Mathematics Professional Development: Critical Features for Developing Leadership Skills and Building Teachers’ Capacity

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This article focuses on three features of professional development (PD) programs that play an important role in developing leadership skills and building teachers’ capacity: (1) fostering a professional learning community, (2) developing teachers’ mathematical knowledge for teaching, and (3) adapting PD to support local needs and interests. We draw from our current research on scaling up the Problem-Solving Cycle (PSC) model of PD to illustrate how we worked with novice teacher leaders to incorporate each of these features as they learned to facilitate the PSC in their schools. In addition, we illustrate how the teacher leaders took each feature into account in a particular PSC workshop. This article contributes to our understanding of PD features that can impact leadership skill and teacher capacity. Further, we conjecture that these features are critical to the scalability and sustainability of a wide variety of mathematics PD efforts.

In recent years, teacher professional development (PD) has achieved a position of prominence in the international educational reform and policy discourse (e.g., Alton-Lee, 2008; Borko, Jacobs, & Koellner, 2010; Knapp, 2003; Villegas-Reimers, 2003). Associated with this increased visibility, there has been a growing demand for PD opportunities for teachers. Mathematics has been at the forefront of both educational reform efforts and calls for PD opportunities, particularly amidst mounting evidence that ongoing support and structured learning opportunities for teachers can lead to significant gains in students’ mathematics achievement (Desimone, 2009; Meiers & Ingvarson, 2005).

The educational community is charged with the task of creating PD programs that are scalable and sustainable – programs that can be enacted in a wide range of local contexts by professional developers other than the program designers. A central factor of a sustainable, scalable PD program is the ability to prepare leaders who can implement the program with integrity, adapting it to local contexts while maintaining consistency with core principles. Regrettably, developing the knowledge base and leadership skills of local instructional leaders is often a missing step in educational reform efforts. As Even (2008) commented:

[I]t is remarkable that the education of teacher educators has been almost neglected until now. Expecting the education of practicing teachers to play a critical role in improving the quality of mathematics teaching and learning at school requires greater attention to educators of practicing teachers. (p. 56)
This paper highlights three features identified in the literature that are critical for effective PD and that we argue are essential in preparing leaders to implement high-quality mathematics PD: (1) fostering a professional learning community, (2) developing teachers’ mathematical knowledge for teaching, and (3) adapting PD to support local goals and interests. Furthermore, we posit that careful attention to these three features is critical to ensuring that the PD will be sustainable and scalable. Here we briefly review the background literature related to these features. The remainder of the paper is devoted to describing our current research project, providing more extensive illustrations of each feature, and highlighting the processes we used to support leader development.

Providing opportunities for teachers to participate actively and collaboratively in a professional community is an essential component of high-quality PD (Darling-Hammond & McLaughlin, 1995; Hawley & Valli, 2000; Knapp, 2003; Putnam & Borko, 2000; Wilson & Berne. 1999). Moreover, PD programs are particularly effective when teachers play a role in developing the learning opportunities, and work collaboratively to inquire and reflect on their practice (Hawley & Valli, 2000; Putnam & Borko, 2000). Trust and respect are important aspects of community development, enabling teachers to engage in productive discussions while maintaining a balance between respecting individual community members and critically analysing issues in their teaching (Darling-Hammond & McLaughlin, 1995; Wilson & Berne, 1999).

The content focus of the PD should be challenging, based on student learning, and situated in the work of teaching and participants’ own practice (Darling-Hammond & McLaughlin, 1995; Ingvarson, 2005; Putnam & Borko, 2000). In mathematics education, Ball and Bass (2000) identified and elucidated the construct “mathematics knowledge for teaching” (MKT)—the mathematical knowledge that teachers must have in order to do the mathematical work of teaching effectively. Within the broader construct of MKT, Ball and colleagues identified and explored four categories that are central to performing the recurrent tasks of teaching mathematics to students: (1) common content knowledge, (2) specialised content knowledge, (3) knowledge of content and teaching, and (4) knowledge of content and students (Ball, Thames, & Phelps, 2008)\(^1\). PD programs that support the development of MKT have shown positive impact on student achievement (Hill & Ball, 2004; Hill, Rowan, & Ball, 2005).

PD should incorporate the needs, interests, and constraints of the participating teachers, schools, and district (Hawley & Valli, 2000). PD programs can build in opportunities for adaptation, enabling facilitators to make decisions that take into account their local context. The most adaptable PD programs not only consider the local context at the initiation of the program, but also have the flexibility to adapt to emerging needs, interests, and constraints as the program progresses (Higgins & Parsons, in press). In addition, PD situated in teachers’

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\(^1\) Ball and colleagues provisionally included two additional categories, horizon content knowledge and knowledge of content and curriculum. To date, they have not explored these categories either theoretically or empirically.
classrooms and focused on specific topics related to teaching and learning contains a high degree of authenticity (Darling-Hammond & McLaughlin, 1995; Goos, Dole, & Makar, 2007; Putnam & Borko, 2000). By taking into account issues related to adaptation and authenticity, PD programs can help to ensure that they will support the goals of the populations they are serving. Ideally, the participating teachers can take part in the identification of these goals and make clear what they want to learn, and when appropriate, participate in the development of the learning opportunity or process to be used (Lee, 2005).

The Problem-Solving Cycle Model

The Problem-Solving Cycle (PSC) is a long-term approach to mathematics PD designed to increase teachers’ MKT, improve their instructional practices, and foster student achievement gains (Borko et al., 2005; Jacobs et al., 2007; Borko et al., 2008). In a number of previous articles we have articulated the theoretical and conceptual underpinnings of the PSC at length (see Borko et al., 2005, Koellner et al., 2007). In brief, the PSC model is strongly influenced by both constructivist and situative theories of learning (Ball & Cohen, 1999; Greeno, 2003; Putnam & Borko, 2000). We share with many teacher educators the view that constructivist and situative theories can be seen as interrelated and that learning involves both construction and enculturation (Cobb, 1994; Driver, Asoko, Leach, Mortimer, & Scott, 1994). Stemming from this framework, three design principles are central to the model: fostering active teacher participation in the learning process, using teachers’ own classrooms as a powerful context for their learning, and enhancing teacher learning by creating a supportive professional community.

The PSC is an iterative model, in which each ‘cycle’ is a series of three interconnected workshops. The three workshops are organised around a rich mathematics task2, enabling teachers to share a common learning, planning, and teaching experience. PSC cycles focus on different mathematics tasks and various topics related to student learning and instructional practices. Because the PSC is designed as an adaptable model of PD, facilitators construct their own specific goals for each workshop to meet the needs of their group.

During Workshop 1, teachers collaboratively engage in a teacher analysis task, which is a modified version of the mathematics task that they will give to their students. The teacher analysis task encourages a critical analysis of the mathematics task, including an understanding of the embedded mathematical concept(s), the different strategies students might use to solve the task, and common misconceptions. The broad goals of Workshop 1 are to promote deeper

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2 We have found that appropriate tasks for the PSC model have the following characteristics. They: (1) address multiple mathematical concepts and skills, (2) are accessible to learners with different levels of mathematical knowledge, (3) have multiple entry and exit points, (4) have an imaginable context, (5) provide a foundation for productive mathematical communication, and (6) are challenging for teachers and appropriate for students (Koellner et al., 2007). Tasks with these characteristics tend to be more readily adaptable and modifiable for use in different contexts.
knowledge of the subject matter and strong planning skills. After the workshop, teachers implement the task with their own students, and their lessons are videotaped. The facilitators then select video clips that highlight key moments in the teachers’ instruction and students’ thinking. Workshops 2 and 3 focus on the group’s collective classroom experiences and rely heavily on the selected video clips to foster productive conversations. The broad goals of these two workshops are to help teachers learn how to elicit and build on student thinking, and to explore a variety of instructional strategies to effectively respond to student thinking.

The iPSC Research Project

Our current research project is titled Toward a Scalable Model of Mathematics Professional Development: A Field Study of Preparing Facilitators to Implement the Problem-Solving Cycle (iPSC)\(^3\). The iPSC project is focused on scaling the PSC PD to all middle schools in one urban school district. A central goal of the iPSC project is to prepare school-based teacher leaders (TLs) to implement the PSC with integrity. The participating TLs were all full-time middle school mathematics teachers. All TLs were nominated by their principals or by the district mathematics coordinator to take part in the project either because they were mathematics department chairs or because they were deemed well suited for this type of leadership position. The role of the TL was to learn to facilitate the PSC, and to facilitate PSC PD on a regular basis with the mathematics teachers at their schools.

Members of the research team provided ongoing structured guidance as TLs facilitated the PSC. All TLs led one iteration of the PSC per academic semester over a period of 1 to 2 study years. As shown in Figure 1, prior to conducting each PSC workshop, TLs attended an Instructional Support Meeting (ISM). These full-day meetings took place at the district headquarters and were co-facilitated by a member of the research team and the school district’s mathematics coordinator. The ISMs were designed to assist the TLs in planning for all aspects of their PSC workshops, including developing and maintaining a professional learning community, orchestrating discussions to help teachers identify and understand the mathematics content embedded in the selected task and develop the specialised content knowledge [SCK] needed to teach a lesson with the task, choosing appropriate video clips from the available lessons, and leading discussions based on the video clips. The ISMs also addressed ways to tailor and adapt the PSC workshops to each TL’s local school context (e.g., cultural/linguistic diversity in the student population, school or district goals, specific workplace norms, constraints on time and scheduling).

\(^3\) This project is funded by the National Science Foundation (Award number DRL 0732212).
Participants

Table 1 shows the number of schools, TLs, and teachers who participated in the project during Years 1–3, broken down by year. In total, 8 middle schools, 12 TLs, and 54 teachers participated. All of the middle schools were part of a large urban school district in the Western United States, with a substantial minority student enrolment. Participation in the study was encouraged by the district but optional; by Year 3 almost all middle schools elected to participate. Some schools had one TL and others had two, depending on their size and preference. In all but one school, all teachers in the mathematics department attended the PSC workshops.

Table 1

<table>
<thead>
<tr>
<th>Participation dates</th>
<th>Middle schools</th>
<th>Teacher Leaders</th>
<th>Teachers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 1 (Winter 2007–Summer 2008)</td>
<td>4</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>Year 2 (Fall 2008–Summer 2009)</td>
<td>3</td>
<td>5</td>
<td>13</td>
</tr>
<tr>
<td>Year 3 (Fall 2009–Summer 2010)</td>
<td>6</td>
<td>8</td>
<td>45</td>
</tr>
<tr>
<td>Totals*</td>
<td>8</td>
<td>12</td>
<td>54</td>
</tr>
</tbody>
</table>

*Owing to a variety of factors, schools, leaders, and teachers participated for either 1, 2, or 3 years. The numbers in this row indicate the total number of schools, Teacher Leaders, and teachers that participated in the project for one or more years. In Year 1, TLs engaged in a series of PSC workshops in their role as classroom teachers. Facilitation of workshops began in Year 2.
Data Sources for This Article

We collected extensive qualitative and quantitative data on the nature of the support provided to TLs, their PSC workshops, and the impact of the program on the TLs and teachers. These data include video records of all ISMs and PSC workshops, interviews with the TLs conducted at the conclusion of each PSC iteration, and a pre/post mathematical knowledge assessment given to the TLs and the teachers with whom they worked. We used parallel forms of the Mathematical Knowledge for Teaching (MKT-MS) assessment for middle school teachers, developed by the Learning Mathematics for Teaching (LMT) Project (Hill, Schilling, & Ball, 2004).

This article focuses on one iteration of the PSC, conducted during Year 3 in Spring 2010. This iteration used the Fuel Gauge task (adapted from Jacob & Fosnot, 2008; see Fig. 2), a rate problem that involves calculating miles per gallon. The examples for this paper are drawn from: (1) the ISM prior to the TLs conducting Workshop 1 of the Fuel Gauge task, and (2) a selected PSC Workshop 1 of the Fuel Gauge task, jointly facilitated by two TLs. Our decision to focus on Workshop 1 is purely pragmatic—to understand the experiences within a given Workshop 2 or 3, it is generally necessary to understand the participants’ prior experiences within a Workshop 1 and teaching the task to their students. We selected the Fuel Gauge cycle because it was the last PSC cycle fully supported by the project, and the focal Fuel Gauge ISM and PSC Workshop 1 contain numerous illustrations of the three features highlighted in this paper. The ISM was led by Karen (the first author of this paper and a co-principal investigator of the iPSC) and Jody4 (the school district’s mathematics coordinator). The PSC Workshop 1 was facilitated by Jason and Kyla, TLs who joined the project in Year 3 with the support of their principal and full participation of the mathematics teachers at their school. Jason and Kyla were also strongly committed to successfully implementing the PSC model at their school. Prior analyses of all Fuel Gauge Workshop 1s indicated that their workshop, overall, was conducted with a high degree of integrity to the core principles of the PSC (Borko et al., 2010).

Data Analysis

First we report on the impact of the iPSC on the MKT-MS scores of the participating TLs and teachers. These data came from a total of 62 participants (10 TLs and 52 teachers) who completed both the pre- and post-assessments; participants with missing data are excluded. We ran paired $t$ tests on the full sample and simple analyses of variance on the sample broken down by the two groups, TLs and teachers.

Next we used vignette analyses to create detailed descriptions of activities in the focal ISM and PSC workshop that illustrate the three features of PD

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4 With the exception of Karen, all names used in this paper are pseudonyms.
Frank runs a business called Frank’s Fresh Farm Produce. Once a week he drives north of the city to farms where he buys the best possible fresh produce for his customers. Frank can travel 600 miles (965.6 km) on a full tank of gas. His truck has a fancy, accurate fuel gauge.

![Fuel Gauge Diagram](image)

Usually Frank has time to visit only one farm on each trip, but one week he decides to visit both Stan’s and Louisa’s farms. When Frank drives from his store to Stan's farm and back, he knows he uses 5/12 of a tank of gas. When he drives to Louisa’s farm and back, he uses 1/3 of a tank. From a map of the area, he learns that there is a road from Stan's farm to Louisa's farm that is 120 miles (193.1 km) long. He realises that he can drive from his store to Stan’s farm, then to Louisa’s farm, and then back to his store in one loop.

Frank can tell by looking at the fuel gauge that he has 5/8 of a tank of gas. Can he drive this loop without having to stop for fuel? Or should he buy gas before he starts his trip?

Figure 2. The Fuel Gauge Problem

highlighted in this paper. The vignettes are intended to reconstruct and authentically represent the events, people, and activities under consideration (Erickson, 1986; LeCompte & Schensul, 1999; Miles & Huberman, 1994). To create the vignettes, the authors examined videotaped records and field notes from the focal ISM and PSC workshop and selected the activities and conversations that were most representative of each feature. Vignettes were then constructed to depict the nature of the events and how TLs thought about them, drawing from the videotapes, field notes, and interviews. Although dialogue is indicated in the vignettes, in the interest of space and readability, the authors have taken some creative licence while striving to remain close to the speakers’ own words and maintain their intention. Vignettes are written in the present tense and set in italics. Interpretive commentary is interwoven using regular font.
Results

Impact of the iPSC on Participants’ Knowledge

As Table 2 indicates, the participants overall showed a significant gain in their math knowledge for teaching, as measured by their scores (percentage correct) on the MKT-MS. The TLs had an average baseline (pre-test) score of 72.4% correct and an average post-test score of 78.1%, whereas the teachers had an average baseline score of 65.4% correct and an average post-test score of 70.7%. Paired t-tests indicate significant gains for the participants as a whole, and for the teachers as a subgroup. The percentage of correct answers for all participants, on average, increased 5.4%, which is similar to the gain for the teachers. The scores of the TLs increased 5.7%, on average, which is not significant (likely due to the small sample size). It is important to note that there was no comparison sample of teachers in the iPSC, so these findings should be interpreted with caution.

Simple analyses of variance did not indicate a significant difference between the two subgroups on either the pre-test or the post-test. The teachers and TLs also did not differ significantly on the degree to which their scores changed over time.

Table 2

<table>
<thead>
<tr>
<th>Sample</th>
<th>Sample size (N)</th>
<th>Pre-test mean (% correct)</th>
<th>Post-test mean (% correct)</th>
<th>Change (% correct)</th>
</tr>
</thead>
<tbody>
<tr>
<td>All participants</td>
<td>62</td>
<td>66.52</td>
<td>71.89</td>
<td>+5.37**</td>
</tr>
<tr>
<td>Teacher Leaders</td>
<td>10</td>
<td>72.40</td>
<td>78.10</td>
<td>+5.70</td>
</tr>
<tr>
<td>Teachers</td>
<td>52</td>
<td>65.38</td>
<td>70.69</td>
<td>+5.31**</td>
</tr>
</tbody>
</table>

**p < .01 indicating significant change from pre-test to post-test.

Developing Leadership Skills: Illustrations from the PSC

In this section we present a series of vignettes to illustrate how we worked with the TLs in the Instructional Support Meetings to guide and support their efforts to: (1) foster community, (2) develop their teachers’ MKT, and (3) adapt the PSC to support their local goals and interests.

ISM Feature 1: Fostering Community

Fostering community among the participating teachers is essential to the success of the PSC model, and a top priority of the iPSC project was supporting TLs’ efforts to establish, maintain and promote a professional learning community within their groups. Collegiality was a theme that we highlighted in each of our ISMs. We wanted TLs to continually consider issues such as promoting a safe environment in which being uncertain or making an error is acceptable.

The ISM facilitators, Karen and Jody, frequently integrated themes or ideas
about community into their work with the TLs. In the ISM held before TLs conducted their Fuel Gauge Workshop 1s, each TL watched a video from their previous iteration of Workshop 1 and reflected on how they orchestrated solving the PSC task and discussed the mathematics with their group of teachers. Karen and Jody asked the TLs to identify aspects of their facilitation that they thought worked well and aspects they would like to change when conducting Fuel Gauge Workshop 1. After each TL had a chance to reflect on their facilitation, Karen asked, “What kinds of things did you notice about how you did math with your teachers the last time? Does it give you any perspective about doing mathematics with them this time?” Several issues the TLs brought up related to maintaining a safe, supportive learning community. Their discussion in response to a difficult situation that Candace shared is illustrative:

Candace describes a point in her previous Workshop 1 when some teachers in the group did the mathematics incorrectly. She asks, “How do you handle that situation?” Jody suggests, “How about modelling it the same way as you would with kids? How would you handle it if kids did the task completely wrong?” Candace responds, “That is what I did.” However, Candace still seems uncomfortable with her role as a facilitator in these circumstances and remarks, “I just thought the teachers should know the mathematics better.”

Karen hones in on Candace’s remark, addressing it directly and carefully: “I think it is really important as a facilitator to never assume that the teachers you are working with know what you know. You should never assume that it is easy for a teacher to do a task in multiple ways because often that’s not how people were taught. I think that some sensitivity to this situation is imperative. And I think Jody’s approach is right. We should try to engage in the conversation in a way that’s productive for the teachers and moves them forward. We have to remember that people are stronger in some areas of math than others.”

Jody adds, “I think it’s so important that you set norms with your teachers. I know we talked about this early on in our ISMs and that most of you already have.” She models a way to do this, “You might say, ‘We’re going to talk about the math. Don’t think that this is meant to be demeaning, it’s so everybody is on the same page.’ That way you’re making it safe for someone who isn’t as comfortable with the math to ask questions and to get that rich learning.” Jody shares her view that when doing professional development, it’s helpful to anticipate that some teachers may struggle with the math.”That may not be the case, but it’s the assumption you should work from all the time. Then you make it safe for the teachers who are in the room, and you’re able to help them learn the math, which is the whole point.”

Karen adds, “Over time you will know that you have been successful as a facilitator when people in your group start to say ‘I don’t get that. Could you go over it again?’ You will know that your PD workshop has become a safe community when people are starting to disclose what they don’t know, when they are not sure, or when they have never thought about the mathematics or a solution strategy in a particular way before. If you never see that and everybody acts like they know everything all the time, it’s probably a cue that you need to work on establishing a safe community.”

The discussion continues, with other TLs contributing strategies for how they try to
create a safe environment and help their teachers feel comfortable. Mandy shares, “I use whiteboards with my teachers. When I notice that a teacher might be confused, I might say, ‘I don’t understand that strategy. Can you explain it again?’” Mandy emphasizes, “That way, I am the person opening up, explaining that I don’t understand, and hopefully this helps to set the stage for others to ask questions or say that they don’t understand something.”

This exchange, although brief (less than 5 minutes), illustrates how Karen and Jody structured a conversation around the TLs’ questions and concerns to bring important issues related to creating a safe mathematics community to the forefront. Karen and Jody modelled specific features of a safe community and encouraged TLs to share strategies they used to create such environments in their own groups, particularly focused on helping teachers be comfortable making mistakes and acknowledging the mathematics that they are struggling with or would like to understand more deeply.

**ISM Feature 2: Developing Mathematical Knowledge for Teaching**

Another essential component of the PSC model is developing participants’ MKT. Throughout our work with TLs, we strove to provide frequent opportunities for them to deepen their MKT, in the service of helping them become both better facilitators and better teachers. One rather subtle, but critical, distinction that we discussed with the TLs is doing mathematics with students versus doing mathematics with teachers. For example, we would tell the TLs to put on either their ‘facilitator hat’ or their ‘teacher hat’ as they thought about the mathematics task. In the ISMs we largely focused on supporting the TLs in their role as PSC facilitators, knowing that in general they had far less experience working with adult learners than they did with middle school students. We stressed the importance of having a clear understanding of why teachers should engage in mathematics in PD, and of designing teacher analysis tasks (see Figure 3) to address specific SCK goals such as analysing the mathematical relationships among different solution strategies (Kazemi et al., 2011).

The PSC model is designed to deepen all aspects of teachers’ MKT, including their SCK, knowledge of content and teaching, and knowledge of content and students. Workshop 1 primarily targets SCK although knowledge of content and students, and knowledge of content and teaching also come into play, especially when teachers plan their PSC lessons. The vignette below illustrates processes used during the ISM to help the TLs further develop their SCK. Karen and Jody requested that the TLs teach the Fuel Gauge task to their students prior to attending the ISM. During the ISM, Karen and Jody had them debrief their classroom experiences and then revisit the teacher analysis task that they would be using during their Workshop 1s (see Figure 3). In this way, Karen and Jody could help the TLs lay the foundation for facilitating the Fuel Gauge task with adults, with an eye towards building their MKT and promoting in-depth discussions around the mathematics.
Below are some strategies to solve this problem. For each strategy explain a student solution method or methods and their interpretation of the method. Describe potential challenges with each method.

1. Using the representation (gas gauge) and marking distances
2. Working with everything in miles
3. Working with fractions of a gas tank
4. Blending approaches, such as combining $\frac{5}{24} + \frac{1}{6}$ with common whole of 24 to get $\frac{9}{24}$.

**Figure 3. Teacher analysis task for the Fuel Gauge problem**

Jody opens the ISM, “We are going to reflect on your teaching of the Fuel Gauge problem. Think about what went well, what was challenging, and what would have helped you plan more effectively to teach that lesson.” Karen adds, “We want to have a really nice, rich conversation, so we can build off everyone’s classroom experiences teaching the problem to prepare for your Workshop 1.”

The TLs share their experiences, including modifications they made to the problem, how their own students solved it, the misconceptions or challenges they faced, and extensions that were helpful for those that could move further within the concept. For example Carla tells the group, “I loved the variety of ways that my students went about solving this problem. Without any prompting, I had some kids using miles, some using fractions. I had kids approach it from so many different directions. It was really cool to see their thinking.”

Several other TLs talk about their lessons. Robert describes his students’ approaches, and offers some insights into their mathematical understanding: “My students solved this problem either one of two ways. They either added fractions of a tank, or they switched fairly quickly to miles to figure out what the maximum number of miles was. I noticed that my stronger problem solvers were the ones who switched to miles, and the ones who left it in fractions tended to be the weaker students. I’m externalizing a little bit, but I think one of the reasons for that is because the better problem solvers take some time to think about the problem before they work on it. On the front page, the information is given in fractions. So the students who jump on it right away take the first information and work with that. The other ones think a bit about what’s going to make the problem easier.”

The conversation continues for almost 30 minutes, providing the TLs with a sense of the experiences of teachers working in different grade levels and at different schools. For the next activity, Karen passes out the teacher analysis task and explains, “We’re going to go through some of the math again and we’re going to try to be really detailed about your planning for the workshop and doing the math with your teachers. Do the first part of the task only, where you’re asked to solve everything in miles. Working with your partner, you’re going to keep a running list: Thinking about your students who solve it in miles, what are the possible roadblocks they might come across? What are the struggles they might have? What are the challenges they might face? What misconceptions or mistakes might they make?”
Jody discusses the importance of going over a wide variety of strategies and the relationships among them, and she offers suggestions about how to support discussions focused on the mathematics. She cautions, “When your teachers see the analysis task, they may glance over the math. The teachers may think they can just whip out the answer and get it really quickly and easily. Because of that, they may not, without prompting, go to a deeper level of thinking about the math that’s actually happening in this problem. So, as you’re working the task, have that in the back of your mind. Think about how a student would think about it, but also think about the richness in the task that you want to deliberately pull out in your Workshop 1. Try to think about both those lenses as the same time.”

Karen and Jody intentionally used the teacher analysis component of the ISM to address some of the challenges the TLs faced during their previous iteration of the PSC. In their prior Workshop 1s, many of the TLs struggled to keep the discussions focused on the mathematics and missed opportunities to deeply explore alternative representations, solution strategies, and potential misconceptions. As indicated in the vignette, Karen and Jody structured the teacher analysis task so that the TLs first had to consider multiple variations of a “miles only” strategy. Once they worked through those strategies, the TLs were able to brainstorm potential roadblocks that might come up for students related to each strategy, and to suggest questions or probes that might help a teacher move students forward. The group then considered solution strategies involving fractions and again discussed potential roadblocks and probes. Karen and Jody encouraged the TLs to reflect on their own understanding of the mathematics in the task, and at the same time consider how to help their teachers gain a deep understanding of the content during Workshop 1. By modelling the use of the teacher analysis task and encouraging self-reflection, Karen and Jody strove to help the TLs become aware of processes that could be used to foster rich conversations and to support the development of MKT in their Workshop 1s.

**ISM Feature 3: Supporting Local Goals and Interests**

The PSC model offers facilitators the flexibility to modify their workshops based on the needs of their participants and the conditions within a given local context (e.g., time allocated to PD workshops, range of grade levels and ability levels of students in participating teachers’ focal classrooms). During the ISMs, Karen and Jody continually encouraged the TLs to identify their participating teachers’ goals and interests, and to carefully consider how to frame their workshops accordingly. Not surprisingly this proved quite challenging for the TLs, as the variation within and across their groups called for attention to a large number of sometimes competing interests. For example, one group comprised the entire mathematics department consisting of 12 teachers from different grade levels, including special education teachers. Another group comprised only three teachers, who taught gifted students at different grade levels.

In each ISM, Karen and Jody provided opportunities for the TLs to generate goals for their upcoming workshops, and they supported each TL to plan a
workshop that best matched his or her goal(s). In addition, Karen and Jody frequently engaged in public, metacognitive reflections on their own goals, and talked to the TLs about why they designed the ISMs in a particular manner and the strategies they were using to meet certain goals. For example, as they transitioned from the teacher analysis task to workshop planning, Karen noted that her goal had been to model facilitating a conversation around anticipating student roadblocks and developing probes to move past those roadblocks.

Karen and Jody noticed that many of the TLs devoted little time, or in some cases no time, to lesson planning during their previous Workshop 1. Therefore, during this ISM, they encouraged the TLs to consider how to include an opportunity for lesson planning in a way that would be consonant with their goals for the workshop. Allocating time for lesson planning is critical to the PSC model, particularly because it allows teachers the opportunity to discuss appropriate modifications to the problem and plan for instructional supports specific to their classroom. In the latter half of the ISM, Karen and Jody passed out two potential protocols, and asked the TLs to think about how the lesson planning time could be used in a way that matched their goals.

Jody launches into the topic of lesson planning by reminding the TLs to keep their goals in the forefront of their minds. She suggests, “Remember what your overall goal is for the different activities you’re doing in the workshop. If you’re very clear what that is, then when you have to make those on-the-spot decisions, you can go back to your overall goal and make the decision that will best support it.” Jody points out that many TLs already have begun writing down and discussing their Workshop 1 plans. She explains that they’re now going to “add some layers to that, focused on ways to support your teachers in planning to teach their Fuel Gauge lessons.” As a resource, she distributes two lesson planning protocols.

Karen mentions that the group has already seen these protocols, although not everyone has used them in their workshops. Karen acknowledges that some of the TLs only have an hour to conduct their Workshop 1, and that it’s easy to work on the math for an hour and not get to the lesson-planning portion of the workshop. She continues, “Lesson planning is such an important aspect of the PSC. We want some planning to happen to help teachers make good instructional decisions when they see what their students are thinking.” Karen explains that they don’t expect the TLs to address all of the questions raised in the planning protocols and directs the group, “Together with the person sitting next to you, think about how you are going to use the protocol so you can be purposeful in helping your teachers plan. Think about how the protocols can help you plan more effectively, and at the same time help you reach your goals as a facilitator.”

After working with a partner, the TLs talk as a whole group about some of their ideas related to lesson planning and their overall workshop goals. Robert comments, “I want to focus on helping teachers reword the problem in a way that fits with their classes and make certain that their students have the background for doing that.” A bit later he adds, “I noticed during the last PSC iteration that I should have moved more carefully through the mathematics of the problem during Workshop 1. The teachers had very different experiences than I did when teaching the problem because our students are very different from grades 6 through 8.” He continues, “For the Fuel Gauge problem, I think we’ll do
all the arithmetic. I want to focus on how we’ll modify this problem for the different classes to make it work for each teacher. They have different needs and I need to make sure that we look at the problem from their different perspectives and levels of students.”

During the latter portion of the ISM, the TLs considered how best to structure the lesson planning component of their Workshop 1s to ensure that the activity would be consonant with their locally constructed goals. Several TLs determined that they wanted to help the teachers in their group successfully differentiate their Fuel Gauge lessons, so that their lessons would take into account students with different levels of mathematical ability. As noted in the vignette, Robert reflected on both his own prior facilitation of PSC workshops and the needs of his group, and determined that he should move through the mathematics of the task more slowly. Doing so, he conjectured, would aid his teachers in their lesson planning process, helping them to better adapt the problem to their grade level and anticipate their students’ challenges. An important component of adapting the PSC model to support local goals is passing ownership of the adaptations to the PD facilitators. Throughout the ISMs Karen and Jody urged the TLs to reflect on the needs of their group and generate their own goals. This process ensures that TLs’ goals will be relevant and responsive to the needs of their teachers, and also serves as motivation for the TLs to enact their goals. In addition, Karen and Jody wanted the TLs to become increasingly purposeful in their planning of each workshop activity and to structure activities, such as the use of lesson planning protocols, in accordance with their goals. While the TLs have a great deal of flexibility in choosing their goals, those goals then play a large role in shaping the nature of their workshop and facilitation decisions.

Building Teachers’ Capacity: Illustrations from the PSC

In this section of the paper we consider how the TLs implemented the PSC model with the mathematics teachers in their schools, highlighting their efforts to foster community, develop mathematical knowledge for teaching, and support the goals and interests of their teachers and school. We draw on selected episodes from the Fuel Gauge Workshop 1 at one school, co-facilitated by Jason and Kyla, to illustrate the enactment of these three features. Typical of many PSC workshops, Jason and Kyla’s Fuel Gauge Workshop 1 took place after school and was just under 2 hours in duration.

PSC Workshop Feature 1: Fostering Community

As noted above, developing and maintaining a professional learning community is an essential feature of effective PD. Successful PSC workshops are characterised by a climate of respect and collaborative working relationships among the teachers. Jason and Kyla’s manner of introducing their group of teachers to the Fuel Gauge task in Workshop 1 illustrates their efforts to engage all participants by making the task personally relevant to their students. Rather than begin by asking the group to work through the task, Jason and Kyla elected
to first have each teacher, individually, consider how they and their students would approach the problem and what types of difficulties their students might encounter when solving it.

Jason and Kyla distribute copies of the Fuel Gauge problem. Jason tells the teachers, “After you read the problem, think about how you would solve it. You do not have to actually solve it, just think about how you would. Also, think about the issues or difficulties your students might encounter when solving this problem.” Teachers silently read the problem and consider the questions that Jason posed for a few minutes. As Jason brings the teachers together for a full group conversation he reminds them, “Think about your particular group of students. If you teach sixth grade, what difficulties might they experience when solving the problem? If you teach eighth grade, what might your students experience?” These questions incite a lively discussion, with most teachers in the group participating. They share ideas about how their students might struggle including misunderstanding the context of the problem, difficulty distinguishing between the amount of miles and the amount of gas, reading the lengthy text, and doing the mathematics using incorrect strategies.

Jason and Kyla’s technique of delving into the task by asking a question about each participant’s students was discussed in many of the ISMs as a way to build community by engaging all participants in a relevant and safe manner. This strategy invites participants to contribute to the professional learning community and take ownership of issues raised in the PD. Jason and Kyla were pleased with the community they saw developing in their math department over the course of the iPSC project. Jason explained in an interview that the PSC workshops enabled teachers in his school to more actively share their ideas.”That was probably the most awesome thing that happened. We brought sharing out of people, and then it got better as we went on. There was more openness as we went on.”

**PSC Workshop Feature 2: Developing Mathematical Knowledge for Teaching**

As we discussed above, there are important differences between doing mathematics with students and doing mathematics with teachers, a point we stressed throughout the ISMs. In the PSC workshops, facilitators must shift from their usual role as classroom teacher – doing mathematics with students – to helping support adults’ learning. Jason and Kyla attempted to develop their teachers’ MKT in Fuel Gauge Workshop 1 by using the teacher analysis task and guiding teachers to work through and discuss the mathematics in a systematic manner, as modelled by Karen and Jody.

*After the teachers have looked through the Fuel Gauge problem and thought about how their students would approach it, Jason and Kyla distribute the teacher analysis task. Kyla explains to the group, “There are two main approaches to solve the Fuel Gauge problem. You can either use miles or fractions of the gas tank. We want all of you to solve the problem first using miles only. Solve it individually and then you can talk to your*
partner about your solutions using miles. Solve it as you would, not like your students.” Jason adds, “Solve it using your own resources, what you know.” The teachers begin solving the problem and discussing their ideas with partners. Teachers can be seen labelling the fuel gauge representation, drawing diagrams and making calculations. Everyone appears to be actively engaging with their partners and explaining their thinking in animated ways.

Requiring teachers to solve the task using the miles approach in multiple ways helped to move the group beyond using just their common mathematical knowledge, to deepen their SCK. This technique enabled Jason and Kyla to assess individual teachers’ understanding of the mathematics entailed in this problem, a critical component of effective facilitation and one discussed in ISMs. Knowing precisely where the teachers in the group are, mathematically, helps TLs structure and adapt the workshop in ways that meet their individual needs. In addition, by getting a wide variety of ideas and approaches on the table, Jason and Kyla were well positioned to move into a discussion about how to support students who use those approaches.

Standing at the overhead projector Kyla shows a chart with three columns across the top labelled: (1) student strategies, (2) how to address roadblocks, and (3) probing questions. In the first column, Kyla shows her documentation of the various mile strategies that she wrote down as each was discussed. Jason asks the group, “If your students solve it using the different mile strategies that we have recorded in the chart, what roadblocks might they run into? And what types of questions or probes might you, as the teacher, use to move students forward?” They begin brainstorming the roadblocks, and one teacher offers, “I think round trip versus one way will be a problem.” Others nod their heads in agreement. Jason encourages further conversation by asking, “Is there anything else that you think might be a problem?” Lilly replies, “I think renaming fractions and finding equivalent fractions will be a problem for my kids. When Henry and I solved the problem, we were using 12ths, 6ths, and 3rds. That would be hard for my kids.” Shana adds, “I think the kids will have to visualize the problem. And some students might not be able to visually see the three parts of the trip.”

Using the language of “roadblocks”, Jason and Kyla encouraged their group to generate potential areas of confusion among their own students, including what they might not understand, what they might do in an incomplete or atypical manner, and aspects of the language or wording of the problem that might be confusing to students. The teachers were able to quickly list a variety of topics they imagined might be problematic. With the list of roadblocks compiled, the group was ready to turn their attention to a consideration of instructional supports. Specifically, Jason and Kyla asked the teachers to brainstorm the types of questions or probes they could ask if their students were struggling in a particular way. In each instance, Jason and Kyla encouraged the group to consider how they could frame questions or probes to address these roadblocks in ways that would support their students’ learning. It is important to note that this entire activity was very much in line with the goals Jason and Kyla had
outlined for their workshop during the ISM: (1) to prepare teachers to be able to interpret their students’ thinking and (2) to consider the types of questions that might be fruitful to move students forward.

This activity of charting and discussing strategies, roadblocks, and probes can be seen as supporting the development of multiple facets of teachers’ MKT, including SCK, knowledge of content and students, and knowledge of content and teaching. The group’s detailed consideration of possible strategies promotes SCK, their consideration of roadblocks promotes knowledge of content and students, and their consideration of questions and probes promotes knowledge of content and teaching. Although these topics are highly interrelated, breaking them down in a PD setting helps to foster a careful and in-depth consideration of each issue in ways that are likely to foster learning for both teachers and their students.

**PSC Workshop Feature 3: Supporting Local Goals and Interests**

The PSC model was developed with broad goals in mind, such as fostering community and developing MKT. In addition, there are specific design features relevant to each of the three workshops to which facilitators are encouraged to adhere. At the same time, TLs are expected to develop goals specifically tailored to their group of teachers, and plan their workshops accordingly. Jason and Kyla were particularly interested in having their teachers take part in the process of determining goals for the group. As part of the previous PSC workshop, Jason and Kyla had asked the teachers to write down their goals for the remainder of the school year. During the ISM prior to their Fuel Gauge Workshop 1, Jason and Kyla reviewed this list, considered which goals to implement, and how to structure workshop activities in ways that highlighted the goals they selected.

Kyla begins the workshop by reminding the group that they previously generated goals for the upcoming semester. Kyla displays a PowerPoint slide with a bulleted list of those goals, which she prepared prior to the workshop. The goals include: developing questioning techniques to encourage all voices to be heard, incorporating more group work and allowing time for investigations, using more inquiry-based problems, getting students more actively engaged in thinking and communicating their thinking, and doing less teacher-directed learning.

Kyla tells the group, “When Jason and I looked through all of your goals, we saw two common themes. People seem to want to use more inquiry-based learning activities and improve their questioning techniques. I think the problem we’re going to focus on today is a great inquiry problem, with lots of avenues to get our kids to think and communicate with each other.”

In her initiation of this iteration of the PSC, Kyla highlighted the fact that the Fuel Gauge task is an “inquiry-based learning problem,” which the teachers at her school expressed a desire to use more often. In addition, as illustrated in the previous vignette focused on developing MKT, Jason and Kyla facilitated a conversation around the teacher analysis task designed to help their group think
through a variety of questions they could use when teaching the Fuel Gauge problem. In their interviews, Jason and Kyla both noted that their Fuel Gauge workshops were better planned than their earlier PSC workshops, and their attention to goal setting aided them in facilitating more in-depth conversations.

Generating and implementing goals for each workshop is a critical component of the PSC model. However, the specific manner in which TEs determine and carry out their goals is flexible. Jason and Kyla had their group brainstorm a set of goals they felt were important to work on during the PSC workshops and in their daily practice. This technique allowed all of the participating teachers to have a voice and a sense of ownership of their PD. Other PSC facilitators identified and used goals in different ways. For instance, one TL had each teacher in her group create a personal goal to work on over the school year. During her PSC workshops, the teachers used their individual lesson videos as a reflection tool and shared progress towards their goal with the group. Another approach was to align the PSC workshop goals with school-identified goals. For example, one TL determined that all of her PSC workshops would be structured to support her school’s goal of reaching diverse learners, including English language learners. In some cases, TEs’ goals for their workshops evolved over time, in order to meet the changing needs and interests of their group members.

**Conclusion**

Supporting TEs to effectively implement high quality mathematics PD presents a significant challenge to the field of mathematics education. Researchers in the field are just beginning to characterise the knowledge and skills that leaders need (e.g., Elliott et al., 2009; Schifter & Lester, 2005), but much more remains to be investigated and unpacked. In this paper, we highlight three features that appear to play an important role in developing leadership skills and building teachers’ capacity, drawing on examples from our current research on the PSC model of mathematics PD. Using the PSC framework, our research team designed experiences to support TEs to facilitate PD that: (1) fostered a professional learning community, (2) developed teachers’ mathematical knowledge for teaching, and (3) matched local needs. Having an articulated PD framework that accommodates these features, such as the PSC, is central to the pursuit of building both leadership skills and teacher capacity.

The vignettes presented in the paper are intended to illustrate how we worked with TEs to incorporate these features as they learned to facilitate the PSC, and how the TEs took each feature into account in a particular PSC workshop. Within the vignettes, we discussed several specific processes, recommended by the research literature, that we used during the course of our work with the TEs including: modelling, fostering discussions, thinking metacognitively, self-reflection, and coaching (Loucks-Horsley, Love, Stiles, Mundy, & Hewson, 2003; Putnam & Borko, 2000). By drawing on a variety of processes, we were able to continually encourage the TEs to take up and implement the features we deemed to be critical. We review these five processes,
briefly, at this point in the paper as they may be instructive to readers who work with TLS, coaches, or teacher educators.

*Modelling* provides leaders with a set of experiences that they can try to recreate in their own PD work. In our ISMs, we not only modelled how to promote community, foster MKT, and adapt PD, but also we were explicit about our modelling. Our intent was not to be perfect models but rather we drew attention to our attempts and encouraged the Tls to similarly be intentional in their behaviours. We strove to *foster discussions* focused on the current (and often changing) needs of the Tls. For example, when Tls encountered specific challenges in their workshops, we facilitated conversations to address those challenges drawing on the collective knowledge of the Tls as a group. We encouraged *thinking metacognitively*, or reflecting on one’s own thought processes, typically as the Tls planned their upcoming workshops. Because the Tls had dual roles, as both leaders and learners, we encouraged them to “wear one hat at a time” and make sure they paid attention to their own learning as well as to their facilitation. On a closely related note, we encouraged the Tls to *self-reflect* on both their learning from the ISMs and their facilitation during their workshops. Because video is an integral component of the PSC model and our research on this model, we videotaped all of the Tls’ workshops and provided opportunities for them to watch and reflect on their video, particularly their efforts to establish community, promote MKT, and adapt to local needs. Finally, we provided *coaching* or one-on-one learning opportunities for individual Tls as needed, generally around areas in which they were struggling. For example, after watching their videos, the Tls often turned to members of our research team, individually, to seek advice on difficult circumstances or areas in their facilitation that they wanted to improve.

We conjecture that implementing these various processes, in combination, was essential to supporting Tls’ ongoing efforts to develop community, foster MKT, and adapt their workshops both to meet the needs of the participating teachers and to fit within their local context. Further research is necessary to more fully explore the features highlighted in this paper and the processes that can be used to promote these features, and to delineate the features and processes that cut across PD programs and leadership roles. Fully articulating the knowledge and skills that leaders need to lead sustainable PD, as well as the processes that best support leader development, is a critical next step in advancing our ability to effectively scale up mathematics PD.
References


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