an enabling prompt even if they did not use it. As two students indicated:

> I feel comfortable knowing I have it there so I strongly agree we keep it there.

> I feel comfortable having it just in case.

A small number of students (approx. 12%) were either ambivalent or negative about using prompts. One student (whose response was coded to the theme ambivalent) indicated that they thought enabling prompts were useful for their learning, but they did experience some stigma around accessing the prompt:

> I think enabling prompts are good because they make me feel like I'm getting smarter, better in my learning. But what I don't like is how people feel embarrassed if they get a prompt.

Another student (response coded to negative) noted that the prompts negatively impacted her self-efficacy as a learner.

> I don't like them. Sometimes a teacher will tell you to get it, and it makes you feel like you did it all wrong and bad.

These comments from two students out of the 132 students who completed the questionnaire were the only evidence from the qualitative aspects of the questionnaire that students felt self-conscious about accessing prompts publicly.

For other students, their ambivalent feelings seemed to arise from the fact that they were internally conflicted as to whether or not to get the prompt. As two of these students indicated:

> Sometimes I want to get an enabling prompt but I don't because I want to figure it out myself.

> I believe that they are sometimes helpful but you shouldn't feel that you need to always use it if you're stuck.

Two of the students who expressed a negative view towards enabling prompt offered a similar rationale:

> I think the enabling prompt is bad because it helps kids sometimes when they don't need it.

> I don't like them. You should challenge yourself - not take shortcuts.

It is notable that this issue about accessing the enabling prompt prematurely, or having too low a threshold to help seek, would likely only be amplified in a context where teachers were instead making decisions about when to administer a prompt, rather than the students themselves.

Only two students with negative views indicated that the enabling prompts were not particularly useful, and they continued to struggle with mathematics even when being able to access such prompts.

> I hate them because I don't get it

> They don't help with the problem itself a good portion of the time.

This can be contrasted with the 113 students that described enabling prompts positively, and the 74 students who specifically noted that prompts are effective for helping students when they are stuck on a problem.

Conclusions and implications

Consistent with other studies (e.g., Cheeseman et al, 2017; Clarke et al., 2014; Sullivan et al., 2015, 2016), our study finds evidence for the notion that enabling prompts are perceived as important for supporting learning mathematics through problem solving. However, whereas previous research has generally relied on the perceptions of teachers, our research adds to the literature in that it suggests that this conclusion is robust when probing the perceptions of students as well. In particular, there was strong evidence from the current study that students perceived enabling prompts as empowering them as learners.

In addition, previous research has generally assumed that teachers should determine when a student should access an enabling prompt (e.g., Sullivan et al., 2006a, 2006b). One of the potential reasons why teachers have played the role of gatekeeper in this regard is an apparent concern that students may be reluctant to access prompts due to embarrassment or social stigma (Minas, 2019). In this paper, we have argued that getting students to access prompts of their own volition should be central to teaching mathematics in this manner.

The classroom teachers involved in the current study allowed students to access enabling prompts when needed, placing the prompts in the same place every lesson (e.g., on the teacher's chair). In general, they did not provide enabling prompts to students, nor did they direct students to go and get a prompt if they noticed a student struggling (although they might have occasionally suggested that a student consider getting a prompt). Students were generally very positive about the power of enabling prompts to support their own and their peers learning. In particular, students valued being able to access prompts when they were stuck on a task, felt that prompts could increase their understanding, and allowed them to approach mathematical tasks with more confidence. Students generally did not associate accessing an enabling prompt as implying that one is 'bad' at mathematics and acknowledged that even strong mathematicians might use a prompt sometimes. There was almost no evidence of any stigma or embarrassment associated with accessing enabling prompts.

Although the study did not explicitly contrast a student-directed approach to accessing enabling prompts with a more teacher-directed model, the implication is that classroom teachers can rapidly establish a culture where students access enabling prompts themselves to support learning mathematics through problem solving. The benefits of a student-directed approach are likely various. First, it reduces the pressure on the classroom teacher to individually monitor the learning progress of all students in relation to the task. Secondly, it reduces the temptation for teachers to tell apparently struggling students how to do the task. Thirdly, and related to this, it reduces the likelihood that an enabling prompt will be inadvertently provided to a student who would rather not have the support, a concern for at least some of the students in our study. Finally, and perhaps most importantly, it enhances students' autonomy as learners, a central psychological need according to self-determination theory (Deci & Ryan, 2012). Enhanced student autonomy in this context is particularly notable given that one of the central objectives of teaching mathematics through problem solving is to foster the belief amongst students that they can become better mathematicians through effort, calculated risk-taking and resourcefulness (Russo, 2018). Empowering students to initiate their own access to enabling prompts is completely consistent with empowering students as learners of mathematics more generally. Given these benefits, mathematics teacher educators should consider encouraging classroom teachers to make students accessing enabling prompts of their volition a central component of teaching mathematics through problem solving.

We conclude on a cautionary note. Although exploring how extending prompts can be used in the classroom when teaching with challenging tasks was beyond the scope of the current paper, we remain circumspect of the claim that students should also access extending prompts of their own volition. It is our view based on our experience in teaching mathematics with challenging tasks that allowing students to get an extending prompt when they determine they are ready can be problematic for at least two reasons. First, it is difficult for students to realise at what stage they have sufficiently exhausted the mathematical potential of the original task. The teacher generally has greater mathematical content knowledge than students in a primary education context, and a deeper understanding of how the task should unfold in vivo based on their prior experience and anticipatory planning. Secondly, it is important that the culture of the classroom does not become pre-occupied with students progressing to the extending prompt; working with the extending prompt is not necessarily a reflection of the depth of mathematical thinking. Most critically, a classroom climate where students consistently aim to 'get up to' the extending prompt risks reinforcing a performance rather than a mastery orientation (Dweck, 2000; Sullivan et al., 2013). Although some pride on the occasions that students engage with the extending prompt might be inevitable and perhaps even healthy, in our view, this needs to be carefully tempered by the teacher. However, we acknowledge that ultimately how to best use extending prompts in classrooms is an empirical question, and we would encourage other research-oriented practitioners and academics to investigate this issue explicitly in future studies.

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