Learning mathematics for teaching mathematics: 
Non-specialist teachers’ mathematics teacher identity

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A non-specialist teacher of mathematics is a school teacher who qualified to teach in a subject other than mathematics yet teaches mathematics to students in secondary school. There is an emerging interest internationally in this population, a brief report of which is given in the paper. Because of concerns about the quality of non-specialists’ mathematics teaching, in-service courses for these teachers have been provided by some educational jurisdictions, including within the United Kingdom (UK). This paper reports on a research project based on designing and teaching such courses in London. The paper operationalises ‘Mathematics Teacher Identity’ by adapting Wenger’s (1998) social ecology of identity to give a framework of ‘Modes of Belonging’ for a teacher of mathematics, which is then used to analyse qualitative data of different types. Analysis of the data identifies (1) aspects of non-specialists’ mathematics teacher identity in terms of indicators and (2) through two case studies, ‘trajectories’ towards development of a mathematics teacher identity.

Keywords · non-specialist teacher of mathematics · mathematics teacher identity · in-service course- teaching across specialisations · ‘out-of-field’ teaching

Introduction

When I first started [the course] I would say; yes, well I teach mathematics, but really I’m a Physical Education (PE) teacher. Whereas now if someone says: what do you teach? I say: mathematics – oh, and I can also teach PE. So it’s like I’ve flipped it because I feel that I have more knowledge and I can actually justify saying it. So I actually feel that I can say that I am a maths teacher, rather than PE playing at maths. (Jessie, end of course interview)

Who teaches mathematics in secondary schools in England?

The demand for mathematics teachers in England has outstripped supply: of mathematics lessons in state schools in England in November 2012, 18% were taught by non-specialists, indicating a shortfall of 5,500 ‘specialist mathematics teachers’ (Hillman, 2014, p. 23). A ‘specialist mathematics teacher’ is a teacher with Qualified Teacher Status (QTS), who had a relevant post A level qualification (where ‘A’ (advanced) level qualification in a specific subject is typically taken by school students aged 16–18 and is the usual requirement for university entrance in England and Wales). The latest available statistics on teacher supply in England
gathered by the Department for Education revealed “79.8 per cent of mathematics lessons taught to pupils in year groups 7-13 were taught by teachers with a relevant qualification; a decrease from 82.7 per cent in 2013” and “75.8 per cent of teachers of mathematics to year groups 7-13 held a relevant post A level qualification (down from 77.6 per cent in 2013)” (Ross, 2015, p. 13).

The distribution of these non-specialists is uneven over different regions in England, and in London, where we are based, there is considerable movement of teachers from school to school and in and out of teaching. Previously, Moor, Jones, Johnson, Martin, Cowell and Bojke’s (2006) survey, commissioned by the Department for Education and Skills, used an economic analysis to show that in regions of the country where there is both a relatively high free school meal eligibility (a UK indicator of social deprivation) and also jobs available locally with higher salaries than those in teaching, schools are more likely to use higher numbers of non-specialist staff; these conditions are satisfied in the London area hence we expect a relatively high percentage of secondary mathematics lessons to be taught by non-specialists in our locale.

To address this shortage of mathematics teachers, serving teachers, qualified in subjects other than mathematics yet teaching secondary mathematics, were eligible to participate in a government-funded course, various versions of which were commissioned and run within 2008-2014 (Teacher Development Agency 2009, 2011). Both authors were involved in the design and running of such courses and the research reported in this paper is mathematics education research arising from our provision of this type of in-service teacher development.

**Context and course design**

We taught four cohorts of non-specialist teachers on two different types of programmes: two 40-day courses (2009-11) and two 20-day courses (2012-14), the length of the programmes being due to government specifications. Our research was on the last three of these cohorts.

The Mathematics Development Programme for Teachers (MDPT) initiative was a 40-day course, of which 30 days were based at the university and 10 days were based in school with specific pedagogical tasks to complete. The course participants were offered a £5000 bursary and a mathematics specialist certificate on completion of the course. In 2011 the MDPT course was decommissioned and the following year it was replaced by a cheaper-to-run 20-day Subject Knowledge Enhancement (SKE) for serving or returning teachers, which offered neither money nor a government approved certificate.

The design principle of our in-service mathematics courses for non-specialist teachers of mathematics was that effective secondary mathematics teaching is founded on sound subject knowledge, together with a thorough knowledge of the curriculum and a sympathetic understanding of pupils’ needs and interests. Thus, the emphasis of our in-service courses was on revisiting and teaching the subject matter (school mathematics); we aimed to develop participating teachers’ technical fluency with some more challenging topics taught at different levels of school education (Key Stage 3: 11 to 14 year old pupils and Key Stage 4: 14 to 16 year olds).

Our decisions about content were informed by evidence from Ofsted (2006) about areas of the curriculum that were often poorly taught. The selected content areas provided many opportunities for attention to pedagogical issues such as: planning, observing and reviewing lessons; use of a rich variety of learning approaches and teaching resources, including digital technologies; developing and adapting personal resourcefulness and creativity; developing questioning strategies; developing a range of assessment strategies; developing an awareness of the connectivity of mathematics and its place in a wider societal context; taking advantage of the opportunities provided by communities of practice and professional associations.
Orientation to the research

Our research was stimulated by the course participants themselves, all of whom were non-specialist teachers of mathematics or aspiring to teach mathematics. In their reflective writing, in-class contributions and interviews, they made remarks concerning changes they experienced, which they attributed to being on the course. An example of such a remark is the initial quotation from Jessie we provided above, or the following from William in which he notes:

How we keep fractions and decimals completely separate and how we represent them in two different ways, when they mean the same thing?! One of the biggest things I'm going to take from the course is actually making the links from different areas. And I want my pupils to realise that links are there for a reason and it's one whole umbrella, rather than broken up into different sections and branches. (William, end-of-course interview)

Both of the previous quotations can be interpreted to indicate the speaker felt change with respect to mathematics teacher identity (Grootenboer & Zevenbergen, 2008): Jessie named herself as a mathematics teacher and William communicated how his new mathematical knowledge would change his practice of teaching mathematics.

A prompt for our research enquiry came from noticing some surprising things, for example, Sue (trained to teach humanities), who was applying for a promotion, told us that she cried when she first had to teach simultaneous equations and, each time that topic came up, she always asked a colleague to teach it for her. On one hand, this teacher wanted to be thought of as an expert mathematics teacher; on the other hand, she was not able either to fluently solve problems on this standard topic in the mathematics curriculum within our class or to contemplate teaching the topic to her students in school. Such a disjunction confirmed our thinking that issues of identity were relevant to our work with non-specialist teachers of mathematics. We became curious about how our non-specialists’ mathematics teacher identities developed, for instance, whether, like Jessie, course participants came to speak of themselves as mathematics teachers or whether, like William, their teaching practices had changed. This involved thinking about how to capture how the participating teachers might transition (or not transition) to having a mathematics teacher identity.

The contribution of this paper to knowledge is:

a. drawing attention to how mathematical knowledge is realised within non-specialist teachers’ mathematics teacher identity; and

b. developing understandings of non-specialist teachers’ experiences in a professional development course.

Literature: Context and Framework

Our use of the term ‘non-specialist teacher of mathematics’ comes from UK government sources interested in tracking the discrepancies between supply and demand of mathematics teachers (Ross, 2015). However, researchers based in other educational jurisdictions have coined different terms to refer to non-specialist teachers of mathematics.

Context: international interest in non-specialist teachers of mathematics and our research questions

The on-going need for mathematics teachers is not unique to England. The current wave of interest in non-specialist teachers of mathematics can be traced back to Ingersoll’s (1999) analysis of statistics provided by the United States (US) Department of Education which
revealed that, at that time, a third of all US high school teachers of mathematics have no higher education qualification in mathematics and so are “teaching subjects for which they have little education or training” (Ingersoll, 1999, p. 26). Ingersoll referred to these teachers as teaching ‘out-of-field’ (OOF), and other researchers also used this terminology (e.g., Becker, 2000; Ingersoll & Curran 2004). Ingersoll’s statistical analysis revealed an uncomfortable state of affairs in US mathematics teacher profiles: there were not enough specialist mathematics teachers so other teachers went out of (their specialist) field in order to help out the mathematics department. In Germany, research on ‘fachfremd’ (meaning ‘non-specialist’ in German) teachers of mathematics, includes Bosse’s (2014) findings that these teachers enjoyed teaching mathematics even though had a narrow view of what mathematics was, often equating mathematics with elementary school mathematics, and they had had little professional development in mathematics teaching. Furthermore, Bosse found that these teachers often perceived themselves to be good mathematics teachers despite researchers judging their teaching performance as poor.

In Eire, a national survey found that 48% of teachers of mathematics at post-primary schools were not mathematics qualified; these teachers were generally teaching mathematics to students in the lower secondary grades or to those with low prior attainment (Ní Riordáin & Hannigan 2011). A not dissimilar situation was found in Korea: Kim and Kim’s (2014) research revealed that the number of non-specialist teachers in five core subjects (including mathematics) increased between 2008 and 2010, with smaller schools and schools in remote areas being more vulnerable to employing non-specialist teachers.

In Australia, Hobbs (2013) found that teachers who were ‘teaching across specialisations’ (TAS) experienced discontinuities which can impact negatively on their confidence and efficiency as a teacher of the new subject. The discontinuities such as the obvious lack of subject content knowledge, as well as those factors of a more personal and contextual nature, arise when crossing the boundary between the familiar practices of the in-field subject and the unfamiliar practices of the non-specialist subject. One of the main findings of Hobbs’ doctoral study was that, although some non-specialist teachers effectively use the challenge to develop new strategies, the average non-specialist teacher finds this a hard thing to accomplish. Her data suggested that it was not a natural process for the teachers to cross the subject boundary, and that they needed support and guidance to do so.

The first TAS Collective was convened in August 2014 in order to share research and practice internationally. Presentations from countries across the world (Australia, England, Germany, Ireland, and South Korea) indicated the extent of the TAS phenomenon and pointed to identity-related factors as important for determining how teachers cope with teaching across specialisation. As Hobbs (2014), one of the organisers of this conference, stated: “Research is needed to establish the key features of effective professional development that leads to such transformation in identity and practice for out-of-field teachers” (p. 46). This paper aims to contribute to their call by investigating the following research questions (RQ) about the participants on our in-service courses for Non-Specialist Teachers of Mathematics, to whom we will refer as NSTMs for the rest of this paper:

RQ1: How can aspects of a NSTM’s mathematics teacher identity be identified?
RQ2: What constitutes a trajectory towards a mathematics teacher identity?

Framework: perspectives on identity relevant to non-specialist teachers of mathematics and our research questions

Identity – as a social science concept – is associated with many different theoretical frameworks; what would be a suitable one as a framework for our research? For example, Hall (1990)
characterises ‘cultural identity’, as “the names we give to the different ways we are positioned by, and position ourselves within, the different narratives of the past” (p. 225). Jessie’s initial quotation can be interpreted in this framework and, more generally, Hall’s notion helps conceptualise identity from a multi-cultural perspective. However, connecting new content to communication with pupils requires a notion of identity that is applicable to teachers’ practices. Such a notion can be found in Wenger’s (1998) social learning theory (more details below). His conceptualisation of identity as a “negotiated experience of self” (p. 150) within a mode of belonging in a community of practice is based on the “profound connection between identity and practice” (p. 149), the ‘practice’ in our research being mathematics teaching.

On teacher learning
Shulman’s (1986) enquiry into the sources of teacher knowledge underpins much subsequent research on teacher knowledge, including our work. In particular, mathematical subject matter knowledge, consisting of knowledge of the subject itself, extent, depth, structure, concepts, procedures and strategies, is a foundation. While an understanding of subject knowledge for oneself is necessary, Wilson, Shulman and Richert (1987) advised that “it is not a sufficient condition for being able to teach” (p. 105). We drew on well-established research into sources of teacher knowledge initiated by Shulman (1986) and adapted and developed further by many other researchers (e.g., Ball, Thames & Phelps, 2008; Ma, 1999) that show that teachers not only need sound subject knowledge, but also mathematics knowledge for teaching in order to develop mathematics-specific pedagogical knowledge.

As such, although government specifications for the delivery of the in-service courses was to address NSTMs’ subject knowledge, another prominent feature of the courses we offered was also the NSTMs’ learning about mathematics pedagogical issues. In our course delivery, the NSTMs often initiated discussions of pedagogical nature. In fact, we found that we could not but engage with the course participants’ enquiries of pedagogical nature of teaching specific mathematics topics (such as: common pupils’ misconceptions, multiple representations of a concept, different teaching approaches). Flexible and friendly teaching of mathematics content was modelled and our NSTMs were encouraged to adapt the approach they had experienced in our course for their classrooms (Wu, 1997).

On mathematics teacher identity
Grootenboer and Zevenbergen (2008) consider it “essential that teachers of mathematics (at all levels) have well-developed personal mathematical identities” (p. 248) and we develop their notion of Mathematics Teacher Identity in this paper. While a variety of frameworks have been employed by researchers to describe teachers’ identity development in mathematics teacher in-service courses (e.g., Fennema & Nelson 1997, Boaler (2001), Graven and Lerman (2003) argue that Wenger’s (1998) social practice perspective of learning is a suitable framework to use to analyse the process of becoming a teacher of mathematics.

Wenger’s theory
Wenger (1998) describes three interlinked modes of belonging worth considering in order to make sense of identity formation in communities of practice: engagement, imagination and alignment. Through engagement “we explore our ability to engage with one another, how we can participate in activities, what we can and cannot do” (p. 192). In this study, engagement refers to our NSTMs doing the mathematics, getting stuck and trying to get unstuck by doing mathematics themselves, asking questions and seeking help and answers from colleagues and tutors, finding resources for teaching mathematics, familiarising themselves with pupils’ misconceptions and using teaching approaches to address them. Imagination is about the kind of pictures of the world and of ourselves that we can build through exploration, taking risks and making connections in order to create new images of the world and of ourselves, hence it is a
source of identity formation. In this study, analysing the NSTMs’ identities meant for us paying attention to their imaginations of themselves beyond the course, contributing to the teaching in a mathematics department. Through alignment “we become part of something big because we do what it takes to play our part” (p. 179). In this study, alignment refers to how teachers saw their learning and achievements on the course part of the practice of (specialist) mathematics teachers. Thus Wenger’s theory focuses in particular on participation or non-participation in relationship to the construction of identity within overlapping communities of practice.

**Related applications of Wenger’s theory**

Graven (2004) investigated the learning of the teachers in a mathematics in-service program in South Africa using Wenger’s social practice theory. One critique of the applicability of Wenger’s framework to mathematics teacher education contexts is the absence of an explicit notion of ‘confidence’. Graven’s analysis of her data suggested that ‘confidence’ emerged as another component, to be added to Wenger’s ‘engagement, imagination and alignment’.

Smith (2006) also used Wenger’s theory to analyse the scenario of Casey, a prospective teacher. The interrelated terms ‘engagement, imagination and alignment’ provided a language that foregrounded identity formation and transformation as crucial aspects of teacher learning. Methodologically, our approach to capturing overlap of communities and changes in practice required a theoretical framework that linked the concepts of practice and community with identity and could include participants’ learning of new mathematical knowledge. This methodological approach, using Wenger’s Modes of Belonging, has recently been used independently by Oppland-Cordell and Martin (2015) in a very similar fashion.

**Our ‘Modes of Belonging’ Mathematics Teacher Identity framework**

We adapted Wenger’s (1998) ‘social ecology of identity’ (p. 190) and operationalised it as an analytic tool in the following way: by drawing on their own teaching experiences at secondary school level and expertise in research-informed teaching of prospective and practicing teachers, the authors developed and replaced Wenger’s general illustrative examples by mathematics education-specific examples of indicators of aspects of identity (Rodd & Crisan, 2012). In this way, Wenger’s notion of identity was adapted to mathematics teacher identity by interpreting the three interlinked ‘Modes of Belonging: engagement, imagination and alignment’ (Wenger, 1998, p. 174) in the two key practices of doing mathematics (Identification with school mathematics) and being a teacher (Negotiability in mathematics teaching), as indicated in Table 1 below.

In this study, Identification with school mathematics refers to how the NSTMs constructed identities as learners of mathematics during our in-service course. Identification through engagement, imagination, and alignment refers to how the NSTMs invested themselves in learning about and doing school mathematics topics, how they constructed images about how pupils learn mathematics and how their views converged towards an increasing connection with how the mathematics teaching community views mathematics as a practice.

Negotiability in mathematics teaching through engagement, imagination, and alignment refers to how the NSTMs negotiated their ways in the mathematics teaching community, how the NSTMs constructed images of themselves as potential specialist mathematics teachers and how their views converged towards an increasing connection with the mathematics teaching community.
Table 1

*Modes of Belonging* Mathematics Teacher Identity framework

<table>
<thead>
<tr>
<th>Identification with school mathematics</th>
<th>Negotiability in mathematics teaching</th>
</tr>
</thead>
<tbody>
<tr>
<td>Identities of participation</td>
<td>Identities of non-participation</td>
</tr>
<tr>
<td>e.g. Enjoy thinking about the mathematics to be taught.</td>
<td>e.g. Avoid mathematical activity.</td>
</tr>
<tr>
<td>e.g. Find new ideas in standard topics.</td>
<td>e.g. Act as if there was only one correct method; avoid thinking about alternative approaches.</td>
</tr>
<tr>
<td>e.g. Want to understand why, expect proof, work detail.</td>
<td>e.g. Routinely get answers to mathematics problems from internet/elsewhere; make errors.</td>
</tr>
</tbody>
</table>
Our participants

Teachers who enrolled on our courses all lived within travelling distance of London. The 47 teachers, in total, who started these courses – completion rates varied and teachers not continuing with their course dropped out at different points – constituted a convenient sample for our empirical approach to researching the experience of being a non-specialist teacher of mathematics on an in-service course. Gray (2014) notes that research that “… tries to understand what is happening … explores the personal construction of the individual’s world [and] studies individuals … using small samples researched in depth or over time” (p. 12). Furthermore, in this sort of qualitative research, it is accepted that researchers do not stand outside, but aim to understand research participants’ experience by gathering multiple forms of data. This sort of research provides insight into the experience of being a non-specialist teacher of mathematics. Miles et al. (2013) in Gray (2014, p. 174) advise the selection of information-rich cases which can be studied in depth. We do this by representing participants’ voices in two ways: (1) though examples of identity-indicators, which are mostly extracted from the analysis of interviews and NSTMs’ written work, and (2) via two case studies, using data from multiple sources.

Ethical issues in practice and for research

Prior to the decision to conduct research alongside teaching our in-service courses, we had a discussion with the NSTMs about the possibility of researching aspects of their experience of this continuing professional development on which they were enrolled. Towards the beginning of each course, we gave a short description of our research interest and invited the teachers to participate in being interviewed if they wished. We again asked for their permission to use their course material and there were no dissenters. Throughout, we have used pseudonyms that reflect gender and all material collected has been anonymised.

Data

Throughout the in-service courses we collected different types of data from the participants:

i. Questionnaire: on participants’ professional background, their routes into teaching, subject specialism of their teacher training, their teaching experience of mathematics (if any) or of their subject specialism (see Table 4 and Table 5 in the Appendix);

ii. Mathematical written work: (1) diagnostic assessment of mathematics subject knowledge and awareness of school students’ errors/misconceptions; (2) collection of on-going mathematics course work;

iii. Written reflections: (1) as part of class/coursework; (2) essay assignments;

iv. Interview: (1) initial narrative style (Hollway & Jefferson, 2008); (2) end of the course semi-structured (e.g. Brown & Dowling, 1998). All interviews lasted between 20 and 40 minutes, were audio-recorded and transcribed;

v. School visit data: (1) lesson observation field notes; (2) post lesson discussion/interview; (3) teaching materials.
Analysis of Data: Indicators of Mathematics Teacher Identity

In this section we answer our first research question, ‘How can aspects of a NSTM’s mathematics teacher identity be identified?’, by locating indicators of mathematics teacher identity through analysis of data using our ‘Modes of Belonging’ Mathematics Teacher Identity framework (Table 1).

Data analysis: refining the meaning of the cells in Table 1

Our framework (Table 1) consists of three modes of belonging central to participation, or non-participation, in the two key practices of doing mathematics and being a teacher. Analysis consisted of refining the meaning of the cells in Table 1 in the context of our study by allocating data items appropriately in cells (exemplified below). This enabled reification of aspects of mathematics teacher identity which we have labelled as ‘indicators’.

Identification with school mathematics, Non-Participation - Alignment

From ‘mathematical written work data’: participants were asked to locate the errors that have been made, offer a view on why the student may have made these errors and, if possible, provide a correct solution.

‘School student’ example 1: \((x+y)^2 = x^2 + y^2\).

Many NSTMs thought that this was always true (rather than only true when either \(x\) or \(y\) are equal to 0). Only a few attempted to check validity by expanding the square \((x+y)^2\) or by substitution of a few actual numbers. Such responses are examples of Identification with school mathematics, Non-participation – Alignment and can be analysed in terms of constituent indicators in the following way: the school student example is recognised as school mathematics, but, for the teachers who did not recognise the answer as incorrect, non-participation with the community-received practices is indicated. However, arguably, because there was what looked like the rule \(2(x+y) = 2x + 2y\), this is an indicator of alignment.

Negotiability in mathematics teaching, Participation – Imagination

From interview data: two excerpts illustrate contrasting ways of a teacher’s imagination being active with respect to mathematics teacher identity.

Example 1: “So I thought it was quite important that I actually had another subject that I could teach, and mathematics is obviously a very stable subject. So almost as if in an insurance kind of way I wanted to move into mathematics as well.” (Nas, first interview).

Example 2: “I got a different teacher and then I passed. From then I have a love for mathematics.” (Madeleine, final interview).

These two snippets from interviews are contrasting indicators of imaginative belonging to a mathematics teaching community: the first is couched in a rationalist discourse that positions Nas as a professional teacher with career options since mathematics teaching offers employment stability due to a shortage of mathematics teachers. The second is a fantasy identification with a former teacher that propels Madeleine to love what that teacher loved and want to do what that teacher did (i.e., teach mathematics).

A set of indicators from one participant

From written reflections: As part of their course, NSTMs were asked to reflect on their development as part of their coursework. One participant, William, was a PE teacher until he had a sports accident, after which his school re-deployed him to teach mathematics and he participated in our course. We illustrate our analysis using an extract from his final assignment:

At the beginning of the course I completed a skills audit to highlight the knowledge I had of mathematics. I was particularly apprehensive while waiting to start and nervous throughout.
Having not thought about mathematics as a subject since leaving school I believed my skills would be lacking. Unfortunately I did not know many of the required answers; I struggled in particular with the processes to find the correct answer. This made me feel inadequate and nervous about accepting the job as a mathematics teacher. Of most concern was that the pupils who I would be teaching would have a greater level of knowledge than me.

As with all mathematics this is a learning process and I have seen that practice and repetition of questions will help to gain knowledge. In order to address the difficulties in this area it has been essential for my development to practice similar questions regularly. I have been able to scrutinise teaching techniques and systematically look at problems that may arise when teaching mathematics and overcome them successfully. This has been an invaluable lesson and one in which I will take forward, continuing to challenge myself to meet the needs of my pupils, linking my work to real life examples, bringing mathematics to life and enthusing the pupils.

The two paragraphs of the extract communicate William’s take on his development towards a mathematics teacher identity over the course. We have used the format of Table 1 (our ‘Modes of Belonging’ Mathematics Teacher Identity framework) to present analysis of William’s reflections from paragraph one in Table 2 and from paragraph two in Table 3.

Table 2
Mathematics Teacher Identity indicators from William’s first paragraph

<table>
<thead>
<tr>
<th>Identification with school mathematics</th>
<th>Negotiability in mathematics teaching</th>
</tr>
</thead>
<tbody>
<tr>
<td>Identities of participation</td>
<td>Identities of non-participation</td>
</tr>
<tr>
<td>“not thought about”</td>
<td>Engagement</td>
</tr>
<tr>
<td>“apprehensive”</td>
<td>Imagination</td>
</tr>
<tr>
<td>“I did not know …answers”</td>
<td>Alignment</td>
</tr>
<tr>
<td></td>
<td>On the course. “skills lacking”</td>
</tr>
<tr>
<td></td>
<td>“concern …[for] inadequate” pupils who I would be teaching”</td>
</tr>
</tbody>
</table>

In Table 2 indicators of William’s non-participation in school mathematics and indicators of his participation and non-participation in negotiability in mathematics teaching are in shaded cells. William’s story is that “at the beginning” he is aware that he does not participate in school mathematics as he indicates lack of engagement (“not thought about”). Furthermore his imagination is negatively positioned (“apprehensive”) and he feels a lack of alignment (“I did not know …answers”). In terms of negotiability in mathematics teaching, he is more of a participant as in his imagination he feels “concern … [for] pupils who I would be teaching” although he feels non-aligned (“inadequate”) without engagement (“skills lacking”).
Table 3
Mathematics Teacher Identity indicators from William’s second paragraph

<table>
<thead>
<tr>
<th>Identification with school mathematics</th>
<th>Negotiability in mathematics teaching</th>
</tr>
</thead>
<tbody>
<tr>
<td>Identities of participation</td>
<td>Identities of participation</td>
</tr>
<tr>
<td>Identities of non-participation</td>
<td>Identities of non-participation</td>
</tr>
<tr>
<td>“practice…regularly”</td>
<td>Engagement</td>
</tr>
<tr>
<td>immersion through repetition</td>
<td>Imagination</td>
</tr>
<tr>
<td>Alignment</td>
<td>“bringing mathematics to life”</td>
</tr>
</tbody>
</table>

Table 3 presents indicators from the second paragraph of William’s story. He considers engagement in school mathematics being imagined as immersion through repetition (“practice…regularly”). He does not make a remark that considers alignment with mathematics specifically, but in “bringing mathematics to life” there is a sense of negotiability within the practice of teaching mathematics as he aligns mathematics with his pupils’ needs and interests; his engagement (“enthusing pupils”) with and imagination (“excited”) of teaching are also evident.

These two subsets of indicators of Mathematics Teacher Identity from William’s reflective writing show that his identity as a teacher of mathematics was changing over the course and that his narrative has a positive sense of increased belonging to a community of teachers of mathematics. Looking at the two tables from William’s reflections, the shift to increased participation is apparent. Thus our second research question arises where we ask, more generally: What constitutes a trajectory towards a mathematics teacher identity? To answer this question we used multiple types of qualitative data from case studies, two of which are presented in the next section, together with the same analytic framework based on ‘Modes of Belonging’.

Two Case Studies

In this section, two case studies, Ewa and Sara, are presented. Our access to Ewa and Sara as part of research, we recognise, is very much bound up with their being successful on our course. Nevertheless, these case studies, viewed with our framework of indicators, give more detail as to how NSTMs might, more generally, develop a mathematics teacher identity (of course, we are not claiming to generalise from two cases).
Table 4
Professional backgrounds of the case study NSTMs

<table>
<thead>
<tr>
<th></th>
<th>Ewa</th>
<th>Sara</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Teacher Education route</td>
<td>Qualified in EU (European Union)</td>
<td>Postgraduate General Certificate of Education (PGCE)</td>
</tr>
<tr>
<td>Subject specialism</td>
<td>PE</td>
<td>Information and Communication Technology (ICT)</td>
</tr>
<tr>
<td>School phase Training</td>
<td>Secondary</td>
<td>Secondary</td>
</tr>
<tr>
<td>Prior to course taught some mathematics for:</td>
<td>1 year: assisting in maths lessons</td>
<td>3 years: two hours per week</td>
</tr>
</tbody>
</table>

Ewa

Ewa’s participation/non-participation is presented here, through a narrative framed by the ‘Modes of Belonging’ mathematics teacher identity framework, using information from her autobiographical data, mathematical work, written assignments and reflections, and observation of her teaching her Year 7 classes (of 11-12 year olds) at her school, together with her university session class notes.

At the beginning of her course Ewa, a qualified PE teacher, had a temporary part-time teaching assistant job at a school that supported her taking our mathematics subject knowledge in-service course. Towards the end of the course Ewa had secured an appointment as a mathematics teacher in the same school.

Ewa’s identification, through engagement, imagination and alignment, with the practice of school mathematics

In our in-service course sessions, Ewa asked questions and worked eagerly with other students, particularly those she judged as having better mathematical skill than her. She wanted to know how to get correct answers and found out how to align to school mathematics from other course participants or tutors. She always handed in her workbook consisting of mathematics exercises at the level of a high attaining 16 year old (other NSTMs’ workbook hand-in rates were as low as 20%), indicating engagement with learning the subject of school mathematics. At her school, the mathematics department is staffed with well-qualified and experienced teachers who (from observation) discuss mathematics in the staffroom, and Ewa uses the resources this environment affords for her mathematical development: “all the mathematics teachers in my school help me get on with mathematics” (post-lesson observation interview). However, when teaching her low prior-attaining 11 and 12 year olds to work with fractions she restricted instruction to rehearsal of standard rules only, for instance, ‘two-fifths of five’ was formally calculated as \( \frac{2 \times 5}{5} = \frac{10}{5} \). She did not exploit linguistic, diagrammatic or scenario representations, suggesting a restricted imagination regarding mathematics as a practice.

Ewa’s negotiability, through engagement, imagination and alignment, in the practice of mathematics teaching

Getting a teacher’s job (to teach both mathematics and PE) at the school where she had been employed as a teaching assistant contributed hugely to Ewa’s continuing engagement with mathematics teaching. Her engagement with mathematics teaching was also evidenced by her
teaching extra tutorial classes for a small group of 16 year olds who were preparing for their public exam, via which she aimed to “prove” that she could teach “more advanced students”:

I think all I do is really valuable for my future development as a Mathematics teacher; being able to attend the course gave me the confidence to become a Mathematics teacher; speaking and observing other Mathematics teachers gave me an idea of how to teach certain topics; receiving constructive feedback helped me to improve as a Mathematics teacher. ‘Mathematics Watch’ is a great resource for not native speakers - it gives you an idea how to explain certain topics. I am hindered only by my teaching only Year 7, bottom set, not getting broader experience narrows my ability to fully develop as a Mathematics teacher, however I hope this will change once I prove I can teach more advanced students. My mentor at school and all the mathematics teachers in my school [and my university tutors] help me teach mathematics. (From Ewa’s reflections on the term following her mathematics teacher appointment).

When about to be observed teaching (for this research), it was evident that Ewa’s general teacher qualities were excellent: as the Year 7 were settling down, Ewa went to speak to them individually, making a remark about their homework, which had been marked with commentary, and placing their exercise on their table in their allotted place. She was fluent with contemporary pedagogies that include classroom technologies like interactive whiteboards and visualisers and her planning was thorough and employed a range of techniques. There was warmth, connection and affection between Ewa and her pupils. Her teacher identity was mature and flourishing. While relationships in the classroom were good, some of the materials, downloaded from internet, were less than ideal. For example, the pupils were shown a picture of a mercury-style thermometer with a scale from 1000°C to 5000°C – an artefact that could not exist – as a context for rounding a three significant figure ‘temperature’ to one with two significant figures. As these resources are downloaded from a mathematics teachers’ site, Ewa could be said to be in alignment with mathematics teacher communities, yet alignment with the practice of mathematics, particularly modelling, was still wanting.

This narrative from Ewa is a story of participation which included her increased ‘belonging’ (Wenger, 1998, p. 181) to her school’s mathematics teacher community. It also indicates how her imagination had been activated towards increasing her connection with that community. Her capacity to work on mathematics and mathematics teaching through her membership of a school mathematics department developed her mathematics teacher identity. Nevertheless, Ewa recognised that she needs a fuller range of mathematics teaching to secure her mathematics teacher identity and, from lesson observation, aspects of her mathematics pedagogy were not aligned with accurate mathematical modelling (the thermometer) or meaning making (the fractions).

Sara

From next year I’ll be a mathematics teacher. It is a bit scary because it is full time, although I will be teaching some programming to my Year 10. (Post-observation interview)

Sara’s identification, through engagement, imagination and alignment, with the practice of school mathematics

Sara came to the UK as an adult having had a French-speaking mathematics education at school, a first degree in Architecture, and a master’s degree in Computer Science. She gained her qualified teacher status in England, specialising in ICT with French. She first became interested in teaching mathematics through helping her own children with the subject. For the three years prior to enrolling on our course, Sara taught ICT across the school in the 11-18 age range, and mathematics to students up to 14 years of age. Right from the beginning of her course, Sara attempted all the questions made available to her and asked for clarifications when needed; she also sought to find out about the mathematics of the National Curriculum (NC) and
joined mathematics teachers’ forums (e.g., the National Centre for Excellence in Teaching Mathematics). For her final assignment of the course, Sara chose to present on the topic of probability which she had not studied prior to the course, ("With probability I learn from the experience of learning myself"), indicating her voluntary engagement with mathematics that was new to her. In preparation for her presentation, Sara researched the mathematics education literature for common misconceptions pupils might have when learning this topic, illustrating this with real life examples. The year after completing the course, Sara was appointed as a mathematics teacher in the same school in which she had been teaching ICT.

*Sara’s negotiability, through engagement, imagination and alignment, in the practice of mathematics teaching*

I’ve learned this from the [this] course: ‘discussion’. Don’t give them the answer immediately, don’t say their answer is wrong. Otherwise I wouldn’t have thought of teaching this way; I wouldn’t feel confident not giving them the answer. The course made me realise this: it is OK, it makes sense, it is the thinking that matters, more than the answer, and that’s why I planned the lesson like this. (Post-observation interview)

Throughout the course, Sara attempted all the questions made available to her and asked for clarifications when needed, ensuring that she knew the mathematics, then she enquired about how else the mathematics could be done. She was genuinely interested in how other NSTMs did the mathematics, as this added to her repertoire of approaches to doing mathematics, which she was keen to develop. Evidence gathered throughout the course showed that Sara was very determined to improve her subject knowledge and familiarity with the school mathematics topics.

Sara was observed teaching a high prior-attaining class of 12-13 year olds in her school six months after she completed her course (quotations used in this section are taken from that visit). At that time, she was still based in the ICT department which was located in a different building from mathematics. This posed practical challenges as “it is difficult to carry two bags of materials and prepare for the beginning of the lesson properly.” This physical separation also lead to her rarely having an opportunity to discuss mathematics teaching with mathematics teachers at her school; the resources for the observed lesson were from an on-line teachers’ forum she had joined. Nevertheless, her negotiability in her mathematics teaching was indicated in several features of the observed lesson. These include her not using formal ‘Learning Objectives’ (a well-established school approach to lesson structure) as “It feels like such an artificial start of the lesson; I like to leave a bit of an element of surprise for the pupils,” her marking of student work which consisted, mainly, of comments or a question pointing the students towards how to improve, and her on-going encouraging of oral contributions: “I wanted the pupils to look at graphs and equations from a different angle. The new NC encourages teaching so pupils look at same things in different ways, building connections”.

When prompted to think about planning for teaching Algebra, the newly gained confidence and personal enjoyment of algebra contributed to Sara feeling confident with teaching the subject and being happy to learn from pupils about their difficulties and misconceptions: “I discover pupils’ difficulties as I teach them, I learn from the experience of teaching.” (Post-lesson observation interview). Through reflection on her teaching experience, Sara told us about her learning how to plan for a range of mathematical abilities. She found that for some pupils, “You need to introduce [algebra] through the back door, without even mentioning the word, through the use of manipulatives, cards, a gentle introduction to the unknown, letting pupils themselves come up with it.” (Post-lesson observation interview). Sara told us that she learned this from the practices of the mathematics teachers in her school whose lessons she had the opportunity to observe.
This narrative from Sara is also a story of participation which included her increased ‘belonging’ (Wenger, 1998, p. 181) to the mathematics teacher community. Sara gained membership to the National Centre for Excellence in Teaching Mathematics and the Science, Technology, Engineering and Mathematics Centre and was very keen to build a pool of resources and ideas for classroom use. Sara also invested time from the beginning of the course in doing school mathematics and this commitment to improving her subject knowledge continued throughout the in-service course and the year after. Just like in Graven’s (2004) study, Sara’s confidence developed as both a product and a process of learning. The positive direction of her trajectory was further evidenced by Sara’s continued effort to develop her knowledge of mathematics at higher level by successfully completing a distance learning course on pure mathematics which, she said, “gave me the confidence” to contribute to the teaching of mathematics in her school even at A level (the standard pre-university course for 16 to 19 year old high attaining students).

Discussion and Conclusion
This section is in two parts: the first reflects on the research presented and the second considers the role of Wenger’s framework.

Summary of findings
Our research has shown that the construct ‘Mathematics Teacher Identity,’ operationalised by adapting Wenger’s ‘Modes of Belonging,’ can be used to track in-service teachers’ Participation in School Mathematics and in Negotiability of Mathematics Teaching. The framework we used to log data from teachers served to clarify what constituted participation as opposed to non-participation. For instance, Ewa’s downloading of resources was considered ‘non-participation’ with regards to engagement in Negotiability of Mathematics Teaching, as in her lesson the materials were used unadapted and were rather inappropriate. Locating such ‘non-participation’ gives mentors or other teacher educators opportunities for contextual discussion, yet this example from Ewa is clearly a small issue when positioned alongside many indicators of her participation.

We found that course participants’ own experiences of learning new mathematics, including getting stuck, engaging with others’ thinking and looking at a piece of mathematics from different perspectives, were indicators of Participation, both in School Mathematics and in Negotiability of Mathematics Teaching, that is, indicators of their Mathematics Teacher Identity.

We also found that by the end of their course, most of the NSTMs were ‘talking the talk’ about what it takes to be a mathematics teacher, influenced by the practices promoted by our in-service course. For example, they talked about the interconnectedness of the mathematics topics, links between topics, use of investigative approaches and group work. This could be interpreted as their alignment with discourses related to Negotiability of Mathematics Teaching. However, such ‘talking the talk’ alignment was, using our framework, distinct from Participation in School Mathematics or ‘walking the walk’ (Crisan & Rodd, 2014).

As discussed and exemplified throughout the paper, the first research question, ‘How can aspects of a NSTM’s mathematics teacher identity be identified?’, was addressed by adapting Wenger’s ‘Modes of Belonging’ (Table 1) for the Mathematics Teacher Identity and data allotted to constituent cells of Table 1 were indicators of participation or of non-participation, respectively.

However, in this paper, there has not been space to analyse or present data collected from the whole class; this is a limitation of this paper as potentially significant community alignment
occurs when engaged in explicit social learning. For example, when Pascal’s triangle emerged for the umpteenth time the whole class became excited. This datum could be classified as Participation in School Mathematics as connections between different mathematical topics were noticed, and engagement was experienced akin to “joy and satisfaction in undertaking mathematical practices” (Grootenboer & Zevenbergen, 2008, p. 246). Furthermore, the positive group atmosphere that occurred can be important in building a community of practice within the class of NSTMs and helping, through development of positive affect, the NSTMs participate in other communities of mathematics teachers. This point about the potential of the community of each class of NSTMs to enhance each individual participant’s trajectory towards a Mathematics Teacher Identity is not one we have brought to the fore yet can help answer the second research question, ‘What constitutes a trajectory towards a mathematics teacher identity?’

In this paper, the narratives of the case studies, Ewa and Sara, include many indicators of participation, both in School Mathematics and in Negotiability of Mathematics Teaching. Nevertheless, there were aspects of their practice which were still edging towards participation, as, for example, Ewa’s resources or Sara’s development of teaching algebra to different ‘abilities’. Indeed, another observation of the whole-class of NSTMs was that, generally, as courses progressed, they became more focussed on the learning and doing of mathematics compared with their focus at the beginning when how to teach a specific mathematical topic was the central concern to them.

Data from NSTMs who attended our in-service courses has been classified in terms of indicating participation or non-participation in the three different ‘Modes of Belonging’ – engagement, imagination and alignment – via the framework tool shown in Table 1. In Wenger’s terms, our NSTMs were newcomers to the mathematics teaching community and as such they negotiated their trajectories towards becoming mathematics teachers in their own ways and their individual tables looked different.

On Wenger’s framework and further work

In their overview of how Wenger’s social theory of learning is used in mathematics education, Roos and Palmér (2015) observe that this theory is used in different ways in different studies. Specifically, they note that there is a variation across the studies they reviewed as to whether communities of practice are viewed as pre-existing or designed and also whether individuals or communities of practice are in the foreground. They also observe that “even though we might think we know what a theory implies in research, if we look beneath the surface we may find that ‘the same’ theory can imply many different things” (p. 172). This comment resonates with the work this paper has presented as our adaptation of Wenger’s theory has developed the ‘Modes of Belonging’ into a Mathematics Teacher Identity framework which was not anticipated in the original social theory of learning (Wenger, 1998). By considering practices central to being a secondary mathematics teacher - Identification with School Mathematics and Negotiability in Mathematics Teaching - we have offered a way of thinking about mathematics teacher development endorsed by other researchers (Oppland-Cordell & Martin, 2015). Furthermore, this Mathematics Teacher Identity framework makes visible to course designers and providers of in-service training how identity interacts with the learning of School Mathematics and with Negotiability in Mathematics Teaching. Using this framework at different points in an in-service course provided a way to evidence Mathematics Teacher Identities emerge and develop. Graven (2005) points to identity transformation seldom being the focus of in-service courses. Rather, identity transformation can happen as a result of the teacher education programs. Graven proposes that identity interacts with teachers’ learning and thus should be a focus of the design and provision of any in-service training. Hobbs (2014)
asserts that identity-related factors contribute to effective professional development of non-specialist teachers of mathematics and our findings contribute to a way of understanding emerging Mathematics Teacher Identity during an in-service mathematics course for non-specialist teachers of mathematics.

References


Appendix

Table 5
Professional backgrounds of the participants mentioned in the paper

<table>
<thead>
<tr>
<th>ITE route</th>
<th>William</th>
<th>Sue</th>
<th>Nas</th>
<th>Lech</th>
<th>Madeleine</th>
<th>Jessie</th>
</tr>
</thead>
<tbody>
<tr>
<td>ITE route</td>
<td>BEd</td>
<td>PGCE</td>
<td>Teach First</td>
<td>Qualified in European Union (EU)</td>
<td>PGCE</td>
<td>BA/QTS</td>
</tr>
<tr>
<td>Subject specialism</td>
<td>PE</td>
<td>Humanities</td>
<td>Citizen ship</td>
<td>PE</td>
<td>General</td>
<td>PE</td>
</tr>
<tr>
<td>School phase Training</td>
<td>Secondary</td>
<td>Secondary</td>
<td>Secondary</td>
<td>Secondary</td>
<td>Primary</td>
<td>Primary</td>
</tr>
<tr>
<td>Prior to course taught some mathematics for:</td>
<td>2 years: one over lesson a week</td>
<td>4 years</td>
<td>2 years: two lessons a week</td>
<td>9 years</td>
<td>2 years</td>
<td>10 years</td>
</tr>
</tbody>
</table>

In this table, the following acronyms are used: BA (Bachelor of Art, a degree awarded for an undergraduate course or program in either the liberal arts, the sciences, or both); BEd (Bachelor of Education, an undergraduate professional degree which prepares students for work as a teacher in schools). A QTS (Qualified Teacher Status) is required in England and Wales to work as a teacher of children in state schools and in special education schools. One of the main routes to achieving QTS, for those already in possession of a degree, involves undertaking a postgraduate teacher training course such as Postgraduate General Certificate of Education (PGCE) or Teach First course. There are also some undergraduate degree qualifications leading to QTS, such as the BEd or a BA. All these routes into qualifying as a teacher could be described more generically as Initial Teacher Education (ITE) routes. On some ITE routes, the training is either subject specific (e.g. Physical Education) or general.

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